Electroweak Baryon Number Violation and Constraints on Left-handed Majorana Neutrino Masses

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Abstract.
During a large period of time, the anomalous baryon number violating interactions are in equilibrium, when the $(B+L)$ asymmetry is washed out. If there is any lepton number violation during this period, that will also erase the $(B-L)$ asymmetry. As a result, survival of the baryon asymmetry of the universe pose strong constraints on lepton number violating interactions. We review here the constraints on the left-handed Majorana neutrino masses arising from this survival requirement of the baryon asymmetry of the universe. We then briefly review models of leptogenesis, where lepton number violation is used to generate a baryon asymmetry of the universe and hence the constraints on the Majorana neutrino mass is relaxed.

Introduction

The generation of the baryon asymmetry of the universe starting from a symmetric universe is one of the very challenging question in cosmology [1]. Sakharov [2] pointed out that for this purpose we need three conditions (A) *Baryon number violation*, (B) *C and CP violation*, and (C) *Departure from thermal equilibrium.* It was then realised that grand unified theories (GUTs) satisfies all these criterion [3, 4]. Because of the quark-lepton unification baryon number is violated in GUTs. Since fermions belong to chiral representation, C is maximally violated in GUTs. Violation of CP

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was the only crucial point, which had to be incorporated in these theories. However, it was not difficult to consider some of the couplings to be complex so that there exist tree level and one loop diagrams which could interfere to give us enough baryon asymmetry in the decays of the heavy gauge and higgs bosons. It was noticed that the masses of these heavy bosons are constrained by the out-of-equilibrium condition to be very high. However, from the gauge coupling unification also the masses of these particles were found to be around the same value. In fact, the proton decay constrains also require the masses of these particles to be around the same scale [4].

This was considered to be one of the major successes of GUTs that it can explain the baryon asymmetry of the universe. After several years it was realised that the chiral nature of the weak interaction also breaks the global baryon and lepton numbers in the standard model [5]. Since these classical global baryon and lepton number symmetries are broken by quantum effects due to the presence of the anomaly, these processes were found to be very weak at the zero temperature. But at finite temperature these baryon and lepton number violating interactions were found to be very strong in the presence of some static topological field configuration - sphalerons [6]. In fact, these interactions are so strong that in no time the particles and anti-particles attain their equilibrium distributions. As a result, since CPT is conserved and hence the masses of the particles and anti-particles are same, the number density of baryons becomes same as that of the anti-baryons and that wash out any primordial baryon asymmetry of the universe. which are generated during the GUT phase transition. This started renewed interest in the subject of the baryon asymmetry of the universe.

Now this problem takes two directions. First, how to generate a baryon asymmetry of the universe, and second, which are the interactions that can wash out the baryon asymmetry of the universe and what constrains they give us on the various parameters of the particle physics models. Since the electroweak anomalous processes breaks both the baryon and the lepton numbers, still conserving the $(B-L)$ quantum number, the baryon asymmetry of the universe is no longer independent of the lepton number violation of the universe [6, 7, 8, 9]. If there is very fast lepton number violation before the electroweak phase transition, then that can erase the $(B-L)$ asymmetry of the universe and hence the baryon asymmetry of the universe. On the other hand, if any lepton asymmetry is generated at some high temperature, that can get converted to a baryon asymmetry of the universe before and during the electroweak phase transition.

The first thing then comes to mind to make use of the baryon number violation of the standard model to generate a baryon asymmetry of the universe. There were several attempts towards this direction [10, 11]. However, in these models one needs to protect the generated baryon asymmetry after the phase transition, which requires the mass of the standard model doublet higgs boson to be substantially light [12]. The present experimental limit of 95 GeV on the mass of the higgs boson *almost* rule out all these possibilities. Then the most interesting scenario remains for the understanding of the baryon number of the universe is through lepton number violation [7, 13, 14, 15], which is also referred to as leptogenesis. In this scenario one generates a lepton asymmetry of the universe, which is the same as the $(B-L)$ asymmetry of the universe. This $(B-L)$ asymmetry of the universe then get converted to the baryon asymmetry of the universe during the period when the sphaleron fields maintain the baryon number violating interactions in equilibrium. On the other hand, if there is vary fast lepton number violation in the universe during this period, that can also wash out the baryon asymmetry of the universe [8, 16, 17, 18].

