CERN–TH/98–333
NORDITA–98–62–HE
hep-ph/9810475

Higher Twist Distribution Amplitudes of Vector Mesons in QCD: Twist-4 Distributions and Meson Mass Corrections

Patricia Ball[1,*] and V.M. Braun[2,†]

[1] *CERN–TH, CH–1211 Geneva 23, Switzerland*

[2] *NORDITA, Blegdamsvej 17, DK-2100 Copenhagen, Denmark*

Abstract

We present a systematic study of twist-4 light-cone distribution amplitudes of vector mesons in QCD, which is based on conformal expansion. The structure of meson mass corrections is studied in detail. A complete set of distribution amplitudes is constructed, which satisfies all (exact) equations of motion and constraints from conformal expansion. Nonperturbative input parameters are estimated from QCD sum rules. Our study suggests that meson mass corrections may present a dominant source of higher twist effects in exclusive processes.

Submitted to Nuclear Physics B

*E-mail: Patricia.Ball@cern.ch
†E-mail: vbraun@nordita.dk

1 Introduction

The notion of distribution amplitudes refers to momentum fraction distributions of partons in a meson, in a particular Fock state, with a fixed number of constituents. For the minimal number of constituents, the distribution amplitude ϕ is related to the meson's Bethe-Salpeter wave function ϕ_{BS} by

$$\phi(x) \sim \int^{|k_\perp|<\mu} d^2k_\perp\, \phi_{BS}(x,k_\perp). \tag{1.1}$$

The standard approach to distribution amplitudes, which is due to Brodsky and Lepage [1], considers the hadron's parton decomposition in the infinite momentum frame. A conceptually different, but mathematically equivalent formalism is the light-cone quantization [2]. Either way, power suppressed contributions to exclusive processes in QCD, which are commonly referred to as higher twist corrections, are thought to originate from three different sources:

- contributions of "bad" components in the wave function and in particular of components with "wrong" spin projection;
- contributions of transverse motion of quarks (antiquarks) in the leading twist components, given for instance by integrals as above with additional factors of $k_\perp^2$;
- contributions of higher Fock states with additional gluons and/or quark-antiquark pairs.

In this paper we continue the systematic study of higher twist light-cone distribution amplitudes started in Ref. [3]. In particular, we extend the analysis of [3] to include twist-4 distribution amplitudes and, most significantly, meson mass corrections. A preliminary account of some of our results has been reported in [4].

Following [3], we define light-cone distribution amplitudes as meson-to-vacuum transition matrix elements of nonlocal gauge invariant light-cone operators. This formalism is perhaps less intuitive than the infinite momentum frame formulation, but it is more convenient for the study of higher twist distributions as it is Lorentz and gauge invariant. It allows all equations of motion to be solved explicitly, relating different higher twist distributions to one another. We will find that, much like in the twist-3 case [3], all dynamical degrees of freedom are those describing contributions of higher Fock states, while all other higher twist effects are given in terms of the latter without any free parameters.

A systematic study of meson mass corrections presents the principal new contribution of this work. By counting dimensions, for any exclusive observable involving a large momentum transfer Q, power suppressed higher twist corrections have the generic structure

$$\frac{1}{Q^2}\Big[m^2 \cdot \langle\!\langle O^{(2)}\rangle\!\rangle + m\cdot \langle\!\langle O^{(3)}\rangle\!\rangle + \langle\!\langle O^{(4)}\rangle\!\rangle\Big].$$

Here m is the meson mass, $\langle\!\langle O^{(2)}\rangle\!\rangle$ and $\langle\!\langle O^{(3)}\rangle\!\rangle$ and $\langle\!\langle O^{(4)}\rangle\!\rangle$ are reduced matrix elements of twist-2, twist-3 and twist-4 operators, which have dimension 0, 1 and 2, respectively. The terms $\sim m^2$ do not involve any new dynamical information about the meson structure as compared to the leading twist terms, and are usually referred to as "kinematic" power corrections.

The structure of such kinematic corrections is well known for deep-inelastic lepton-hadron scattering, in which case they can be absorbed into a redefinition of the scaling variable [5]. The crucial observation leading to this "Nachtmann scaling" is that hadron mass corrections ("target mass corrections" in this context) arise exclusively from the definition of the relevant leading twist matrix elements and do not involve new (higher twist) operators. This simplification does not hold in exclusive processes because there are additional contributions of operators containing total derivatives. Specifically, to twist-4 accuracy, in addition to Nachtmann's corrections, there are also contributions of operators of type

$$\partial^2 O^{(2)}_{\mu_1\mu_2\ldots\mu_n}$$

and

$$\partial_{\mu_1} O^{(2)}_{\mu_1\mu_2\ldots\mu_n},$$

where $O^{(2)}$ is a leading twist operator. We find that contributions of the first type can be taken into account consistently for all moments, while contributions of the second type are more complicated and can be unravelled only order by order in the conformal expansion.

The outline of this paper is as follows: definitions of and notations for distribution amplitudes are presented in Sec. 2 together with general remarks about specific features of the operator product expansion (OPE) for exclusive processes and about conformal expansion. Section 3 gives a general discussion of meson mass corrections for a simple example. The subsequent Secs. 4 and 5 contain a detailed derivation of chiral-even and chiral-odd distribution amplitudes, respectively. We take into account contributions of the leading and next-to-leading conformal spin and derive a self-consistent approximation for the distribution amplitudes, which respects the exact QCD equations of motion. The chiral-even and chiral-odd asymptotic distribution amplitudes involve three nonperturbative parameters, and four additional parameters are required for the description of the leading corrections. The corresponding estimates are worked out using the QCD sum rule approach [6]. On the basis of these estimates, we suggest that higher twist effects in exclusive processes are in many cases dominated by meson mass corrections alone. The final Sec. 6 contains a summary and conclusions. The paper also contains three appendices in which we derive equations of motion for nonlocal operators, and derive and estimate the independent nonperturbative parameters that enter the twist-4 distributions discussed here.

Throughout this paper we denote the meson momentum by P_μ and introduce light-like vectors p and z such that

$$p_\mu = P_\mu - \frac{1}{2} z_\mu \frac{m_\rho^2}{pz}. \tag{1.2}$$

The meson polarization vector $e^{(\lambda)}_\mu$ is decomposed into projections onto the two light-like vectors and the orthogonal plane as

$$e^{(\lambda)}_\mu = \frac{(e^{(\lambda)}z)}{pz}\left(p_\mu - \frac{m_\rho^2}{2pz} z_\mu\right) + e^{(\lambda)}_{\perp\mu}. \tag{1.3}$$

Some useful scalar products are

$$z\cdot P = z\cdot p = \sqrt{(x\cdot P)^2 - x^2 m_\rho^2},$$

$$p\cdot e^{(\lambda)} = -\frac{m_\rho^2}{2pz}\, z\cdot e^{(\lambda)},$$

$$e^{(\lambda)}\cdot z = e^{(\lambda)}\cdot x. \tag{1.4}$$

We also need the projector onto the directions orthogonal to p and z:

$$g^{\perp}_{\mu\nu} = g_{\mu\nu} - \frac{1}{pz}(p_\mu z_\nu + p_\nu z_\mu), \tag{1.5}$$

and will use the notations

$$a. \equiv a_\mu z^\mu, \qquad a_* \equiv a_\mu p^\mu/(pz), \tag{1.6}$$

for an arbitrary Lorentz vector a_μ.

We use the standard Bjorken–Drell convention [7] for the metric and the Dirac matrices; in particular $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and the Levi–Civita tensor $\epsilon_{\mu\nu\lambda\sigma}$ is defined as the totally anti-symmetric tensor with $\epsilon_{0123} = 1$. The covariant derivative is defined as $D_\mu \equiv \overrightarrow{D}_\mu = \partial_\mu - igA_\mu$, and we also use the notation $\overleftarrow{D}_\mu = \overleftarrow{\partial}_\mu + igA_\mu$ in later sections. The dual gluon field strength tensor is defined as $\widetilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$.

2 General Framework

Amplitudes of light-cone-dominated processes involving vector mesons can be expressed in terms of matrix elements of gauge invariant nonlocal operators sandwiched between the vacuum and the vector meson state, e.g. a matrix element over a two-particle operator,

$$\langle 0|\bar u(x)\Gamma[x,-x]d(-x)|\rho^-(P,\lambda)\rangle, \tag{2.1}$$

where Γ is a generic Dirac matrix structure and we use the notation $[x,y]$ for the path-ordered gauge factor along the straight line connecting the points x and y:

$$[x,y] = \mathrm{P}\exp\left[ig\int_0^1 dt\,(x-y)_\mu A^\mu(tx+(1-t)y)\right]. \tag{2.2}$$

To simplify the notation, we will explicitly consider charged ρ mesons; the distribution amplitudes of ρ^0 can be obtained by choosing appropriate isospin currents.

The asymptotic expansion of exclusive amplitudes in powers of large momentum transfer corresponds to the expansion of amplitudes like (2.1) in powers of the deviation from the light-cone $x^2 = 0$. As always in a quantum field theory, such an expansion generates divergences and has to be understood as an OPE in terms of renormalized light-cone nonlocal operators whose matrix elements define meson distribution amplitudes of increasing twist. To leading logarithmic accuracy, the coefficient functions are just taken at tree-level, and the distributions have to be evaluated at the scale $\mu^2 \sim x^{-2}$. In this section we present the necessary expansions and introduce a complete set of meson distribution amplitudes to twist-4 accuracy. This set is, in fact, overcomplete, and different distributions are related to one another via the QCD equations of motion, as detailed in later sections.

2.1 Chiral-Even Distribution Amplitudes

We start with the matrix elements involving an odd number of γ matrices, which we refer to as chiral-even in what follows. For the vector and axial vector operators the light-cone expansion to twist-4 accuracy reads:

$$\begin{aligned}\langle 0|\bar{u}(x)\gamma_\mu d(-x)|\rho^-(P,\lambda)\rangle &= f_\rho m_\rho \Bigg\{ \frac{e^{(\lambda)}x}{Px}\, P_\mu \int_0^1 du\, e^{i\xi Px}\Big[\phi_\parallel(u) + \frac{m_\rho^2 x^2}{4}\mathbb{A}(u)\Big] \\ &\quad + \left(e_\mu^{(\lambda)} - P_\mu \frac{e^{(\lambda)}x}{Px}\right)\int_0^1 du\, e^{i\xi Px}\,\mathbb{B}(u) \\ &\quad - \frac{1}{2}x_\mu \frac{e^{(\lambda)}x}{(Px)^2} m_\rho^2 \int_0^1 du\, e^{i\xi Px}\mathbb{C}(u)\Bigg\}, \qquad (2.3)\end{aligned}$$

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\rho^-(P,\lambda)\rangle = \frac{1}{2}\left(f_\rho - f_\rho^T \frac{m_u+m_d}{m_\rho}\right) m_\rho \epsilon_\mu{}^{\nu\alpha\beta} e_\nu^{(\lambda)} P_\alpha x_\beta \int_0^1 du\, e^{i\xi Px}\mathbb{D}(u). \qquad (2.4)$$

Notice that in order to calculate exclusive amplitudes to $O(1/Q^2)$ accuracy, terms of $O(x^2)$ have to be kept in the vector matrix element, but can be neglected in the axial vector one. For brevity, here and below we do not show gauge factors between the quark and the antiquark fields; we also use the short-hand notation

$$\xi = 2u - 1.$$

The vector and tensor decay constants f_ρ and f_ρ^T are defined, as usual, as

$$\langle 0|\bar{u}(0)\gamma_\mu d(0)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho e_\mu^{(\lambda)}, \qquad (2.5)$$

$$\langle 0|\bar{u}(0)\sigma_{\mu\nu} d(0)|\rho^-(P,\lambda)\rangle = i f_\rho^T (e_\mu^{(\lambda)} P_\nu - e_\nu^{(\lambda)} P_\mu). \qquad (2.6)$$

The coupling f_ρ^T is scale-dependent, with

$$f_\rho^T(Q^2) = f_\rho^T(\mu^2)\left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right)^{C_F/b}, \qquad (2.7)$$

with the standard notation $C_F = (N_c^2-1)/(2N_c)$ and $b = (11N_c - 2n_f)/3$.

The expansions in (2.3), (2.4) involve several Lorentz invariant amplitudes, which we have to interpret in terms of meson parton distributions. Definitions of the latter involve nonlocal operators at strictly light-like separations and can most conveniently be written using the light-cone variables (1.2), and for longitudinal and transverse meson polarizations separately.

Following [3], we define chiral-even two-particle distribution amplitudes of the ρ meson as

$$\langle 0|\bar{u}(z)\gamma_\mu d(-z)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho \left[p_\mu \frac{e^{(\lambda)}z}{pz} \int_0^1 du\, e^{i\xi pz} \phi_\parallel(u,\mu^2) \right.$$

$$\left. + \, e^{(\lambda)}_{\perp\mu} \int_0^1 du\, e^{i\xi pz} g_\perp^{(v)}(u,\mu^2) - \frac{1}{2} z_\mu \frac{e^{(\lambda)}z}{(pz)^2} m_\rho^2 \int_0^1 du\, e^{i\xi pz} g_3(u,\mu^2) \right], \tag{2.8}$$

$$\langle 0|\bar{u}(z)\gamma_\mu\gamma_5 d(-z)|\rho^-(P,\lambda)\rangle =$$

$$= \frac{1}{2}\left(f_\rho - f_\rho^T \frac{m_u+m_d}{m_\rho}\right) m_\rho \epsilon_\mu{}^{\nu\alpha\beta} e^{(\lambda)}_{\perp\nu} p_\alpha z_\beta \int_0^1 du\, e^{i\xi pz} g_\perp^{(a)}(u,\mu^2). \tag{2.9}$$

The distribution amplitude $\phi_\parallel$ is of twist-2, $g_\perp^{(v)}$ and $g_\perp^{(a)}$ of twist-3 and g_3 of twist-4. All four functions $\phi = \{\phi_\parallel, g_\perp^{(v)}, g_\perp^{(a)}, g_3\}$ are normalized as

$$\int_0^1 du\, \phi(u) = 1, \tag{2.10}$$

which can be checked by comparing the two sides of the defining equations in the limit $z_\mu \to 0$ and using the equations of motion.

Comparing (2.8), (2.9) with the light-cone expansions in (2.3), (2.4), we easily find

$$\begin{aligned} \mathbb{B}(u) &= g_\perp^{(v)}(u), \\ \mathbb{C}(u) &= g_3(u) + \phi_\parallel(u) - 2g_\perp^{(v)}(u), \\ \mathbb{D}(u) &= g_\perp^{(a)}(u), \end{aligned} \tag{2.11}$$

which is nothing but the tree-level OPE of the invariant amplitudes $\mathbb{B}, \mathbb{C}, \mathbb{D}$ in terms of meson distribution amplitudes. We will find (see also [3]) that all higher twist two-particle distribution amplitudes do not present genuine independent degrees of freedom, but can be expressed in terms of three-particle distribution amplitudes. The same analysis will allow us to calculate the remaining invariant amplitude $\mathbb{A}$, which accounts for the transverse momentum distribution in the valence component of the wave function.

Three-particle chiral-even distributions are rather numerous and can be defined by the following matrix elements:

$$\langle 0|\bar{u}(z) g\widetilde{G}_{\mu\nu}(vz)\gamma_\alpha\gamma_5 d(-z)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho p_\alpha [p_\nu e^{(\lambda)}_{\perp\mu} - p_\mu e^{(\lambda)}_{\perp\nu}]\mathcal{A}(v,pz)$$

Twist	$(\mu\nu\alpha)$	$\bar\psi\widetilde{G}_{\mu\nu}\gamma_\alpha\gamma_5\psi$	$\bar\psi G_{\mu\nu}\gamma_\alpha\psi$	$(\mu\nu\alpha\beta)$	$\bar\psi G_{\mu\nu}\sigma_{\alpha\beta}\psi$	$(\mu\nu)$	$\bar\psi G_{\mu\nu}\psi$	$\bar\psi\widetilde{G}_{\mu\nu}\gamma_5\psi$
3	$\cdot\perp\cdot$	$\mathcal{A}$	$\mathcal{V}$	$\cdot\perp\cdot\perp$	$\mathcal{T}$			
4	$\cdot\perp\perp$	$\widetilde\Phi$	Φ	$\perp\perp\cdot\perp$	$T_1^{(4)}$	$\cdot\perp$	S	$\widetilde S$
	$\cdot * \cdot$	$\widetilde\Psi$	Ψ	$\cdot\perp\perp\perp$	$T_2^{(4)}$			
				$\cdot * \cdot\perp$	$T_3^{(4)}$			
				$\cdot\perp\cdot *$	$T_4^{(4)}$			

Table 1: Identification of three-particle distribution amplitudes with projections onto different light-cone components of the nonlocal operators. For example, $\cdot\perp\perp$ refers to $\bar\psi\widetilde{G}_{\cdot\perp}\gamma_\perp\gamma_5\psi$.

$$+ f_\rho m_\rho^3\frac{e^{(\lambda)}z}{pz}[p_\mu g^\perp_{\alpha\nu} - p_\nu g^\perp_{\alpha\mu}]\widetilde\Phi(v,pz)$$

$$+ f_\rho m_\rho^3\frac{e^{(\lambda)}z}{(pz)^2}p_\alpha[p_\mu z_\nu - p_\nu z_\mu]\widetilde\Psi(v,pz), \quad (2.12)$$

$$\langle 0|\bar u(z)gG_{\mu\nu}(vz)i\gamma_\alpha d(-z)|\rho^-(P)\rangle = f_\rho m_\rho p_\alpha[p_\nu e^{(\lambda)}_{\perp\mu} - p_\mu e^{(\lambda)}_{\perp\nu}]\mathcal{V}(v,pz)$$

$$+ f_\rho m_\rho^3\frac{e^{(\lambda)}z}{pz}[p_\mu g^\perp_{\alpha\nu} - p_\nu g^\perp_{\alpha\mu}]\Phi(v,pz)$$

$$+ f_\rho m_\rho^3\frac{e^{(\lambda)}z}{(pz)^2}p_\alpha[p_\mu z_\nu - p_\nu z_\mu]\Psi(v,pz), \quad (2.13)$$

where

$$\mathcal{A}(v,pz) = \int \mathcal{D}\underline{\alpha}\, e^{-ipz(\alpha_u-\alpha_d+v\alpha_g)}\mathcal{A}(\underline{\alpha}), \quad (2.14)$$

etc., and $\underline{\alpha}$ is the set of three momentum fractions $\underline{\alpha} = \{\alpha_d, \alpha_u, \alpha_g\}$. The integration measure is defined as

$$\int \mathcal{D}\underline{\alpha} \equiv \int_0^1 d\alpha_d \int_0^1 d\alpha_u \int_0^1 d\alpha_g\, \delta\left(1-\sum\alpha_i\right). \quad (2.15)$$

The distribution amplitudes $\mathcal{V}$ and $\mathcal{A}$ are of twist-3, while the rest is of twist-4; we have not shown further Lorentz structures corresponding to twist-5 contributions[1]. Different distribution amplitudes can be separated by projecting onto particular light-cone components, as summarized in Table 1.

