FREQUENCY-DOMAIN CONSIDERATIONS IN MAGNET SORTING

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Summary

For large superconducting synchrotrons such as the Tevatron or the proposed SSC, the random normal sextupole component of dipoles is the most important factor in limiting their linear aperture. Since one selects the operating point in the tune diagram such that the third-integer resonances driven by particular harmonic components are not serious, the linear aperture cannot be enlarged by reducing these harmonic components alone. The magnet sorting scheme described in this note still depends on the harmonic description of the aperture-limiting effects but it tries to control a large number of harmonic components in the most troublesome regions. The scheme assumes that the sextupole component of each magnet is known and that, for each sorting group, there will be enough magnets to cover more than one betatron oscillation period. The "global" scheme is combined with the "local" sorting scheme such that the effectiveness of the scheme does not depend too strongly on a small change in tunes. An example is given to show that it is possible to achieve an order-of-magnitude reduction in the phase-space distortion ("smear") when some forty magnets are sorted as a group.

Introduction

If multipole components of all the dipoles to be used in the ring are measured and the data show nontrivial amount of fluctuations from magnet to magnet, as would be the case for superconducting magnets, one cannot afford to ignore the possible benefits expected from magnet sorting. The most attractive feature of any sorting is the fact that it is practically cost-free.

One must realize, however, that there is no unique way of sorting magnets. Factors that may influence the choice of sorting schemes are: total number of magnets in the ring, number in each cell, natural partition of the ring such as cryoloops and power supply stations, magnet installation schedules, magnet storage capacity, type and scope of correction systems, and allowance or non-allowance of magnets when one or two multipole components exceed the pre-determined values. It is also obvious that magnet sorting cannot cope with problems arising from very large systematic multipoles such as the normal sextupoles at injection induced by the persistent current in superconducting filaments. For these, there must be several families of correction packages and the system recently proposed by David Neuffer[1] seems to be the most promising one. Another weakness of magnet sorting is that it is difficult to take into account more than one component and, at the same time, to control more than a handful of harmonic terms which drive resonances. One might perhaps depend on sorting for one component and use other compensating schemes such as "binning" for the second component. Binning, as proposed by R. Talman for the SSC, classifies all magnets in the ring into a number of groups ("bins"); magnets belonging to the same group are then all connected to the same power supply to excite the correction windings. Depending on how many bins one is willing to consider, the effective rms value of multipole component can be reduced substantilly. A real two-component sorting for the SSC has been tried by L. Schachinger[2] to reduce the effects of skew quadrupole and normal sextupole errors simultaneously but the improvement over the one-component sorting is not clear from her report alone.

Since the Tevatron is the first superconducting machine, it is understandable that the first (recorded) magnet sorting was performed for it with a limited but well-defined goal in mind.[3] The goal was simply to minimize the magnitude of several isolated resonance-driving terms, these resonances arising from sextupole components b_2 and a_2 and skew octupole component a_3. The dimensionless figure-of-merit was the magnitude of each term relative to what one would expect from the distribution of these components if the sorting were not done. Since this sorting involves only one particular harmonic component for each resonance (altogether six), it is the simplest case of what one might call the "global" compensation.[4] Although the effectiveness of this scheme is self-evident when the tune is very close to one of the resonances considered, it will be impossible to say how much the beam lifetime is enhanced by the sorting. One must be satisfied with the often-expressed feeling of Fermilab people that the Tevatron is a very "linear" machine. In "global" compensation schemes, it is important to note that cancellation of errors in two widely separated magnets would be destroyed by small changes in the tune or uncertainties in the lattice functions. Magnets to be sorted as a group should not cover too much betatron phase advance. For the Tevatron, thirty or so magnets were sorted as a group and they covered approximately 280^o.

