Determining the Local Coordinate System Parameters of Detectors

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Parameters determining the position of detectors in some general Cartesian coordinate system are necessary for solving a track reconstruction problem. As a rule, the parameters are determined by geodesic methods but their precision in this case is much lower than that of detectors. To perfom a high-quality reconstruction of detected events, the geodesic values must be improved. The results of the registration of particles by detectors are the input information for the solving of this problem. The problem is studied in a wide number of papers [1 – 7]. The present paper proposes an economical method for difining the consistency estimates of the local coordinate system parameters of discrete detectors. The theoretical basis is given. The method has been successfully tested using the Monte-Carlo method.

Let some general Cartesian coordinate system be defined where m detectors have been placed for measuring x coordinates and m detectors have been placed for measuring y coordinates of a particle's track. Let all the detectors be combined in pairs so that each pair measures x and y coordinates and the distance between the detectors of the i-th pair is equal to Δ_i and the angle is equal to $\pi/2$. A local coordinate system X_iY_i $(i = 1, 2, ..., m)$ has been connected with each pair. Let the angle between the perpendicular to the plane X_iOY_i and the axis OZ be equal to 0. Let each local coordinate system be turned through an angle α_i relative to XOY and the origin O_i have coordinates (S_i^x, S_i^y, z_i). The parameters z_i can be replaced by $k_i (k_i = (z_i - z_1)/(z_m - z_1)$. The parameters $\alpha_i, S_i^x, S_i^y, k_i$ -taking into consideration measuring errors - can be found from the minimum of the function

$$
\begin{aligned}
F = & \textstyle\sum_{j=1}^{N} \sum_{i=2}^{m-1} \{y_{ij}cos(\alpha_i) + x_{ij}sin(\alpha_i) + T_{ij}cos(\alpha_i) + S_i^y - \\
& -(1-k_i)[(y_{1j} + \varepsilon_{1j}^y)cos(\alpha_1) + (x_{ij} + \varepsilon_{1j}^x)sin(\alpha_1) + T_{1j}cos(\alpha_1) + S_1^y] - \\
& -k_i[(y_{mj} + \varepsilon_{mj}^y)cos(\alpha_m) + (x_{mj} + \varepsilon_{mj}^x)sin(\alpha_m) + T_{mj}cos(\alpha_m) + S_m^y]\}^2 + \\
& + \textstyle\sum_{j=1}^{N} \sum_{i=2}^{m-1} \{x_{ij}cos(\alpha_i) - y_{ij}sin(\alpha_i) - T_{ij}sin(\alpha_i) + S_i^x - \\
& -(1-k_i)[(x_{1j} + \varepsilon_{1j}^x)cos(\alpha_1) - (y_{ij} + \varepsilon_{1j}^y)sin(\alpha_1) - T_{1j}sin(\alpha_1) + S_1^x] - \\
& -k_i[(x_{mj} + \varepsilon_{mj}^x)cos(\alpha_m) - (y_{mj} + \varepsilon_{mj}^y)sin(\alpha_m) - T_{mj}sin(\alpha_m) + S_m^x]\}^2,
\end{aligned}
\tag{1}
$$

where N-a number of straightforward tracks that intersect m local coordinate systems, $x_{ij}(y_{ij})$-measuring of a coordinate (j-track number, i-detector number), $\varepsilon^x(\varepsilon^y)$- uncorrelated random errors arising from measuring a coordinate with zero mean and dispersion $D_x(D_y)$, $T_{ij} \equiv (a_{xj}sin(\alpha_i) - a_{yj}cos(\alpha_i))\Delta_i$, $a_{xj}(a_{yj})$-tangent of the inclination angle of the projection of the i-th track on the plane $X0Z(Y0Z)$. Derivation of F with respect to α, S^x, S^y, k qives

