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Effects of SM Kaluza-Klein excitations on electroweak observables

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Abstract

The presence of an extra dimension of size $R \approx \text{TeV}^{-1}$ introduces a tower of Kaluza-Klein gauge boson excitations that affects the standard model (SM) relations between electroweak observables. The mixing of the W and Z bosons with their excitations changes their masses and couplings to fermions. This effect depends on the Higgs field, which may live in the bulk of the extra dimension, on its boundary, or may be a combination of both types of fields. We use high-precision electroweak data to constrain $1/R$. We find limits from 1 to 3 TeV from different observables, with a model independent bound of 2.5 TeV.

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1 Introduction

It is widely believed that the standard model (SM) is the low-energy limit of a more fundamental theory including gravity. It is also believed that one of the requirements for this fundamental theory is the existence of more than three spatial dimensions, which would be compact and with a radius R of Planckean size. Recently, however, it has been suggested that the extra dimensions can appear at much lower energies. A first possibility was given in Refs. [1, 2] in the context of string theory. It was shown that large extra dimensions do not necessarily spoil the gauge coupling unification of the 4D theory. A more radical possibility, proposed in Ref. [3], is to decrease the scale of unification of gravity with the gauge interactions down to the TeV. This can be realized by means of two sub-millimeter extra dimensions in which only gravity propagates. Although the gauge interactions would not feel these sub-millimeter dimensions, a fundamental scale (string scale) in the TeV region suggests the possibility of compact dimensions of this size where the SM fields do propagate.

Large ($\approx$ TeV^{-1}) extra dimensions find also an interesting motivation as a framework to break supersymmetry. This has been studied in detail in Refs. [4, 5, 6], where it was predicted a compactification scale around $3-20$ TeV. Also recently, it was discussed [7, 8] how an extra dimension could lead to the unification of the gauge forces at the TeV-scale.

In this letter we study the effects of extra dimensions on electroweak observables. If the SM gauge bosons can propagate in a compact dimension, their (quantized) momentum along this dimension can be associated to the mass n/R ($n = 1, .., \infty$) of a tower of Kaluza-Klein (KK) excitations. As a consequence the relations between electroweak observables will be modified with respect to those of the 4D SM. There are two kinds of effects. The first one is due to the presence of mixing between the zero and the n-modes of the W and Z bosons. This leads to a modification of the W and Z masses and their couplings to the fermions. The second effect arises from the exchange of KK-excitations of the W, Z and γ vector bosons, which induces extra contributions to four-fermion interactions. We calculate these effects and show how to put bounds on the size of an extra dimension from high-precision electroweak data. We find limits on $1/R$ from 1 to 3 TeV from different observables, with a model independent bound of ≈ 2.5 TeV.

2 Framework

The model that we want to study is based on an extension of the SM to 5D [4]. The fifth dimension x^5 is compactified on the segment S^1/Z_2, a circle of radius R with the identification $x^5 \to -x^5$. This segment, of length πR, has two 4D boundaries at $x^5 = 0$ and $x^5 = \pi R$ (the two fixed points of the orbifold S^1/Z_2). The SM gauge fields live in the 5D bulk, while the SM fermions are localized on the 4D boundaries. The Higgs fields can be either in the 5D bulk or on the 4D boundaries. Models with the Higgs in the bulk have been considered in Refs. [4, 8], while models with Higgs on the boundary have been considered in Refs. [7, 6]. The most general case consists of a SM Higgs which is a combination of both types of fields. We will then assume the presence of two Higgs doublets, ϕ_1 and ϕ_2, living respectively in the bulk and on the boundary.

