CYCLOTRON MAGNET CALCULATIONS

Y. Jongen and S. Zaremba
Ion Beam Applications s.a.
Avenue Jean Lenoir 6
1348 Louvain-la-Neuve, Belgium

Abstract
Different aspects of a cyclotron magnet design, and not only calculations, are reviewed. The design is an iterative process starting from a simple model which requires a vision of the complete cyclotron and an integration of all subsystems. Finally detailed cyclotron magnet calculations are described.

1 INTRODUCTION

Many accelerator physicists tend to merely reduce the cyclotron design to cyclotron magnet calculations. This quite common error should be avoided. Therefore major issues of cyclotron magnet design, not only calculations, will be discussed in this paper. The type of cyclotrons discussed here are fixed field, fixed frequency machines with azimuthally varying magnetic field (hills and valleys) provided by sectors (poles) and such as described in textbooks, for example, in [1].

Cyclotron magnet design requires a vision of the complete cyclotron. Different subsystems of cyclotron (RF (radiofrequency) system, vacuum, injection, extraction) interact with each other and always interact with the magnet. Constraints imposed by a previously completed design of the cyclotron magnet can create enormous difficulties later when designing other elements.

Like any other design it is natural to start with a crude and simple model which is iteratively refined as the design process converges. Finally very detailed and time-consuming calculations and an optimization of the cyclotron magnet are performed.

2 CYCLOTRON MAGNET QUALITIES AND FUNCTIONS

It seems that the time is over for cyclotrons to be designed and constructed mainly for nuclear physics experiments. The last few years confirm that the majority of currently constructed cyclotrons and probably all future cyclotrons will be designed for specific applications, and generally, for a single, unique task.

The most important consequences of this trend are that:

- versatility of cyclotrons is no longer an important issue;
- cost, simplicity, reliability and low radiation doses are very important for the cyclotron users and for the cyclotron designers;
- weight, occupied space and consumed power are often less important to cyclotron users than presumed by designers who see the minimization of these parameters as an interesting challenge.

Radioisotope production is the most frequent application of recently constructed cyclotrons. Low kinetic energy protons or deuterons ($\leq$ 40 MeV) are usually required by radioisotope producers. On the other hand the therapeutic applications require high kinetic energies of accelerated particles (up to hundreds of MeV).

A well designed cyclotron magnet should ensure:

- isochronous conditions during acceleration;
- axial (vertical) and radial focusing of the beam ;
- an operation point away from dangerous resonances or a fast passage of the beam through the resonance(s) zone(s).

2.1 Isochronism and magnetic field shape

The maximum kinetic energy of accelerated ions determines the magnetic rigidity of particles. Magnetic rigidity indicates values of a bending radius and the magnetic field at this radius. A reasonable choice for the magnetic field and the bending radius allows a first estimation of a cyclotron magnet radius. The magnetic rigidity as a function of the kinetic energy is expressed by the equation:

$$B \cdot \rho = \frac{1}{300 \cdot Z} \cdot \left[T^2 + 2 \cdot T \cdot E_0\right]^{1/2} \tag{1}$$

where:
B - magnetic field (Tesla);
ρ - bending radius (m);
Z - charge state of particles, with respect to electron charge;
T, E_0 - kinetic energy and the rest mass (MeV).

Ions should stay synchronized with a given phase of the cyclotron RF system during acceleration. Therefore the magnetic field should ensure a constant rotation period of ions in the cyclotron. The magnetic field with this feature is called **isochronous**.

The isochronous average magnetic field increases with radius r.

$$B(r) = B_0 \cdot \gamma(r) = B_0 \cdot \sqrt{1 + \frac{\beta^2(r)}{1 - \beta^2(r)}} \tag{2}$$

where:
$\beta(r) = v(r)/c$; c - light velocity and $\gamma(r) = 1 + T(r)/E_0$;
B_0 - the magnetic field in the center of the cyclotron.

The requested shape of the magnetic field can be obtained by different methods:

- using steel;
  The increase of a ratio hill/valley with radius r (increase of the azimuthal length of sectors) is the easiest way to obtain the isochronous magnetic field. Another possibility is to decrease the gap between poles with radius r (azimuthal shims) but it is not recommended when a high beam intensity is required.

