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COMPARISON OF SCALING DEVIATIONS IN NEUTRINO, ELECTRON AND MUON INELASTIC SCATTERING

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ABSTRACT

The energy dependence of the $x = q^2/2M\nu$ distribution in νp and νN inelastic scattering from ~ 2 GeV to ~ 60 GeV is found to be consistent with the deviations from Bjorken scaling observed in inelastic $\mu N/eN$ scattering. A simple parametrization of the deviations is given.

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This letter describes an investigation of departures from Bjorken scaling in inelastic charged-current neutrino-nucleon scattering. The data employed are from experiments in hydrogen in the Argonne 12-foot bubble chamber [1] and in the Fermilab 15-foot chamber [2], and from experiments with nuclear targets in Gargamelle, CERN [3] and by the HPWF group at Fermilab [4]. A comparison is made between the normalized distributions in the Bjorken variable $x = q^2/2M\nu$ in two energy ranges: for νp experiments, at $E = 1\text{-}6$ GeV (ANL) and $E = 15\text{-}200$ GeV (FNAL); and for the νN experiments, at $E = 3\text{-}5$ GeV (CERN) and $E = 10\text{-}300$ GeV (FNAL). It is important to emphasize that the type of scaling deviations discussed here are not to be confused with the very large anomalies reported in antineutrino scattering at very high energies [5]. Antineutrino reactions are not considered in this letter.

The first clear evidence that the structure factors $F_i(x, q^2)$, in the region $q^2 \sim 1\text{-}20$ GeV^2 show departures from true scaling ($F_i(x, q^2) = F_i(x)$) was presented in a muon-nucleus scattering experiment at Fermilab [6] in the region of predominantly small x. Similar effects were subsequently reported in the scattering of electrons [7] and muons from hydrogen and deuterium targets [8,9]. The trend of the scale-breaking is that $F(x, q^2)$ increases with q^2 at small $x (< 0.2)$ and decreases at large $x (> 0.2)$.

In order to compare the neutrino data with that from electron/muon scattering, it is necessary to link the q^2 dependence, measured in the latter, with the E-dependence, measured in the former. Some parametrization of the data is therefore required. In the paper on SLAC ep and ed data by Riordan et al., [7], the scaling deviations in F_2 and xF_1, for $x > 0.1$, $q^2 > 1$ are described in terms of the factorization

$$F_i(x, q^2) = G_i(x)(1 + q^2/\Lambda^2)^{-2}, \qquad (1)$$

where $1/\Lambda^2 \simeq 0.02$ GeV^{-2}. Such a parametrization does not allow $F(x)$ to increase with q^2, as it is observed to do at small x. In this region, the high energy muon data of Watanabe et al., [6] and Anderson et al., [8] have been described by scaling deviations of the form

$$\frac{\partial \ln F_2(x, q^2)}{\partial \ln(q^2/q_o^2)\ \partial \ln(x/x_o)} = -a, \qquad (2)$$

where $a \approx 0.07 - 0.10$, $q_o^2 = 3$ GeV^2 and $x_o \simeq 0.17$. This expression has the property that, as q^2 increases, $F_2(x, q^2)$ increases for $x \leqslant x_o$.

We have found that the data on F_2 from electron and muon scattering in hydrogen and deuterium, in the q^2 range 1-40 GeV^2, can be quite well fitted by the formula

$$F_2(x, q^2) = F_2(x, q_o^2)(q^2/q_o^2)^{f(x)}, \qquad (3)$$

where $$f(x) = \frac{\partial \ln F_2(x, q^2)}{\partial \ln q^2} \simeq (0.25-x) \qquad (4)$$

over the range $0 \leqslant x \leqslant 0.8$ investigated ($q_o^2 = 3$ GeV^2, as before). These data are shown in figs 1(a) and (b), the points indicating least-square fits. The range of q^2 covered by the electron data depends on x, varying from $q^2 \sim 1$-5 GeV^2 for $x < 0.3$ to $q^2 \sim 5$-15 GeV^2 for $x > 0.5$. In addition to this data from Riordan et al., there is less complete electron data from Atwood [10], mainly for $x > 0.5$ and large q^2 (up to 30 GeV^2), showing a similar q^2 dependence.

The results for the other structure function, $F_1(x, q^2)$, are less accurate than those for $F_2(x, q^2)$; the form of the scaling deviations in F_1 is within errors, the same as that in F_2, as given in eq. (3) and (4).