For completeness, let us mention that also four-particle twist-4 distribution amplitudes exist, corresponding to contributions of Fock states with two gluons or an additional $q\bar q$ pair,

[1]Note that we use a different normalization of three-particle twist-3 distributions compared to [3].

of type

$$\bar\psi\gamma_{.}(\gamma_5)\psi\,\bar\psi\gamma_{.}(\gamma_5)\psi,\qquad \bar\psi\, G_{.\perp}\, G_{.\perp}\gamma_{.}\psi.$$

Such distributions will not be considered in this paper for two reasons: first, it is well known [8] that four-particle twist-4 operators do not allow the factorization of vacuum condensates such as $\langle\bar\psi\psi\rangle$, $\langle G^2\rangle$. Because of this, their matrix elements cannot be estimated reliably by existing methods (e.g. QCD sum rules), although they are generally expected to be small. Second, and more importantly, the four-particle distributions decouple from the QCD equations of motion in the two lowest conformal partial waves. To this accuracy, therefore, it is consistent to put them to zero. Vice versa, nonvanishing four-particle distributions necessitate the inclusion of higher conformal spin corrections to distributions with less particles, which are beyond the approximation adopted in this paper.

2.2 Chiral-Odd Distribution Amplitudes

For chiral-odd operators involving $\sigma_{\mu\nu}$ and $\mathbf{1}$, the light-cone expansion to twist-4 accuracy reads:

$$\begin{aligned}\langle 0|\bar u(x)\sigma_{\mu\nu}d(-x)|\rho^-(P,\lambda)\rangle = {}& if_\rho^T\left[(e^{(\lambda)}_\mu P_\nu - e^{(\lambda)}_\nu P_\mu)\int_0^1 du\, e^{i\xi Px}\left[\phi_\perp(u)+\frac{m_\rho^2x^2}{4}\mathbb{A}_T(u)\right]\right.\\ &+(P_\mu x_\nu - P_\nu x_\mu)\frac{e^{(\lambda)}x}{(Px)^2}m_\rho^2\int_0^1 du\, e^{i\xi Px}\mathbb{B}_T(u)\\ &\left.+\frac{1}{2}(e^{(\lambda)}_\mu x_\nu - e^{(\lambda)}_\nu x_\mu)\frac{m_\rho^2}{Px}\int_0^1 du\, e^{i\xi Px}\mathbb{C}_T(u)\right],\end{aligned}\tag{2.16}$$

$$\langle 0|\bar u(x)d(-x)|\rho^-(P,\lambda)\rangle = -i\left(f_\rho^T - f_\rho\frac{m_u+m_d}{m_\rho}\right)\left(e^{(\lambda)}x\right)m_\rho^2\int_0^1 du\, e^{i\xi Px}\mathbb{D}_T(u).\tag{2.17}$$

The couplings f_ρ and f_ρ^T are defined in (2.5) and (2.6).

The corresponding distribution amplitudes on the light-cone are defined as

$$\begin{aligned}\langle 0|\bar u(z)\sigma_{\mu\nu}d(-z)|\rho^-(P,\lambda)\rangle = {}& if_\rho^T\left[(e^{(\lambda)}_{\perp\mu} p_\nu - e^{(\lambda)}_{\perp\nu} p_\mu)\int_0^1 du\, e^{i\xi pz}\phi_\perp(u,\mu^2)\right.\\ &+(p_\mu z_\nu - p_\nu z_\mu)\frac{e^{(\lambda)}z}{(pz)^2}m_\rho^2\int_0^1 du\, e^{i\xi pz}h_\parallel^{(t)}(u,\mu^2)\\ &\left.+\frac{1}{2}(e^{(\lambda)}_{\perp\mu} z_\nu - e^{(\lambda)}_{\perp\nu} z_\mu)\frac{m_\rho^2}{pz}\int_0^1 du\, e^{i\xi pz}h_3(u,\mu^2)\right],\end{aligned}\tag{2.18}$$

$$\langle 0|\bar u(z)d(-z)|\rho^-(P,\lambda)\rangle = -i\left(f_\rho^T - f_\rho\frac{m_u+m_d}{m_\rho}\right)(e^{(\lambda)}z)m_\rho^2\int_0^1 du\, e^{i\xi pz}h_\parallel^{(s)}(u,\mu^2).\tag{2.19}$$

The distribution amplitude $\phi_\perp$ is of twist-2, $h_\parallel^{(s,t)}$ are of twist-3 and h_3 is of twist-4. All four functions $\phi = \{\phi_\perp, h_\parallel^{(s,t)}, h_3\}$ are normalized as

$$\int_0^1 du\, \phi(u) = 1.$$

Comparing (2.18) and (2.19) with the light-cone expansion (2.16) and (2.17), we easily find

$$\begin{aligned}\mathbb{B}_T(u) &= h_\parallel^{(t)}(u) - \frac{1}{2}\phi_\perp(u) - \frac{1}{2}h_3(u),\\ \mathbb{C}_T(u) &= h_3(u) - \phi_\perp(u).\end{aligned} \tag{2.20}$$

As for chiral-even distribution amplitudes, only the twist-2 distribution $\phi_\perp$ represents genuinely independent degrees of freedom, the others can be expressed in terms of three-particle distribution amplitudes.

The three-particle distribution amplitudes are even more numerous than in the chiral-even case and can be defined as:

$$\begin{aligned}
\langle 0|\bar{u}(z)\sigma_{\alpha\beta} gG_{\mu\nu}(vz)d(-z)|\rho^-(P,\lambda)\rangle \;=& \\
= f_\rho^T m_\rho^2 \frac{e^{(\lambda)}z}{2(pz)}&[p_\alpha p_\mu g^\perp_{\beta\nu} - p_\beta p_\mu g^\perp_{\alpha\nu} - p_\alpha p_\nu g^\perp_{\beta\mu} + p_\beta p_\nu g^\perp_{\alpha\mu}]\mathcal{T}(v,pz)\\
+ f_\rho^T m_\rho^2 &[p_\alpha e^{(\lambda)}_{\perp\mu} g^\perp_{\beta\nu} - p_\beta e^{(\lambda)}_{\perp\mu} g^\perp_{\alpha\nu} - p_\alpha e^{(\lambda)}_{\perp\nu} g^\perp_{\beta\mu} + p_\beta e^{(\lambda)}_{\perp\nu} g^\perp_{\alpha\mu}]T_1^{(4)}(v,pz)\\
+ f_\rho^T m_\rho^2 &[p_\mu e^{(\lambda)}_{\perp\alpha} g^\perp_{\beta\nu} - p_\mu e^{(\lambda)}_{\perp\beta} g^\perp_{\alpha\nu} - p_\nu e^{(\lambda)}_{\perp\alpha} g^\perp_{\beta\mu} + p_\nu e^{(\lambda)}_{\perp\beta} g^\perp_{\alpha\mu}]T_2^{(4)}(v,pz)\\
+ \frac{f_\rho^T m_\rho^2}{pz}&[p_\alpha p_\mu e^{(\lambda)}_{\perp\beta} z_\nu - p_\beta p_\mu e^{(\lambda)}_{\perp\alpha} z_\nu - p_\alpha p_\nu e^{(\lambda)}_{\perp\beta} z_\mu + p_\beta p_\nu e^{(\lambda)}_{\perp\alpha} z_\mu]T_3^{(4)}(v,pz)\\
+ \frac{f_\rho^T m_\rho^2}{pz}&[p_\alpha p_\mu e^{(\lambda)}_{\perp\nu} z_\beta - p_\beta p_\mu e^{(\lambda)}_{\perp\nu} z_\alpha - p_\alpha p_\nu e^{(\lambda)}_{\perp\mu} z_\beta + p_\beta p_\nu e^{(\lambda)}_{\perp\mu} z_\alpha]T_4^{(4)}(v,pz),
\end{aligned} \tag{2.21}$$

$$\begin{aligned}
\langle 0|\bar{u}(z) gG_{\mu\nu}(vz)d(-z)|\rho^-(P,\lambda)\rangle \;&=\; i f_\rho^T m_\rho^2 [e^{(\lambda)}_{\perp\mu} p_\nu - e^{(\lambda)}_{\perp\nu} p_\mu] S(v,pz),\\
\langle 0|\bar{u}(z) ig\widetilde{G}_{\mu\nu}(vz)\gamma_5 d(-z)|\rho^-(P,\lambda)\rangle \;&=\; i f_\rho^T m_\rho^2 [e^{(\lambda)}_{\perp\mu} p_\nu - e^{(\lambda)}_{\perp\nu} p_\mu] \widetilde{S}(v,pz).
\end{aligned} \tag{2.22}$$

Of these seven amplitudes, $\mathcal{T}$ is of twist-3 and the other six of twist-4; higher twist terms are suppressed. The relation of these distribution amplitudes to specific light-cone projections of the matrix elements was made explicit in Table 1.

Also in this case there exist four-particle twist-4 distribution amplitudes which we do not consider for the reasons mentioned at the end of Sec. 2.1.

2.3 Conformal Partial Wave Expansion

Conformal partial wave expansion in QCD [9, 10, 11, 12, 13] parallels the partial wave expansion of wave functions in standard quantum mechanics, which allows the separation of the dependence on angular coordinates from that on radial ones. The basic idea is to write distribution amplitudes as a sum of contributions from different conformal spins. For a given spin, the dependence on the momentum fractions is fixed by the symmetry. To specify the function, one has to fix the coefficients in this expansion at some scale; conformal invariance of the QCD Lagrangian then guarantees that there is no mixing between contributions of different spin to leading logarithmic accuracy. For leading twist distributions the mixing matrix becomes diagonal in the conformal basis and the anomalous dimensions are ordered with spin. Thus, only the first few "harmonics" contribute at sufficiently large scales (for sufficiently hard processes).

For higher twist distributions, the use of the conformal basis offers the crucial advantage of "diagonalizing" the equations of motion: since conformal transformations commute with the QCD equations of motion, the corresponding constraints can be solved order by order in the conformal expansion. Note that relations between different distributions obtained in this way are exact: despite the fact that conformal symmetry is broken by quantum corrections, equations of motion are not renormalized and remain the same as in free theory.

The general procedure to construct the conformal expansion for arbitrary multi-particle distributions was developed in [11, 12]. To this end, each constituent field has to be decomposed (using projection operators, if necessary) into components with fixed (Lorentz) spin projection onto the light-cone.

Each such component corresponds to a so-called quasi-primary field in the language of conformal field theories, and has conformal spin

$$j = \frac{1}{2}\,(l + s), \tag{2.23}$$

where l is the canonical dimension and s the (Lorentz) spin projection. In particular, $l = 3/2$ for quarks and $l = 2$ for gluons. The quark field is decomposed as $\psi_+ \equiv (1/2)\not{z}\not{p}\psi$ and $\psi_- = (1/2)\not{p}\not{z}\psi$ with spin projections $s = +1/2$ and $s = -1/2$, respectively. For the gluon field strength there are three possibilities: $G_{\cdot\perp}$ corresponds to $s = +1$, $G_{*\perp}$ to $s = -1$ and both $G_{\perp\perp}$ and $G_{\cdot *}$ correspond to $s = 0$.

Multi-particle states built of quasi-primary fields can be expanded in irreducible unitary representations with increasing conformal spin. An explicit expression for the distribution amplitude of a multi-particle state with the lowest conformal spin $j = j_1 + \ldots + j_m$ built of m primary fields with spins j_k is

$$\phi_{as}(\alpha_1, \alpha_2, \ldots, \alpha_m) = \frac{\Gamma[2j_1 + \ldots + 2j_m]}{\Gamma[2j_1]\ldots\Gamma[2j_m]}\alpha_1^{2j_1-1}\alpha_2^{2j_2-1}\ldots\alpha_m^{2j_m-1}. \tag{2.24}$$

Here α_k are the corresponding momentum fractions. This state is nondegenerate and cannot mix with other states because of conformal symmetry. Multi-particle irreducible representations with higher spin $j + n, n = 1, 2, \ldots$, are given by polynomials of m variables (with the constraint $\sum_{k=1}^{m} \alpha_k = 1$), which are orthogonal over the weight-function (2.24).

3 Meson Mass Corrections

The structure of meson mass corrections in exclusive processes is in general more complicated than that of target mass corrections in deep inelastic scattering where they can be resummed using the Nachtmann variable [5]. For illustration, consider the simplest matrix element

$$\langle 0|\bar{u}(x)\not{x}d(-x)|\rho^-(P,\lambda)\rangle =$$
$$= f_\rho m_\rho (e^{(\lambda)}x)\int_0^1 du\, e^{i(2u-1)Px}\Big[\phi_\parallel(u) + \frac{x^2}{4}\Phi(u) + O(x^4)\Big]. \tag{3.1}$$

We assume that $x^2 \ll \Lambda_{\rm QCD}^{-2}$, but nonzero, $\phi_\parallel(u)$ is the twist-2 chiral-even distribution amplitude and $\Phi(u) = m_\rho^2[\mathbb{A}(u) + (1/2)\int_0^u dv \int_0^v dw\, \mathbb{C}(w)]$ (c.f. (2.3)) describes higher twist corrections, the "kinematic" contributions to which, due to the massive ρ meson, we want to calculate.

Experience with inclusive distributions tells us that meson mass corrections are related to contributions of leading twist operators. Indeed, the conditions of symmetry and zero traces for twist-2 local operators imply

$$\langle 0|\Big[\bar{u}\not{x}(i \stackrel{\leftrightarrow}{D} x)^n d\Big]_{\rm tw.2}|\rho^-(P,\lambda)\rangle =$$
$$= f_\rho m_\rho (e^{(\lambda)}x)\Big[(Px)^n - \frac{x^2 m_\rho^2}{4}\frac{n(n-1)}{n+1}(Px)^{n-2}\Big]\langle\!\langle O_n\rangle\!\rangle, \tag{3.2}$$

where $[\ldots]_{\rm tw.2}$ denotes taking the leading twist part (subtraction of traces, in this case) and $\langle\!\langle O_n\rangle\!\rangle$ is the reduced matrix element related to the n-th moment of the leading twist distribution

$$M_n^{(\parallel)} \equiv \int_0^1 du\,(2u-1)^n \phi_\parallel(u) = \langle\!\langle O_n\rangle\!\rangle. \tag{3.3}$$

Expanding (3.1) at short distances $x \to 0$ and comparing it with (3.2), we find that the same reduced matrix element gives a contribution to the twist-4 distribution amplitude:

$$M_n^{(\Phi)} \equiv \int_0^1 du\,(2u-1)^n \Phi(u) = \frac{1}{n+3} m_\rho^2 \langle\!\langle O_{n+2}\rangle\!\rangle, \tag{3.4}$$

which is the direct analogue of Nachtmann's correction in deep inelastic scattering.

As pointed out in [14], there exists an alternative possibility to describe the mass corrections by modification of the exponential factor in (3.1) rather than a contribution to the twist-4 distribution amplitude. To this end, we write

$$\langle 0|\Big[\bar{u}(x)\not{x}d(-x)\Big]_{\rm tw.2}|\rho^-(P,\lambda)\rangle = f_\rho m_\rho \int_0^1 du\,\Big[(e^{(\lambda)}x)e^{i\xi Px}\Big]_{\rm tw.2}\phi_\parallel(u), \tag{3.5}$$

where $[\ldots]_{\rm tw.2}$ on the left-hand side correspond by definition to a subtraction of traces in all local operators in the Taylor expansion of the nonlocal operator at short distances. As shown in [15], this definition implies that the nonlocal operator satisfies the homogeneous Laplace equation

$$\frac{\partial^2}{\partial x_\eta \partial x^\eta}\Big[\bar{u}(x)\not{x}d(-x)\Big]_{\rm tw.2} = 0, \tag{3.6}$$

and the same condition has to be fulfilled by the function $\Big[(e^{(\lambda)}x)e^{i\xi Px}\Big]_{\rm tw.2}$ in order that Eq. (3.5) be satisfied. The solution can easily be constructed order by order in the $(m^2x^2)^k$ expansion [14]. To twist-4 accuracy, we obtain

$$\Big[(e^{(\lambda)}x)e^{i\xi Px}\Big]_{\rm tw.2} = (e^{(\lambda)}x)\Big[e^{i\xi Px} + \frac{m^2x^2\xi^2}{4}\int_0^1 dt\, t^2\, e^{it\xi Px} + O(x^4)\Big]. \tag{3.7}$$

By taking moments, it is easy to check that Eq. (3.1), with the higher twist distribution function $\Phi(u)$ given in (3.4), is equivalent to Eq. (3.5) with the substitution (3.7).