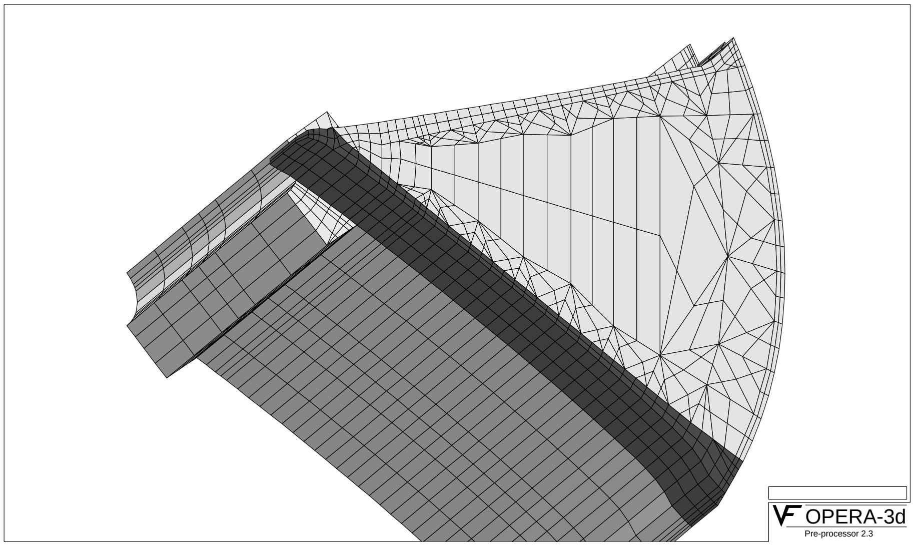


Figure 1: The removable pole edge attached to the magnet sector of the IBA C30 cyclotron. The mesh grid size is smaller in regions where high magnetic field gradients are present.

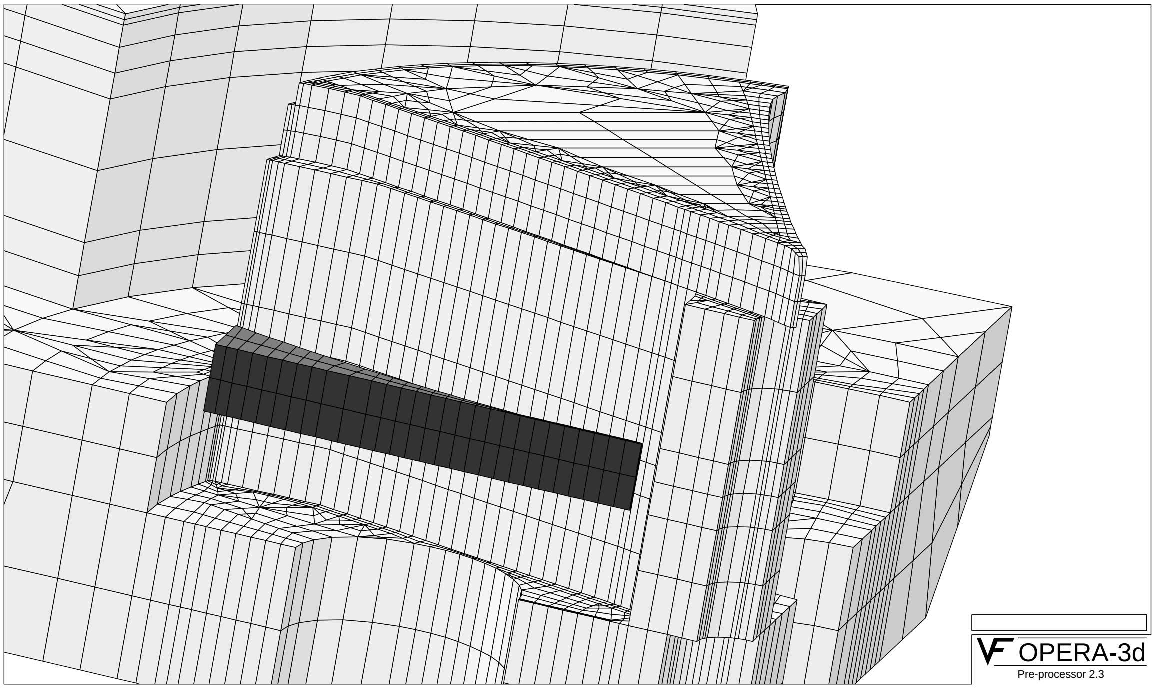
Magnet pole edges that can be dismounted, mechanically corrected and once again assembled, provide a very effective way to produce the isochronous magnetic field shape, as in figure 1. This method almost always guarantees the convergence to the isochronous magnetic field when applied as an iterative process consisting of three steps: (1) magnetic field measurements, (2) correction calculations, (3) pole edge mechanical correction. The correction of pole edges at the given radius r is based on the relation between the relative frequency error of the particles and the relative tolerance of the magnetic field

$$\Delta B(r)/B(r) = \gamma^2(r) \cdot \Delta f_p(r)/f_p(r) \tag{3}$$

where: $\Delta B(r)$ - the magnetic field error, $B(r)$ - the magnetic field value, $\Delta f_p(r)$ - frequency error and $f_p(r)$ - the rotation frequency of particles in the cyclotron.