The present neutrino hydrogen experiments are carried out with wide-band beams and it is not feasible either to separate the structure functions $F_1^{\nu p}$, $F_2^{\nu p}$ and $F_3^{\nu p}$, or to exhibit their q^2 dependence directly. However, it can be shown that, if all three structure functions behave according to eq. (3) then

$$\partial \ln \left[\frac{1}{\sigma}\frac{d\sigma}{dx}\right] \Big/ \partial \ln E = f(x). \qquad (5)$$

In the case of neutrino reactions, the F_2 term alone contributes more than half the total cross section and eq. (5) can therefore be expected to hold to good approximation.

We have first compared the normalized x distributions (1/N) dN/dx observed in the ANL νp data (258 events, $1 < E \leqslant 6$ GeV) with those of the Fermilab νp data (450 events, $15 < E \leqslant 200$ GeV)$^{(*)}$. Both sets of data have been corrected for the known effects of finite energy resolution, neutral current background, neutron-induced background, and muon acceptance. The x distributions are shown in figs 3(a) and (b). Comparing these, and using the fact that the mean value of the logarithmic energy difference in the two experiments is $\Delta \ln E = 3.05$, we obtain the points shown in fig. 2(a). These are in quite close agreement with the trend observed in the electron and muon data.

We have also compared the CERN Gargamelle neutrino-freon data with the HPWF collaboration data for neutrinos on a nuclear target. For the CERN data [3] events in the energy range 3-5 GeV (total 450) were selected; elastic events, which only contribute for $x \geqslant 0.8$, have been removed. The HPWF data [4] consists of 2474 events in the range E = 10-300 GeV. The x distributions are given in figs 4(a) and (b) and the comparison in fig. 2(b). Again, there is good agreement with the trends in electron and muon scattering. A similar analysis can be carried out using the FNAL νp data set, or the FNAL νN data set, alone, and dividing each into two or more energy bands. The leverage $\Delta \ln E$ is however much reduced and the x dependence observed, although consistent with the results of fig. 1, is hardly significant statistically.

In the neutrino experiments, the mean value of q^2 at energy E is given by $\langle q^2 \rangle \sim 0.2E$ GeV^2 (E in GeV), so that for x = 0.25 (the average

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(*) The neutrino data sets are compared under the assumption of equal area, i.e. that σ/E is constant or slowly varying. If the equal area assumption is relaxed, the effect is to change the intercept (0.25) but not the slope in the expression $f(x) = 0.25 - x$.

value) the ANL and Gargamelle data refer to the range $q^2 \sim 0.5 - 1$ GeV^2, below that of the electron/muon data. The two FNAL neutrino experiments cover the range $q^2 \sim 3$-30 GeV^2.

The actual shapes of the neutrino x-distributions can easily be predicted from the SLAC data and the form (3) for the q^2-dependence. Assuming strange quarks make a negligible contribution to both neutrino and electron cross sections for $x > 0.1$, and that the sea of quark-antiquark pairs is charge-symmetric, the structure functions for neutrino scattering on a proton and nucleon are

$$F_2^{\nu p}(x,\ q_o^2) = (24/5)F_2^{ed}(x,\ q_o^2) - 6F_2^{ep}(x,\ q_o^2), \tag{6a}$$

$$F_2^{\nu N}(x,\ q_o^2) = (9/5)F_2^{ed}(x,\ q_o^2). \tag{6b}$$

In all four neutrino experiments, the observed distributions in $y = \nu/E$ are approximately flat. Little error is made by assuming $d\sigma/dy$ = constant and therefore, since $q^2 = 2MExy$,

$$\frac{dN^{\nu p,\ \nu N}}{dx}(x,\ E) \propto \frac{F_2^{\nu p,\ \nu N}(x,\ q_o^2)}{[1 + f(x)]}\left(\frac{2MEx}{q_o^2}\right)^{f(x)} . \tag{7}$$

This expression was evaluated for each experiment assuming $f(x) = 0.25 - x$, and the results are given in figs 3 and 4. The agreement between the observed and predicted shapes is remarkably good. The dashed curve in each case shows the prediction for

$$F_2^{\nu p,\ \nu N}(x) = F_2^{\nu p,\ \nu N}(x,\ q_o^2 = 3),$$

independent of energy i.e. no scaling deviations.