In this article I shall discuss the constraints on the left-handed Majorana neutrino mass, which comes from the survival of the baryon asymmetry of the universe. Since, in models of leptogenesis these constraints are weakened, we shall also discuss models of leptogenesis briefly. The limitations of these constraints will be mentioned.

Sphaleron processes in thermal equilibrium and relation between baryon and lepton numbers

Anomaly breaks any classical symmetry of the lagrangian at the classical level. So, all local gauge theories should be free of anomalies. However, there may be anomalies corresponding to any global current, *i.e.*, in the triangle loop while two gauge bosons would couple to two vertices, a global current will be associated to the third vertex. That will simply mean that such global symmetries of the classical lagrangian are broken through quantum effects.

In the standard model the chiral nature of the weak interaction makes the baryon and lepton number anomalous. If we associate the $SU(2)_L$ or the $U(1)_Y$ gauge bosons at the two vertices of a triangle diagram and associate a global current corresponding to baryon or lepton numbers at the third vertex, then sum over all the fermions in the standard model will give us non vanishing axial current [5]

$$\delta_\mu j^{\mu 5}_{(B+L)} = 6[\frac{\alpha_2}{8\pi} W_a^{\mu\nu} \tilde{W}_{a\mu\nu} + \frac{\alpha_1}{8\pi} Y^{\mu\nu} \tilde{Y}_{\mu\nu}]$$

which will break the $(B+L)$ symmetry. However, the anomaly corresponding to the baryon and lepton number are same and as a result there is no anomaly corresponding to the $(B-L)$ charge.

Because of the anomaly [5], baryon and lepton numbers are broken during the electroweak phase transition,

$$\Delta(B+L) = 2N_g \frac{\alpha_2}{8\pi} \int d^4x W_a^{\mu\nu} \tilde{W}_{a\mu\nu} = 2N_g \nu,$$

but their rate is very small at zero temperature, since they are suppressed by quantum tunnelling probability, $\exp[-\frac{2\pi}{\alpha_2}\nu]$, where ν is the Chern-Simmons number.

At finite temperature, however, it has been shown that there exists non-trivial static topological soliton configuration, called the sphalerons, which enhances the baryon number violating transition rate [6]. As a result, at finite temperature these baryon and lepton number violating processes are no longer suppressed by quantum tunneling factor, rather the suppression factor is now replaced by the Boltzmann factor

$$\exp[-\frac{V_0}{T}\nu]$$

where the potential or the free energy V_0 is related to the mass of the sphaleron field, which is about TeV. As a result, at temperatures between

$$10^{12}GeV > T > 10^2 GeV \qquad (1)$$

the sphaleron mediated baryon and lepton number violating processes are in equilibrium. For the simplest scenario of $\nu = 1$, the sphaleron induced processes are $\Delta B = \Delta L = 3$, given by,

$$|vac> \longrightarrow [u_L u_L d_L e_L^- + c_L c_L s_L \mu_L^- + t_L t_L b_L \tau_L^-]. \qquad (2)$$

These baryon and lepton number violating fast processes can, in general, wash out any pre-existing baryon or lepton number asymmetry of the universe. However, if there are any $(B-L)$ asymmetry of the universe, that will not be washed out. In fact, any $(B-L)$ asymmetry before the electroweak phase transition will get converted to a baryon and lepton asymmetry of the universe during this phase transition, which can be seen from an analysis of the chemical potential [9].

We consider all the particles to be ultrarelativistic, which is the case above the electroweak scale. At lower energies, a careful analysis has to include the mass corrections, but since they are anyway small we ignore them for our present discussion. The particle asymmetry, *i.e.* the difference between the number of particles (n_+) and the number of antiparticles (n_-) can be given in terms of the chemical potential of the particle species μ (for antiparticles the chemical potential is $-\mu$) as

$$n_+ - n_- = n_d \frac{gT^3}{6}\left(\frac{\mu}{T}\right), \qquad (3)$$

where $n_d = 2$ for bosons and $n_d = 1$ for fermions.

In the standard model the quarks and leptons transform under $SU(3)_C \times SU(2)_L \times U(1)_Y$ as,

$$\begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \sim (3,2,1/6), \quad u_{iR} \sim (3,1,2/3), \quad d_{iR} \sim (3,1,-1/3); \qquad (4)$$

$$\begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L \sim (1,2,-1/2), \quad e_{iR} \sim (1,1,-1). \tag{5}$$

where, $i = 1,2,3$ corresponds to three generations. In addition, the scalar sector consists of the usual Higgs doublet,

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (1,2,1/2) \tag{6}$$

which breaks the electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ down to $U(1)_{em}$. When these leptons interact with other particles in equilibrium, the chemical potentials get related by simple additive relations, and that will allow us to relate the lepton asymmetry n_L to the baryon asymmetry during the electroweak phase transition.