An entirely different scheme has been tried for the Relativistic Heavy Ion Collider (RHIC) in which the effect of normal and skew quadrupole components is considered to be serious.[5] Since there are only two dipoles in each cell and the phase advance per cell is 90^o, one is forced to sort a very small number (8 to 12 used) of magnets in order to avoid covering too much phase advance. With such a small number of magnets, global compensations controlling many harmonic components are not suitable and a "local" sorting scheme has been used. Local scheme is more appropriate when the source of errors is within a relatively small area of the ring. One tries to confine the effect of errors within this area. If the compensation is perfect, there will be no effect outside the area although the effect may not be too small inside.

Frequency-Domain Consideration

If it is allowed to ignore variations of machine parameters which are slow compared with the revolution time, any field perturbations in the ring are periodic with the period 2π in θ = path length/(average machine radius). One can express the resulting change in quantities such as closed orbit, betatron amplitude functions, dispersion functions, the transition energy of the ring and the distorted functions[7] associated with nonlinear perturbations in a Fourier expansion, i.e., frequency-domain formulas.[4,6,8] The particular advantage of frequency-domain formulas is that they show explicitly the relative importance of each harmonic component by the denominators of the n-th harmonic term of the form

$$Q^2 - n^2, \quad (2Q)^2 - n^2, \quad 3Q - n, \quad \text{and } (Q_1 \pm 2Q_2) - n, \text{ etc.}$$

Mathematically speaking, these frequency-domain expressions are of course completely equivalent to some closed forms, i.e., time-domain formulas. The time-domain formalism is convenient when the perturbation is confined within a small region of the ring and one tries to confine its effect within that region, or at least to minimize the disturbance propagating out of the region. Localized closed orbit bumps and beta-function bumps are typical examples.

When one or two well-identifiable resonances dominate the effect under consideration, it is easy to see from frequency-domain expressions what particular harmonic components must be controlled. The sorting performed for the Tevatron is based on the assumption that this is indeed what one must do in order to make the machine more manageable than otherwise. Since there was no attempt to reduce the magnitudes of more than a handful of harmonic components, it is not really surprising that, according to a tracking study by N. Gelfand,[9] the dynamic aperture of the sorted Tevatron does not exhibit any improvement over randomly arranged rings when the tune is away from third-integer resonances. One must conclude that a large number of harmonic components contribute to the aperture limitation and it is generally not enough to manipulate a few terms only. Harmonic correction magnets will be incapable of handling this situation unless one is willing to install and operate many independently controllable families. The sorting scheme proposed by Gluckstern and Ohnuma aims to control approximately M components ($M \simeq Q$, tune value) around $n \simeq Q$ and $3Q$ (actually around $n \simeq$ any integer times Q where integers are either even or odd) when sextupole effects are considered.[10] It is assumed here that two tunes Q_1 and Q_2 are close to each other so that $|Q_1 - Q_2|$ is much less than Q_1 and Q_2.

Assume that there are N measured magnets available for installation at N consecutive locations. These N magnets should cover the betatron phase advance of 360°. For example, if the phase advance is 90° per cell and there are five dipoles in each half cell, N is 40. Magnets are numbered in the order of their sextupole contents, 1 for the most negative to N for the most positive. Note that the average part is not considered here. The arrangement of N magnets in N locations will be discussed later but it is by no means unique. In the next group of N magnets, they are numbered such that 1 is now the most positive and N the most negative. Their arrangement should be identical as far as the numbers are concerned; if the arrangement chosen for the first N magnets is 3,9,6,2,..., for example, the order for the second group should be identical although the numbering is now in descending order. It is obvious that the phase advance from the magnet n in the first group to the magnet n in the second group is 360° for all n's. It is also clear from this arrangement that one should like to have an even number of groups in each superperiod. In reality, this is not always possible and there will be some number of cells which are not paired. One of the easiest solution is to take aside good magnets from each group and use them in the unbalanced cells. For example, if there are $60\frac{1}{2}$ cells in each superperiod, fourteen groups of 40 magnets will be balanced but the remaining 45 magnets will not be balanced. One requests 43 or 44 magnets in each group, instead of the exact number 40, and set aside the extra three or four best magnets in each group. They can then be used in the unbalanced cells randomly or in some suitable arrangement. It can be shown[10] that the expected average magnitude of harmonic component n is zero unless $n = M/2$, $3M/2$, $5M/2$,.... and the rms values are all reduced by a factor

$$\left(\frac{2}{N+1}\right)^{\frac{1}{2}}$$

compared with the value expected for random arrangement.