The result in (3.4) is, however, incomplete. The reason is that in exclusive processes one has to take into account higher twist operators containing total derivatives, and vacuum-to-meson matrix elements of such operators reduce, in certain cases, to powers of the meson mass times reduced matrix elements of leading twist operators. In the present case, write [15]

$$\begin{aligned}\bar{u}(x)\not{x}d(-x) &= \Big[\bar{u}(x)\not{x}d(-x)\Big]_{\rm tw.2} + \frac{x^2}{4}\int_0^1 dt\,\frac{\partial^2}{\partial x_\alpha \partial x^\alpha}\bar{u}(tx)\not{x}d(-tx) + O(x^4)\\ &= \Big[\bar{u}(x)\not{x}d(-x)\Big]_{\rm tw.2} - \frac{x^2}{4}\int_0^1 dt\, t^2\, \partial^2[\bar{u}(tx)\not{x}d(-tx)]\\ &\qquad + \text{contributions of operators with gluons} + O(x^4),\end{aligned} \tag{3.8}$$

where we used Eq. (A.9) to obtain the last line. In the matrix element we can make the substitution $\partial^2 \to -m_\rho^2$. Expanding, again, at short distances, and comparing with the short-distance expansion of (3.1), we get an additional contribution to $M_n^{(\Phi)}$, so that the corrected version of (3.4) becomes

$$M_n^{(\Phi)} = \frac{1}{n+3}m_\rho^2\,[\langle\!\langle O_{n+2}\rangle\!\rangle + \langle\!\langle O_n\rangle\!\rangle] + \text{ gluons}. \tag{3.9}$$

Assuming the asymptotic form of the leading twist distribution amplitude ϕ, $\phi(u) = 6u(1-u)$, so that $\langle\!\langle O_n\rangle\!\rangle = 3/[2(n+1)(n+3)]$, this equation for moments is easily solved and gives

$$\Phi(u) = 30u^2(1-u)^2\left[\frac{2}{5}m_\rho^2 + \frac{4}{3}m_\rho^2\zeta_4\right], \tag{3.10}$$

where we have included the "genuine" twist-4 correction (term in ζ_4) due to the twist-4 quark–gluon operator, see definition in Eq. (4.6). The QCD sum rule estimate is $\zeta_4 \sim 0.15$ [16], so that the meson mass effect on the twist-4 distribution function is by a factor 2 larger than the "genuine" twist-4 correction. This is an important difference to deep inelastic scattering, where the target mass corrections are small.

The above discussion is still oversimplified and does not provide us with a complete separation of meson mass effects. The major complication arises because of contributions of operators of the type

$$\partial_{\mu_1}\Big[\bar{u}\gamma_{\mu_1}(i\overleftrightarrow{D}_{\mu_2})\ldots(i\overleftrightarrow{D}_{\mu_n})d\Big]_{\rm tw.2}. \tag{3.11}$$

Such operators can be expressed in terms of operators with extra gluon fields, which means, conversely, that certain combinations of $\bar{q}Gq$ operators reduce to divergences of leading twist operators and give rise to extra meson mass correction terms. The corresponding corrections to twist-4 distributions involve, however, higher order contributions in the conformal expansion of the distribution amplitudes of leading twist and do not affect the result in (3.10), which only includes leading conformal spin[2]. A calculation of the next-to-leading corrections will be presented below.

4 Chiral-Even Distribution Amplitudes

In this section we derive explicit expressions for chiral-even distribution amplitudes of twist-4 including the leading and next-to-leading contributions in the conformal expansion. We first give a short summary of the relevant results of [3] to twist-3 accuracy. We then discuss the conformal expansion of twist-4 three-particle distribution amplitudes and relate the coefficients to matrix elements of local operators. Only a few operators turn out to be independent, so that the number of nonperturbative parameters is reduced considerably. Finally, we calculate the twist-4 two-particle distribution amplitudes from the equations of motion (EOM). The quark mass corrections will be neglected throughout this section.

4.1 Twist-3 Distributions

A comprehensive study of ρ meson distribution amplitudes to twist-3 accuracy was carried out in [17, 18, 3], and we begin this section by quoting the results relevant to the present paper.

The leading twist-2 distribution amplitude for the longitudinally polarized ρ mesons, $\phi_\parallel$, is expanded as [17, 18]

$$\phi_\parallel(u) = 6u\bar{u}\left[1 + a_2^\parallel\,\frac{3}{2}(5\xi^2-1)\right]. \tag{4.1}$$

The parameter $a_2^\parallel$ is defined by the matrix element of a twist-2 conformal operator with conformal spin 3:

$$\langle 0|\bar{u}\not{z}(i\overleftrightarrow{D}z)^2 d - \frac{1}{5}(i\partial z)^2\bar{u}\not{z}d|\rho^-(P,\lambda)\rangle = \frac{12}{35}(e^{(\lambda)}z)(pz)^2 f_\rho m_\rho\, a_2^\parallel, \tag{4.2}$$

[2]The reason why leading conformal spin is not affected is that the divergence of a conformal operator vanishes in free theory.

and is scale-dependent:

$$a_2^\parallel(Q^2) = L^{\gamma_2^\parallel/b} a_2^\parallel(\mu^2), \qquad \gamma_2^\parallel = \frac{25}{6}C_F, \tag{4.3}$$

where $L \equiv \alpha_s(Q^2)/\alpha_s(\mu^2)$ and $C_F = (N_c^2-1)/(2N_c)$, $b = (11N_c - 2n_f)/3$. The parameter $a_2^\parallel$ has been estimated from QCD sum rules; its value at the reference scale $\mu = 1\,\text{GeV}$ is given in Table 2.

The three-particle distributions of twist-3 read [17, 3][3]:

$$\mathcal{V}(\underline{\alpha}) = 540\,\zeta_3\omega_3^V(\alpha_d - \alpha_u)\alpha_d\alpha_u\alpha_g^2, \tag{4.4}$$

$$\mathcal{A}(\underline{\alpha}) = 360\,\zeta_3\alpha_d\alpha_u\alpha_g^2\left[1 + \omega_3^A\,\frac{1}{2}\,(7\alpha_g - 3)\right]. \tag{4.5}$$

The dimensionless coupling ζ_3 is defined by the matrix element

$$\langle 0|\bar u g\tilde G_{\mu\nu}\gamma_\alpha\gamma_5 d|\rho^-(P,\lambda)\rangle = f_\rho m_\rho\zeta_3\left[e_\mu^{(\lambda)}\left(P_\alpha P_\nu - \frac{1}{3}m_\rho^2\,g_{\alpha\nu}\right) - e_\nu^{(\lambda)}\left(P_\alpha P_\mu - \frac{1}{3}m_\rho^2\,g_{\alpha\mu}\right)\right]$$
$$+\frac{1}{3}f_\rho m_\rho^3\zeta_4\left[e_\mu^{(\lambda)}g_{\alpha\nu} - e_\nu^{(\lambda)}g_{\alpha\mu}\right], \tag{4.6}$$

where ζ_4 is a matrix element of twist-4, which we will need below, while ω_3^V and ω_3^A are defined as

$$\langle 0|\bar u \not z(gG_{\alpha\nu}z^\alpha(i\overrightarrow{D}\,z) - (i\overleftarrow{D}\,z)gG_{\alpha\nu}z^\alpha)d|\rho^-(P,\lambda)\rangle = i(pz)^3 e_{\perp\nu}^{(\lambda)}m_\rho f_\rho\frac{3}{28}\,\zeta_3\,\omega_3^V + O(z_\nu) \tag{4.7}$$

and

$$\langle 0|\bar u\not z\gamma_5\left[iDz, g\tilde G_{\mu\nu}z^\mu\right]d - \frac{3}{7}(i\partial z)\bar u\not z\gamma_5 g\tilde G_{\mu\nu}z^\mu d|\rho^-(P,\lambda)\rangle = -(pz)^3 e_{\perp\nu}^{(\lambda)}m_\rho f_\rho\frac{3}{28}\zeta_3\,\omega_3^A + O(z_\nu), \tag{4.8}$$

respectively, where [,] stands for the commutator.

The scale-dependence of the twist-3 parameters is given by [3] (with $C_A = N_c$):

$$\zeta_3(Q^2) = L^{\gamma_3^\zeta/b}\zeta_3(\mu^2), \quad \gamma_3^\zeta = -\frac{1}{3}C_F + 3C_A, \tag{4.9}$$

and

$$\begin{pmatrix}\omega_3^V - \omega_3^A\\ \omega_3^V + \omega_3^A\end{pmatrix}^{Q^2} = L^{\Gamma_3^\omega/b}\begin{pmatrix}\omega_3^V - \omega_3^A\\ \omega_3^V + \omega_3^A\end{pmatrix}^{\mu^2},$$
$$\Gamma_3^\omega = \begin{pmatrix}3C_F - \frac{2}{3}C_A & \frac{2}{3}C_F - \frac{2}{3}C_A\\ \frac{5}{3}C_F - \frac{4}{3}C_A & \frac{1}{2}C_F + C_A\end{pmatrix}. \tag{4.10}$$

[3]Note that we use a normalization of distribution amplitudes different from that in [17, 3]. In the notation of Ref. [3], $\omega_{1,0}^A \equiv \omega_3^A$, $\zeta_3^A \equiv \zeta_3$, and $\zeta_3^V \equiv (3/28)\zeta_3\omega_3^V$.

Numerical estimates are given in Table 2.

Finally, the two-particle distributions of twist-3 are determined from the EOM [3]:

$$g_\perp^{(a)}(u) = 6u\bar{u}\left[1+\left\{\frac{1}{4}a_2^\parallel+\frac{5}{3}\,\zeta_3\left(1-\frac{3}{16}\,\omega_3^A+\frac{9}{16}\omega_3^V\right)\right\}(5\xi^2-1)\right],$$

$$g_\perp^{(v)}(u) = \frac{3}{4}(1+\xi^2)+\left(\frac{3}{7}\,a_2^\parallel+5\zeta_3\right)(3\xi^2-1)$$
$$+\left[\frac{9}{112}\,a_2^\parallel+\frac{15}{64}\,\zeta_3\Big(3\,\omega_3^V-\omega_3^A\Big)\right](3-30\xi^2+35\xi^4)\,. \qquad (4.11)$$

4.2 Twist-4 Distributions

Due to the odd G-parity of the operator in (2.13), the distribution amplitudes Φ and Ψ are antisymmetric under the exchange of α_d and α_u, whereas $\widetilde{\Phi}$ and $\widetilde{\Psi}$ are symmetric. The distributions $\widetilde{\Psi}$, Ψ correspond to the light-cone projection $\gamma.G_{.*}$ (see Table 1) and have the conformal expansion

$$\widetilde{\Psi}(\underline{\alpha}) = 120\alpha_u\alpha_d\alpha_g\left[\widetilde{\psi}_{00}+\widetilde{\psi}_{10}(3\alpha_g-1)+\ldots\right],$$

$$\Psi(\underline{\alpha}) = 120\alpha_u\alpha_d\alpha_g\left[\;0\;+\psi_{10}(\alpha_d-\alpha_u)+\ldots\right], \qquad (4.12)$$

respectively. Note that the leading spin contribution to Ψ vanishes because of G-parity (for massless quarks).

In turn, the distribution amplitudes $\widetilde{\Phi}$, Φ correspond to the $\gamma_\perp G_{.\perp}$ light-cone component, and before a conformal expansion can be performed, we first have to separate the different quark spin projections. To this end, we define auxiliary amplitudes:

$$\langle 0|\bar{u}(z)g\widetilde{G}_{\mu\nu}(vz)\gamma_.\gamma_\alpha\gamma_5\gamma_* d(-z)|\rho\rangle = f_\rho m_\rho^3\,\frac{ez}{pz}\left(p_\mu g^\perp_{\alpha\nu}-p_\nu g^\perp_{\alpha\mu}\right)\Phi^{\uparrow\downarrow}(v,pz),$$

$$\langle 0|\bar{u}(z)g\widetilde{G}_{\mu\nu}(vz)\gamma_*\gamma_\alpha\gamma_5\gamma_. d(-z)|\rho\rangle = f_\rho m_\rho^3\,\frac{ez}{pz}\left(p_\mu g^\perp_{\alpha\nu}-p_\nu g^\perp_{\alpha\mu}\right)\Phi^{\downarrow\uparrow}(v,pz). \qquad (4.13)$$

The relation of $\Phi^{\uparrow\downarrow}, \Phi^{\downarrow\uparrow}$ to the original amplitudes is given by:

$$\widetilde{\Phi}(\underline{\alpha}) = \frac{1}{2}\left[\Phi^{\uparrow\downarrow}+\Phi^{\downarrow\uparrow}\right](\underline{\alpha}),$$

$$\Phi(\underline{\alpha}) = \frac{1}{2}\left[\Phi^{\uparrow\downarrow}-\Phi^{\downarrow\uparrow}\right](\underline{\alpha}), \qquad (4.14)$$

and their conformal expansion goes in terms of Appell polynomials:

$$\Phi^{\uparrow\downarrow}(\underline{\alpha}) = 60\alpha_u\alpha_g^2\left[\phi_{00}+\phi_{01}(\alpha_g-3\alpha_d)+\phi_{10}\left(\alpha_g-\frac{3}{2}\alpha_u\right)\right],$$

$$\Phi^{\downarrow\uparrow}(\underline{\alpha}) = 60\alpha_d\alpha_g^2\left[\phi_{00} + \phi_{01}(\alpha_g - 3\alpha_u) + \phi_{10}\left(\alpha_g - \frac{3}{2}\alpha_d\right)\right], \tag{4.15}$$

where we have taken into account the symmetry properties, i.e. $\phi_{00}^{\uparrow\downarrow} = \phi_{00}^{\downarrow\uparrow}$, etc. Combining everything, we obtain

$$\widetilde{\Phi}(\underline{\alpha}) = 30\alpha_g^2\left[\phi_{00}(1-\alpha_g) + \phi_{01}\Big[\alpha_g(1-\alpha_g) - 6\alpha_u\alpha_d\Big] + \phi_{10}\Big[\alpha_g(1-\alpha_g) - \frac{3}{2}(\alpha_u^2+\alpha_d^2)\Big]\right],$$

$$\Phi(\underline{\alpha}) = 30\alpha_g^2(\alpha_u - \alpha_d)\left[\phi_{00} + \phi_{01}\alpha_g + \frac{1}{2}\phi_{10}(5\alpha_g - 3)\right]. \tag{4.16}$$

At this point, the expansion involves two parameters, ϕ_{00} and $\widetilde{\psi}_{00}$, to leading conformal twist accuracy, and four more $(\phi_{10}, \phi_{01}, \psi_{10}, \widetilde{\psi}_{10})$ for the corrections. Our next task will be to relate them to matrix elements of local operators and find out how many coefficients are independent.

For leading spin the answer is easily obtained by taking the relevant light-cone projections of the matrix element in (4.6):

$$\widetilde{\psi}_{00} = \frac{2}{3}\zeta_3 + \frac{1}{3}\zeta_4,$$

$$\phi_{00} = -\frac{1}{3}\zeta_3 + \frac{1}{3}\zeta_4. \tag{4.17}$$

Note that the "twist-4" distribution amplitudes receive contributions of *both* twist-3 and twist-4 operators. This is due to the fact that the standard counting of twist in terms of "good" and "bad" components as introduced in [20] is at variance with the definition of twist as spin minus dimension of an operator. See also the discussion in Sec. 2.2. of Ref. [3]. The parameter ζ_4 in (4.6) can be explicitly defined as the matrix element of a pure twist-4 operator:

$$\langle 0|\bar{u}g\tilde{G}_{\mu\nu}\gamma_\nu\gamma_5 d|\rho^-(P,\lambda)\rangle = f_\rho m_\rho^3 e_\mu^{(\lambda)}\zeta_4. \tag{4.18}$$

Its scale-dependence is given by [8]

$$\zeta_4(Q^2) = L^{\gamma_4^\zeta/b}\zeta_4(\mu^2), \quad \gamma_4^\zeta = \frac{8}{3}C_F, \tag{4.19}$$

and the numerical value was estimated in [16] from QCD sum rules, see Table 2 and App. C.

