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Neutron Radii Determinations from the Ratio of π^- Elastic Scattering from $^{12,13}C$ and $^{16,18}O$

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Abstract

Differential elastic cross section ratios and absolute cross sections have been measured for $^{12,13}C$ at 29.2 and 49.5 MeV average π^- energy and for $^{16,18}O$ at 29.2 MeV. Range telescopes detected the scattered pions. The ratio data were compared with different optical potential calculations to extract neutron radii of 2.35 ± 0.03 fm for ^{13}C and 2.81 ± 0.03 fm for ^{18}O, relative to the neutron radii of ^{12}C (2.31) and ^{16}O (2.60), respectively. Our studies indicate little sensitivity to the optical model used. The ratio measurements determine the neutron radius itself rather than the difference between the neutron and proton radii.

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A longstanding question of nuclear structure concerns the neutron density distribution which is not nearly as well known as the proton distribution. Various methods have been applied but there are still major discrepancies between the results obtained by different techniques.1) Here we consider a new method that involves the measurement of the angular distributions of elastically scattered π^- at low energy from a pair of isotopes, one of which has reasonably well-established neutron and proton density distributions. We measured the ratio of the differential cross sections for the pair since both systematic errors in the data and uncertainties in the theory cancel to a large extent. In the low energy region the π^-n elastic scattering amplitude is much larger than that for π^-p mainly because of a cancellation in the π-nucleon p wave. This feature occurs in any reasonable low-energy pion-nucleus optical model and we shall see that this makes the results insensitive to the precise form of the model employed. A similar experiment has been carried out on $^{6,7}Li$ and $^{12,13}C$ using 50 MeV π^+.2) However, the results showed little sensitivity to the neutron density distribution.

At low energies, where the π-N interaction is relatively weak, one can develop an optical potential in terms of the density of nucleons and medium-corrected π-N scattering amplitudes.3-7) Two different π-nucleus optical potentials were used in our analysis, that of Stricker, McManus and Carr (SMC)3) and that of DiGiacomo, Rosenthal, Rost and Sparrow5) (DRRS).

The SMC form is closely related to that derived by Ericson and Ericson8,9) for pionic atoms. It contains, in addition to the usual impulse (or Kisslinger potential) term, corrections due to Fermi motion, Pauli blocking, true pion S and P wave absorption and a P-wave Lorentz-Lorenz effect. Details are described in the SMC paper.3) We have used the SMC parameter set 1 unless otherwise stated.

The DRRS form5) is also taken from the theory of pi-mesic atoms extended to positive energy. The most significant differences are the inclusion of πN phase shifts rather than scattering lengths, a different relativistic reduction (angle transformation), and a phenomenological extraction10) of the S-wave absorption term using an analysis of 50 MeV π^+ scattering from many-target nuclei. We have taken the optical parameter set from Ref. 5 in which there is no P-wave absorption and the Lorentz-Lorenz parameter is zero (DRRS-A).

Each of the π-nucleus potentials, SMC set 1 and DRRS-A, fit π^+ elastic scattering data quite well for nuclei from ^{12}C to ^{208}Pb. This strongly suggests that variations in parameters between neighbouring nuclei will be negligible. Neither code includes spin flip, but an impulse approximation gave a maximum (incoherent) contribution for ^{13}C of less than 0.02 mb/sr. For the matter density we used a modified harmonic oscillator form fitted to electron scattering.11) For the N = Z nuclei ^{12}C and ^{16}O we assumed equal neutron and proton density distributions as is reasonable from their closed shell character. It should be noted that low-energy pions determine only low q features of the neutron distribution and thus a smooth density form should be adequate.

As an additional test of model dependency, we have allowed the S- and P-wave absorption parameters, the Lorentz-Lorenz term, and (for SMC only) the π-N phase parameters to vary freely to obtain a best fit to the ^{12}C and ^{16}O π^- angular distributions. Then, using these sets of parameters, the ratio data were fitted by varying the neutron and proton radii of

^{13}C and ^{18}O.