Unfortunately these methods can only be applied to cyclotrons accelerating a single type of particles to a fixed final kinetic energy. Different kinetic energies of ions are provided by the beam extraction at different radii. In this case the extraction of particles by stripping method is more suitable than by electrostatic deflection method.

- using movable iron shims;
  Moveable iron shims can be used when two different fixed magnetic fields for two different types of ions are required. The final shape of iron shims is obtained using the same procedure as described above. IBA Cyclone 18/9 and Cyclone 30/15 cyclotrons operate according to this principle, see figure 2.



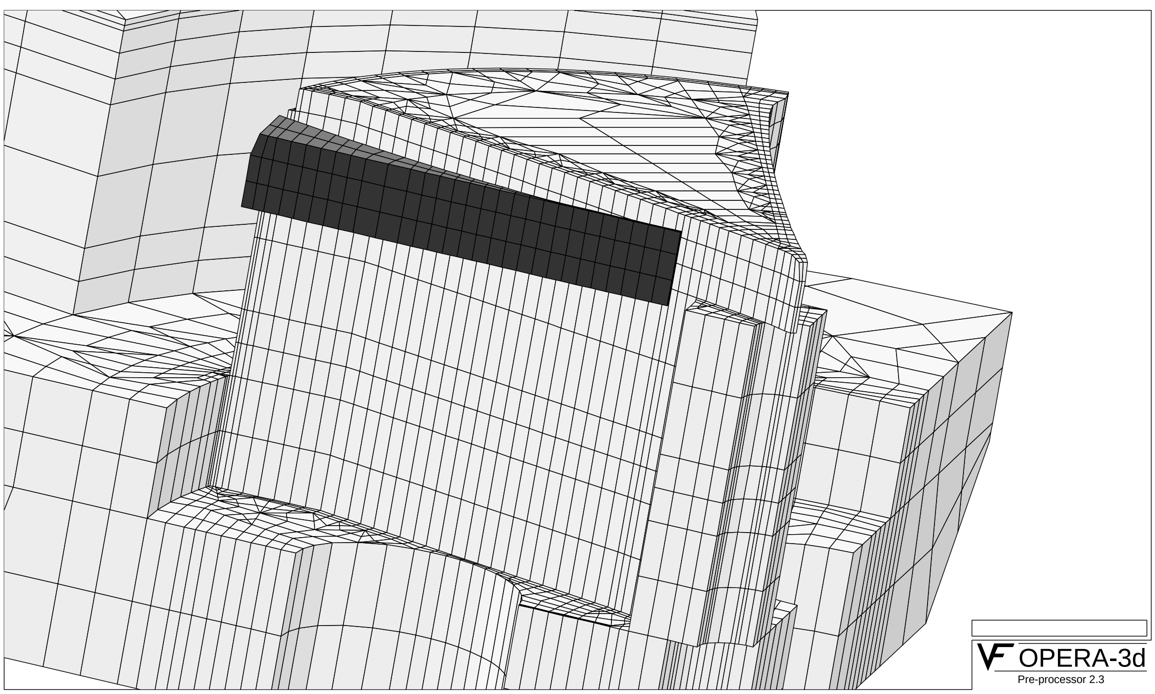


Figure 2: Moveable radial shims are placed in the middle of two opposite valleys, parallel to the cyclotron median plane. Shims close to or far from the cyclotron median plane produce the magnetic field for H^- or D^- ions respectively.

- using trim-coils;
  The flexibility of the magnetic field modified by trim-coils permits the acceleration of different types of ions to different final kinetic energies with the beam extraction at the same final radius. The extraction of particles by the electrostatic deflection is often preferred in this case.

  Trim-coils require careful tuning using an adequate number of power supplies. The cyclotron is more complicated so its reliability diminishes. It is also more expensive and usually consumes more power.