To illustrate how to obtain the SM in such a framework (for more details see Refs. [9, 4, 6]), let us consider a U(1) gauge theory in 5D with two scalars, ϕ_1 in the bulk and ϕ_2 localized on the $x^5 = 0$ boundary, together with a fermion q living on the same boundary. We assume that all these fields have U(1) charges equal to 1. The 5D lagrangian is given by

$$\mathcal{L}_5 = -\frac{1}{4g_5^2}F_{MN}^2 + |D_M\phi_1|^2 + \Big[i\bar{q}\sigma^\mu D_\mu q + |D_\mu\phi_2|^2\Big]\delta(x^5)\,, \tag{1}$$

where $D_M = \partial_M + iV_M$, $M = (\mu, 5) = (1, ..., 5)$, and g_5 is the 5D gauge coupling. The fields living in the bulk are defined to be even under the Z_2-parity, i.e., $\Phi(x^5) = \Phi(-x^5)$ for $\Phi = V_M, \phi_1$. They can be Fourier-expanded as

$$\Phi(x^\mu, x^5) = \sum_{n=0}^{\infty} \cos\frac{nx^5}{R}\Phi^{(n)}(x^\mu)\,. \tag{2}$$

Using Eq. (2) and integrating over the fifth dimension, the resulting 4D theory (in the unitary gauge [6]) is given by

$$\begin{aligned}\mathcal{L}_4 \;=\; & \sum_{n=0}^{\infty}\left[-\frac{1}{4}F_{\mu\nu}^{(n)\,2} + \frac{1}{2}\left(\frac{n^2}{R^2} + 2g^2|\phi_1|^2\right)V_\mu^{(n)}V^{(n)\,\mu}\right] \\ +\; & g^2|\phi_2|^2\left(V_\mu^{(0)} + \sqrt{2}\sum_{n=1}^{\infty}V_\mu^{(n)}\right)^2 + i\bar{q}\sigma^\mu\left[\partial_\mu + igV_\mu^{(0)} + ig\sqrt{2}\sum_{n=1}^{\infty}V_\mu^{(n)}\right]q + \dots\,,\end{aligned} \tag{3}$$

where g is now the 4D gauge coupling, related to the 5D coupling by $g = g_5/\sqrt{\pi R}$. We are only writing the terms which are relevant to generate gauge boson masses via Higgs vacuum expectation values (VEVs) and the couplings of the gauge KK-excitations $V_\mu^{(n)}$ to the fermions on the boundary. These are the only types of terms that will be needed in our analysis. Several comments are in order. Due to the presence of the boundary ϕ_2 field and its VEV, the zero and n-mode of the gauge boson will mix. These mixing terms are allowed due to the breaking of x^5-translational invariance by the boundaries. Second, the coupling of the KK-excitations to the fermion is enhanced by a factor of $\sqrt{2}$ due to the different normalization of the zero and the n-modes in the KK-tower.

The generalization of the above lagrangian to the SM is straightforward. Following the standard notation, we will parametrize the VEVs of the Higgs by $\langle\phi_1\rangle = v\cos\beta \equiv vc_\beta$ and $\langle\phi_2\rangle = v\sin\beta \equiv vs_\beta$ [1]. For $s_\beta = 0$ the SM Higgs lives in the bulk and has KK-excitations, whereas for $s_\beta = 1$ it is a boundary field. The W gauge boson mass matrix is given by

$$\mathcal{M}_W^2 \simeq \begin{pmatrix} m_W^2 & \sqrt{2}m_W^2 s_\beta^2 & \sqrt{2}m_W^2 s_\beta^2 & \dots \\ \sqrt{2}m_W^2 s_\beta^2 & M_c^2 & & \\ \sqrt{2}m_W^2 s_\beta^2 & & (2M_c)^2 & \\ \vdots & & & \ddots \end{pmatrix}\,, \tag{4}$$

[1]We do not specify the couplings of ϕ_1 or ϕ_2 to the fermions since it is not needed here. However, the fact that the coupling of ϕ_1 to the boundary is suppressed by a factor $\sqrt{\pi R}$ suggests that ϕ_1 (ϕ_2) is the responsible for giving a mass to the bottom (top).

where $M_c \equiv 1/R$, $m_W^2 = g^2v^2/2$, and g is the SU(2)$_L$ gauge coupling. In Eq. (4) we have neglected terms of $\mathcal{O}(m_W^2)$ for the KK-excitation masses, since they are subleading in the limit $M_c \gg m_W$ considered here. From now on we will only consider the leading corrections, of $\mathcal{O}(m_W^2/M_c^2)$, to the masses and couplings of the lightest modes. The eigenvalues of the matrix (4) can be obtained at this order by the rotation $\mathcal{R}\mathcal{M}_W^2\mathcal{R}^\dagger$, with

$$\mathcal{R} \simeq \begin{pmatrix} 1 & \theta_1 & \theta_2 & \dots \\ -\theta_1 & 1 & & \\ -\theta_2 & & 1 & \\ \vdots & & & \ddots \end{pmatrix}, \quad \text{and} \quad \theta_n = -\frac{\sqrt{2}m_W^2 s_\beta^2}{n^2 M_c^2}. \tag{5}$$