The calculation of the next-to-leading order spin corrections is more involved and is presented in detail in App. B. The main observation is that the four coefficients $\phi_{10}, \phi_{01}, \psi_{10}, \widetilde{\psi}_{10}$ in fact involve only one new nonperturbative parameter, in addition to the ones defined above. We find:

$$\phi_{01} = \frac{1}{12}a_2^{\parallel} - \frac{5}{12}\zeta_3 + \frac{3}{16}\zeta_3(\omega_3^A + \omega_3^V) + \frac{7}{2}\zeta_4\,\omega_4^A,$$

f_ρ [MeV]	$a_2^\parallel$	ζ_3	ω_3^A	ω_3^V	ζ_4	ω_4^A
198 ± 7	0.18 ± 0.10	0.032 ± 0.010	-2.1 ± 1.0	3.8 ± 1.8	0.15 ± 0.10	0.8 ± 0.8

Table 2: Parameters of chiral-even distribution amplitudes. Renormalization scale is $\mu = 1\,\text{GeV}$.

$$\begin{aligned}
\phi_{10} &= -\frac{1}{12}a_2^\parallel + \frac{3}{4}\zeta_3 + \frac{3}{16}\zeta_3(\omega_3^A - \omega_3^V) + 7\zeta_4\,\omega_4^A, \\
\psi_{10} &= -\frac{1}{4}a_2^\parallel - \frac{7}{12}\zeta_3 + \frac{9}{16}\zeta_3\,\omega_3^V - \frac{21}{4}\zeta_4\,\omega_4^A, \\
\widetilde{\psi}_{10} &= \ \ \frac{2}{3}\zeta_3 - \frac{9}{16}\zeta_3\,\omega_3^A + \frac{21}{4}\zeta_4\,\omega_4^A,
\end{aligned} \tag{4.20}$$

where the new parameter ω_4^A is defined as

$$\langle 0|\bar u\left[iD_\mu, g\tilde G_{\nu\xi}\right]\gamma_\xi\gamma_5 d - \frac{4}{9}(i\partial_\mu)\bar u g\tilde G_{\nu\xi}\gamma_\xi\gamma_5 d|\rho^-(P,\lambda)\rangle + (\mu\leftrightarrow\nu) =$$

$$= 2f_\rho m_\rho^3\,\zeta_4\,\omega_4^A\left(e_\mu^{(\lambda)}P_\nu + e_\nu^{(\lambda)}P_\mu\right). \tag{4.21}$$

ω_4^A is estimated from QCD sum rules in App. C, with the result given in Table 2. The one-loop anomalous dimension of the operator on the left-hand side of (4.21) is not known.

A few comments on the structure of (4.20) are in order. First, as already mentioned, twist-4 distribution amplitudes contain contributions of operators of twist-3. Note that the twist-4 chiral-even distributions considered here correspond to longitudinally polarized ρ mesons, while the twist-3 parts appearing in (4.20) formally correspond to transversely polarized mesons. The physical reason why distributions with different polarization appear to be related is Lorentz symmetry: a longitudinally polarized ρ meson can be made transversely polarized by going over to the meson rest frame, rotating the spin and boosting back to the infinite momentum frame. The spin rotation, however, is not a member of the collinear conformal group. Because of this, the conformal structure of twist-3 additions to twist-4 amplitudes is rather complicated and does not match the naive expansion, similar to the case considered in App. B of [3]. Formally, this is yet another complication of having a nonzero meson mass.

Secondly, we find a term in $a_2^\parallel$ that corresponds to the next-to-leading correction in the conformal expansion of the leading twist distribution amplitude. This contribution thus presents an additional meson mass correction and appears, in technical terms, through the operator identity (see App. B) relating the divergence of a two-particle conformal operator to operators involving gluon fields. In this respect distribution amplitudes in exclusive reactions are fundamentally different from inclusive distributions, which involve only forward-scattering matrix elements, so that matrix elements of operators with total derivatives vanish.

Third, an inspection of the numerical size of the entries in Table 2 reveals that the coefficients in (4.20) are grossly dominated by the term in ω_A^4, which is a genuine twist-4 effect. We do not see any physical reasons for this dominance, but, if correct, it suggests that the above-mentioned complications may have an only marginal effect on phenomenology.

Finally, we have to specify the two-particle twist-4 distributions g_3 and $\mathbb{A}$ defined in Sec. 2. They are not independent, but can be expressed in terms of Φ and Ψ by using the EOM, see Eqs. (A.10), (A.12). To next-to-leading accuracy, we obtain:

$$\begin{aligned} g_3(u) = {} & 1 + \left(-1 - \frac{2}{7}\, a_2^\parallel + \frac{40}{3}\,\zeta_3 - \frac{20}{3}\,\zeta_4\right) C_2^{1/2}(\xi) \\ & + \left(-\frac{27}{28}\, a_2^\parallel + \frac{5}{4}\,\zeta_3 - \frac{15}{16}\,\zeta_3\left\{\omega_3^A + 3\omega_3^V\right\}\right) C_4^{1/2}(\xi), \end{aligned} \tag{4.22}$$

$$\begin{aligned} \mathbb{A}(u) = {} & 30u^2\bar{u}^2 \left\{\frac{4}{5}\left(1 + \frac{1}{21}\, a_2^\parallel + \frac{10}{9}\,\zeta_3 + \frac{25}{9}\,\zeta_4\right)\right. \\ & \left. + \frac{1}{5}\left(\frac{9}{14}\, a_2^\parallel + \frac{1}{18}\,\zeta_3 + \frac{3}{8}\,\zeta_3\left[\frac{7}{3}\,\omega_3^V - \omega_3^A\right]\right) C_2^{5/2}(\xi)\right\} \\ & + 10\left(-2a_2^\parallel - \frac{14}{3}\,\zeta_3 + \frac{9}{2}\,\zeta_3\omega_3^V - 42\zeta_4\omega_4^A\right) \\ & \times \int_0^u dv \int_0^v dw \left\{1 + C_2^{1/2}(\xi_w) - 3\xi_w(1-\xi_w^2)\ln\frac{1+\xi_w}{1-\xi_w}\right\} \end{aligned} \tag{4.23}$$

with $\xi_w = 2w-1$. The double-integral can of course be taken analytically:

$$\begin{aligned} & \int_0^u dv \int_0^v dw \left\{1 + C_2^{1/2}(\xi_w) - 3\xi_w(1-\xi_w^2)\ln\frac{1+\xi_w}{1-\xi_w}\right\} = \\ & = \frac{1}{10}u\bar{u}(2+13u\bar{u}) + \frac{1}{5}u^3(10-15u+6u^2)\ln u + \frac{1}{5}\bar{u}^3(10-15\bar{u}+6\bar{u}^2)\ln\bar{u}. \end{aligned} \tag{4.24}$$

The resulting functions $g_3(u)$ and $\mathbb{A}(u)$ are shown in Fig. 1 by solid lines. The dashed curves are obtained by omitting the next-to-leading spin corrections (which is the approximation adopted in [4, 21]), and the dash-dotted curves correspond to taking into account the meson mass corrections only and neglecting all twist-3 and twist-4 matrix elements. It is evident that the contributions from next-to-leading order conformal spin are small in both cases. The mass terms clearly dominate $g_3(u)$ and constitute about half of $\mathbb{A}(u)$.

We stress that the given expressions are exact, provided the three-particle distributions are taken in the above approximation. This means, in particular, that (4.22) and (4.23) reproduce the exact second moments of g_3 and $d^2/du^2\mathbb{A}$, i.e. the normalization of $\mathbb{A}$, but the fourth moment of g_3 (second of $\mathbb{A}$) also includes (uncalculated) contributions from even

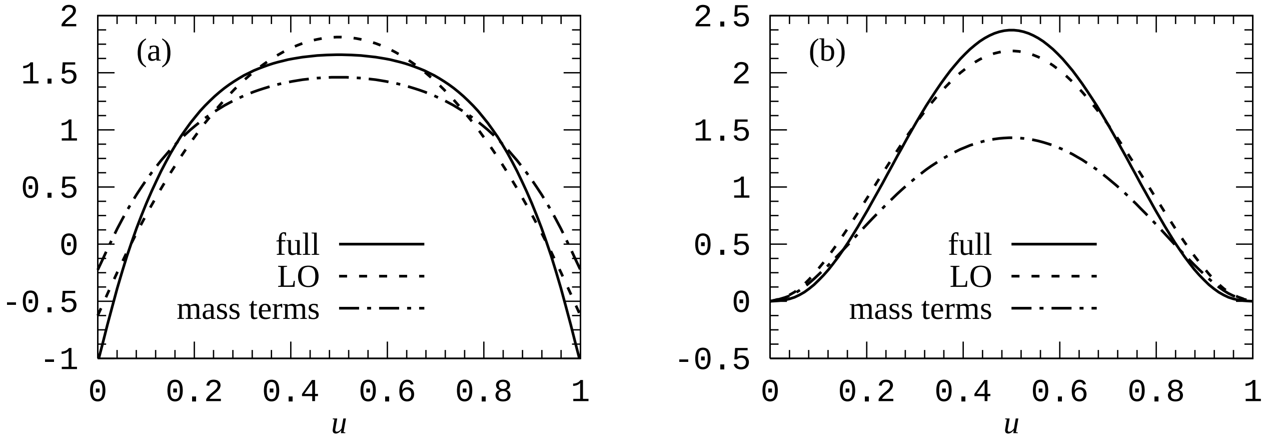

Figure 1: Two-particle twist-4 chiral-even distribution amplitudes of the ρ meson: g_3 (a) and $\mathbb{A}$ (b). LO means neglecting contributions of higher conformal spin for twist-3 and twist-4 operators and the mass terms correspond to pure meson mass corrections.

higher conformal spin operators. We have checked that the second moments agree with those obtained from Taylor expanding (2.3).

Note that g_3 corresponds to the spin projection $s = -1/2$ for both the quark and the antiquark, and thus has a conformal expansion in Gegenbauer polynomials $C^{1/2}(2u-1)$, cf. (2.24):

$$g_3(u,\mu^2) = 1 + \sum_{k=2,4,\ldots}^{\infty} g_3^{(k)}(\mu^2) C_k^{1/2}(2u-1).$$

The coefficients $g_3^{(2)}$ and $g_3^{(4)}$ can be read off (4.22). The conformal expansion of $\mathbb{A}$ is not straightforward and contains for instance logarithms.

5 Chiral-Odd Distribution Amplitudes

The construction of twist-4 chiral-odd distribution amplitudes parallels that for chiral-even distributions in the previous section. We first recall the results for distribution amplitudes of twist-3. Next, we derive the conformal expansion of three-particle distribution amplitudes to next-to-leading order in conformal spin and relate the expansion coefficients to matrix elements of local operators. The two-particle distribution amplitudes are then obtained from the EOM as derived in App. A.

5.1 Twist-3 Distributions

The leading twist-2 distribution amplitude for transversely polarized ρ mesons, $\phi_\perp$, is expanded as [17, 18]

$$\phi_\perp(u) = 6u\bar{u}\left[1 + a_2^\perp \frac{3}{2}(5\xi^2 - 1)\right]. \qquad (5.1)$$

The parameter $a_2^\perp$ is defined by the matrix element of a twist-2 conformal operator with conformal spin 3,

$$\langle 0|\bar{u}\sigma_{\perp .}(i\overset{\leftrightarrow}{D} z)^2 d - \frac{1}{5}(i\partial z)^2 \bar{u}\sigma_{\perp .} d|\rho^-(P,\lambda)\rangle = \frac{12}{35}\, e_\perp^{(\lambda)}(pz) i f_\rho^T\, a_2^\perp, \qquad (5.2)$$

and is scale-dependent:

$$a_2^\perp(Q^2) = L^{\gamma_2^\perp/b} a_2^\perp(\mu^2), \qquad \gamma_2^\perp = \frac{10}{3} C_F. \qquad (5.3)$$

The numerical value of $a_2^\perp$ has been estimated from QCD sum rules and is given in Table 3 (at the reference scale $\mu = 1\,\mathrm{GeV}$).

The only existing three-particle distribution amplitude of twist-3, $\mathcal{T}$, is given by:

$$\mathcal{T}(\underline{\alpha}) = 540\,\zeta_3\,\omega_3^T(\alpha_d - \alpha_u)\alpha_d\alpha_u\alpha_g^2, \qquad (5.4)$$

with ω_3^T defined as

$$\langle 0|\bar{u}\sigma_{\mu\nu} z^\nu (gG^{\mu\beta} z_\beta (i\vec{D} z) - (i\overset{\leftarrow}{D} z) gG^{\mu\beta} z_\beta) d|\rho^-(P,\lambda)\rangle = (pz)^2 (e^{(\lambda)} z) m_\rho^2 f_\rho^T \frac{3}{28}\,\zeta_3\omega_3^T. \qquad (5.5)$$

The parameter ζ_3 was already defined in (4.6). The scale-dependence of ω_3^T is given by [3]

$$\omega_3^T(Q^2) = L^{\gamma_3^{\omega^T}/b}\omega_3^T(\mu^2), \quad \gamma_3^{\omega^T} = \frac{25}{6}\, C_F - 2C_A. \qquad (5.6)$$

Finally, we also quote the two-particle distributions of twist-3 as obtained from the EOM [3]:

$$h_\parallel^{(s)}(u) = 6u\bar{u}\left[1 + \left(\frac{1}{4}a_2^\perp + \frac{5}{8}\,\zeta_3\omega_3^T\right)(5\xi^2 - 1)\right], \qquad (5.7)$$

$$h_\parallel^{(t)}(u) = 3\xi^2 + \frac{3}{2}a_2^\perp\,\xi^2\,(5\xi^2 - 3) + \frac{15}{16}\zeta_3\omega_3^T(3 - 30\xi^2 + 35\xi^4). \qquad (5.8)$$

Numerical values of the input parameters are collected in Table 3.

5.2 Twist-4 Distributions

Due to the odd G-parity of the operator in (2.21), the distribution amplitudes $T_i^{(4)}$ are antisymmetric under the exchange of α_u and α_d, whereas S and $\widetilde{S}$ are symmetric. In order to resolve the conformal structure of $T_i^{(4)}$, it is advantageous to exploit the fact that $\sigma_{\mu\nu}\gamma_5$ is not independent of $\sigma_{\mu\nu}$, and to define a "dual" matrix element

$$\langle 0|\bar{u}(z)i\sigma_{\alpha\beta}\gamma_5 g\widetilde{G}_{\mu\nu}(vz)d(-z)|\rho\rangle = \text{r.h.s. of (2.21) with } T\to\widetilde{T}. \tag{5.9}$$

One easily finds

$$\begin{aligned}\widetilde{\mathcal{T}}^{(3)} = \mathcal{T}^{(3)}, \qquad \widetilde{T}_1^{(4)} &= -T_3^{(4)}, \quad \widetilde{T}_2^{(4)} = -T_4^{(4)},\\ \widetilde{T}_3^{(4)} = -T_1^{(4)}, \quad \widetilde{T}_4^{(4)} &= -T_2^{(4)}.\end{aligned} \tag{5.10}$$

We next note that the distributions $T_1^{(4)}$ and $\widetilde{T}_1^{(4)} = -T_3^{(4)}$ correspond to the Lorentz spin projection $s=+1/2$ for both quark fields and the spin projection zero for the gluon. Hence

$$\begin{aligned}T_1^{(4)}(\underline{\alpha}) &= 120t_{10}(\alpha_u-\alpha_d)\alpha_u\alpha_d\alpha_g,\\ T_3^{(4)}(\underline{\alpha}) &= -120\widetilde{t}_{10}(\alpha_u-\alpha_d)\alpha_u\alpha_d\alpha_g.\end{aligned} \tag{5.11}$$

For the distribution amplitudes S, $\widetilde{S}$, $T_1^{(4)}$ and $T_4^{(4)}$, on the other hand, one has to separate different quark spin projections. To this end, we define auxiliary amplitudes

$$\begin{aligned}\langle 0|\bar{u}(z)\gamma.\gamma_* gG_{\mu\nu}(vz)d(-z)|\rho^-(P,\lambda)\rangle &= if_\rho^T m_\rho^2[e_\mu^{(\lambda)\perp}p_\nu - e_\nu^{(\lambda)\perp}p_\mu]S^{\uparrow\downarrow}(v,pz),\\ \langle 0|\bar{u}(z)\gamma.\gamma_* i\gamma_5 g\widetilde{G}_{\mu\nu}(vz)d(-z)|\rho^-(P,\lambda)\rangle &= if_\rho^T m_\rho^2[e_\mu^{(\lambda)\perp}p_\nu - e_\nu^{(\lambda)\perp}p_\mu]\widetilde{S}^{\uparrow\downarrow}(v,pz),\end{aligned} \tag{5.12}$$

and, similarly, two more distributions $S^{\downarrow\uparrow}$ and $\widetilde{S}^{\downarrow\uparrow}$ by replacing $\gamma.\gamma_*\to\gamma_*\gamma.$. The relations to the distribution amplitudes in (2.21), (2.22) are given by:

$$\begin{aligned}S(\underline{\alpha}) &= \frac{1}{2}\,(S^{\uparrow\downarrow}(\underline{\alpha}) + S^{\downarrow\uparrow}(\underline{\alpha})),\\ \widetilde{S}(\underline{\alpha}) &= \frac{1}{2}\,(\widetilde{S}^{\uparrow\downarrow}(\underline{\alpha}) + \widetilde{S}^{\downarrow\uparrow}(\underline{\alpha})),\\ T_4^{(4)}(\underline{\alpha}) &= \frac{1}{2}\,(S^{\uparrow\downarrow}(\underline{\alpha}) - S^{\downarrow\uparrow}(\underline{\alpha})),\\ -T_2^{(4)}(\underline{\alpha}) \;=\; \widetilde{T}_4^{(4)}(\underline{\alpha}) &= \frac{1}{2}\,(\widetilde{S}^{\uparrow\downarrow}(\underline{\alpha}) - \widetilde{S}^{\downarrow\uparrow}(\underline{\alpha})).\end{aligned} \tag{5.13}$$

The auxiliary amplitudes are expanded in Appell polynomials as

$$\begin{aligned}S^{\uparrow\downarrow}(\underline{\alpha}) &= 60\alpha_u\alpha_g^2\left[s_{00} + s_{10}\left(\alpha_g - \frac{3}{2}\,\alpha_u\right) + s_{01}(\alpha_g - 3\alpha_d)\right],\\ S^{\downarrow\uparrow}(\underline{\alpha}) &= 60\alpha_d\alpha_g^2\left[s_{00} + s_{10}\left(\alpha_g - \frac{3}{2}\,\alpha_d\right) + s_{01}(\alpha_g - 3\alpha_u)\right],\end{aligned} \tag{5.14}$$

and similarly for $\widetilde{S}^{\uparrow\downarrow}$ and $\widetilde{S}^{\downarrow\uparrow}$. Here we made use of the symmetry of S under the exchange of the u and d quarks, i.e. $s_{00}^{\uparrow\downarrow} = s_{00}^{\downarrow\uparrow}$, etc.