2.2 Tolerances and imperfections

Any discrepancy of a real cyclotron magnetic field and the isochronous magnetic field results in a shift of accelerated particles with respect to the chosen phase of the cyclotron RF system. An acceptable phase shift of the particles determines the tolerance of the magnetic field isochronism. The equation below gives a relation between the phase shift and the relative magnetic field imperfection:

$$d(\sin\phi) = 2 \cdot \pi \cdot h \cdot \frac{dB}{B_{iso}} \cdot n \tag{4}$$

where:
ϕ - phase shift of particles;
h - harmonic mode of the cyclotron operation;
n - number of turns;
dB - amplitude of the magnetic field imperfection;
B_{iso} - nominal value of the isochronous magnetic field

The phase shift is given by the integral of equation 4. Therefore small, often opposite signs and local imperfections of the magnetic field can be accepted. They can not be accepted when they create local, positive gradients of the magnetic field resulting in imaginary values of the axial betatron frequency ν_z and consequently in axial blow-up of the beam, see section 2.3.

The acceptable maximum value of the phase shift depends on the type of particle extraction, the geometry of the accelerating electrodes (azimuthal length of the dees) and the harmonic mode of operation.

Multiturn extraction allows larger phase shift. Values of the maximum accepted phase shift are between $20 - 45°$ in currently constructed cyclotrons, assuming that particles are still sufficiently accelerated in all accelerating gaps. The tolerance on the magnetic field $\Delta B/B \approx 10^{-4}$ is frequently obtained by substitution of the accepted phase shift to equation 4. Such requirements are possible to fulfill for the cyclotron magnet and its power supply.

For a single turn extraction without flat-topping, the accepted values of the phase shift are much smaller, in the range of few degrees. The substitution of such phase shift gives the tolerance of the magnetic field $\Delta B/B \approx 10^{-7}$. Clearly it will be difficult to create the magnetic field with such stability and to find the magnet power supply providing these characteristics.

The cyclotron magnetic field B changes due to changes of the excitation current I

of the main coil(s). Generally in measurements $\Delta B/B$ are smaller than $\Delta I/I$ due to the saturation of the magnet steel. A value of $\Delta B/B$ greater than $\Delta I/I$ more than likely indicates problems with the cyclotron magnet and/or the measurements system.

2.3 Axial (vertical) and radial focusing of the beam avoiding dangerous resonances

Axial (vertical) focusing of the beam can be provided by:

- negative field index i.e., the magnetic field decreasing with the radius;
- azimuthally varying magnetic field i.e., magnet sectors creating hills and valleys of the magnetic field;
- spiralization of magnet sectors.

The betatron frequencies can be approximated by the following equations [2] :

$$\nu_z^2 = -k + \frac{N^2}{N^2 - 1} \cdot F \cdot \left(1 + 2 \cdot \tan^2 \xi\right) \tag{5}$$

$$\nu_r^2 = 1 + k + \frac{3 \cdot N^2}{(N^2 - 1) \cdot (N^2 - 4)} \cdot F \cdot \left(1 + \tan^2 \xi\right) \tag{6}$$

where:
ν_z - axial (vertical) betatron frequency;
ν_r - radial betatron frequency;
$k = r/\langle B \rangle \cdot d\langle B \rangle/dr$ - radial field index;
N - number of symmetry periods of the cyclotron;
ξ - sector spiral angle;
F - magnetic field flutter at the radius r which can be defined using the calculated averages of the magnetic field $\langle B \rangle$ and of the magnetic field square $\langle B^2 \rangle$ at the radius r. The cyclotron magnetic field at the given radius r, having the N-fold rotational symmetry, can be presented as Fourier series. The flutter of the magnetic field can also be defined using coefficients of Fourier series $A_i, B_i, i = 1, 2, ...$:

$$F = \frac{\langle B^2 \rangle - \langle B \rangle^2}{\langle B \rangle^2} = \frac{1}{2} \cdot \sum_{i=1}^{\infty} (A_i^2 + B_i^2) \tag{7}$$

The presented analytical formulas 5–7 are approximative. Correct values of the betatron frequencies have to be calculated by programs tracking particle trajectories.

2.4 Resonances

Resonance is encountered when

$$K \cdot \nu_r + L \cdot \nu_z = P \tag{8}$$

where K, L and P are integers. P is the symmetry of the driving term and the sum $|K| + |L|$ is the order of the resonance. Resonances of order 1, 2 or 3 are driven by a dipolar, quadrupolar or sextupolar component of the guiding magnetic field respectively.