The mass eigenvalues are

$$m_W^{(ph)\,2} = m_W^2 \left[1 - 2s_\beta^4 \sum_{n=1}^{\infty} \frac{m_W^2}{n^2 M_c^2}\right], \tag{6}$$

$$M_{KK}^{(n)\,2} = n^2 M_c^2 + \mathcal{O}\left(m_W^2\right), \quad n = 1, 2, ..., \infty\,. \tag{7}$$

The lightest mode, of mass $m_W^{(ph)\,2}$, is the one to be associated with the SM W boson. Its coupling to the fermions is affected by the rotation (5). We obtain

$$g^{(ph)} = g \left[1 - 2s_\beta^2 \sum_{n=1}^{\infty} \frac{m_W^2}{n^2 M_c^2}\right]. \tag{8}$$

For the neutral SM gauge bosons, W_3 and B, the situation is analogous. After the usual rotation by the electroweak angle θ_W, the states are split into the massless γ plus its KK-excitations (with masses nM_c), and the KK-tower of Zs, whose mass matrix is identical to Eq. (4) with the replacement $m_W \to m_Z$. The lightest Z boson has a mass and a gauge coupling to the fermions given by

$$m_Z^{(ph)\,2} = m_Z^2 \left[1 - 2s_\beta^4 \sum_{n=1}^{\infty} \frac{m_Z^2}{n^2 M_c^2}\right], \tag{9}$$

$$g_Z^{(ph)} = \frac{g}{\cos\theta_W} \left[1 - 2s_\beta^2 \sum_{n=1}^{\infty} \frac{m_Z^2}{n^2 M_c^2}\right], \tag{10}$$

where m_Z and $g/\cos\theta_W$ would be the mass and the coupling in the case with no mixing of the Z with its KK-excitations.

3 Electroweak observables and constraints on M_c

Let us start considering the effect of the KK-tower to the SM tree-level relation

$$G_F^{SM} = \frac{\pi\alpha}{\sqrt{2}m_W^{(ph)\,2}[1 - m_W^{(ph)\,2}/m_Z^{(ph)\,2}]}, \tag{11}$$

where by G_F^{SM} we refer to the SM prediction for the Fermi constant measured in the μ-decay, and $m_W^{(ph)}$ and $m_Z^{(ph)}$ are the measured (physical) masses. In our model the μ-decay can be also mediated by the W excitations. Therefore we have

$$\frac{G_F}{\sqrt{2}} = \frac{g^{(ph)\,2}}{8m_W^{(ph)\,2}} + \sum_{n=1}^{\infty} \frac{(\sqrt{2}g)^2}{8n^2M_c^2}\,, \tag{12}$$

where now $g^{(ph)}$ and $m_W^{(ph)\,2}$ are given in Eqs. (8) and (6), respectively. On the other hand, since α is experimentally obtained at zero momentum, the KK-contribution is negligible and we have

$$\alpha = \frac{g^2}{4\pi}\left(1 - \frac{m_W^2}{m_Z^2}\right) = \frac{g^{(ph)\,2}}{4\pi}\left(1 - \frac{m_W^{(ph)\,2}}{m_Z^{(ph)\,2}}\right)\left[1 + (2s_\beta^4 + 4s_\beta^2)\sum_{n=1}^{\infty}\frac{m_W^{(ph)\,2}}{n^2M_c^2}\right]. \tag{13}$$

From Eqs. (12), (13) and (11), we obtain

$$G_F = G_F^{SM}\left[1 - 2(s_\beta^4 + 2s_\beta^2 - 1)\sum_{n=1}^{\infty}\frac{m_W^{(ph)\,2}}{n^2M_c^2}\right], \tag{14}$$

that expresses the deviation versus the SM prediction due to the KK-excitations.