During the period $10^2 GeV < T < 10^{12} GeV$, the sphaleron [6] induced electroweak $B+L$ violating interaction arising due to the nonperturbative axial-vector anomaly [5] will be in equilibrium along with the other interactions. In Table 1, we present all other interactions and the corresponding relations between the chemical potentials. In the third column we give the chemical potential which we eliminate using the given relation. We start with chemical potentials of all the quarks $(\mu_{uL}, \mu_{dL}, \mu_{uR}, \mu_{dR})$; leptons $(\mu_{aL}, \mu_{\nu aL}, \mu_{aR}$, where $a = e, \mu, \tau)$; gauge bosons (μ_W for W^-, and 0 for all others); and the Higgs scalars (μ_-^ϕ, μ_0^ϕ).

Table 1. Relations among the chemical potentials

Interactions	μ relations	μ eliminated
$D_\mu \phi^\dagger D_\mu \phi$	$\mu_W = \mu_-^\phi + \mu_0^\phi$	μ_-^ϕ
$\overline{q_L} \gamma_\mu q_L W^\mu$	$\mu_{dL} = \mu_{uL} + \mu_W$	μ_{dL}
$\overline{l_L} \gamma_\mu l_L W^\mu$	$\mu_{iL} = \mu_{\nu iL} + \mu_W$	μ_{iL}
$\overline{q_L} u_R \phi^\dagger$	$\mu_{uR} = \mu_0 + \mu_{uL}$	μ_{uR}
$\overline{q_L} d_R \phi$	$\mu_{dR} = -\mu_0 + \mu_{dL}$	μ_{dR}
$\overline{l_{iL}} e_{iR} \phi$	$\mu_{iR} = -\mu_0 + \mu_{iL}$	μ_{iR}

The chemical potentials of the neutrinos always enter as a sum and for that reason we can consider it as one parameter. We can then express all the chemical potentials in terms of the following independent chemical potentials only,

$$\mu_0 = \mu_0^\phi; \quad \mu_W; \quad \mu_u = \mu_{uL}; \quad \mu = \sum_i \mu_i = \sum_i \mu_{\nu iL}. \tag{7}$$

We can further eliminate one of these four potentials by making use of the relation given by the sphaleron processes (2). Since the sphaleron interactions are in equilibrium, we can write down the following $B+L$ violating relation among the chemical potentials for three generations,

$$3\mu_u + 2\mu_W + \mu = 0. \tag{8}$$

We then express the baryon number, lepton numbers and the electric charge and the hypercharge number densities in terms of these independent chemical potentials,

$$\begin{aligned} B &= 12\mu_u + 6\mu_W && (9)\\ L_i &= 3\mu + 2\mu_W - \mu_0 && (10)\\ Q &= 24\mu_u + (12+2m)\mu_0 - (4+2m)\mu_W && (11)\\ Q_3 &= -(10+m)\mu_W && (12)\end{aligned}$$

where m is the number of Higgs doublets ϕ.

At temperatures above the electroweak phase transition, $T > T_c$, both Q and Q_3 must vanish. These conditions and the sphaleron induced $B-L$ conserving, $B+L$ violating condition can be expressed as

$$\begin{aligned} < Q >= 0 &\implies \mu_0 = \frac{-12}{6+m}\mu_u && (13)\\ < Q_3 >= 0 &\implies \mu_W = 0 && (14)\\ \text{Sphaleron transition} &\implies \mu_a = -5\mu_u && (15)\end{aligned}$$

Using these relations we can now write down the baryon number, lepton number, and their combinations in terms of the $B-L$ number density as,

$$\begin{aligned} B &= \frac{24+4m}{66+13m}\,(B-L) && (16)\\ L &= \frac{-42-9m}{66+13m}\,(B-L) && (17)\\ B+L &= \frac{-18-5m}{66+13m}\,(B-L) && (18)\end{aligned}$$