From (5.13) and (5.14) it now follows immediately that

$$
\begin{aligned}
S(\underline{\alpha}) &= 30\alpha_g^2\Big[s_{00}\,(1-\alpha_g) + s_{10}\left\{\alpha_g(1-\alpha_g) - \frac{3}{2}\,(\alpha_u^2+\alpha_d^2)\right\} + s_{01}\,\{\alpha_g(1-\alpha_g) - 6\alpha_u\alpha_d\}\Big]\,,\\
\widetilde{S}(\underline{\alpha}) &= 30\alpha_g^2\Big[\widetilde{s}_{00}\,(1-\alpha_g) + \widetilde{s}_{10}\left\{\alpha_g(1-\alpha_g) - \frac{3}{2}\,(\alpha_u^2+\alpha_d^2)\right\} + \widetilde{s}_{01}\,\{\alpha_g(1-\alpha_g) - 6\alpha_u\alpha_d\}\Big]\,,\\
T_2^{(4)}(\underline{\alpha}) &= -30\alpha_g^2(\alpha_u-\alpha_d)\left[\widetilde{s}_{00} + \frac{1}{2}\,\widetilde{s}_{10}\,(5\alpha_g-3) + \widetilde{s}_{01}\alpha_g\right],\\
T_4^{(4)}(\underline{\alpha}) &= \quad 30\alpha_g^2(\alpha_u-\alpha_d)\left[s_{00} + \frac{1}{2}\,s_{10}\,(5\alpha_g-3) + s_{01}\alpha_g\right].
\end{aligned}
\qquad (5.15)
$$

At this point, the expansion involves two parameters of leading conformal spin, s_{00} and $\widetilde{s}_{00}$, and six more $(s_{10}, s_{01}, \widetilde{s}_{10}, \widetilde{s}_{01}, t_{10}, \widetilde{t}_{10})$ for the corrections. Our next task will be to relate them to matrix elements of local operators and find out how many coefficients are independent.

For the leading spin, the answer is easily obtained by taking the local limit $z \to 0$ of (2.22), so that

$$
s_{00} = \zeta_4^T, \quad \widetilde{s}_{00} = \widetilde{\zeta}_4^T \qquad (5.16)
$$

with

$$
\begin{aligned}
\langle 0|\bar{u}gG_{\mu\nu}d|\rho^-(P,\lambda)\rangle &= if_\rho^T m_\rho^2 \zeta_4^T (e_\mu^{(\lambda)}P_\nu - e_\nu^{(\lambda)}P_\mu),\\
\langle 0|\bar{u}g\widetilde{G}_{\mu\nu}i\gamma_5 d|\rho^-(P,\lambda)\rangle &= if_\rho^T m_\rho^2 \widetilde{\zeta}_4^T (e_\mu^{(\lambda)}P_\nu - e_\nu^{(\lambda)}P_\mu).
\end{aligned}
\qquad (5.17)
$$

The parameters ζ_4^T, $\widetilde{\zeta}_4^T$ renormalize multiplicatively [19]:

$$
\begin{aligned}
\left(\zeta_4^T + \widetilde{\zeta}_4^T\right)(Q^2) &= L^{\gamma^+/b}\left(\zeta_4^T + \widetilde{\zeta}_4^T\right)(\mu^2), \;\; \gamma_+ = 3C_A - \frac{8}{3}\,C_F,\\
\left(\zeta_4^T - \widetilde{\zeta}_4^T\right)(Q^2) &= L^{\gamma^-/b}\left(\zeta_4^T - \widetilde{\zeta}_4^T\right)(\mu^2), \;\; \gamma_- = 4C_A - 4C_F.
\end{aligned}
\qquad (5.18)
$$

The numerical values can be estimated from QCD sum rules, see Table 4 and App. C.

The calculation of the next-to-leading order spin corrections is involved and presented in detail in App. B. The main observation is that the six coefficients s_{10}, s_{01}, $\widetilde{s}_{10}$, $\widetilde{s}_{01}$, t_{10} and $\widetilde{t}_{10}$ involve three new nonperturbative parameters. We find:

$$
\begin{aligned}
s_{10} &= -\frac{3}{22}\,a_2^\perp - \frac{1}{8}\,\zeta_3\omega_3^T + \frac{28}{55}\,\langle\!\langle Q^{(1)}\rangle\!\rangle + \frac{7}{11}\,\langle\!\langle Q^{(3)}\rangle\!\rangle + \frac{14}{3}\,\langle\!\langle Q^{(5)}\rangle\!\rangle,\\
\widetilde{s}_{10} &= \quad \frac{3}{22}\,a_2^\perp - \frac{1}{8}\,\zeta_3\omega_3^T - \frac{28}{55}\,\langle\!\langle Q^{(1)}\rangle\!\rangle - \frac{7}{11}\,\langle\!\langle Q^{(3)}\rangle\!\rangle + \frac{14}{3}\,\langle\!\langle Q^{(5)}\rangle\!\rangle,
\end{aligned}
$$

$$
\begin{aligned}
s_{01} &= \frac{3}{44}\, a_2^\perp + \frac{1}{8}\,\zeta_3\omega_3^T + \frac{49}{110}\,\langle\!\langle Q^{(1)}\rangle\!\rangle - \frac{7}{22}\,\langle\!\langle Q^{(3)}\rangle\!\rangle + \frac{7}{3}\,\langle\!\langle Q^{(5)}\rangle\!\rangle,\\
\widetilde{s}_{01} &= -\frac{3}{44}\, a_2^\perp + \frac{1}{8}\,\zeta_3\omega_3^T - \frac{49}{110}\,\langle\!\langle Q^{(1)}\rangle\!\rangle + \frac{7}{22}\,\langle\!\langle Q^{(3)}\rangle\!\rangle + \frac{7}{3}\,\langle\!\langle Q^{(5)}\rangle\!\rangle,\\
t_{10} &= -\frac{9}{44}\, a_2^\perp - \frac{3}{16}\,\zeta_3\omega_3^T - \frac{63}{220}\,\langle\!\langle Q^{(1)}\rangle\!\rangle + \frac{119}{44}\,\langle\!\langle Q^{(3)}\rangle\!\rangle,\\
\widetilde{t}_{10} &= \frac{9}{44}\, a_2^\perp - \frac{3}{16}\,\zeta_3\omega_3^T + \frac{63}{220}\,\langle\!\langle Q^{(1)}\rangle\!\rangle + \frac{35}{44}\,\langle\!\langle Q^{(3)}\rangle\!\rangle.
\end{aligned}
\tag{5.19}
$$

The above relations involve the three parameters $\langle\!\langle Q^{(1)}\rangle\!\rangle$, $\langle\!\langle Q^{(3)}\rangle\!\rangle$ and $\langle\!\langle Q^{(5)}\rangle\!\rangle$, which can be defined as matrix elements of the following operators:

$$
\begin{aligned}
Q^{(1)}_{\alpha,\xi\eta} &= -i\bar u \overleftrightarrow{\nabla}_\alpha\, (\sigma_{\xi\rho} g G_{\eta\rho} - \sigma_{\eta\rho} g G_{\xi\rho}) d + 7\bar u \mathcal{D}_\alpha g[G - i\gamma_5\widetilde{G}]_{\xi\eta} d - \frac{11}{3}\,\partial_\alpha \bar u g[G - i\gamma_5\widetilde{G}]_{\xi\eta} d,\\
Q^{(3)}_{\alpha,\xi\eta} &= \Big\{ i\bar u \overleftrightarrow{\nabla}_\xi\, \sigma_{\eta\rho} g[G + i\gamma_5\widetilde{G}]_{\alpha\beta}\, d - \frac{1}{3}\,\bar u \mathcal{D}_\alpha g[G + i\gamma_5\widetilde{G}]_{\xi\eta}\, d - \frac{1}{3}\,\bar u \mathcal{D}_\eta g[G + i\gamma_5\widetilde{G}]_{\xi\alpha}\, d\\
&\qquad + \frac{1}{3}\,\partial_\alpha \bar u g[G + i\gamma_5\widetilde{G}]_{\xi\eta}\, d + \frac{1}{3}\,\partial_\eta \bar u g[G + i\gamma_5\widetilde{G}]_{\xi\alpha}\, d \Big\} - \{\xi \leftrightarrow \eta\},\\
Q^{(5)}_{\alpha,\xi\eta} &= \bar u \mathcal{D}_\alpha g[G + i\gamma_5\widetilde{G}]_{\xi\eta} d - \frac{1}{2}\,\partial_\alpha \bar u g[G + i\gamma_5\widetilde{G}]_{\xi\eta} d,
\end{aligned}
\tag{5.20}
$$

where we used convenient short-hand notations (see [19]) for the covariant derivatives: $\overleftrightarrow{\nabla}_\alpha\, G_{\mu\nu} \equiv G_{\mu\nu}\, \overrightarrow{D}_\alpha - \overleftarrow{D}_\alpha\, G_{\mu\nu}$ acting on quark fields only, and $\mathcal{D}_\alpha G_{\mu\nu} \equiv [D_\alpha, G_{\mu\nu}]$ acting on gluon fields only. The reduced matrix elements $\langle\!\langle Q^{(i)}\rangle\!\rangle$ of these operators are defined as

$$
\begin{aligned}
\langle 0|Q^{(i)}_{\alpha,\xi\eta}|\rho^-(P,\lambda)\rangle &= \left[e^{(\lambda)}_\xi \left(P_\alpha P_\eta - \frac{1}{3}\, m_\rho^2 g_{\alpha\eta}\right) - e^{(\lambda)}_\eta \left(P_\alpha P_\xi - \frac{1}{3}\, m_\rho^2 g_{\alpha\xi}\right)\right] f_\rho^T m_\rho^2 \langle\!\langle Q^{(i)}\rangle\!\rangle\\
&\qquad + (e^{(\lambda)}_\xi g_{\alpha\eta} - e^{(\lambda)}_\eta g_{\alpha\xi}) \langle\!\langle R^{(i)}\rangle\!\rangle,
\end{aligned}
\tag{5.21}
$$

where $\langle\!\langle Q^{(i)}\rangle\!\rangle$ is of twist-4 and $\langle\!\langle R^{(i)}\rangle\!\rangle$ of twist-5. The operators renormalize multiplicatively and their one-loop anomalous dimensions are known [19]; to obtain the scale-dependence of the matrix elements $\langle\!\langle Q^{(i)}\rangle\!\rangle$, one has to subtract the anomalous dimension of f_ρ^T, Eq. (2.7), so that

$$
\begin{aligned}
&\langle\!\langle Q^{(i)}\rangle\!\rangle(Q^2) = L^{\gamma_{Q^{(i)}}/b}\, \langle\!\langle Q^{(i)}\rangle\!\rangle(\mu^2)\\
&\qquad \gamma_{Q^{(1)}} = -4C_F + \frac{11}{2}\, C_A, \quad \gamma_{Q^{(3)}} = \frac{10}{3}\, C_F, \quad \gamma_{Q^{(5)}} = -\frac{5}{3}\, C_F + 5C_A.
\end{aligned}
\tag{5.22}
$$

f_ρ^T [MeV]	$a_2^\perp$	ζ_3	ω_3^T
160 ± 10	0.20 ± 0.10	0.032 ± 0.010	7.0 ± 7.0

Table 3: Parameters of twist-2 and twist-3 chiral-odd distribution amplitudes. Renormalization scale is $\mu = 1\,\text{GeV}$.

ζ_4^T	$\widetilde{\zeta}_4^T$	$\langle\langle Q^{(1)}\rangle\rangle$	$\langle\langle Q^{(3)}\rangle\rangle$	$\langle\langle Q^{(5)}\rangle\rangle$
0.10 ± 0.05	-0.10 ± 0.05	-0.15 ± 0.15	0	0

Table 4: Parameters of twist-4 chiral-odd distribution amplitudes. Renormalization scale as above.

Numerical estimates for these matrix elements were obtained in [19] and App. C and are collected in Table 4.

Finally, we have to specify the two-particle twist-4 distributions h_3 and $\mathbb{A}_T$ defined in Sec. 2. They are not independent, but can be expressed in terms of S, $T_2^{(4)}$ and $T_4^{(4)}$ by using the EOM, see Eqs. (A.13), (A.14). To next-to-leading accuracy, we obtain:

$$
\begin{aligned}
h_3(u) &= 1 + \left\{-1 + \frac{3}{7}\,a_2^\perp - 10(\zeta_4^T + \widetilde{\zeta}_4^T)\right\} C_2^{1/2}(\xi) \\
&\quad + \left\{-\frac{3}{7}\,a_2^\perp - \frac{15}{8}\,\zeta_3\omega_3^T\right\} C_4^{1/2}(\xi), \qquad (5.23)
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{A}_T(u) &= 30u^2\bar{u}^2 \left\{\frac{2}{5}\left(1 + \frac{2}{7}\,a_2^\perp + \frac{10}{3}\,\zeta_4^T - \frac{20}{3}\,\widetilde{\zeta}_4^T\right) + \left(\frac{3}{35}\,a_2^\perp + \frac{1}{40}\,\zeta_3\omega_3^T\right) C_2^{5/2}(\xi)\right\} \\
&\quad - \left(\frac{18}{11}\,a_2^\perp - \frac{3}{2}\,\zeta_3\omega_3^T + \frac{126}{55}\,\langle\langle Q^{(1)}\rangle\rangle + \frac{70}{11}\,\langle\langle Q^{(3)}\rangle\rangle\right) \\
&\quad \times \left(u\bar{u}(2 + 13u\bar{u}) + 2u^3(10 - 15u + 6u^2)\ln u + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln\bar{u}\right). \qquad (5.24)
\end{aligned}
$$

In Fig. 2, we plot h_3 and $\mathbb{A}_T$ as functions of u, showing full results and the contributions from leading order conformal spin and mass correction terms separately. Like in the chiral-even case, the mass terms dominate $h_3(u)$ and constitute approximately one half of $\mathbb{A}_T(u)$.

We stress that the given expressions are exact provided the three-particle distributions are taken in the above approximation. This means, in particular, that (5.23) and (5.24) reproduce the exact second moments of h_3 and $d^2/du^2\mathbb{A}_T$, i.e. the normalization of $\mathbb{A}_T$, but the fourth moment of h_3 (second of $\mathbb{A}_T$) also includes (uncalculated) contributions from even higher conformal spin operators. We have checked that the second moments agree with those obtained from Taylor expanding (2.16).

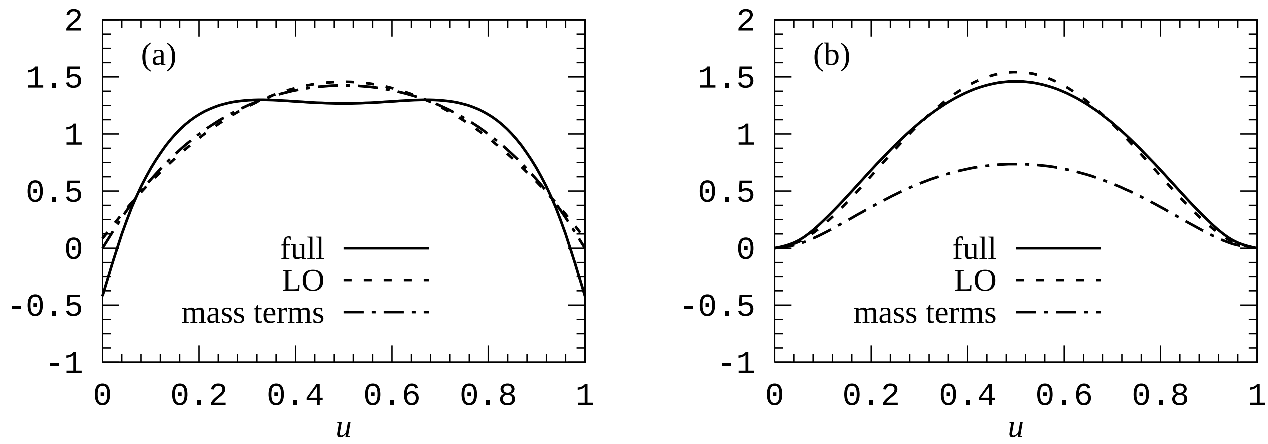

Figure 2: Two-particle twist-4 chiral-odd distribution amplitudes of the ρ meson: h_3 (a) and $\mathbb{A}_T$ (b). LO means neglecting contributions of higher conformal spin for twist-3 and twist-4 operators and the mass terms correspond to retaining meson mass corrections only.

Note that, like g_3, h_3 corresponds to the spin projection $s = -1/2$ for both the quark and the antiquark, and thus has a conformal expansion in Gegenbauer polynomials $C^{1/2}(2u-1)$, cf. (2.24):

$$h_3(u,\mu^2) = 1 + \sum_{k=2,4,\dots}^{\infty} h_3^{(k)}(\mu^2) C_k^{1/2}(2u-1).$$

The coefficients $h_3^{(2)}$ and $h_3^{(4)}$ can be read off (5.23). The conformal expansion of $\mathbb{A}_T$ is more complicated.