In order to compare with the high-precision electroweak data, we must include radiative corrections. The loop effects of the KK-excitations can be neglected in the limit $M_c \gg m_W$ [2]. In consequence we must only consider the ordinary SM radiative corrections. These can be easily incorporated by replacing the tree-level relation (11) by the loop-corrected one, that can be extracted from Ref. [10]. The excellent agreement between G_F^{SM} and the observed value leads to a severe constraint on the ratio $G_F/G_F^{SM} - 1$. Actually, since the experimental determination of G_F is still more precise than $m_W^{(ph)\,2}$, the analysis of electroweak observables uses G_F, $m_Z^{(ph)\,2}$ and α as input parameters, and takes the relation in (11), corrected by radiative corrections (see Eq. (10.6a) of Ref. [10]), as a SM prediction for the W physical mass m_W^{SM}. This must be compared with the experimental value [11] $m_W^{(ph)} = 80.39 \pm 0.06$ GeV. Using Eq. (14) and the relation $\sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$, we derive at the 2σ level

$$\frac{m_W^{SM\,2}[m_Z^{(ph)\,2} - m_W^{SM\,2}]}{m_W^{(ph)\,2}[m_Z^{(ph)\,2} - m_W^{(ph)\,2}]} = \left[1 - (s_\beta^4 + 2s_\beta^2 - 1)\frac{\pi^2 m_W^{(ph)\,2}}{3M_c^2}\right] = 1^{+0.0088}_{-0.0083}\,. \tag{15}$$

This translates into the bound on M_c given in Fig. 1. For $s_\beta = 0$ we have $M_c \gtrsim 1.6$ TeV. A similar bound was derived (in this limit $s_\beta = 0$) in Ref. [12]. Notice that the bound depends strongly on s_β and goes to zero for $s_\beta^2 = \sqrt{2} - 1$. Therefore, we find that this is not a good observable to constrain M_c in a model independent way.

We can proceed as above to obtain predictions for other electroweak observables. We have considered three more quantities: (1) Q_W, obtained in atomic parity violating experiments [10], (2) $\Gamma(l^+l^-)$, the leptonic width of the Z, and (3) the ρ-parameter defined as

[2] These loop effects modify the gauge coupling of the KK-excitations. We estimate a $\lesssim 10\%$ of variation on the KK-contribution to G_F.

$\rho = m_W^{(ph)\,2}/(m_Z^{(ph)\,2}\cos^2\hat{\theta}_W)$ [10]. The SM prediction for these observables can be found in Ref. [10]. In order to compare with the SM values of Ref. [10], our prediction for Q_W and $\Gamma(l^+l^-)$ must be expressed as a function of G_F (instead of $m_W^{(ph)}$) and $m_Z^{(ph)\,2}$. We obtain [3]

$$
\begin{aligned}
Q_W &= Q_W^{SM}\left[1+2(s_\beta^2-1)^2\sin^2\theta_W\sum_{n=1}^{\infty}\frac{m_Z^{(ph)\,2}}{n^2M_c^2}\right], && (16)\\
\Gamma(l^+l^-) &= \Gamma^{SM}(l^+l^-)\left[1+2\left[(s_\beta^2-1)^2\sin^2\theta_W-1\right]\sum_{n=1}^{\infty}\frac{m_Z^{(ph)\,2}}{n^2M_c^2}\right], && (17)\\
\rho &= \rho^{SM}\left[1-2s_\beta^4\sin^2\theta_W\sum_{n=1}^{\infty}\frac{m_Z^{(ph)\,2}}{n^2M_c^2}\right]. && (18)
\end{aligned}
$$