The experiments were performed at TRIUMF on the M8 and M9 pion channels. The details of the range telescopes used in this experiment, as well as the beam-monitoring devices, have been reported elsewhere.12,13) The major emphasis in this experiment was to determine the ratio of the differential cross section for the two isotopes of interest as a function of angle. For this purpose data were accumulated on identically shaped targets of natural ^{12}C (water) immediately followed by runs with a 99.7% ^{13}C target (99.5% H_2O^{18}) and then with an empty target frame. The ratios hence involved essentially none of the uncertainties associated with the absolute normalization of the data and removed most systematic errors as well.

Corrections for scattering to excited states were based on the measured efficiency for detection of lower energy pions ($\leq$ 10%), and used differential cross sections determined with a DWBA calculation. These corrections were smaller than 1.5%. Corrections for π^- elastic scattering from hydrogen in the water target were found to be negligible, as were the corrections for scattering to the 6.6 MeV ^{16}O state. Corrections for scattering to the 1.98 MeV ^{18}O state reached a maximum of 3.5% at 150°.

The measured differential cross sections and their ratios are shown in Figs. 1-3. The absolute normalization uncertainties in the differential cross sections are ±15%. Relative errors include statistics and a 7% systematic error. Our previously published ^{12}C results12) have been folded in with the new results to give the ^{12}C differential cross section. The errors in the ratios are statistical only.

The optical potential parameters DRRS-A and SMC set 1 were each used to calculate differential cross sections. The cross section ratios were calculated by varying the neutron radii for ^{13}C and ^{18}O until a best fit was obtained (typically $\chi^2/N \leq 2$). The neutron radii found to give the best fit are given in Table I.

The absolute differential cross sections predicted by these parameters were as much as 40% low at back angles for carbon at 50 MeV and for oxygen at 30 MeV. To remedy this, and to examine model dependencies, the ^{12}C and ^{16}O absolute differential cross sections were fitted by allowing the absorption and Lorentz-Lorenz parameters (and the π-N phases for the SMC code) to vary freely until good fits to the angular distributions were obtained. S- and P-wave absorption parameters were varied about ±30% for both codes. The Lorentz-Lorenz parameter varied from 0.5 to 1.0 for SMC and from 0 to 0.6 for DRRS.

The analysis of the ratio data was repeated and statistically very well defined values for the neutron radii of the neutron rich isotopes were obtained. The radii determined from DRRS-A and SMC set 1 as well as from the parameter sets found with the best fits at each energy are listed in Table I. For carbon, radii found by combining both the 30 and the 50 MeV data are also listed. All radii determinations assume the neutron radius of the comparison isotope (^{12}C or ^{16}O) equal to its proton radius. We feel that a comparison of the neutron radii extracted from an analysis in these two models should give a fair indication of the model independence. The absence of the Lorentz-Lorenz effect in all but one of the DRRS calculations is particularly significant because Gibbs et al[14]) have shown that this correction changes the effective nuclear size!

A particularly interesting feature of the analysis with both models is that even if the proton radii are allowed to vary far outside the region consistent with electron scattering experiments for ^{13}C and ^{18}O, the neutron radius which gives the best fit to the ratio data does not change appreciably. We are consequently making direct measurements of the neutron radius relative to ^{12}C and ^{16}O respectively, and not the difference between a neutron and a proton radius. The statistical uncertainty in the determination of a neutron radius for each set of parameters is very small. A more meaningful estimate of both the statistical errors and the parameter dependence is given by the standard deviation of these radii from the mean. This is quoted as our final error.

There are no known experimental determinations of the neutron rms radius for ^{13}C. For ^{18}O there have been several recent determinations. A total cross section measurement reported a difference of rms neutron and proton radii for ^{18}O of 0.19 ± 0.02 fm.[15]) Note that they assumed $r_p(^{18}O) = r_p(^{16}O) = r_n(^{16}O)$. This corresponds to a difference in $\langle r_n \rangle^{\frac{1}{2}}$ of 0.12 when the $^{18}O\langle r_p^2 \rangle^{\frac{1}{2}}$ of 2.68 is used, and is in good agreement with our results. The elastic scattering results of Iversen *et al.*[16]) at 164 and 230 MeV indicate that the neutron-proton radius difference for ^{18}O is $\leq 0.03 \pm 0.04$ fm. The results of Jansen *et al.*[17]) at 160 MeV are not inconsistent[18]) with ours. It should be noted that the various experiments were done at energies varying from 30 to 230 MeV and consequently sampled different regions of the nuclear density.[19])

Our method for determining neutron radii from the ratio of π^- elastic scattering differential cross sections is relatively insensitive to the proton radius of the isotope and to the model used for the calculation.