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$$\begin{cases} \sum_{j=1}^{N}\sum_{i=1}^{m}[(y_{ij}+d_{ij}^{y}+T_{ij})cos(\alpha_i)+(x_{ij}+d_{ij}^{x})sin(\alpha_i)+S_i^y] \\ [(x_{lj}+d_{lj}^{x}+T'_{lj})cos(\alpha_l)-(y_{lj}+d_{lj}^{y}+T_{lj})sin(\alpha_l)]c_{li}- \\ -\sum_{j=1}^{N}\sum_{i=1}^{m}[(x_{ij}+d_{ij}^{x})cos(\alpha_i)-(y_{ij}+d_{ij}^{y}+T_{ij})sin(\alpha_i)+S_i^x] \\ [(x_{lj}+d_{lj}^{x}+T_{lj})sin(\alpha_l)-(y_{lj}+d_{lj}^{y}+T_{lj})cos(\alpha_l)c_{li}=0, \\ \sum_{j=1}^{N}\sum_{i=1}^{m}[(x_{ij}+d_{ij}^{x})cos(\alpha_i)-(y_{ij}+d_{ij}^{y}+T_{ij})sin(\alpha_i)+S_i^x)]c_{li}=0, \\ \sum_{j=1}^{N}\sum_{i=1}^{m}[(x_{ij}+d_{ij}^{x})sin(\alpha_i)+(y_{ij}+d_{ij}^{y}+T_{ij})cos(\alpha_i)+S_i^y)]c_{li}=0, \\ \sum_{j=1}^{N}\sum_{i=1}^{m}[(y_{ij}+d_{ij}^{y}+T_{ij})cos(\alpha_i)+(x_{ij}+d_{ij}^{x})sin(\alpha_i)+S_i^y] \\ [(y_{1j}+d_{1j}^{y}+T_{1j})cos(\alpha_1)+(x_{1j}+d_{1j}^{x})sin(\alpha_1)-S_1^y- \\ -(y_{mj}+d_{mj}^{y}+T_{mj})cos(\alpha_m)+(x_{mj}+d_{mj}^{x})sin(\alpha_m)-S_m^y]c_{li}+ \\ +\sum_{j=1}^{N}\sum_{i=1}^{m}[(x_{ij}+d_{ij}^{x})cos(\alpha_i)-(y_{ij}+d_{ij}^{y}+T_{ij})sin(\alpha_i)+S_i^x] \\ [(x_{1j}+d_{1j}^{x})cos(\alpha_1)-(y_{1j}+d_{1j}^{y}+T_{1j})sin(\alpha_1)+S_1^x- \\ -(x_{mj}+d_{mj}^{x})cos(\alpha_m)+(y_{mj}+d_{mj}^{y}+T_{mj})sin(\alpha_m)-S_m^x]c_{li}=0, \end{cases} \quad (2)$$

where $l=1,2,...,m,\ d_{ij}^{x}=(\delta_{1i}+\delta_{mi})\varepsilon_{ij}^{x},\ d_{ij}^{y}=(\delta_{1i}+\delta_{mi})\varepsilon_{ij}^{y},\ c_{li}-$ are the elements of the matrix C,

$$C=\begin{pmatrix} -\sum_{i=2}^{m-1}(1-k_i)^2 & (1-k_2) & (1-k_3) & \dots & (1-k_{m-1}) & -\sum_{i=2}^{m-1}k_i(1-k_i) \\ -(1-k_2) & 1 & 0 & \dots & 0 & -k_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -(1-k_{m-1}) & 0 & 0 & \dots & 1 & -k_{m-1} \\ -\sum_{i=2}^{m-1}(1-k_i)k_i & k_2 & k_3 & \dots & k_{m-1} & -\sum_{i=2}^{m-1}k_i^2 \end{pmatrix}.$$

The parts of equations 1 and 4 of the system (2) that include the members with S^x, S^y can be destroyed by means of equations 2 and 3. In view of the fact that all coordinate measurements errors are uncorrelated and the values $\mid \alpha_i-\alpha_l \mid$ are small, the system (2) can be transformed

$$\begin{cases} \sum_{i=1}^{m}c_{li}(\alpha_i-\alpha_l)[\sum_{k=1}^{N}\sum_{j=1}^{N}(x_{lj}x_{ij}+y_{lj}y_{ij}-x_{lk}x_{ij}-y_{lk}y_{ij}+ \\ +T_{ij}y_{lj}+T_{lj}y_{ij}+T'_{lj}x_{ij}+T_{ij}T_{lj}-T_{ij}y_{lk}-T_{lk}y_{ij}-T'_{lk}x_{ij}-T_{ij}T_{lk})- \\ -(\delta_{1l}+\delta_{ml})(\delta_{1i}+\delta_{mi})N(N-1)(D_i^x+D_i^y)]= \\ =\sum_{i=1}^{m}\sum_{k=1}^{N}\sum_{j=1}^{N}c_{li}(x_{ij}y_{lj}-x_{lj}y_{ij}+x_{lk}y_{ij}-x_{ij}y_{lk}- \\ -T_{ij}x_{lj}+T_{lj}x_{ij}-T'_{lj}y_{ij}-T_{ij}T'_{lj}+T_{ij}x_{lk}-T_{lk}x_{ij}+T'_{lk}y_{ij}+T_{ij}T'_{lk}), \\ \sum_{i=1}^{m}\sum_{j=1}^{N}[x_{ij}cos(\alpha_i)-(y_{ij}+T_{ij})sin(\alpha_i)+S_i^x]c_{li}=0, \\ \sum_{i=1}^{m}\sum_{j=1}^{N}[x_{ij}sin(\alpha_i)+(y_{ij}+T_{ij})cos(\alpha_i)+S_i^y]c_{li}=0, \\ \sum_{i=1}^{m}\sum_{j=1}^{N}\sum_{k=1}^{N}c_{li}[(\alpha_i-\alpha_l)(x_{ij}y_{1j}-x_{1j}y_{ij}+x_{1k}y_{ij}-x_{ij}y_{1k}+x_{ij}T_{1j}- \\ -x_{1j}T_{ij}+x_{1k}T_{ij}-x_{ij1k})-(\alpha_i-\alpha_m)(x_{ij}y_{mj}-x_{mj}y_{ij}+x_{mk}y_{ij}-x_{ij}y_{mk}+ \\ +x_{ij}T_{mj}-x_{mj}T_{ij}+x_{mk}T_{ij}-x_{ij}T_{mk})]= \\ =\sum_{i=1}^{m}c_{li}[\sum_{j=1}^{N}\sum_{k=1}^{N}(x_{ij}x_{1j}+y_{1j}y_{ij}-x_{1k}x_{ij}-y_{ij}y_{1k}+y_{ij}T_{1j}+ \\ +y_{1j}T_{ij}+T_{ij}T_{1j}-y_{ij}T_{1k}-y_{1k}T_{ij})-T_{1k}T_{ij}-\delta_{1i}N(N-1)(D_1^x+D_1^y)]- \\ -\sum_{i=1}^{m}c_{li}[\sum_{j=1}^{N}\sum_{k=1}^{N}(x_{ij}x_{mj}+y_{mj}y_{ij}-x_{mk}x_{ij}-y_{ij}y_{mk}+x_{ij}T_{mj}+ \\ +y_{mj}T_{ij}+T_{ij}T_{mj}-y_{mk}T_{ij}-y_{ij}T_{mk}-T_{mk}T_{ij}-\delta_{mi}N(N-1)(D_m^x+D_m^y)], \end{cases} \quad (3)$$