We conclude therefore that the magnitude and form of the changes in the x distributions in neutrino inelastic scattering with energy are consistent with the observed q^2 dependence of the structure functions in electron and muon scattering. This implies that all three structure functions F_2, $x\ F_1$ and $x\ F_3$ in neutrino scattering must have a similar q^2-dependence to those (F_2^e and $x\ F_1^e$) in electron and muon scattering.

Finally, the results presented here may be compared with theoretical expectations on the approach to Bjorken scaling. As is well known, the magnitude of the deviations from scaling at finite q^2 depends on the choice of scaling variable. For example, in the SLAC energy range, the deviations (for $q^2 > 1\ \mathrm{GeV}^2$, $W^2 > 4\ \mathrm{GeV}^2$) using the Bloom-Gilman variable [11] $x' = q^2/(2M\nu + M^2)$ are only about one third of those in the Bjorken variable x. Some modification of the scaling variable is expected theoretically because of target mass effects [12-16], amounting to a change $-\Delta x/x \sim M^2x^2/q^2$. At present, there seems to be no general agreement on how to calculate such effects exactly over the whole kinematic range $x = 0 \to 1$. According to recent calculations, mass corrections are unable to account for the bulk of the observed deviations from scaling [13].

Deviations from scaling may also occur because of the effects of (heavy quark) thresholds, associated with new quantum numbers. Such effects should be quite different in neutrino and in electron/muon scattering. We conclude that they are not detectable within the error limits of the present data.

If new quark thresholds are unimportant and mass correction terms cannot account for the effects observed, the dominant source of the scaling deviations must be the q^2-dependence of the quark-gluon coupling constant, $\alpha(q^2)$. In asymptotically free field theories (AFFT), $\alpha(q^2)$ goes to zero as $(\ln q^2/\mu^2)^{-1}$ for sufficiently large q^2 [17]. The unknown parameter μ^2 is the arbitrary renormalization point of the theory. The moments of the structure functions are predicted to fall as $(\ln q^2/\mu^2)^{-\gamma_i}$ where γ_i are positive constants of order unity. Clearly the simple power-law dependence eq. (3) with $f(x) = 0.25 - x$ independent of q^2 would predict moments increasing without limit as $q^2 \to \infty$, in disagreement with AFFT. However, the SLAC data are also compatible with values of $\partial \ln F_2/\partial \ln q^2$ which show a weak q^2-dependence at fixed x. In AFFT, this would imply a small value for μ^2 ($\ll 1$ GeV^2). Small values for μ^2 have previously been suggested by Johnson and Tung [18] in their analysis of FNAL muon data at small x.

In summary, our main conclusion is that the x distributions in electron, muon and neutrino scattering can be described in terms of a simple and universal parametrization of the deviations from exact scaling - whatever their origins. This parametrization holds over a substantial range in q^2 ($\sim 0.5 - 40$ GeV^2).