The amplitude of betatron oscillations grows at a resonance. Imperfection resonance ($K \cdot \nu_r = P$, $L \cdot \nu_z = P$) effects are dependent on the amplitude and the radial extension of imperfection in the cyclotron. Coupling resonances ($K, L \neq 0$) are important when large radial oscillations are generated. The axial acceptance of the cyclotron is generally much smaller than the radial and the beam is easily lost. The third order coupling Walkinshaw resonance $\nu_r - 2\nu_z = 0$ frequently encountered in cyclotrons is considered as one of the most destructive.

The operation point (ν_r, ν_z) moves as the particle kinetic energy increases and can cross the resonance line(s). Resonances are dangerous when the crossing is slow (low kinetic energy gain per turn) and/or when the driving term is large but the fast passage through the resonance zone does not greatly deteriorate a beam quality.

It is possible to show (e.g. [1]) that the field index is $k = \gamma^2 - 1$, when the magnetic field is isochronous. Therefore the radial betatron frequency ν_r (equation 6) can be expressed as $\nu_r \approx \gamma + \cdots$, where dots replace terms dependent on N, F and ξ.

Fundamental resonances given by the equation $\nu_r = N/2$ determine the minimum number of symmetry periods (sectors) of the cyclotron. The frequency of radial betatron oscillation starts from the value of 1 ($\gamma = 1$) at the cyclotron center. Cyclotrons with $N = 2$ sectors will be unstable from the center and therefore they do not exist.

Due to the same fundamental resonance the theoretical maximum kinetic energy of protons is limited to 469 ($\gamma = 3/2$) and 938 MeV ($\gamma = 4/2$) for cyclotrons with 3 or 4 symmetry periods (sectors) respectively. Practically the maximum kinetic energy of particles will be smaller because terms dependent on N, F and ξ increase the radial betatron frequency ν_r.

Very high kinetic energies require more symmetry periods in the cyclotron. To avoid the fundamental resonance $\nu_r = 3/2$, it was necessary to mill a deep groove in each of three sectors of the AGOR cyclotron [3]. This ingenious solution multiplied by two the number of sectors, without a significant decrease of the average magnetic field in the cyclotron.

3 MAGNET DESIGN

3.1 Decision strategies

Decisions need to be taken concerning the choice of:

- gap between magnet poles (hills of the magnetic field);
- magnetic field level between magnet poles (hills of the magnetic field);
- magnet pole radius;
- gap in the valleys;
- number of symmetry periods (sectors).

3.2 Gap between magnet poles

Very often the choice of the gap between magnet poles is a compromise between the two situations presented below.

A small gap between magnet poles reduces the required number of ampere-turns in the cyclotron main coil and allows the last orbit of accelerated particles to be brought closer to the pole edge. This reduces the radius of the pole and consequently diminishes the overall weight of the magnet which is approximatively proportional to r_{pole}^3. A small axial gap limits the radial extension of the magnetic field hole in the cyclotron center created for the central region elements or the axial injection system. The defocusing radial gradient of the magnetic field in this region can deteriorate the beam by axial blow-up.

A large gap between magnet poles increases the available space for the beam oscillating in axial (vertical) plane. Also such a gap facilitates the installation of different elements of the cyclotron:

- diagnostic probes
- stripper(s)
- central region components: ion source(s) or inflector, dee bridge
- electrostatic deflector(s) in superconducting cyclotrons

It is never possible to keep the cyclotron field isochronous up to the limit of the magnet pole. The magnetic flux beyond the last orbit is useless but has to be guided through the return yoke. An elliptic gap between the poles (closed beyond the last orbit), allows the last orbit to be brought within a few millimeters of the pole edge according to the magnetostatic theorem that the ellipsoid void in an infinite bloc of uniformly magnetized steel will exhibit a constant magnetic field, dependent on the shape of the ellipsoid.

In this case the magnetic field is isochronous to the magnetic shunt closing the gap. Unfortunately, a small tunnel has to be made for the extracted beam. The magnetic field in the tunnel is difficult to calculate and even more difficult to measure. Calculations show that gradients of this magnetic field are strong and nonlinear.

3.3 Number of sectors

The choice of the number of sectors is quite controversial. At first sight three sectors (symmetry periods) seem to be the best choice for low kinetic energy cyclotrons. Such structure was often used in old cyclotrons where the large gap between magnet poles provided many possibilities for the installation of the necessary subsystems. Also it was used in recently constructed super-conducting cyclotrons where the gap was small. Super-conducting magnet valleys were filled with the RF system cavities and the small axially extraction system was installed between magnet poles. A disadvantage of this solution is that three coupled resonators usually oscillate in phase so only harmonic modes $H = 3, 6, \ldots$ are possible.