6 Summary and Conclusions

In the present paper we have studied the twist-4 two- and three-particle distribution amplitudes of vector mesons in QCD and expressed them in a model-independent way by a minimal number of nonperturbative parameters. The work reported here is an extension of our earlier paper on twist-3 distribution amplitudes [3]. The one ingredient in the approach is the use of the QCD equations of motion, which allow us to reveal interrelations between different distribution amplitudes of a given twist and to obtain exact integral representations for distribution amplitudes that are not dynamically independent. The other ingredient is the use of conformal expansion: analogously to partial wave decomposition in quantum mechanics, it allows one to separate transverse and longitudinal variables in the wave function. The dependence on transverse coordinates is represented as scale-dependence of the relevant operators and is governed by renormalization-group equations; the dependence on the longitudinal momentum fraction is described in terms of irreducible representations of the corresponding symmetry group, the collinear conformal group SL(2,R). The conformal partial wave expansion is explicitly consistent with the equations of motion since the latter are not renormalized. The expansion thus makes maximum use of the symmetry of the theory in order to simplify the dynamics, which is related, in the perturbative domain, to renormalization properties of the relevant operators.

The analysis of twist-4 distribution amplitudes is complicated by the fact that the twist-4 terms are of different origin: there are, first, "intrinsic" twist-4 corrections from matrix elements of twist-4 operators. There are, second, admixtures of matrix elements of twist-3 operators, as the counting of twist in terms of "good" and "bad" projections on light-cone coordinates does not exactly match the definition of twist as "dimension minus spin" of an operator. There are, third, meson mass corrections, which one may term kinematical corrections, that come, on the one hand, from the subtraction of traces in the leading twist operators and, on the other hand, from higher twist operators containing total derivatives of twist-2 operators. Meson mass corrections of the first kind are formally analogous to Nachtmann corrections in inclusive processes, while the contribution of operators with total derivatives is a specific new feature of exclusive processes, which makes the structure of these corrections much more complex.

Our final results are collected in Secs. 4 and 5. We present a complete set of distribution amplitudes that is consistent with the QCD equations of motion and has a minimum number of nonperturbative parameters whose numerical values are estimated from QCD sum rules. It turns out that the meson mass corrections are the dominant ones in all two-particle twist-4 distributions, which is in contrast to what is observed in deep-inelastic scattering and welcome from the phenomenological point of view, as the higher twist matrix elements, $\zeta_{3,4}$ etc., come with considerable numerical uncertainties.

The results of our study are immediately applicable — and, in fact, have already been applied [21] — to processes such as exclusive or radiative B decays and hard electroproduction of vector mesons at HERA.

Acknowledgements

P.B. is supported by DFG through a Heisenberg fellowship. She also acknowledges hospitality and financial support from NORDITA during a visit there when this work was initiated. We thank G. Stoll for collaboration on part of the subjects covered in this work.

Appendices

A Equations of Motion

A.1 Operator Identities

In this appendix we collect exact operator identities, which can be derived using the approach of [15] and which present a nonlocal equivalent to the equations of motion for Wilson local operators. The basic idea is to study the response of nonlocal operators to total translations and/or the change of the interquark separation along the light-cone. For convenience, we work in the Fock–Schwinger gauge $x_\mu A_\mu(x) = 0$, so that

$$[x,-x]=1, \qquad A_\mu(x) = \int_0^1 dv\, v x_\alpha G_{\alpha\mu}(vx).$$

All operator relations can be made manifestly gauge invariant by restoring the path-ordered gauge factor between field operators at different points in space-time.

For the general two-particle operator, we can write

$$\frac{\partial}{\partial x_\mu}\,\bar u(x)\Gamma d(-x) = -\bar u(x)\Gamma \overset{\leftrightarrow}{D}_\mu d(-x) - i\int_{-1}^1 dv\, v\bar u(x)x_\alpha g G_{\alpha\mu}(vx)\Gamma d(-x), \tag{A.1}$$

where Γ is an arbitrary Dirac matrix and $\overset{\leftrightarrow}{D}_\mu = \vec{D}_\mu - \overset{\leftarrow}{D}_\mu = (\overset{\rightarrow}{\partial} - iA(-x))_\mu - (\overset{\leftarrow}{\partial} + iA(x))_\mu$. The derivatives act on the arguments of the quark operators.

In a similar way we can calculate the derivative with respect to the total translation:

$$\partial_\mu\{\bar u(x)\Gamma d(-x)\} = \bar u(x)(\overset{\leftarrow}{D}_\mu + \vec{D}_\mu)\Gamma d(-x) - i\int_{-1}^1 dv\, \bar u(x)x_\alpha G_{\alpha\mu}(vx)\Gamma d(-x), \tag{A.2}$$

where, by definition,

$$\partial_\mu\{\bar u(x)\Gamma d(-x)\} \equiv \frac{\partial}{\partial y_\mu}\,\{\bar u(x+y)[x+y,-x+y]\Gamma d(-x+y)\}\Big|_{y\to 0}. \tag{A.3}$$

Here it is important to keep the gauge factors, which give a nonvanishing contribution:

$$\partial_\mu[x,-x] = iA_\mu(x)[x,-x] - i[x,-x]A_\mu(-x) - i\int_{-1}^1 dv\,[x,vx]x_\alpha G_{\alpha\mu}(vx)[vx,-x].$$

For chiral-even operators, $\Gamma = \{\gamma_\mu, \gamma_\mu\gamma_5\}$, the first terms on the right-hand side of Eqs. (A.1), (A.2) vanish by virtue of the massless Dirac equation, so that

$$\frac{\partial}{\partial x_\mu}\,\bar u(x)\gamma_\mu(\gamma_5)d(-x) \;=\; -\,i\int_{-1}^1 dv\, v\bar u(x)x_\alpha g G_{\alpha\mu}(vx)\gamma_\mu(\gamma_5)d(-x), \tag{A.4}$$

$$\partial_\mu\{\bar u(x)\gamma_\mu(\gamma_5)d(-x)\} \;=\; -\,i\int_{-1}^1 dv\, \bar u(x)x_\alpha G_{\alpha\mu}(vx)\gamma_\mu(\gamma_5)d(-x). \tag{A.5}$$

For chiral-odd operators, $\Gamma = \{\mathbf{1}(\gamma_5), \sigma_{\mu\nu}(\gamma_5)\}$, on the other hand, we can use the identities

$$\sigma_{\mu\nu}\overset{\leftrightarrow}{D}_\mu = i(\vec{D}_\nu + \overset{\leftarrow}{D}_\nu), \qquad (\vec{D}_\mu + \overset{\leftarrow}{D}_\mu)\sigma_{\mu\nu} = i\overset{\leftrightarrow}{D}_\nu, \tag{A.6}$$

and get, combining Eqs. (A.1) and (A.2):

$$\partial_\mu \bar{u}(x)\sigma_{\mu\nu}(\gamma_5)d(-x) = -i\,\frac{\partial}{\partial x_\nu}\,\bar{u}(x)(\gamma_5)d(-x) + \int_{-1}^{1} dv\, v\bar{u}(x)x_\rho g G_{\rho\nu}(vx)(\gamma_5)d(-x)$$

$$- i\int_{-1}^{1} dv\,\bar{u}(x)x_\rho g G_{\rho\mu}(vx)\sigma_{\mu\nu}(\gamma_5)d(-x), \qquad \text{(A.7)}$$

$$\frac{\partial}{\partial x_\mu}\,\bar{u}(x)\sigma_{\mu\nu}(\gamma_5)d(-x) = -i\partial_\nu\bar{u}(x)(\gamma_5)d(-x) + \int_{-1}^{1} dv\,\bar{u}(x)x_\rho g G_{\rho\nu}(vx)(\gamma_5)d(-x)$$

$$- i\int_{-1}^{1} dv\, v\bar{u}(x)x_\rho g G_{\rho\mu}(vx)\sigma_{\mu\nu}(\gamma_5)d(-x). \qquad \text{(A.8)}$$

This method is general and can also be used for calculating the second derivative. In particular, the following formula is useful:

$$\frac{\partial^2}{\partial x_\alpha \partial x^\alpha}\bar{u}(x)\Gamma\, d(-x) = -\partial^2\bar{u}(x)\Gamma\, d(-x) + \bar{u}(x)[\Gamma\sigma G + \sigma G\,\Gamma]d(-x)$$

$$- 2ix^\nu\frac{\partial}{\partial x_\mu}\int_{-1}^{1} dv\, v\,\bar{u}(x)\Gamma G_{\nu\mu}(vx)d(-x) - 2ix^\nu\partial_\mu\int_{-1}^{1} dv\,\bar{u}(x)\Gamma G_{\nu\mu}(vx)d(-x)$$

$$+ 2\int_{-1}^{1} dv\int_{-1}^{v} dt\,(1+vt)\bar{u}(x)\Gamma x^\mu x^\nu G_{\mu\rho}(vx)G^\rho{}_\nu(tx)d(-x)$$

$$+ ix^\nu\int_{-1}^{1} dv\,(1+v^2)\,\bar{u}(x)\Gamma[D_\mu, G^\mu{}_\nu](vx)d(-x), \qquad \text{(A.9)}$$

where $[D_\mu, G^\mu{}_\nu] = -t^A(\bar{\psi}\gamma_\nu t^A\psi)$, assuming summation over light flavours ψ.

A.2 Relations Between Distribution Amplitudes

We are now in a position to derive relations between two- and three-particle amplitudes. Sandwiching (A.4) between the vacuum and the ρ meson state, we find

$$g_3(u) = \phi_\parallel(u) - 2\,\frac{d}{du}\int_0^u d\alpha_d\int_0^{\bar{u}} d\alpha_u\,\frac{1}{\alpha_g}\,\{2\Phi(\underline{\alpha}) + \Psi(\underline{\alpha})\}\,. \qquad \text{(A.10)}$$

To arrive at Eq. (A.10), the following formula proves useful:

$$\int_{-1}^{1} dv\, v^k\int \mathcal{D}\alpha\,\mathcal{F}(\underline{\alpha})\exp[-ipx\{\alpha_u - \alpha_d + v\alpha_g\}] =$$

$$= \int_0^1 du\,\exp[i\xi px]\int_0^u d\alpha_d\int_0^{\bar{u}} d\alpha_u\,\frac{2}{\alpha_g}\,\left[\frac{1}{\alpha_g}\,(\alpha_d - \alpha_u - \xi)\right]^k \mathcal{F}(\underline{\alpha}), \qquad \text{(A.11)}$$

which is valid for an arbitrary function $\mathcal{F}(\underline{\alpha})$.

On the other hand, taking the matrix element of (A.5), and eliminating g_3 by virtue of (A.10), we obtain

$$\frac{1}{8}\frac{d^2}{du^2}\mathbb{A}(u) = 4(g_\perp^v(u)-\phi_\parallel(u)) + 4\frac{d}{du}\int_0^u d\alpha_d\int_0^{\bar u} d\alpha_u\,\frac{1}{\alpha_g}\{2\Phi(\underline{\alpha})+\Psi(\underline{\alpha})\}$$
$$+\frac{d^2}{du^2}\int_0^u d\alpha_d\int_0^{\bar u} d\alpha_u\,\frac{1}{\alpha_g^2}(\alpha_u u-\alpha_d\bar u)\{2\Phi(\underline{\alpha})+\Psi(\underline{\alpha})\}\,. \qquad \text{(A.12)}$$

$\mathbb{A}$ can then be obtained as

$$\mathbb{A}(u) = \int_0^u dv\int_0^v dw\,\frac{d^2}{dw^2}\mathbb{A}(w).$$

Two more relations are derived in a similar manner between chiral-odd distribution amplitudes using the operator identities in (A.7) and (A.8). From (A.7):

$$h_\parallel^{(s)}(u)-\frac{1}{2}(h_3(u)+\phi_\perp(u)) =$$
$$= \frac{d}{du}\int_0^u d\alpha_d\int_0^{\bar u} d\alpha_u\left[\frac{\alpha_d-\alpha_u-\xi}{\alpha_g^2}\,S(\underline{\alpha})-\frac{1}{\alpha_g}\left(T_2^{(4)}(\underline{\alpha})-T_3^{(4)}(\underline{\alpha})\right)\right], \qquad \text{(A.13)}$$

and from (A.8):

$$4\int_0^u dv\int_0^v dw\,[h_3(w)-\phi_\perp(w)]-\frac{1}{2}\mathbb{A}_T(u)-\int_0^u dv\,(2v-1)[\phi_\perp(v)+h_3(v)] =$$
$$=\int_0^u d\alpha_d\int_0^{\bar u} d\alpha_u\,\frac{2}{\alpha_g}\left\{\frac{\alpha_d-\alpha_u-\xi}{\alpha_g}\left(T_2^{(4)}(\underline{\alpha})-T_3^{(4)}(\underline{\alpha})\right)-S(\underline{\alpha})\right\}. \qquad \text{(A.14)}$$

B Short-Distance Expansion of Distribution Amplitudes: Relation to Local Operators

In this appendix we calculate the next-to-leading corrections to the conformal expansion of twist-4 three-particle distribution amplitudes as given in Eq. (4.20) and (5.19).

B.1 Chiral-Even

Our general strategy will be to consider matrix elements of the relevant operators with all Lorentz indices open. Taking different light-cone projections we will relate coefficients in the conformal expansion of distribution amplitudes to the invariant Lorentz structures and then identify the relevant contractions of indices that relate ϕ_{10}, ϕ_{01}, ψ_{10} and $\widetilde{\psi}_{10}$ to matrix elements of independent twist-4 operators.

To the next-to-leading conformal spin accuracy, we need local operators of dimension 6 with one quark-antiquark pair, one gluon field and one additional covariant derivative. Taking into account G-parity, only two operators can contribute, apart from operators with total derivatives:

$$\begin{aligned} O^{(1)}_{\alpha\beta\mu\nu} &= \bar{u}(i\overleftarrow{D}_\beta\, g\widetilde{G}_{\mu\nu} + g\widetilde{G}_{\mu\nu} i\overrightarrow{D}_\beta)\gamma_\alpha\gamma_5 d, \\ O^{(2)}_{\alpha\beta\mu\nu} &= \bar{u}(-\overleftarrow{D}_\beta\, gG_{\mu\nu} + gG_{\mu\nu}\overrightarrow{D}_\beta)\gamma_\alpha d. \end{aligned} \tag{B.1}$$

For each of them we write down a general Lorentz decomposition:

$$\begin{aligned} \langle 0|O^{(i)}_{\alpha\beta\mu\nu}|\rho^-(P,\lambda)\rangle = &\Big\{ e^{(\lambda)}_\mu \Big[P_\alpha P_\beta P_\nu - \frac{5}{24}\, m_\rho^2 (P_\alpha g_{\beta\nu} + P_\beta g_{\alpha\nu}) - \frac{1}{6}\, m_\rho^2 P_\nu g_{\alpha\beta}\Big] \\ &- e^{(\lambda)}_\nu \Big[P_\alpha P_\beta P_\mu - \frac{5}{24}\, m_\rho^2 (P_\alpha g_{\beta\mu} + P_\beta g_{\alpha\mu}) - \frac{1}{6}\, m_\rho^2 P_\mu g_{\alpha\beta}\Big] \\ &- \frac{1}{24}\, m_\rho^2 \Big[e^{(\lambda)}_\alpha (g_{\beta\nu}P_\mu - g_{\beta\mu}P_\nu) + e^{(\lambda)}_\beta (g_{\alpha\nu}P_\mu - g_{\alpha\mu}P_\nu)\Big]\Big\} A^{(i)} f_\rho m_\rho \\ &+ P_\alpha (e^{(\lambda)}_\mu g_{\beta\nu} - e^{(\lambda)}_\nu g_{\beta\mu}) B^{(i)} + P_\beta (e^{(\lambda)}_\mu g_{\alpha\nu} - e^{(\lambda)}_\nu g_{\alpha\mu}) C^{(i)} \\ &+ e^{(\lambda)}_\alpha (P_\mu g_{\beta\nu} - P_\nu g_{\beta\mu}) D^{(i)} + e^{(\lambda)}_\beta (P_\mu g_{\alpha\nu} - P_\nu g_{\alpha\mu}) E^{(i)} \\ &- g_{\alpha\beta}(P_\mu e^{(\lambda)}_\nu - P_\nu e^{(\lambda)}_\mu) F^{(i)}. \end{aligned} \tag{B.2}$$

Here $A^{(i)}$ is of twist-3 and can easily be related to an integral over the twist-3 distribution amplitudes $\mathcal{A}$ and $\mathcal{V}$, respectively. Using (4.7) and (4.8), we find

$$-(pz)^3 e^{(\lambda)}_\perp m_\rho f_\rho A^{(1)} = \langle 0|O^{(1)}_{\cdots\perp}|\rho\rangle \;=\; -(pz)^3 e^{(\lambda)}_\perp m_\rho f_\rho \zeta_3 \left(\frac{3}{7} + \frac{3}{28}\,\omega_3^A\right), \tag{B.3}$$

$$(pz)^3 e^{(\lambda)}_\perp m_\rho f_\rho\, A^{(2)} = \langle 0|O^{(2)}_{\cdots\perp}|\rho\rangle \;=\; (pz)^3 e^{(\lambda)}_\perp m_\rho f_\rho\, \frac{3}{28}\,\zeta_3\omega_3^V. \tag{B.4}$$

To project onto the intrinsic twist-4 contributions, we must replace one "dot" projection by a "perp" projection in (B.3) and (B.4), which yields

$$\begin{aligned} \langle 0|O^{(1)}_{\perp\cdot\cdot\perp}|\rho\rangle &= f_\rho m_\rho^3 (ez)(pz) g^\perp_{\perp\perp}\left\{\frac{1}{2}\left(-\frac{1}{3}\,\zeta_3 + \frac{1}{3}\,\zeta_4\right) - \frac{1}{14}\,(\phi_{01}+\phi_{10})\right\}, \\ \langle 0|O^{(2)}_{\perp\cdot\cdot\perp}|\rho\rangle &= f_\rho m_\rho^3 (ez)(pz) g^\perp_{\perp\perp}\left\{\frac{1}{6}\left(-\frac{1}{3}\,\zeta_3 + \frac{1}{3}\,\zeta_4\right) + \frac{1}{14}\,(\phi_{01}-\phi_{10})\right\}, \end{aligned} \tag{B.5}$$

whereas direct contraction of (B.2) gives

$$\langle 0|O^{(i)}_{\perp\cdot\cdot\perp}|\rho\rangle = -\frac{1}{4}\, m_\rho^3 f_\rho (ez)(pz) g^\perp_{\perp\perp} A^{(i)} + (ez)(pz) g^\perp_{\perp\perp}\left(C^{(i)} + E^{(i)}\right). \tag{B.6}$$

Once $C^{(i)} + E^{(i)}$ are known in terms of ζ_3, ζ_4 and $\omega^{V,A}_{3,4}$, these two equations serve to determine ϕ_{01} and ϕ_{10}.