The arrangement within each group can be arbitrary as long as the same order is maintained in all the groups. This statement of course ignores the importance of local cancellation and, for some arrangement, the effectiveness of sorting may depend too much on the choice of tune values. Uncertainties in the lattice parameters may also affect the performance. Common sense dictates that a large positive and a large negative sextupoles should be placed next to each other. Two large positive(negative) ones should be installed with phase advance of 180° so that they will contribute little to $n = M$ and $n = 3M$ which are still the most important harmonics. Even then, there are many equally effective arrangements and this flexibility can be used to control harmful effects arising from <u>other</u> multipole components in a few magnets.

An Example

Since the smears used by the SSC Central Design Group are proportional to distortion functions defined by Tom Collins,[7,8] their rms magnitudes are used to compare the sorted ring with random ones. There are five pairs of functions and in each pair, one is cos-like and the other is sin-like. Cos-terms are continuous (as beta functions) but sin-terms change their values at each sextupole components (as alphas functions with thin lens). Their behavior is completely analogous to a closed orbit, for example; cos-term is like x or y and sin-term is like x' or y'. The difference is the phase angle one must use in moving from one sextupole to the next. For a closed orbit, the phase angle to be used from one dipole error to the next is the betatron phase angle itself, ψ_x or ψ_y. For distortion functions,

cos-term	B_3	B_1	$\bar{B}$	$B_S(B_+)$	$B_D(B_-)$
sin-term	A_3	A_1	$\bar{A}$	$A_S(A_+)$	$A_D(A_-)$
phase angle	$3\psi_x$	ψ_x	ψ_x	$\psi_x + 2\psi_y$	$\psi_x - 2\psi_y$

All ten functions are evaluated at each quadrupole location in the example given below. The magnitude of each pair is

$$C = (A^2 + B^2)^{\frac{1}{2}} .$$

Two figure of merits are used: MAX = maximum values of C's in the ring,

$\mathrm{SIGMA} = (\Sigma\ C^2/N_Q)^{\frac{1}{2}}$ where the summation is for N_Q quadrupole locations in the ring. For ten rings with different magnets, the ratio of MAX(sorted)/MAX(unsorted) is averaged over ten rings. For SIGMA(sorted)/SIGMA(unsorted), the summation is taken for all ten machines before taking the square-root. The test lattice used is

90^o/cell, $64\frac{1}{2}$ cells/period, six superperiods with insertions,

5 dipoles/half cell, 40 dipoles/360^o,

tune = 118 + (four different values of fraction)

<u>arrangement within each group</u>

(QD)19,23,15,27,11(QF)31, 7,35, 3,39 /(QD)1,37, 5,33, 9(QF)29,13,25,17,21/

(QD)20,24,16,28,12(QF)32, 8,36, 4,40 /(QD)2,38, 6,34,10(QF)30,14,26,18,22/

tune		C_3	C_1	$\bar{C}$	C_S	C_D
118.10	MAX	.11	.16	.11	.13	.16
	SIGMA	.087	.095	.072	.083	.068
118.25	MAX	.10	.14	.14	.10	.16
	SIGMA	.079	.11	.095	.070	.099
118.32	MAX	.067	.14	.14	.058	.17
	SIGMA	.056	.11	.10	.044	.10
118.40	MAX	.088	.14	.14	.10	.17
	SIGMA	.076	.11	.10	.068	.11

It would be interesting to find the amount of smear from tracking but this has has not been done.

References

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