Below the critical temperature, Q should vanish since the universe is neutral with respect to all conserved charges. However, since $SU(2)_L$ is now broken we can consider $\mu_0^\phi = 0$ and $Q_3 \neq 0$. This gives us,

$$\begin{aligned} < Q >= 0 &\implies \mu_W = \frac{12}{2+m}\mu_u && (19)\\ \langle\phi\rangle \neq 0 &\implies \mu_0 = 0 && (20)\\ \text{Sphaleron transition} &\implies \mu_a = -3\mu_W - \frac{9}{2}\mu_u && (21)\end{aligned}$$

which then let us write the baryon and lepton numbers as some combinations of $B-L$ as

$$B = \frac{32+4m}{98+13m}\,(B-L) \tag{22}$$

$$L = \frac{-66 - 9m}{98 + 13m} \; (B - L) \tag{23}$$

$$B + L = \frac{-34 - 5m}{98 + 13m} \; (B - L) \tag{24}$$

Thus the baryon and lepton number asymmetry of the universe after the electroweak phase transition will depend on the primordial $(B - L)$ asymmetry of the universe. If there are very fast lepton number violation during the period (1), that would erase the $(B - L)$ asymmetry of the universe, and hence we will be left with a baryon symmetric universe at the end [8, 9]. On the other hand, if there is enough lepton asymmetry of the universe at some high temperature, then that will get converted to a baryon asymmetry of the universe [7, 9].

Before proceeding further, we shall briefly discuss what do we mean when we say that some interaction is fast and that will erase some asymmetry [1, 2, 20]. In equilibrium the number density of particles with non-zero charge Q would be same as the antiparticle number density since the expectation value of the conserved charge vanishes. A mathematical formulation of this statement reads that the expectation value of any conserved charge Q is given by,

$$< Q >= \frac{\mathrm{Tr}\left[Qe^{-\beta H}\right]}{\mathrm{Tr}\left[e^{-\beta H}\right]}$$

and since any conserved charge Q is odd while H is even under CPT transformation this expectation value vanishes. So for the generation of the baryon asymmetry of the universe we have to circumvent this theorem either by including nonzero chemical potential, or go away from equilibrium or violate CPT. In most of the popular models CPT conservation is assumed and one starts with vanishing chemical potential for all the fields which ensures that the entropy is maximum in chemical equilibrium. Then to generate the baryon asymmetry of the universe one needs to satisfy the out-of-equilibrium condition [1, 2, 20].

The requirement for the out-of-equilibrium condition may also be stated in a different way [1]. If we assume that the chemical potential associated with B is zero and CPT is conserved, then in thermal equilibrium the phase space density of baryons and antibaryons, given by $[1 + exp(\sqrt{p^2 + m^2}/kT)]^{-1}$ are identical and hence there cannot be any baryon asymmetry.

Whether a system is is equilibrium or not can be understood by solving the Boltzmann equations. But a crude way to put the out-of-equilibrium condition is to say that the universe expands faster than some interaction rate. For example, if some B-violating interaction is slower than the expansion rate of the universe, this interaction may not bring the distribution of baryons and antibaryons of the universe in equilibrium. In other words, before the chemical potentials of the two states gets equal, they move apart from each other. Thus we may state the out-of-equilibrium condition as

$$\Gamma < \sqrt{1.7 g_*} \frac{T^2}{M_P} \tag{25}$$

where, Γ is the interaction rate under discussion, g_* is the effective number of degrees of freedom available at that temperature T, and M_P is the Planck scale.

Bounds on Left-handed neutrino mass

In this section we shall discuss the constraints on the left-handed neutrino mass arising from the constraints of baryogenesis. If there is Majorana type interactions, which explains the masses of the neutrinos, then that gives a measure of lepton number violation. If this lepton number violation is too large before the electroweak phase transition is over, then that can erase all L asymmetry and hence B asymmetry of the universe.

The first attempt to constraint the neutrino mass was made in a fairly general framework [8]. In the standard model there is no lepton number violation. However, one can consider a higher dimensional effective operator which violates $(B-L)$, given by

$$L = \frac{2}{M} l_L l_L \phi\phi + h.c. \tag{26}$$

There is no origin of such interactions within the standard model. So one expects that some new interaction at some high energy will give us this effective interaction at low energy. The scale of the new interaction M, which gives us this interaction, is also the scale of lepton number violation in this scenario.