Our personal opinion is that four sector (four-fold rotational symmetry) geometry of the cyclotron seems to be more practical. Two opposite valleys can be used by RF cavities operated in phase in harmonic mode $H = 2, 4, 6, \ldots$ or at 180° in $H = 1, 3, 5, \ldots$ Two other valleys can be used for other devices: ion source, electrostatic deflector, internal target or movable iron shims.

3.4 Initial magnet calculations

Initial calculations start from the estimation of the cyclotron magnet pole dimensions using equation 1. The chosen average magnetic field value $\langle B \rangle$ at the extraction radius is the basis of the first estimation of the magnetic field in the hill B_{hill}, in the valley B_{valley} and the fraction of the hill α on one symmetry period according to the equation:

$$\langle B \rangle = \alpha B_{hill} + (1 - \alpha) B_{valley} \tag{9}$$

After adding the chosen number of sectors N one can verify the focusing properties of the cyclotron. The flutter F and slightly optimistic the betatron frequencies ν_r, ν_z can be determined from the analytical equations 5–7. The reasonable minimum value of the axial betatron frequency ν_z is about $0.10 - 0.15$ when one wants to keep a small gap between magnet poles. It is necessary to change one or more parameters or introduce a spiralization of sectors when the value of ν_z is too small. Then new iteration of analytical calculations can follow.

Schematic drawings can facilitate the next step of magnet calculations. First it is necessary to estimate a total magnetic flux in the cyclotron using the average magnetic field $\langle B \rangle$ and the pole radius. The pole dimensions are extended beyond the last orbit. The part of magnetic flux of this zone can be redirected by adding a chamfer on the outer pole edge. It should be remembered that in the median plane the effective field boundary is about 0.6 gap width outside the pole edge. Then, the value of the magnetic field in the flux return (return yoke) has to be determined taking into consideration the magnetic properties of the used steel. The flux return shape can be designed when cross-sections of the flux return and the coil are known.

The coil cross-section is calculated from the number of ampere-turns in the cyclotron. The simple equation $n \cdot I \approx 8 \cdot 10^5 \cdot B \cdot l$ where $n \cdot I$ is the number of ampere-turns (A), B is the magnetic field (T) and l is the gap width (m) which determines the number of ampere-turns in the cyclotron gap. Then a certain number of ampere-turns for the steel should be added. Added ampere-turns can be calculated assuming the constant value of the magnetic permeability in the magnet steel, or just estimated. Our experience confirms that it is difficult at this moment to find a correct value of the magnetic permeability due to strong non-linearities of the magnetic field in magnet steel. An estimation can be made by adding a generous percentage of ampere-turns to avoid later problems with the coil power supply (e.g. $30\% - 50\%$ of ampere-turns). The total number of ampere-turns gives the choice of coil cross-section and the current density. From the current density one can estimate the coil cooling requirements. Coil shape design is possible with the last information.

After finishing this step of magnet calculations, it is necessary to develop a complete preliminary shape of the cyclotron with all subsystems included. Holes and ports required by other cyclotron elements have to be taken into account to evaluate changes in the cyclotron magnet structure.

The calculations described above are simple, analytical and allow an initial layout of the cyclotron magnet. More sophisticated tools are required for further optimization of the cyclotron magnet.