In order to determine $C^{(i)} + E^{(i)}$, we first have to introduce some more matrix elements:

$$\begin{aligned}
\langle 0|O^{(i)}_{\xi\beta\xi\nu}|\rho\rangle &= (e_\beta P_\nu + e_\nu P_\beta) X^{(i)}_+ + (e_\beta P_\nu - e_\nu P_\beta) X^{(i)}_-,\\
\langle 0|O^{(i)}_{\alpha\xi\xi\nu}|\rho\rangle &= (e_\alpha P_\nu + e_\nu P_\alpha) Y^{(i)}_+ + (e_\alpha P_\nu - e_\nu P_\alpha) Y^{(i)}_-.
\end{aligned} \tag{B.7}$$

By construction, X_+ and Y_+ are of twist-4 and X_- and Y_- of twist-5; thus, to our accuracy:

$$X^{(i)}_- = Y^{(i)}_- = 0.$$

Also note that, by definition, Eq. (4.21):

$$X^{(1)}_+ = \zeta_4\left(\omega^A_4 - \frac{5}{18}\right). \tag{B.8}$$

Now, by contracting (B.2) with $g_{\alpha\beta}$, etc., we find a set of linear equations relating A, B, ... to X_+ and Y_+, which can be solved to give

$$\begin{aligned}
B = D &= \frac{1}{4}\left(X_+ - 3Y_+\right) f_\rho m_\rho^3,\\
C = E &= -\frac{1}{4}\left(X_+ - 3Y_+\right) f_\rho m_\rho^3,\\
F &= 0.
\end{aligned} \tag{B.9}$$

We recall that contributions from twist-5 operators are neglected in these solutions.

Of the remaining three unknowns $X^{(2)}_+$, $Y^{(1)}_+$ and $Y^{(2)}_+$, $Y^{(1)}_+$ can be obtained rather easily by observing that $D_\mu \widetilde{G}_{\mu\nu} = 0$, so that

$$\begin{aligned}
\langle 0|O^{(1)}_{\alpha\xi\xi\nu} + O^{(1)}_{\nu\xi\xi\alpha}|\rho\rangle &= P_\xi \langle 0|\bar u(\widetilde{G}_{\xi\nu}\gamma_\alpha\gamma_5 + \widetilde{G}_{\xi\alpha}\gamma_\nu\gamma_5) d|\rho\rangle\\
&= -(e_\nu P_\alpha + e_\alpha P_\nu) f_\rho m_\rho^3 \left(\frac{2}{3}\,\zeta_3 + \frac{1}{3}\,\zeta_4\right),
\end{aligned}$$

which means

$$Y^{(1)}_+ = -\frac{1}{3}\,\zeta_3 - \frac{1}{6}\,\zeta_4. \tag{B.10}$$

In order to determine the remaining parameters $X_+^{(2)}$ and $Y_+^{(2)}$, we make use of the operator identities

$$O^{(2)}_{\xi\beta\xi\alpha} - O^{(2)}_{\beta\xi\xi\alpha} = O^{(1)}_{\xi\alpha\xi\beta} - O^{(1)}_{\alpha\xi\xi\beta} + g_{\alpha\beta}\, O^{(1)}_{\sigma\xi\xi\sigma} \tag{B.11}$$

and

$$\begin{aligned}\frac{4}{5}\partial_\mu E_{\mu\alpha\beta} &= -12i\bar{u}\gamma_\rho \left\{G_{\rho\beta}\, \overrightarrow{D}_\alpha - \overleftarrow{D}_\alpha\, G_{\rho\beta} + (\alpha \leftrightarrow \beta)\right\} d - 4\partial_\rho\bar{u}(\gamma_\beta\widetilde{G}_{\alpha\rho} + \gamma_\alpha\widetilde{G}_{\beta\rho})\gamma_5 d \\ &\quad - \frac{8}{3}\,\partial_\beta\bar{u}\gamma_\sigma\widetilde{G}_{\sigma\alpha}\gamma_5 d - \frac{8}{3}\,\partial_\alpha\bar{u}\gamma_\sigma\widetilde{G}_{\sigma\beta}\gamma_5 d + \frac{28}{3}\, g_{\alpha\beta}\partial_\rho\bar{u}\gamma_\sigma\widetilde{G}_{\sigma\rho} d,\end{aligned} \tag{B.12}$$

where

$$E_{\mu\alpha\beta} = \left[\frac{15}{2}\bar{u}\gamma_\mu\, \overleftrightarrow{D}_\alpha \overleftrightarrow{D}_\beta\, d - \frac{3}{2}\partial_\alpha\partial_\beta\bar{u}\gamma_\mu d - \text{traces}\right]_{\text{symmetrized}}$$

is a leading twist-2 conformal operator. Taking matrix elements, we find

$$X_+^{(2)} - Y_+^{(2)} = X_+^{(1)} - Y_+^{(1)}, \tag{B.13}$$

and

$$\frac{4}{7}\, m_\rho^2 a_2^\parallel = -24 f_\rho m_\rho^3 X_+^{(2)} + 4 f_\rho m_\rho^3 \left(\frac{2}{3}\,\zeta_3 + \frac{1}{3}\,\zeta_4\right) - \frac{8}{3}\, f_\rho m_\rho^3 \zeta_4. \tag{B.14}$$

With $X^{(i)}$ and $Y^{(i)}$ from Eqs. (B.8), (B.10) and (B.14), we get $C + E$ from (B.9), and thus ϕ_{10} and ϕ_{01} from (B.5) and (B.6), see the first two lines of Eq. (4.20).

Note that it is precisely operator relations of type (B.12), where the divergence of a leading twist conformal operator is expressed as a certain combination of quark–quark–gluon operators, that make the analysis of meson mass corrections to twist-4 distribution amplitudes so complicated. This divergence vanishes in a free theory, as expected.

The determination of the remaining parameters ψ_{10} and $\widetilde{\psi}_{10}$ is now fairly easy: introducing a different "bad" component in (B.3) and (B.4), we find

$$\langle 0|O^{(i)}_{\dots *}|\rho\rangle = f_\rho m_\rho^3 (ez)(pz)\,\frac{1}{2}\left\{A^{(i)} - \left(X_+^{(i)} + Y_+^{(i)}\right)\right\}. \tag{B.15}$$

On the other hand, taking proper integrals over distribution amplitudes:

$$\begin{aligned}\langle 0|O^{(1)}_{\dots *}|\rho\rangle &= (ez)(pz) f_\rho m_\rho^3 \left(\frac{2}{3}\,\widetilde{\psi}_{00} - \frac{2}{21}\,\widetilde{\psi}_{10}\right), \\ \langle 0|O^{(2)}_{\dots *}|\rho\rangle &= -\frac{2}{21}\, f_\rho m_\rho^3 (ez)(pz)\psi_{10}.\end{aligned} \tag{B.16}$$

By equating (B.15) and (B.16), we obtain the last two lines of Eq. (4.20).

B.2 Chiral-Odd

The calculation of next-to-leading order spin corrections to chiral-odd distribution amplitudes essentially parallels the calculation of similar corrections to the photon distribution amplitude in Ref. [19]. To follow this analogy, it is convenient to express the corrections in terms of

$$\eta_1 = -\frac{3}{4}(s_{10} + 2s_{01}), \quad \eta_2 = -\frac{1}{4}(s_{10} - 2s_{01}) \tag{B.17}$$

and the corresponding "dual" quantities $\widetilde{\eta}_1$, $\widetilde{\eta}_2$, instead of the coefficients $s_{01}, s_{10}, \widetilde{s}_{10}, \widetilde{s}_{01}$ in the expansion over Appell polynomials.

Expanding (2.21) and (2.22) in powers of (pz) to first order, we obtain

$$\begin{aligned}\langle 0|\bar{u} \overset{\leftrightarrow}{\nabla}\!. \,\sigma_{\alpha\beta} g G_{\mu\nu} d|\rho^-\rangle \;&=\; f_\rho^T m_\rho^2 \frac{ez}{2pz}\,\left[p_\alpha p_\mu g^\perp_{\beta\nu} - \ldots\right] ipz \left(-\frac{3}{28}\,\zeta_3\omega_3^T\right)\\
&+ f_\rho^T m_\rho^2 ipz \Bigg\{ \left[p_\alpha e^\perp_\mu g^\perp_{\beta\nu} - \ldots\right] \frac{2}{21}\, t_{10} + \left[e^\perp_\alpha p_\mu g^\perp_{\beta\nu} - \ldots\right] \left(-\frac{1}{6}\,\widetilde{\zeta}_4^T - \frac{1}{42}\,\widetilde{\eta}_1 - \frac{3}{14}\,\widetilde{\eta}_2\right)\\
&- \left[p_\alpha p_\mu e^\perp_\beta z_\nu - \ldots\right] \frac{1}{pz}\,\frac{2}{21}\widetilde{t}_{10} + \left[p_\alpha p_\mu z_\beta e^\perp_\nu - \ldots\right] \frac{1}{pz}\left(\frac{1}{6}\,\zeta_4^T + \frac{1}{42}\,\eta_1 + \frac{3}{14}\,\eta_2\right)\Bigg\} + O(\text{twist-5}),\end{aligned} \tag{B.18}$$

$$\begin{aligned}\langle 0|\bar{u} g\mathcal{D}.G_{\mu\nu} d|\rho^-\rangle \;&=\; f_\rho^T m_\rho^2 pz (e^\perp_\mu p_\nu - e^\perp_\nu p_\mu)\left\{\frac{1}{2}\,\zeta_4 - \frac{1}{14}\,(\eta_1+\eta_2)\right\},\\
\langle 0|\bar{u} ig\mathcal{D}.\widetilde{G}_{\mu\nu} d|\rho^-\rangle \;&=\; f_\rho^T m_\rho^2 pz (e^\perp_\mu p_\nu - e^\perp_\nu p_\mu)\left\{\frac{1}{2}\,\widetilde{\zeta}_4 - \frac{1}{14}\,(\widetilde{\eta}_1+\widetilde{\eta}_2)\right\}.\end{aligned} \tag{B.19}$$

Comparing these expressions with the reduced matrix elements of the conformal operators, Eq. (5.21), we immediately find

$$\langle\!\langle Q^{(5)}\rangle\!\rangle = -\frac{1}{14}\,(\eta_1+\eta_2) - \frac{1}{14}\,(\widetilde{\eta}_1+\widetilde{\eta}_2), \tag{B.20}$$

while contracting (B.18) over $g_{\beta\nu}$, we find

$$\langle\!\langle Q^{(1)}\rangle\!\rangle = -\frac{2}{21}\,(t_{10}-\widetilde{t}_{10}) - \frac{10}{21}(\eta_1-\widetilde{\eta}_1) - \frac{2}{7}\,(\eta_2-\widetilde{\eta}_2). \tag{B.21}$$

Next, we introduce one more operator

$$Q^{(2)}_{\alpha,\xi\eta} = i\bar{u} \overset{\leftrightarrow}{\nabla}_\alpha (\sigma_{\xi\rho} g G_{\eta\rho} - \sigma_{\eta\rho} g G_{\xi\rho}) d + \frac{1}{3}\,\bar{u}\mathcal{D}_\alpha (gG_{\xi\eta} - ig\widetilde{G}_{\xi\eta}\gamma_5) d. \tag{B.22}$$

Using (B.18) and (B.19), we easily find

$$\langle\!\langle Q^{(2)}\rangle\!\rangle = \frac{2}{21}\,(t_{10}-\widetilde{t}_{10}) - \frac{1}{21}\,(\eta_1-\widetilde{\eta}_1) - \frac{5}{21}\,(\eta_2-\widetilde{\eta}_2). \tag{B.23}$$

$Q^{(2)}$ is actually not independent, but related to $Q^{(3)}$ via an important operator identity derived in [19]:[4]

$$Q^{(2)}_{\alpha,\xi\eta} = \frac{1}{3}\,Q^{(3)}_{\alpha,\xi\eta} + \frac{1}{30}\,\partial_\rho\left(O^2_{\xi,\eta\alpha\rho} - O^2_{\eta,\xi\alpha\rho}\right), \tag{B.24}$$

where O^2 is the leading twist-2 conformal operator

$$O^2_{\alpha\cdots} = \frac{15}{2}\,\bar u\sigma_{\alpha\cdot}\stackrel{\leftrightarrow}{\nabla}_{\cdot}\stackrel{\leftrightarrow}{\nabla}_{\cdot}d - \frac{3}{2}\,\partial^2_{\cdot}\bar u\sigma_{\alpha\cdot}d. \tag{B.25}$$

The matrix element of the conformal operator on the right-hand side of Eq. (B.24) is equal to

$$\begin{aligned}\langle 0|O^2_{\xi,\eta\alpha\rho} - O^2_{\eta,\xi\alpha\rho}|\rho^-\rangle = if_\rho^T\left(-\frac{24}{7}\,a_2^\perp\right)\Big\{&P_\alpha P_\rho(e_\xi P_\eta - e_\eta P_\xi) - \frac{1}{6}\,m_\rho^2 g_{\alpha\rho}(e_\xi P_\eta - e_\eta P_\xi)\\ &+\frac{1}{24}\,m_\rho^2\left[e_\alpha(g_{\xi\rho}P_\eta - g_{\eta\rho}P_\xi) + e_\rho(g_{\xi\alpha}P_\eta - g_{\eta\alpha}P_\xi)\right]\\ &+\frac{5}{24}\,m_\rho^2\left[P_\alpha(g_{\xi\rho}e_\eta - g_{\eta\rho}e_\xi) + P_\rho(g_{\xi\alpha}e_\eta - g_{\eta\alpha}e_\xi)\right]\Big\},\end{aligned} \tag{B.26}$$

so that

$$\langle\!\langle Q^{(2)}\rangle\!\rangle = \frac{1}{3}\,\langle\!\langle Q^{(3)}\rangle\!\rangle - \frac{1}{14}\,a_2^\perp. \tag{B.27}$$

At this point we have established three relations for the six independent parameters. Three more relations follow from the analysis of the most general matrix element

$$\begin{aligned}\langle 0|O_{\alpha,\mu\nu,\xi\eta}|\rho^-\rangle &= \langle 0|\bar u\stackrel{\leftrightarrow}{\nabla}_\alpha\sigma_{\mu\nu}igG_{\xi\eta}d|\rho^-\rangle\\ &= P_\alpha\left[e_\mu(P_\xi g_{\nu\eta} - P_\eta g_{\nu\xi}) - e_\nu(P_\xi g_{\mu\eta} - P_\eta g_{\mu\xi})\right]A\\ &\quad + P_\alpha\left[P_\mu(e_\xi g_{\nu\eta} - e_\eta g_{\nu\xi}) - P_\nu(e_\xi g_{\mu\eta} - e_\eta g_{\mu\xi})\right]B\\ &\quad + e_\alpha\left[P_\mu(P_\xi g_{\nu\eta} - P_\eta g_{\nu\xi}) - P_\nu(P_\xi g_{\mu\eta} - P_\eta g_{\mu\xi})\right]C\\ &\quad + (g_{\alpha\mu}P_\nu - g_{\alpha\nu}P_\mu)(e_\xi P_\eta - e_\eta P_\xi)D + (g_{\alpha\xi}P_\eta - g_{\alpha\eta}P_\xi)(e_\mu P_\nu - e_\nu P_\mu)E\\ &\quad + e_\alpha(g_{\mu\xi}g_{\nu\eta} - g_{\mu\eta}g_{\nu\xi})F + \left[g_{\alpha\mu}(g_{\xi\nu}e_\eta - g_{\eta\nu}e_\xi) - g_{\alpha\nu}(g_{\xi\mu}e_\eta - g_{\eta\mu}e_\xi)\right]G\\ &\quad + \left[g_{\alpha\xi}(g_{\eta\mu}e_\nu - g_{\eta\nu}e_\mu) - g_{\alpha\eta}(g_{\xi\mu}e_\nu - g_{\xi\nu}e_\mu)\right]H.\end{aligned} \tag{B.28}$$