Comparing the above with the experimental values we can place bounds on M_c . These are shown in Fig. 1. The experimental values and SM predictions for Q_W and $\Gamma(l^+l^-)$ have been taken from the Table 10.3 of Ref. [10]. The experimental value of ρ has been taken from Ref. [11], and for the SM prediction we have used [10] $\rho^{SM} = 1.0109 \pm 0.0006$. We find that the strongest bound on M_c comes from the leptonic Z width, an observable that seems to be very appropriate to constrain models with extra gauge bosons. This is because (a) it is measured at the level of 0.1%, (b) the SM loop corrections are calculated with an even better precision [4], and (c) its dependence on s_β is very mild. We find an absolute bound of $M_c \gtrsim 2.5$ TeV. The bound coming from Q_W is much weaker. This disagrees with Ref. [12], where a stronger bound from Q_W was obtained in the limit $s_\beta = 0$. We think that the reason of this disagreement is that in Ref. [12] Q_W was derived without taking into account the KK contribution to G_F. The bound from ρ is not very strong either. This is due to the fact that the gauge boson sector has an approximate SU(2)-custodial symmetry only broken by the difference $(m_Z^2 - m_W^2)/m_Z^2 \simeq .23$.

One can look for other observables that would lead to analogous bounds. For example, the total width of the Z or $\sin^2\hat{\theta}_W$ from the relation in Eq. (10.9a) of Ref. [10]. The later also gives bounds around 2.5 TeV, but with a strong dependence on s_β.

KK-excitations also affect the differential cross-sections for $e^+e^- \to f^+f^-$ measured at high energies, $q^2 > m_Z^2$. These experiments can be used to test four-fermion contact interactions, and consequently to put bounds on the masses of the KK-excitations [2]. We find that the largest bound comes from limits on the vector four-fermion interaction, $\epsilon_V[e^+\gamma^\mu e^-][f^+\gamma_\mu f^-]$. In our model these are mediated (predominantly) by the KK-tower of the photon and gives $\epsilon_V = -2q_f^2e^2\sum_{n=1}^{\infty}1/(n^2M_c^2)$. The minus sign indicates that the contribution interferes destructively with the SM one. The strongest constraint on ϵ_V is found in the LEP2 experiment, that gives $\epsilon_V < 4\pi/(9.3)^2$ TeV^{-2} for leptons at the 95% CL [15]. This implies $M_c \gtrsim 1.5$ TeV. Constraints on M_c can also be obtained from direct searches for Z' at Tevatron [13]. The present limit for a SM-like Z' is $M_{Z'} > 690$ GeV. In our model, however, we must consider that the coupling of the KK-excitations to fermions is a factor $\sqrt{2}$ larger than that of the Z, and the cross-section

[3] We have neglected possible KK-contributions to the ratio $g_V^2/g_A^2 = (1-4\sin^2\theta_W)^2 \approx 0.006$.

[4] We must stress that this is true if $\Gamma(l^+l^-)$ is expressed as a function of G_F and $m_Z^{(ph)\,2}$. The reason is that the dominant SM correction is suppressed by a factor $\tan^2\theta_W$ with respect to that in G_F^{SM}.

production is enhanced by a factor of 4. From Ref. [13], we get the limit $M_c \gtrsim 820$ GeV. Similarly, from searches for W' [14] we obtain the bound $M_c \gtrsim 780$ GeV.

Finally, we want to comment on models with more than one extra dimension. Our analysis can be easily extended just by replacing the sum $\sum_{n=1}^{\infty} 1/n^2$ appearing in the above equations by the sum over all the KK-excitations of the theory. This sum, however, depends on the manifold on which the theory is compactified [2]. In addition, for more than one extra dimension it is not finite. For two extra dimensions, for example, the sum diverges logarithmically $\sim \ln(\Lambda/M_c)$ and therefore depends also on the cutoff of the theory Λ. Consequently, the bounds on M_c for more than one extra dimension will be stronger but very model dependent.

4 Conclusions

There are well motivated theoretical arguments that imply the existence of more than three spatial dimensions. In order to be consistent with all observations, of course, the extra dimensions must be compactified at some high-energy scale. If this scale is around the TeV, their presence must affect the SM electroweak predictions currently being tested at high precision experiments.

In this letter we have analyzed these effects. We have shown how the associated tower of KK-excitations of the SM fields modify the relations between different electroweak observables. We have considered the most general case by taking the SM Higgs doublet as a combination of a field living in the 5D bulk and another living on the 4D boundary of the manifold. We have compared with the present electroweak data and have put constraints on the compactification scale. We have shown that, if an extra dimension exists, it must be compactified at a scale larger than ≈ 2.5 TeV. This bound will be improved with a better experimental determination of, for example, the W mass. Also new LEP2 data on differential cross-sections for $e^+e^- \to f^+f^-$ can, as discussed above, be very useful to establish the maximum length of an extra dimension.