For the cases studied, this experiment gives the best measurement of neutron radii of neighbouring isotopes known to date.

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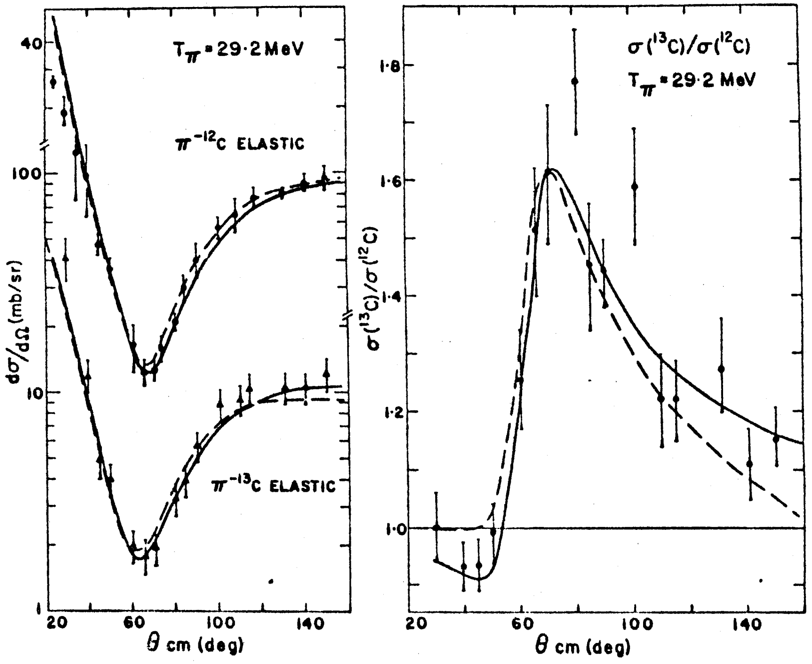
Table I. Neutron rms radii for ^{13}C and ^{18}O obtained for various optical model calculations — assuming $r_n (=r_p) = 2.31$ fm for ^{12}C, and $r_p (=r_n) = 2.60$ fm for ^{16}O.

Parameter set	^{13}C	^{18}O
DRRS-A	2.32 ± 0.02	2.82 ± 0.02
DRRS, 30 MeV fit	2.32 ± 0.02[a]	2.76 ± 0.02
DRRS, 50 MeV fit	2.35 ± 0.01	---
DRRS, combined	2.34 ± 0.01	---
DRRS, mean	2.34 ± 0.01	---
SMC - set 1	2.33 ± 0.02	2.84 ± 0.01
SMC, 30 MeV fit	2.41 ± 0.02	2.83 ± 0.01
SMC, 50 MeV fit	2.35 ± 0.01	---
SMC, combined	2.36 ± 0.01	---
SMC, mean	2.36 ± 0.03	---
Overall mean	2.35 ± 0.03	2.81 ± 0.03

[a] Only this DRRS fit had Lorentz-Lorenz and P-wave absorption fit different from 0. All others had best fits with these parameters equal zero.

Figure Captions

1. Angular distributions for the elastic scattering of π^- from ^{12}C and ^{13}C (on the left) at 29.2 MeV. The ratio of the ^{13}C to the ^{12}C cross sections are shown on the right. The curves are the best fit calculations described in the text with the DRRS code (solid line) and SMC code (dashed line).
2. Angular distributions and ratio for $^{12,13}C$ at 49.5 MeV. The curves are as described in Fig. 1.
3. Angular distributions and ratio for the elastic scattering of π^- from ^{16}O and ^{18}O at 29.2 MeV. The curves are the best fit calculations described in the text with the DRRS code (solid line) and the SMC code (dashed line).