By projecting onto different light-cone variables, we find a set of linear equations for the coefficients A,..., E (F, G, H are of twist-5 and thus not relevant for the following discussion):

$$A + B + C = f_\rho^T m_\rho^2\,\frac{3}{56}\,\zeta_3\omega_3^T,$$

[4]Note that we obtain a different sign in front of the total derivative operator on the right-hand side as compared to Eq. (4.24) in [19]. We thank G. Stoll for checking this equation.

$$B = -\frac{2}{21} f_\rho^T m_\rho^2 t_{10},$$

$$A = f_\rho^T m_\rho^2 \left(\frac{1}{6} \widetilde{\zeta}_4^T + \frac{1}{42} \widetilde{\eta}_1 + \frac{3}{14} \widetilde{\eta}_2 \right),$$

$$A - E = -\frac{2}{21} f_\rho^T m_\rho^2 \widetilde{t}_{10},$$

$$B - D = f_\rho^T m_\rho^2 \left(\frac{1}{6} \zeta_4^T + \frac{1}{42} \eta_1 + \frac{3}{14} \eta_2 \right). \tag{B.29}$$

Contracting (B.28) with $g_{\alpha\mu}$, we find

$$\langle 0|\bar{u}\mathcal{D}_\nu G_{\xi\eta} d - \partial_\nu \bar{u} G_{\xi\eta} d|\rho^-\rangle = \langle 0|O_{\rho,\rho\nu,\xi\eta}|\rho^-\rangle$$

$$= P_\nu [P_\xi e_\eta - P_\eta e_\xi](B - C - 3D - E) + [e_\xi g_{\nu\xi} - e_\eta g_{\nu\xi}](m_\rho^2 B + F - 3G - H). \tag{B.30}$$

Using (B.19), this can be translated into

$$\zeta_4^T - \left(\frac{1}{2} \zeta_4^T - \frac{1}{14} (\eta_1 + \eta_2) \right) = \frac{2}{21} (t_{10} - \widetilde{t}_{10}) - \frac{3}{56} \zeta_3 \omega_3^T + 3 \left(\frac{1}{6} \zeta_4^T + \frac{1}{42} \eta_1 + \frac{3}{14} \eta_2 \right), \tag{B.31}$$

so that finally

$$\frac{2}{3} (t_{10} - \widetilde{t}_{10}) = \frac{3}{8} \zeta_3 \omega_3^T - 4\eta_2. \tag{B.32}$$

The same analysis can be performed for the matrix element of the dual operator

$$\widetilde{O}_{\alpha,\mu\nu,\xi\eta} = \bar{u} i \overleftrightarrow{\nabla}_\alpha \widetilde{\sigma}_{\mu\nu} \widetilde{G}_{\xi\eta} d, \tag{B.33}$$

which results in the "dual" version of the relation (B.32):

$$\frac{2}{3} (\widetilde{t}_{10} - t_{10}) = \frac{3}{8} \zeta_3 \omega_3^T - 4\widetilde{\eta}_2. \tag{B.34}$$

To obtain the last relation from which $\eta_1, \eta_2, \widetilde{\eta}_1, \widetilde{\eta}_2, t_{10}, \widetilde{t}_{10}$ can be extracted, we use the identity:

$$Q^{(3)}_{\alpha,\xi\eta} = -\frac{2}{3} \left(Q^{(5)}_{\alpha,\xi\eta} - \frac{1}{2} \partial_\alpha O^+_{\xi\eta} \right) - \frac{1}{3} \left(Q^{(5)}_{\eta,\xi\alpha} - \frac{1}{2} \partial_\eta O^+_{\xi\alpha} \right) + \frac{1}{3} \left(Q^{(5)}_{\xi,\eta\alpha} - \frac{1}{2} \partial_\xi O^+_{\eta\alpha} \right)$$

$$+ \bar{u} i \left[\overleftrightarrow{\nabla}_\xi \sigma_{\eta\rho} g (G + i\gamma_5 \widetilde{G})_{\alpha\rho} - \overleftrightarrow{\nabla}_\eta \sigma_{\xi\rho} g (G + i\gamma_5 \widetilde{G})_{\alpha\rho} \right] d, \tag{B.35}$$

where $O^+_{\xi\eta} = \bar{u}(G + i\gamma_5 \widetilde{G})_{\xi\eta} d$. The matrix element of the first three terms on the right-hand side is:

$$\langle 0| -\frac{2}{3} (\ldots) - \frac{1}{3} (\ldots) + \frac{1}{3} (\ldots) |\rho^-\rangle = P_\alpha (e_\eta P_\xi - e_\xi P_\eta) f_\rho^T m_\rho^2 \left\{ \langle\langle Q^{(5)} \rangle\rangle - \frac{1}{2} (\zeta_4^T + \widetilde{\zeta}_4^T) \right\}. \tag{B.36}$$

For the remaining term on the right-hand side we use that

$$\sigma_{\eta\rho} i\gamma_5 \widetilde{G}_{\alpha\rho} = \sigma_{\alpha\rho} G_{\eta\rho} - \frac{1}{2}\, g_{\alpha\eta}\sigma G,$$

and observe that the term in σG has zero matrix element over the ρ meson. Thus, using (B.18), we find

$$\langle 0|\bar{u} i \overset{\leftrightarrow}{\nabla}_\xi (\sigma_{\eta\rho} g G_{\alpha\rho} + \sigma_{\alpha\rho} g G_{\eta\rho}) d - (\xi \leftrightarrow \eta)|\rho^-\rangle = P_\alpha (P_\xi e_\eta - P_\eta e_\xi)[2A + 2B - 4C - 3D - 3E]. \tag{B.37}$$

This gives

$$-\langle\!\langle Q^{(3)} \rangle\!\rangle = 2A + 2B - 4C - 3D - 3E + \langle\!\langle Q^{(5)} \rangle\!\rangle - \frac{1}{2}\,(\zeta_4^T + \widetilde{\zeta}_4^T), \tag{B.38}$$

and with $A, \ldots, E$ from (B.29), finally:

$$\langle\!\langle Q^{(3)} \rangle\!\rangle + \langle\!\langle Q^{(5)} \rangle\!\rangle = \frac{3}{14}\,\zeta_3\omega_3^T + \frac{2}{7}\,(t_{10} + \widetilde{t}_{10}) - \frac{1}{14}\,(\eta_1 + 9\eta_2) - \frac{1}{14}\,(\widetilde{\eta}_1 + 9\widetilde{\eta}_2). \tag{B.39}$$

The eight relations (B.17), (B.20), (B.21), (B.23), (B.27), (B.32), (B.34), (B.39) yield s_{10}, s_{01}, $\widetilde{s}_{10}$, $\widetilde{s}_{01}$, t_{10}, $\widetilde{t}_{10}$ as given in (5.19).

C Numerical Estimates: QCD Sum Rules

In this appendix we estimate the independent nonperturbative parameters from QCD sum rules. The sum rules for chiral-odd matrix elements can be adapted from the analysis of the photon distribution amplitudes in Ref. [19], while those for chiral-even matrix elements are partly available from Ref. [16], partly new. The numerical results are collected in Tables 2, 3 and 4.

C.1 Chiral-Even

To leading conformal spin accuracy, we need the single parameter ζ_4 (4.18). This matrix element was discussed at great length in [16]. The best estimate comes from considering the correlation function

$$i \int d^4x\, e^{iqx}\, \langle 0|T\bar{u}(x) g\widetilde{G}_{\mu\alpha}\gamma^\alpha\gamma_5 d(x)\, \bar{d}(0)\gamma_\nu u(0)|0\rangle = (q_\mu q_\nu - g_{\mu\nu}q^2)\Pi_{\zeta_4}(q^2), \tag{C.1}$$

which yields the following sum rule [16]:

$$f_\rho^2\,\frac{m_\rho^2}{M^2}\,\zeta_4 e^{-m_\rho^2/M^2} = $$

$$= -\frac{\alpha_s}{18\pi^3}\, M^2 \left\{1 - e^{-s_0/M^2}\left(1 + \frac{s_0}{M^2}\right)\right\} + \frac{\langle(\alpha_s/\pi)G^2\rangle}{6M^2} - \frac{32}{27M^4}\,\pi\alpha_s\langle\bar{q}q\rangle^2. \tag{C.2}$$

In the Borel window $1\,\mathrm{GeV}^2 \le M^2 \le 2\,\mathrm{GeV}^2$ and with $s_0 \approx 1.5\,\mathrm{GeV}^2$ and the condensates $\langle(\alpha_s/\pi)G^2\rangle = (0.012 \pm 0.006)\,\mathrm{GeV}^4$ and $\langle\sqrt{\alpha_s}\bar{q}q\rangle^2 = 0.56 \cdot (-0.25\,\mathrm{GeV})^6$, we obtain

$$\zeta_4(\mu = 1\,\mathrm{GeV}) = 0.15 \pm 0.10. \tag{C.3}$$

The calculation of ω_4^A involves dimension 6 operators for which the QCD sum rule approach becomes rather unreliable. Because of this, we choose to make a simple estimate by considering the leading contribution to a correlation function that vanishes in perturbation theory:

$$\begin{aligned} CF_X &= i\int d^4y\, e^{iqy}\langle 0|T\bar{d}(y)\sigma_{\kappa\lambda}u(y)O^{(1)}_{\xi\beta\xi\nu}(0)|0\rangle \\ &= \frac{i}{2}\,\langle\bar{q}\sigma gGq\rangle\,\frac{q_\beta}{q^2}(q_\kappa g_{\lambda\nu} - q_\lambda g_{\kappa\nu}) + O(1/q^4). \end{aligned} \tag{C.4}$$

The contribution of the ρ meson to this correlation function is

$$CF_X = \frac{i}{m_\rho^2 - q^2}\, f_\rho f_\rho^T m_\rho^3 \left[q_\nu(q_\kappa g_{\lambda\beta} - q_\lambda g_{\kappa\beta})(X_+^{(1)} + X_-^{(1)}) + q_\beta(q_\kappa g_{\lambda\nu} - q_\lambda g_{\kappa\nu})(X_+^{(1)} - X_-^{(1)})\right], \tag{C.5}$$

from which we obtain in the local duality limit $q^2 \to -\infty$:

$$X_+^{(1)} \simeq -X_-^{(1)} \simeq -\frac{1}{4 f_\rho f_\rho^T m_\rho^3}\,\langle\bar{q}\sigma gGq\rangle \tag{C.6}$$

at a hadronic scale $\mu \approx 1\,\mathrm{GeV}$. This has to be compared with the estimate for ζ_4 obtained in the same approximation by considering a similar correlation function with the operator (4.18):

$$\zeta_4 \simeq -\frac{1}{2 f_\rho f_\rho^T m_\rho^3}\,\langle\bar{q}\sigma gGq\rangle. \tag{C.7}$$

Putting in numbers, we get $\zeta_4 \approx 0.3$, which is a factor two larger than what comes from the more accurate (and laborious) analysis in [16], see above. Using the definition (B.8) we get, finally

$$\omega_4^A(1\,\mathrm{GeV}) \approx 7/9 \tag{C.8}$$

with, probably, a 100% error.

C.2 Chiral-Odd

The calculation of the matrix elements ζ_4^T and $\widetilde{\zeta}_4^T$, defined in (5.17), and the matrix elements $\langle\!\langle Q^{(i)}\rangle\!\rangle$ of the operators $Q^{(i)}_{\alpha,\xi\eta}$, $i = 1, 3, 5$, defined in (5.21), is analogous to calculation of the parameters of the photon distribution function in Ref. [19], and the sum rules obtained in this paper can be adapted to the present case.

To leading conformal spin accuracy, we need to estimate two parameters, ζ_4^T and $\widetilde{\zeta}_4^T$. To this end, we consider the correlation functions

$$CF_{\pm} = i\int d^4y\, e^{iqy}\langle 0|T\bar{d}(y)\gamma_\mu u(y)\,\bar{u}(0)g[G(0)\pm i\gamma_5\widetilde{G}(0)]_{\alpha\beta}d(0)|0\rangle\,, \tag{C.9}$$

which vanish in perturbation theory. The leading power corrections were calculated in [19], yielding

$$\begin{aligned} CF_+ &= i(q_\beta g_{\mu\alpha} - q_\alpha g_{\mu\beta})\,O(1/q^4), \\ CF_- &= i(q_\beta g_{\mu\alpha} - q_\alpha g_{\mu\beta})\left\{\frac{\langle\bar{q}\sigma gGq\rangle}{3q^2} + O(1/q^4)\right\}. \end{aligned} \tag{C.10}$$

Saturation with a ρ meson gives, on the other hand,

$$CF_{\pm} = \frac{1}{m_\rho^2 - q^2}\, m_\rho^3 f_\rho f_\rho^T(\zeta_4^T \pm \widetilde{\zeta}_4^T) i(g_{\mu\alpha}P_\beta - g_{\beta\mu}P_\alpha), \tag{C.11}$$

so that, taking into account only the leading $1/q^2$ terms, we have

$$\begin{aligned} \zeta_4^T + \widetilde{\zeta}_4^T &= 0, \\ \zeta_4^T - \widetilde{\zeta}_4^T &= -\frac{1}{3}\,\frac{\langle\bar{q}\sigma gGq\rangle}{m_\rho^3 f_\rho f_\rho^T}. \end{aligned} \tag{C.12}$$

Using the same numerical input as in the last section, we obtain

$$\zeta_4^T(1\,\text{GeV}) = -\widetilde{\zeta}_4^T(1\,\text{GeV}) = 0.10 \pm 0.05 \tag{C.13}$$

with a rather conservative large error.

We use the same method to estimate also the $\langle\!\langle Q^{(i)}\rangle\!\rangle$, and consider the correlation functions

$$CF^{(i)} = i\int d^4y\, e^{iqy}\langle 0|T\bar{d}(y)\gamma_\kappa u(y)\, Q^{(i)}_{\alpha,\xi\eta}(0)|0\rangle. \tag{C.14}$$

As shown in [19], the lowest order power correction to $CF^{(3,5)}$ vanishes, so that

$$\langle\!\langle Q^{(3)}\rangle\!\rangle = \langle\!\langle Q^{(5)}\rangle\!\rangle = 0 \tag{C.15}$$

to that accuracy. For $\langle\!\langle Q^{(1)}\rangle\!\rangle$, on the other hand, the mixed condensate gives a nonzero contribution and we obtain

$$\langle\!\langle Q^{(1)}\rangle\!\rangle(1\,\text{GeV}) \simeq \frac{5}{9}\,\frac{\langle\bar{q}\sigma gGq\rangle(1\,\text{GeV})}{m_\rho^3 f_\rho f_\rho^T} \simeq -0.30 \tag{C.16}$$

This value is likely to be overestimated since the mass scale in the correlation function is much larger than the ρ meson mass, see the discussion in [19]. We thus prefer to give

$$\langle\!\langle Q^{(1)}\rangle\!\rangle(1\,\text{GeV}) = -0.15 \pm 0.15 \tag{C.17}$$

as a conservative estimate.

References

[1] S.J. Brodsky and G.P. Lepage, in: *Perturbative Quantum Chromodynamics*, ed. by A.H. Mueller, p. 93, World Scientific (Singapore) 1989.

[2] S.J. Brodsky, H.-C. Pauli and S.S. Pinsky, Phys. Rept. **301** (1998) 299.

[3] P. Ball *et al.*, Nucl. Phys. **B529** (1998) 323.

[4] P. Ball and V.M. Braun, Preprint hep–ph/9808229, to appear in the Proceedings of the 3rd Workshop on *"Continuous Advances in QCD"*, Minneapolis (MN), USA, April 1998.

[5] O. Nachtmann, Nucl. Phys. **B63** (1973) 237.

[6] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. **B147** (1979) 385, 448, 519.

[7] J.D. Bjorken and S.D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

[8] E.V. Shuryak and A.I. Vainshtein, Nucl. Phys. **B199** (1982) 451 and **B201** (1982) 141.

[9] S.J. Brodsky *et al.*, Phys. Lett. B **91** (1980) 239, Phys. Rev. D **33** (1986) 1881 and Phys. Lett. B **167** (1986) 347.

[10] Yu.M. Makeenko, Sov. J. Nucl. Phys. **33** (1981) 440 [Yad. Fiz. **33** (1981) 842].

[11] Th. Ohrndorf, Nucl. Phys. **B198** (1982) 26.

[12] V.M. Braun and I.E. Filyanov, Z. Phys. C **48** (1990) 239.

[13] D. Müller, Phys. Rev. D **51** (1995) 3855 and D **58** (1998) 054005;
A. Belitsky and D. Müller, Phys. Lett. B **417** (1998) 129 and Preprint hep–ph/9804379.

[14] I.I. Balitsky and V.M. Braun, Nucl. Phys. **B361** (1991) 93.

[15] I.I. Balitsky and V.M. Braun, Nucl. Phys. **B311** (1989) 541.

[16] V.M. Braun and A.V. Kolesnichenko, Sov. J. Nucl. Phys. **44** (1986) 489 [Yad. Fiz. **44** (1986) 756].

[17] V.L. Chernyak and A.R. Zhitnitsky, Phys. Rept. **112** (1984) 173.

[18] P. Ball and V.M. Braun, Phys. Rev. D **54** (1996) 2182.

[19] I.I. Balitsky, V.M. Braun and A.V. Kolesnichenko, Nucl. Phys. **B312** (1989) 509.

[20] J. Kogut and D. Soper, Phys. Rev. D **1** (1970) 2901.

[21] P. Ball and V.M. Braun, Phys. Rev. D **58** (1998) 094016.