If this interaction is strong enough, it can bring the neutrinos in thermal equilibrium with the physical higgs scalars, which can wash out any lepton asymmetry of the universe. The survival of the baryon asymmetry of the universe will then require this interaction to be slower than the expansion rate of the universe,

$$\Gamma_{L\neq 0} \sim \frac{0.122}{\pi} \frac{T^3}{M^2} < 1.7\sqrt{g_*} \frac{T^2}{M_P} \qquad \text{at } T \sim 100 GeV \tag{27}$$

which gives a bound on the mass of the heavy scale to be, $M > 10^9 GeV$. When the higgs doublets ϕ acquires a vev, the higher dimensional operator will induce a Majorana mass of the left-handed neutrinos. This bound on the heavy scale M will then imply,

$$m_\nu < 50 keV.$$

In specific models one may give stronger bounds on the mass of the neutrinos [16]. In models with right handed neutrinos ($N_{Ri}, i = e, \mu, \tau$), the neutrino masses comes from the see-saw mechanism [21]. The lagrangian for the lepton sector containing the mass terms of the singlet right handed neutrinos N_i and the Yukawa couplings of these fields with the light leptons is given by,

$$\begin{aligned}
\mathcal{L}_{int} &= \sum_i M_i[\overline{(N_{Ri})^c} N_{Ri} + \overline{N_{Ri}} (N_{Ri})^c] \\
&+ \sum_{\alpha,i} h^*_{\alpha i}\, \overline{N_{Ri}}\, \phi^\dagger\, \ell_{L\,\alpha} + \sum_{\alpha,i} h_{\alpha i}\, \overline{\ell_{L\,\alpha}} \phi\, N_{Ri} \\
&+ \sum_{\alpha,i} h^*_{\alpha i}\, \overline{(\ell_{L\,\alpha})}^c\, \phi^\dagger\, (N_{Ri})^c + \sum_{\alpha,i} h_{\alpha i} \overline{(N_{Ri})^c}\, \phi\, (\ell_{L\,\alpha})^c
\end{aligned} \tag{28}$$

where $\phi^T = (-\overline{\phi^\circ}, \phi^-) \equiv (1, 2, -1/2)$ is the usual higgs doublet of the standard mode; $l_{L\alpha}$ are the light leptons, $h_{\alpha i}$ are the complex Yukawa couplings and α is the generation index. Without loss of generality we work in a basis in which the Majorana mass matrix of the right handed neutrinos is real and diagonal with eigenvalues M_i.

Once the higgs doublet ϕ acquires a vev, the masses of the neutrinos in the basis $[\nu_{L\alpha} \quad N_{Ri}]$ is given by,

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \tag{29}$$

where, $m \equiv h_{\alpha i} < \phi >$ and $M \equiv M_{ij}$ are 3×3 matrices. In the limit when all eigenvalues of M are much heavier than those of m, and the matrix M is not singular, this matrix may be block diagonalised. It then gives three heavy right handed Majorana neutrinos with masses $\sim M$ and the Majorana mass matrix of the left-handed neutrinos will be given by,

$$m_\nu = m \frac{1}{M} m^T. \tag{30}$$

In this scenario the see-saw masses of the left-handed neutrinos explain naturally why they are so light. In addition, the decay of the heavy right handed Majorana neutrinos can generate enough lepton asymmetry of the universe, which can then get converted to a baryon asymmetry of the universe, which we shall discuss in the next section.

The decay of N_{Ri} into a lepton and an antilepton,

$$\begin{aligned}
N_{Ri} &\to \ell_{jL} + \bar{\phi}, \\
&\to {\ell_{jL}}^c + \phi.
\end{aligned} \tag{31}$$

breaks lepton number. Since the lightest of the right handed neutrinos (say N_1) will decay at the end, this interaction (N_1 decay) should be slow enough so as not to erase the baryon asymmetry of the universe, which now implies

$$\frac{|h_{\alpha 1}|^2}{16\pi} M_1 < 1.7\sqrt{g_*}\frac{T^2}{M_P} \qquad \text{at } \ T = M_1 \tag{32}$$

which can then give a very strong bound [16] on the mass of the lightgest of the left-handed neutrinos to be

$$m_\nu < 4 \times 10^{-3} eV.$$

In models [24, 25, 26], where the left-handed neutrino mass is not related to any heavy neutrinos through see-saw mechanism, the abovementioned bounds may not be valid. In addition, there are several specific cases even within the framework of see-saw models, where these bounds are not applicable. These bounds are also not valid if some global U(1) symmetry is exactly conserved up to an electroweak anomaly [17]. Furthermore, in some very specific models where a baryon asymmetry of the universe is generated after the electroweak phase transition [22], or there are some extra baryon number carrying singlets which decays after the electroweak phase transition [23], it is possible to avoid all the bounds from constraints of survival of the baryon asymmetry of the universe. This issue will be discussed in another talk in this meeting [23] in details.