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FIGURE CAPTIONS

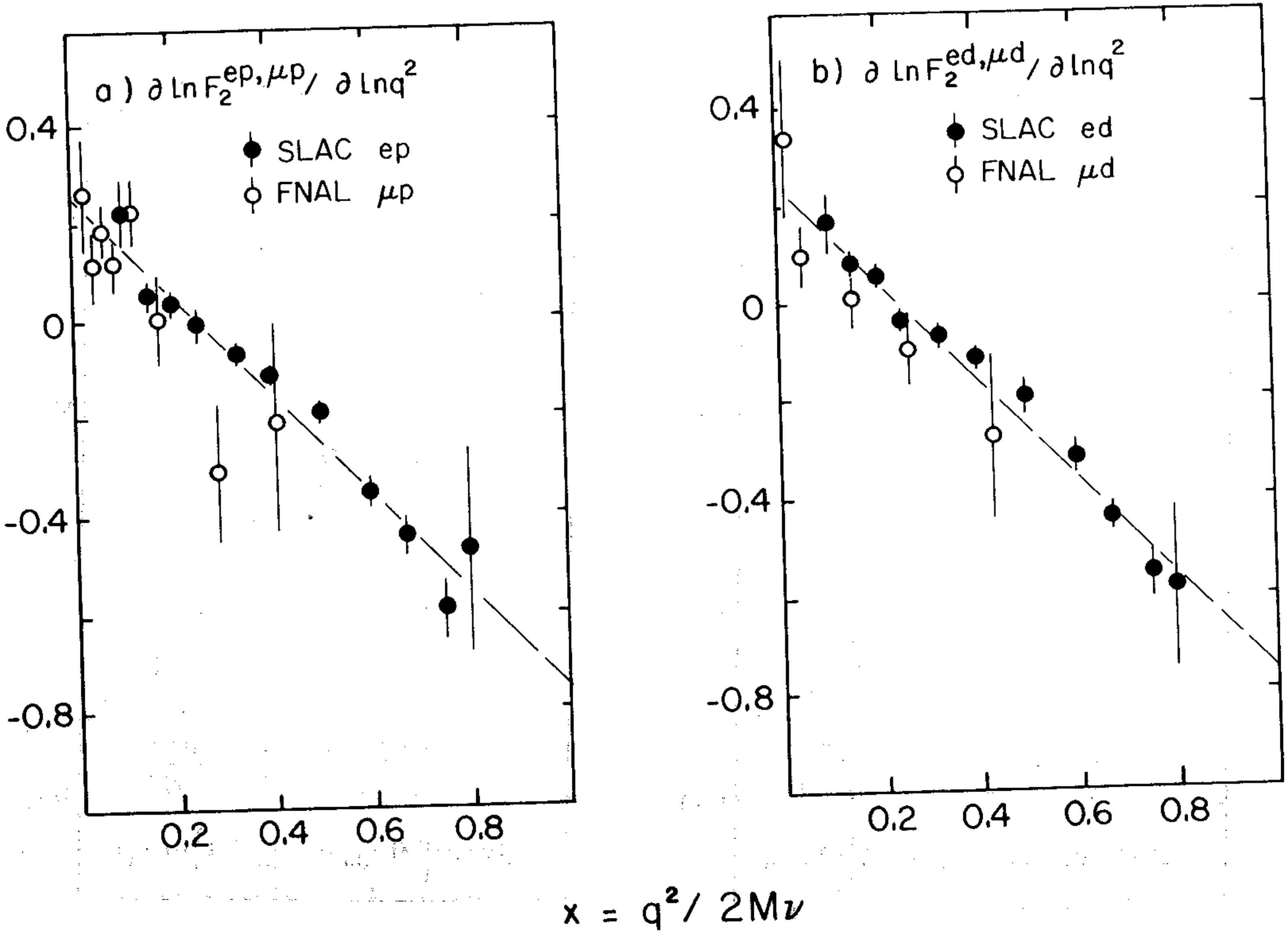
Fig. 1 The slopes $\partial \ln F_2(x, q^2)/\partial \ln q^2$ of the power-law dependence of the structure functions on q^2. Electron and muon scattering data are shown for: (a) hydrogen and (b) deuterium. The points in all cases show least square fits. The dashed line shows the simple (0.25 - x) dependence.

Fig. 2(a) The logarithmic intensity ratio $\ln\left(\frac{1}{N_1}\frac{dN_1}{dx}\bigg/\frac{1}{N_2}\frac{dN_2}{dx}\right)$ of x distributions in νp collisions observed in the FNAL 15' chamber and the ANL 12' chamber, divided by the logarithmic energy ratio ($\Delta \ln E = 3.05$). If the scaling deviations have the same origin as for fig. 1 and the neutrino cross sections are linear with energy, a similar x dependence is expected.

(b) A similar comparison for νN collisions. The HPWF counter data (taken at FNAL) are compared with the Gargamelle heavy liquid chamber data (taken at CERN).

Fig. 3 Observed x distributions for the neutrino events, compared with predictions from SLAC electron data, using eqs (3), (6) and (7): (a) νp ANL, $\langle \ln E \rangle = 0.663$ and (b) νp FNAL, $\langle \ln E \rangle = 3.71$. The dashed line indicates the variation expected for fixed $q^2 = q_o^2 = 3$ GeV^2 i.e. no scaling deviations. The observed and predicted distributions are normalized to equal area in the range $x = 0.1 \to 0.8$.