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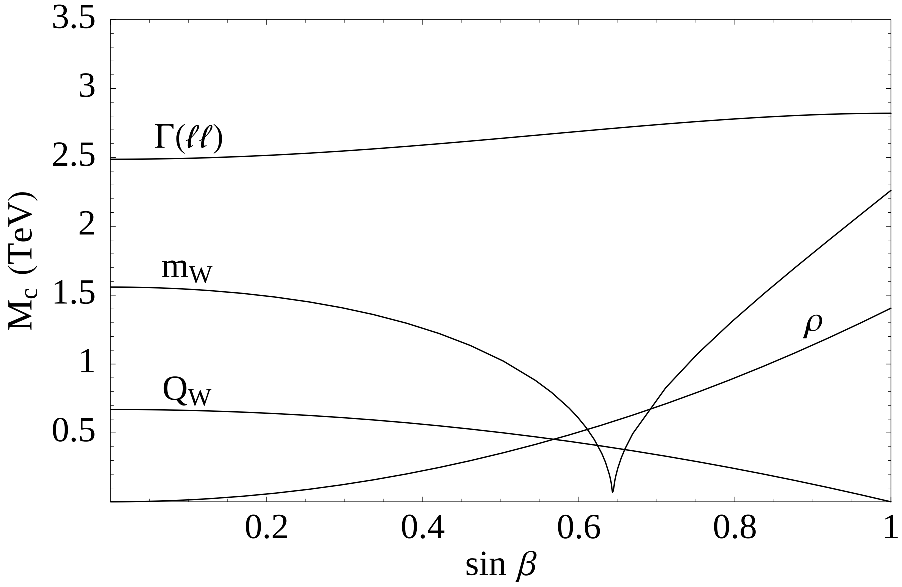
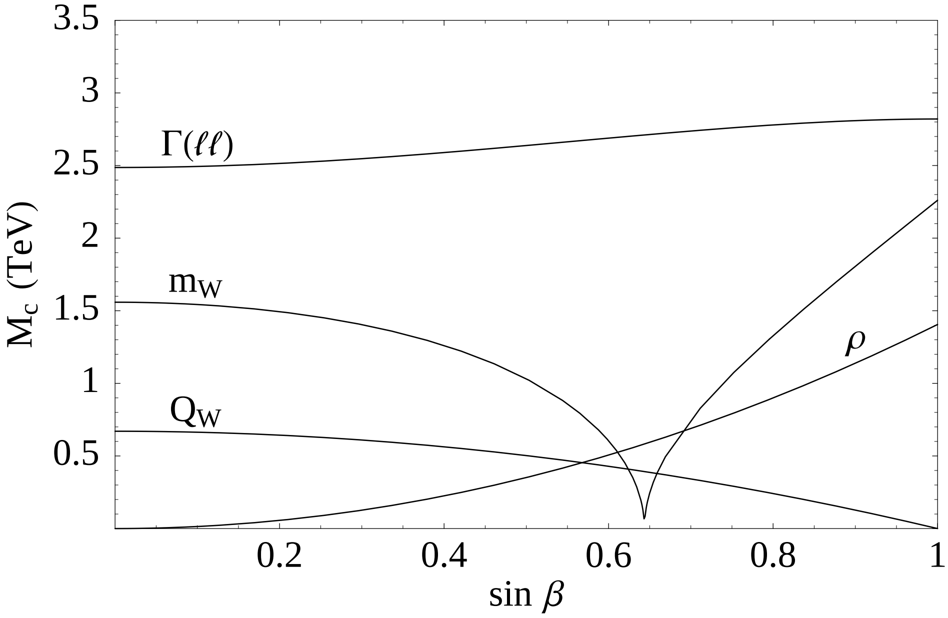


Figure 1: Bounds on the compactification scale M_c from electroweak observables.


3.5
3
2.5
2
1.5
1
0.5
0.2
0.4
0.6
0.8
1
M_c (TeV)
sin β
$\Gamma(\ell\ell)$
m_W
Q_W
ρ