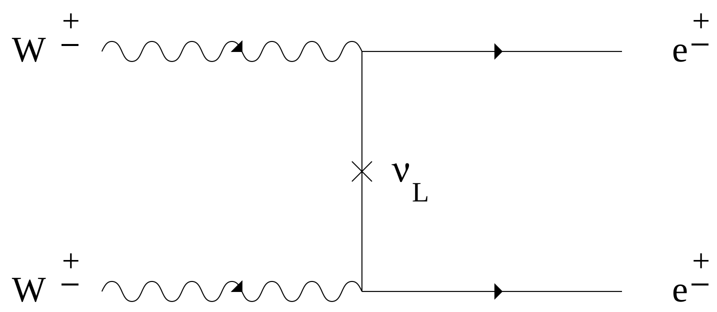

Figure 1. Lepton number violating processes $W^\pm + W^\pm \to e^\pm + e^\pm$ mediated by the left handed Majorana neutrinos.

Since some of the bounds derived indirectly from an bound on the scale of lepton number violation may be circumvented in some models, we now try to discuss the question if one can constrain the Majorana mass of the left-handed neutrinos directly. This is possible since during the electroweak phase transition, after the higgs doublets acquires a vacuum expectation value (vev) and the $SU(2)_L$ group is broken, there is no symmetry which can prevent the mass of the left-handed neutrinos. So if lepton number is broken before the electroweak symmetry, then as soon as ϕ acquires a vev the left-handed neutrinos will get a mass (m_ν). This can, in principle, induce very fast lepton number violating processes, which can wash out any primordial $(B-L)$ asymmetry and hence the baryon asymmetry of the universe.

There are several lepton number violating processes, which are active at the time of electroweak phase transition. The process,

$$W^+ + W^+ \to e_i^+ + e_j^+ \qquad \text{and} \qquad W^- + W^- \to e_i^- + e_j^- \tag{33}$$

mediated by a virtual left-handed neutrino exchange as shown in figure 1 is a lepton number violating interaction active during the electroweak phase transition. Here i and j are the generation indices. Depending on the physical mass (and also on the elements of the mass matrix) of the left-handed Majorana neutrinos these processes can wash out any baryon asymmetry between the time when the higgs acquires a vev and the $W^\pm$ freeze out, *i.e.*, between the energy scales 250 GeV and 80 GeV. The condition that these processes will be slower than the expansion rate of the universe,

$$\Gamma(WW \to e_i e_j) = \frac{\alpha_W^2 (m_\nu)_{ij}^2 T^3}{m_W^4} < 1.7\sqrt{g_*}\frac{T^2}{M_p} \qquad \text{at} \;\; T = M_W \tag{34}$$

gives a bound on the Majorana mass of the left-handed neutrinos to be,

$$(m_\nu)_{ij} < 20 keV. \tag{35}$$

This bound is on each and every element of the mass matrix and not on the physical states and independent on the existence of any right handed neutrinos.

In general, a Majorana particle can be described by a four component real field,

$$\Psi_M = \sqrt{\frac{m_\nu}{E_\nu}}[u_\nu(b_\nu + d_\nu^*)\mathrm{e}^{(-ip.x)} + v_\nu(b_\nu^* + d_\nu)\mathrm{e}^{(ip.x)}] \tag{36}$$

and hence the charged current containing a Majorana field,

$$j_\mu = \overline{\Psi}_l \gamma_\mu (1-\gamma_5)\Psi_M$$

will have a lepton number violating part. However, this lepton number violating contribution will always be suppressed by a factor (m_ν/E_ν) and hence the rate of such processes will be suppressed by a factor $(m_\nu/E_\nu)^2$. Thus even the decay of the $W^\pm$ to e and ν will have lepton number violation at a rate,

$$\Gamma(W \to e\nu) = \frac{\alpha_W}{4} \frac{m_\nu^2 M_W^2}{T^2(T^2 + M_W^2)^{1/2}}.$$

The survival of baryon asymmetry of the universe after the electroweak phase transition again requires this process to be slow enough,

$$\Gamma(W \to e\nu) < H.$$

This translates to a bound on the Majorana mass of the left handed neutrino,

$$m_\nu < 30 \quad \text{keV}. \tag{37}$$

Similarly, the decay of the higgs doublet to an electron and an anti-neutrino will also have lepton number violating contribution, but they will be suppressed by the Yukawa coupling constants and cannot give stronger bounds. Similarly scattering processes involving the higgs, like $\phi + \phi \to l_i + l_j$ (mediated by a virtual left-handed neutrino) will contribute to the evolution of the lepton number asymmtry of the universe, but it will be much suppressed compared to the charged current interactions and hence cannot give stronger bound to the Majorana mass of the left-handed neutrinos.