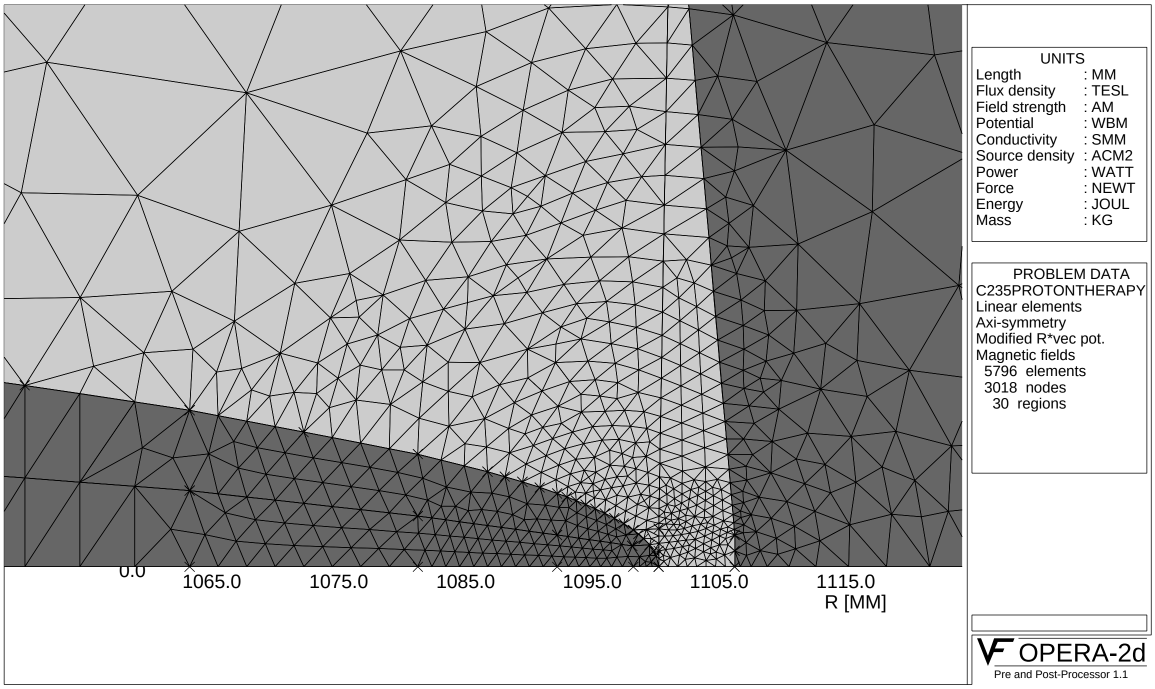


Figure 3: The detailed discretization of the elliptic gap between cyclotron magnet poles in the 2-D model. The cyclotron median plane determines a lower boundary line of the model.

3.5 Detailed magnet calculations

Detailed magnet calculations, using a finite elements method — or a similar method with the same kind of accuracy — must first confirm the validity of the initial design. They are then mainly used to uncover weak points of the design and help correcting them. Finally, they can serve to optimize the design taking into account all important and requested particular characteristics of the magnet. Detailed magnet design and calculations require the use of two-dimensional (2-D) and three-dimensional (3-D) computer codes. The list of available programs is presented in [4],[5].

The maximum number of elements in two- and three-dimensional calculations is of the same order. The two-dimensional model is only the plane and the three-dimensional model should be a volume comprising the symmetry period of the cyclotron (a part of it, when mirror symmetries are possible). Hence two-dimensional calculations offer the possibility of creating elements having much smaller grid size than three-dimensional ones and ensure a better accuracy of cyclotron magnet details. Even with smaller grid size of the 2-D model fewer nodes will be used than in the 3-D model.

Figure 3 presents the part of the two-dimensional model mesh in the IBA C235 protontherapy cyclotron with the closed elliptic gap. The same geometry model was developed and calculated in three dimensions. The analysis of the same plane in the 3-D model shows that one element only represented the part of the cyclotron magnet pole between radii 1100 and 1105 mm and up to 6 mm from the cyclotron median plane.

The execution time of programs 2-D and 3-D is proportional to n^3 where n is the number of created node points. Only application of all possible symmetries of the cyclotron magnet significantly contributes to a decrease of the total number of nodes in the 3-D model. Sometimes the construction of 3-D models which do not respect strictly model symmetries can decrease the number of created nodes and the computing time but the solution should be treated with limited confidence. Often excessively long computing times (tens of hours) and the impossibility to create more accurate model due to a lack of computer memory, encourage designers to develop more exactly two-dimensional models.

Conductors created in two-dimensional calculations are very simple and their magnetic field is calculated very quickly. Programs 3-D offer, except for a library of the standard shape conductors, practically unlimited possibilities for creating shapes of conductors as a sequence of bricks with variable cross-section. The magnetic field of coils having very exotic shapes (e.g. particular trim-coils) can be calculated this way but take a hopelessly long time (tens of hours).

2-D codes essentially solve x-y problems in which the third dimension is infinite or at least long enough to be considered as infinite. Programs 2-D serve also to solve axisymmetric (cylindrical) three-dimensional problems. Cyclotrons with an azimuthally varying field are hardly axisymmetric, but the method of stacking factors can be used in regions where the magnet structure differs from axisymmetric (sectors and valleys, important holes in the yoke). Figure 4 shows the application of this method. The stacking factor SF is defined as the fraction of the circle occupied by the real ferromagnetic material. Properties of the pseudo-material filling regions with the stacking factor are given by the B-H curve equation [6]:

$$B_{pseudo} = \mu_0 \cdot H + (B - \mu_0 \cdot H) * SF \tag{10}$$

where B and H are related by the curve of B-H for the real ferromagnetic material.