Fig. 4 Similar distributions to those of fig. 3, for νN data from: (a) CERN Gargamelle, $\langle \ln E \rangle = 1.39$ and (b) HPWF (FNAL), $\langle \ln E \rangle = 3.90$.



a) $\partial \ln F_2^{ep,\mu p} / \partial \ln q^2$
● SLAC ep
○ FNAL μp
b) $\partial \ln F_2^{ed,\mu d} / \partial \ln q^2$
● SLAC ed
○ FNAL μd
0.4
0
-0.4
-0.8
0.2
0.4
0.6
0.8
$x = q^2 / 2M\nu$

fig. 1

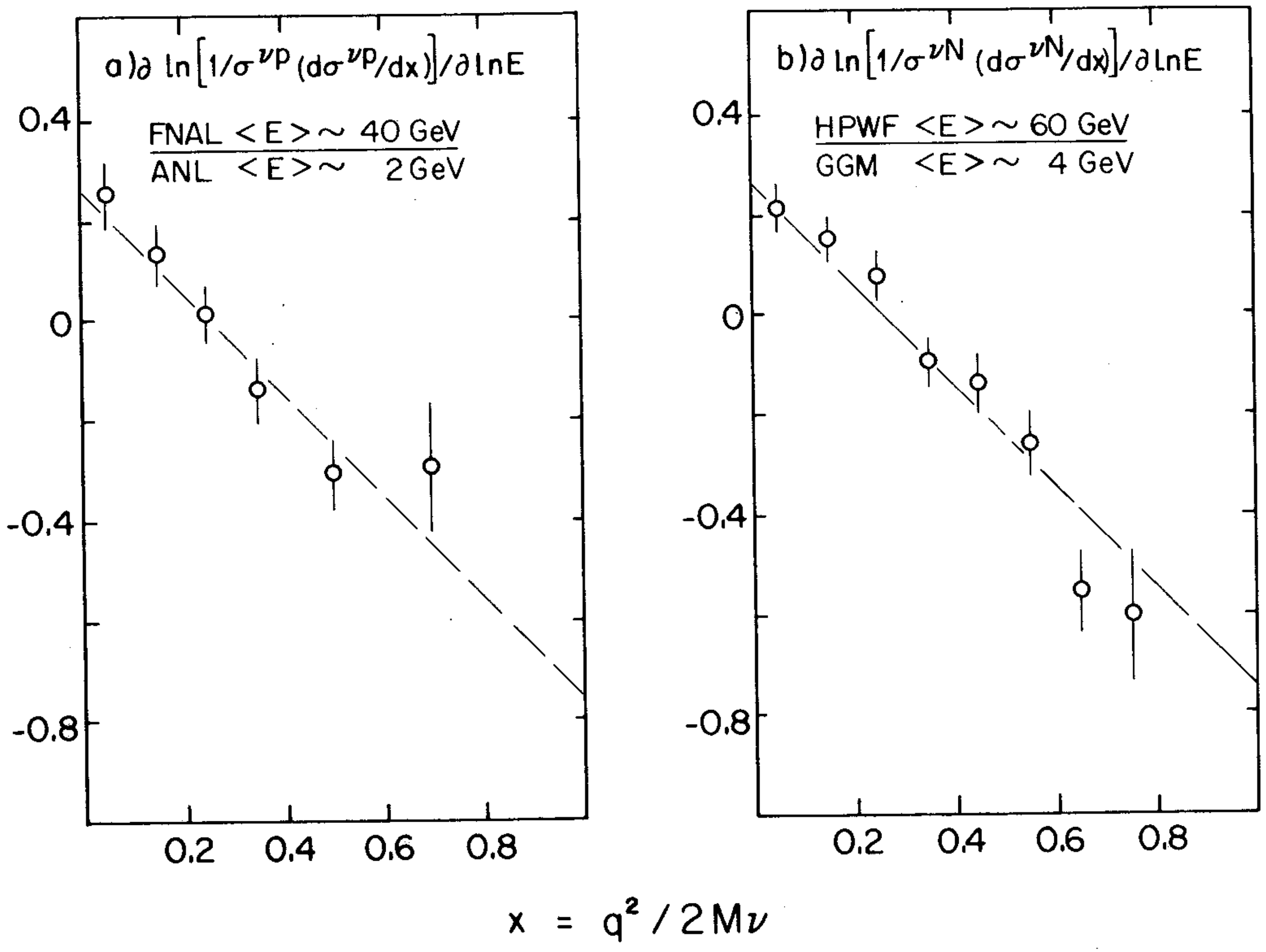

a) $\partial \ln\left[1/\sigma^{\nu p}(d\sigma^{\nu p}/dx)\right]/\partial \ln E$
FNAL <E> ~ 40 GeV
ANL <E> ~ 2 GeV
b) $\partial \ln\left[1/\sigma^{\nu N}(d\sigma^{\nu N}/dx)\right]/\partial \ln E$
HPWF <E> ~ 60 GeV
GGM <E> ~ 4 GeV
0.4
0
-0.4
-0.8
0.2
0.4
0.6
0.8
$x = q^2/2M\nu$