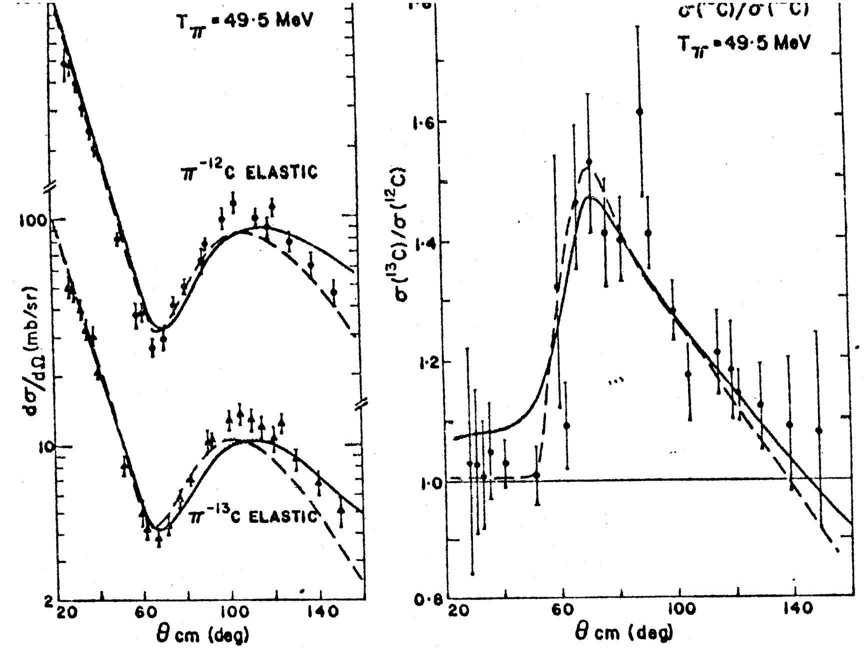

T_π = 49·5 MeV
π-^{12}C ELASTIC
π-^{13}C ELASTIC
$d\sigma/d\Omega$ (mb/sr)
100
10
2
20
60
100
140
θ cm (deg)
$\sigma(^{13}C)/\sigma(^{12}C)$
T_π = 49·5 MeV
1·6
1·4
1·2
1·0
0·8
20
60
100
140
θ cm (deg)

Fig. 2

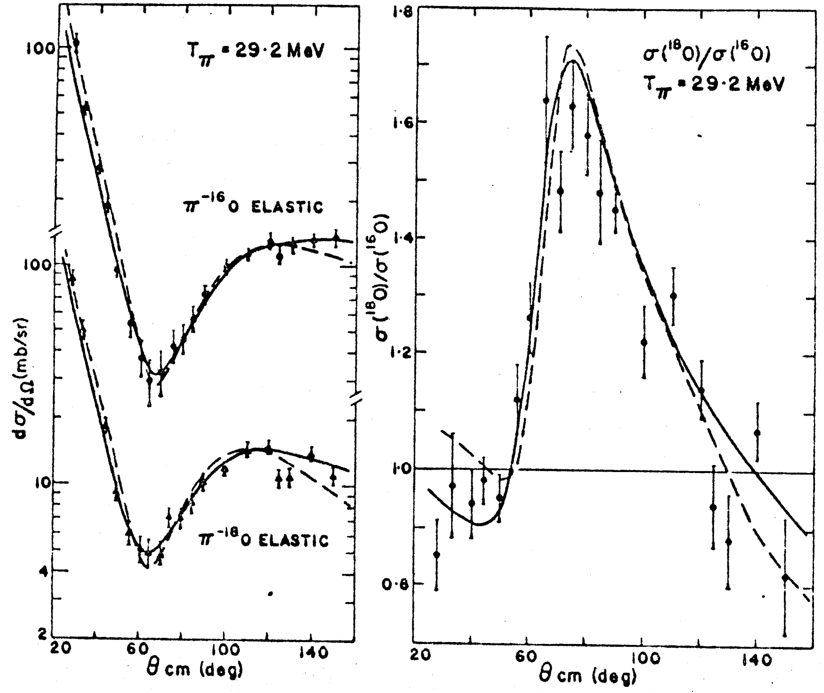

T_π = 29·2 MeV
π-^{16}O ELASTIC
π-^{18}O ELASTIC
$d\sigma/d\Omega$ (mb/sr)
100
100
10
4
2
20
60
100
140
θ cm (deg)
$\sigma(^{18}O)/\sigma(^{16}O)$
T_π = 29·2 MeV
1·8
1·6
1·4
1·2
1·0
0·8
20
60
100
140
θ cm (deg)

Fig. 3