Programs 2-D are useful to study details or local modifications which produce local effects. At first the large model solution is calculated. Then a zoom of the area of interest from the large model is made (as e.g. in figure 3). Vector potential values corresponding to the large model are assigned to boundary points of the zoomed model. The analysis of the effects of perturbations has to take into account the conservation of the total magnetic flux which results in a balance of a magnetic field increase at some places and a magnetic field decrease somewhere else in the model.

Three-dimensional calculations are more time and memory-consuming than two-dimensional. Three-dimensional models are the closest to reality. Therefore results of measurements are expected to be close to results of calculations. It was observed in many cases that absolute results of calculations and measurements differ by $2-3\%$. This difference can be explained by the absolute numerical precision of the software, by differences of the B-H curves and still some geometrical differences between the model and the real cyclotron magnet. Grid size dimensions are smaller in regions where high magnetic field gradients are expected (see e.g. figures 1, 2). Despite grid size variations small discontinuities of the magnetic fields are sometimes observed due to the limited number of nodes in the model mesh.

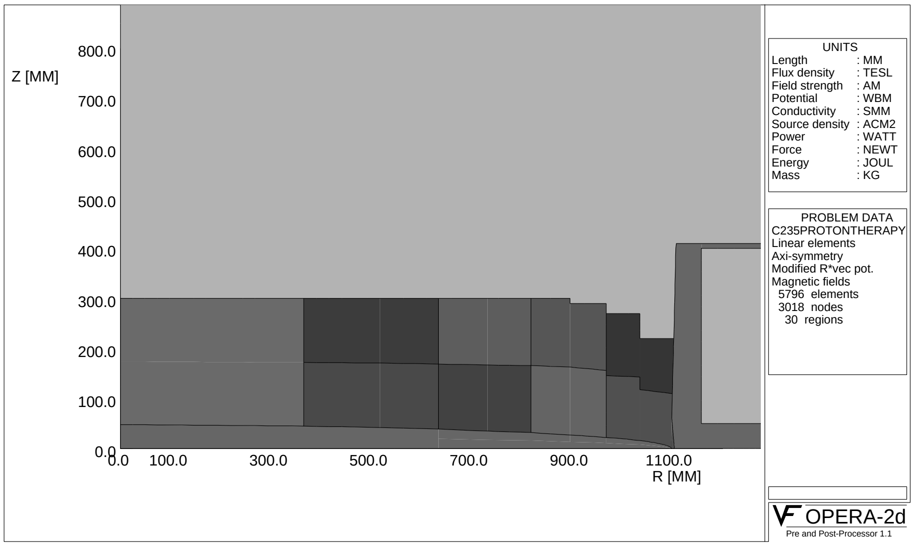


Figure 4: Different stacking factors in distinct regions simulate the variation of the hill/valley ratio of the IBA 235 MeV protontherapy cyclotron featuring an elliptical gap between its poles.

The comparison of differences between calculations of two similar models and two corresponding measurements (e.g. different coil current, shift of movable shims) very often shows a perfect agreement [7],[8].

Results of 2-D and 3-D magnetic field calculations are used to create maps of the magnetic field in the cyclotron median plane or in three-dimensions around this plane. Then the map of the magnetic field becomes a part of input data to programs calculating trajectories of particles and properties of the magnetic field [5], [9]. The most important part of the output of these programs present values of the frequency error, the integrated phase shift of particles and betatron frequencies. The magnet model has to be changed when one or more parameters are not acceptable. Our experience shows that during the creation of the 3-D model one should foresee possibilities of unexpected changes of the model geometry without the necessity to construct the model from scratch once again.

Fully acceptable values of the frequency error and the integrated phase shift of particles ensures the isochronism of the magnetic field. Correct values of betatron frequencies confirm the focusing properties of the magnetic field and the absence of dangerous resonances during acceleration.

Magnetic calculations are followed by the analysis of mechanical deformations due to the weight of magnet elements, magnetic forces between them and forces of the air pressure when the vacuum is produced in a cyclotron vacuum chamber. The request for minimal deformation and practical aspects of handling determine the partition of the magnet.

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