fig. 2

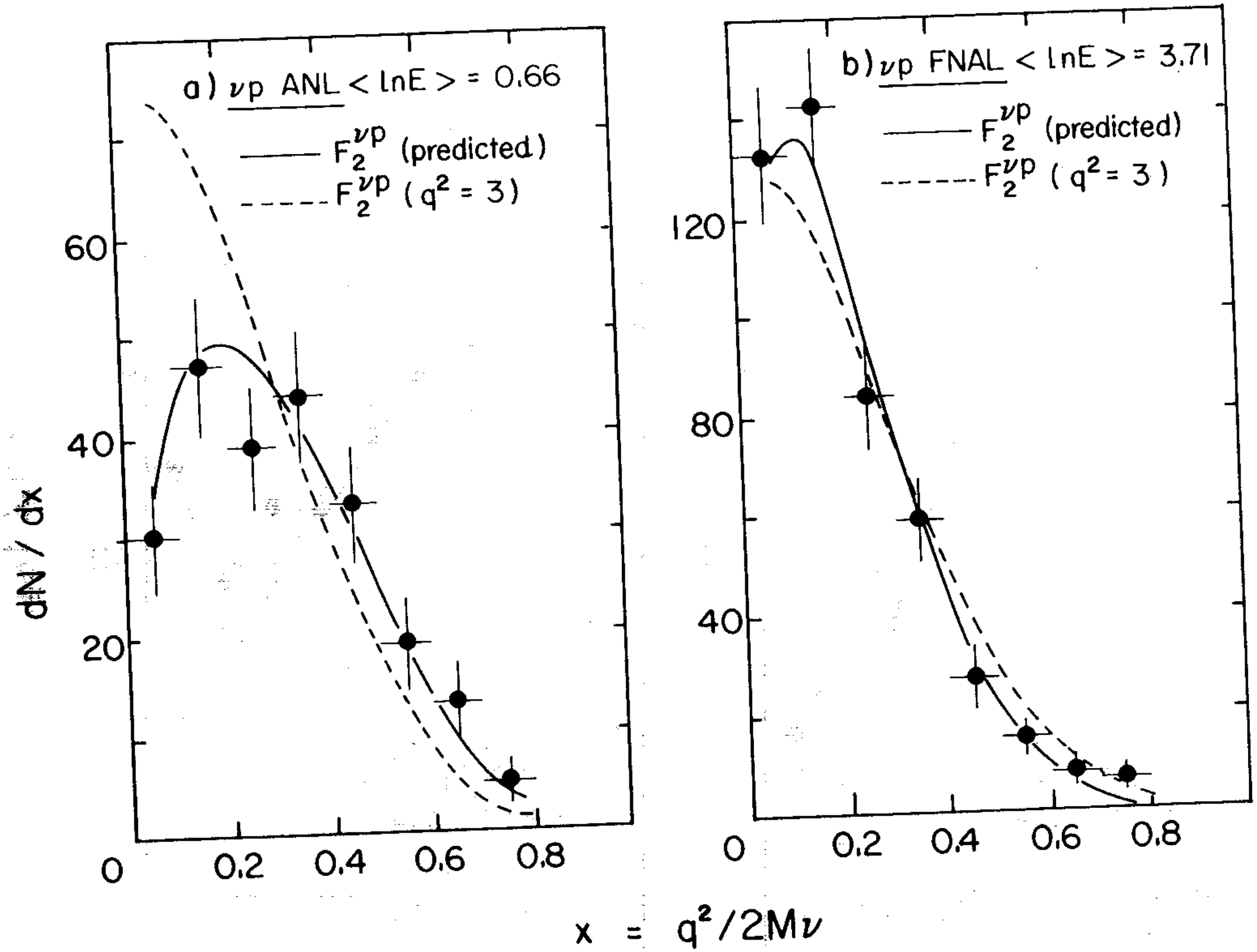

a) νp ANL $< \ln E > = 0.66$
$F_2^{\nu p}$ (predicted)
$F_2^{\nu p}$ $(q^2 = 3)$
b) νp FNAL $< \ln E > = 3.71$
$F_2^{\nu p}$ (predicted)
$F_2^{\nu p}$ $(q^2 = 3)$
dN / dx
0
20
40
60
80
120
0.2
0.4
0.6
0.8
$x = q^2/2M\nu$