The necessary parameters may be calculated by the iterative method with each step solving the system of equations (3) under the assumption that all T_{ij} are known (on the first step the value of T_{ij} is calculated from the results of geodesic measuring, further from the results of the previous step). The analysis of the matrix of system (3) shows that for a single-valued calculation of all values of the angles α_l $(l = 1, 2, .., m)$ it is necessary firstly to set one value, secondly it is necessary that all dispersions of coordinates are not zero. All the values $S_i^x(S_i^y)$ may be calculated if two of them are set. All values of z_l $(l = 1, 2, ..., m)$ may be calculated if two of them are set too.

To test the method, a problem of calculating α_i, S_i^x, S_i^y has been solved for the conditions $m = 6$, $D^x = D^y = 1mm^2$, $z_{1-6} = (0,\ 80,\ 180,\ 250,\ 300,\ 400)cm$, $\Delta = 2cm$, $S_{1-6}^x = (3,\ 2,\ 3,\ 4,\ 1,\ 3)cm$, $S_{1-6}^y = (1,\ 2,\ 3,\ 4,\ 1,\ 3)cm$, $\alpha_{1-6} = (.01,\ .015,\ .02,\ .015,\ .01,\ .02)$. The absolutly precision .0001 was reached on the third step of the iterative process, the details are presented in the figure.

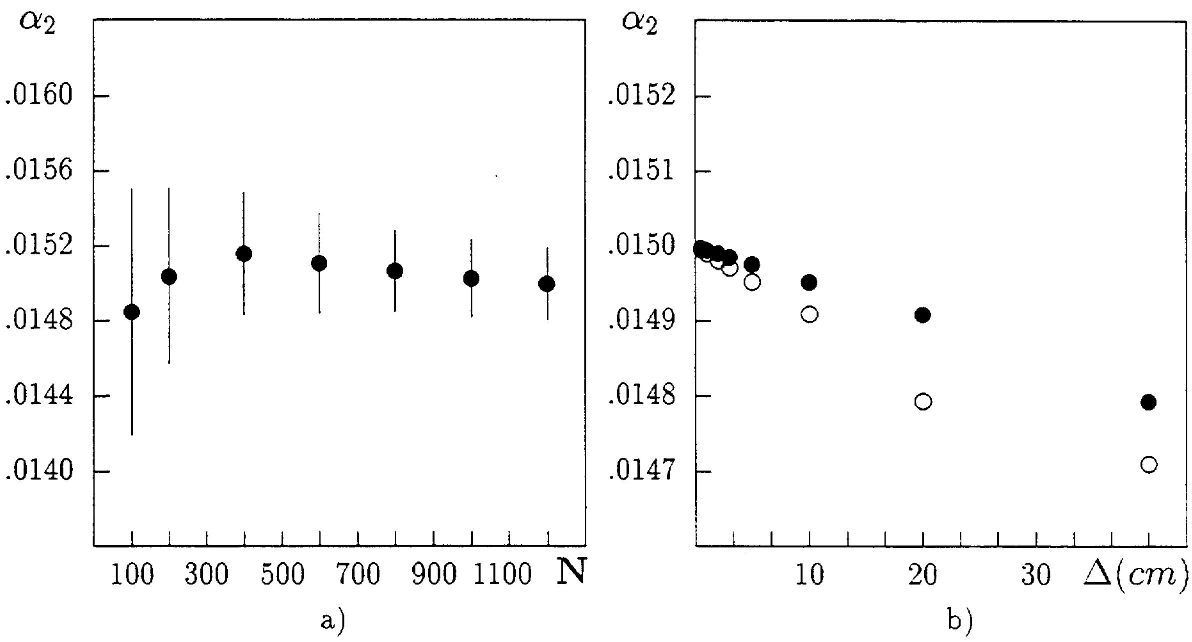

Fig. Precision of α_2 as function of a)number of tracks, b)distance Δ between the detectors of each pair without regard for it in the system (3) (● -the base of chambers is 800 cm, ○ -the base of chambers is 400 cm)

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