fig. 3

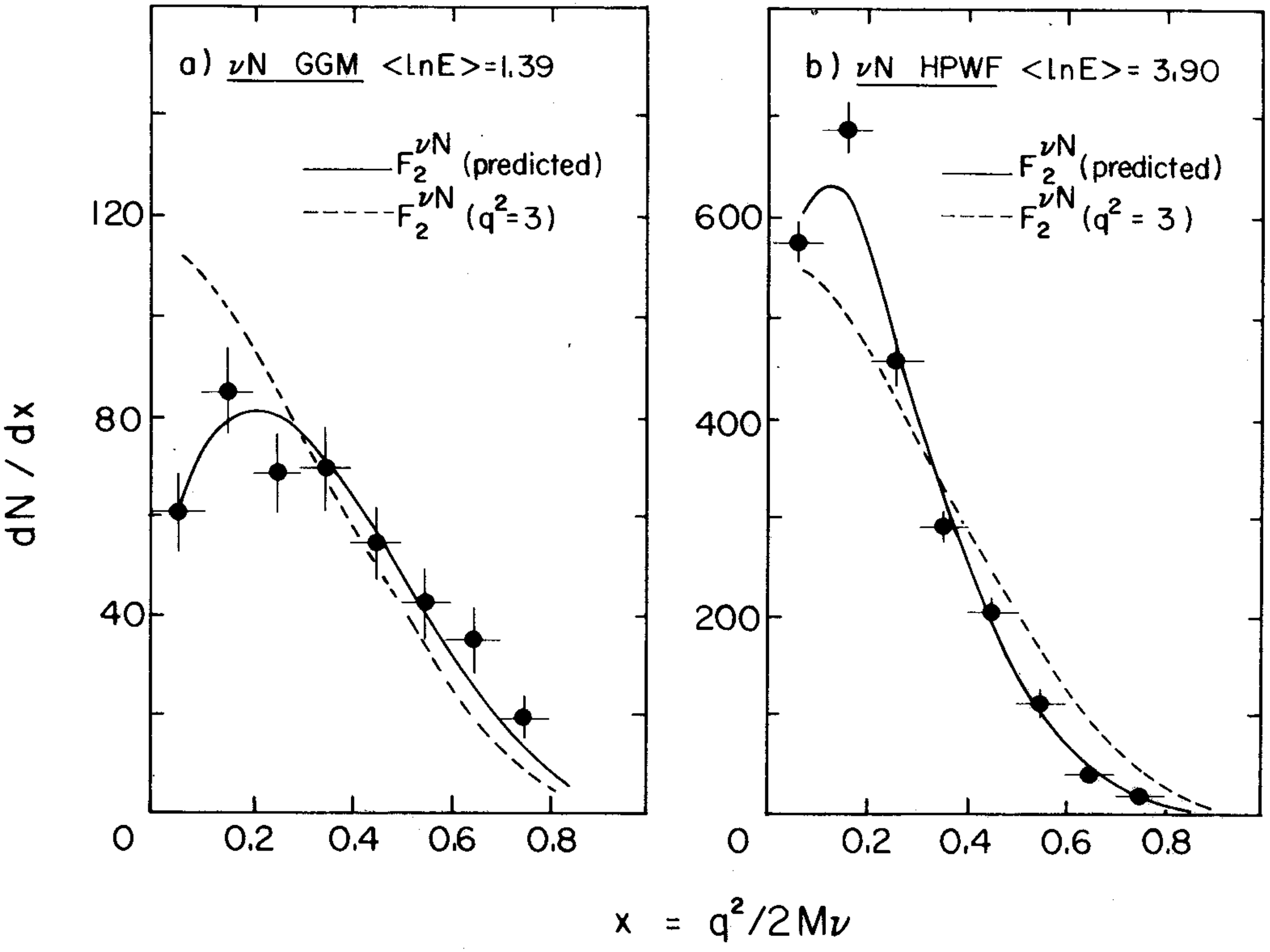

a) νN GGM <lnE>=1.39
F₂^νN (predicted)
F₂^νN (q²=3)
b) νN HPWF <lnE>= 3.90
F₂^νN (predicted)
F₂^νN (q² = 3)
dN / dx
120
80
40
0
600
400
200
0
0.2
0.4
0.6
0.8
x = q²/2Mν

fig. 4