Bounds on neutrino mass in models of Leptogenesis

In this section we shall discuss two scenarios of leptogenesis. In the first, one starts with the standard model and add to it three right handed neutrinos [7, 13, 14, 15]. In total this will add 12 degrees of freedom. In the other scenario [26] one adds two complex $SU(2)_L$ triplet higgs scalars ($\xi_a \equiv (1,3,-1); a = 1,2$), which also adds in total 12 degrees of freedom. At present these two scenarios are indistinguishable, except that the particle contents are different. In the left-right symmetric models [27, 28] both these scenarios are present, which we shall not discuss here. The CP violation required to generate a lepton asymmetry of the universe are different for the two scenarios.

In the standard model neutrinos are massless. In models with right handed neutrinos the left-handed neutrinos acquire a Majorana mass through see-saw mechanism as we mentioned earlier. In the triplet higgs scenario the *vev*s of the triplet higgses can give small Majorana masses to the neutrinos through the interaction

$$f_{ij}[\xi^0 \nu_i \nu_j + \xi^+ (\nu_i l_j + l_i \nu_j)/\sqrt{2} + \xi^{++} l_i l_j] + h.c. \tag{38}$$

If the triplet higgs acquires a *vev* and break lepton number spontaneously [29], then there will be Majorons in the problem. Then from the precision measurements of the $Z-$width that model is ruled out [30]. However, in a variant of this model [26] lepton number is broken explicitly through an interaction of the triplet with the higgs doublet

$$V = \mu(\bar{\xi}^0 \phi^0 \phi^0 + \sqrt{2}\xi^- \phi^+ \phi^0 + \xi^{--} \phi^+ \phi^+) + h.c. \tag{39}$$

Let $\langle \phi^0 \rangle = v$ and $\langle \xi^0 \rangle = u$, then the conditions for the minimum of the potential relates the *vev* of the two scalars by

$$u \simeq \frac{-\mu v^2}{M^2}, \tag{40}$$

where M is the mass of the triplet higgs scalar. This is analogous to the usual seesaw mechanism for obtaining small Majorana neutrino masses, except that here we do not have any right-handed neutrinos.

Another way of handling the heavy Higgs triplet is to integrate it out. From the couplings of the triplet scalar, we obtain the effective nonrenormalizable term

$$\frac{-f_{ij}\mu}{M^2}[\phi^0 \phi^0 \nu_i \nu_j - \phi^+ \phi^0 (\nu_i l_j + l_i \nu_j) + \phi^+ \phi^+ l_i l_j] + h.c. \tag{41}$$

The reduced Higgs potential involving only the doublet higgs scalar is

$$V = m^2 \Phi^\dagger \Phi + \frac{1}{2}\left(\lambda_1 - \frac{2\mu^2}{M^2}\right)(\Phi^\dagger \Phi)^2, \tag{42}$$

where m is the mass of the higgs doublet. The last term comes from the exchange of ξ. As ϕ^0 acquires a nonzero vacuum expectation value v, we obtain Eq. (40) as we should, and the neutrino mass matrix becomes

$$-2 f_{ij} \mu v^2 / M^2 = 2 f_{ij} u$$

.

In models with right handed neutrinos lepton number is violated when the right handed Majorana neutrinos decay as we discussed earlier [equation (31)]. The out-of-equilibrium condition is satisfied if the masses of the lightest of the right handed neutrinos satisfy $M_1 > 10^7$ GeV, which is obtained by solving the Boltzmann equations. There are two sources of CP violation in this scenario :

(*i*) vertex type diagrams which interferes with the tree level diagram given by figure 2.

(*ii*) self energy diagrams could interfere with the tree level diagrams to produce CP violation analogous to CP violation in the $K^\circ\overline{K^\circ}$ oscillation as shown in figure 3. This type of CP violation has several interesting features, which will be discussed in another talk at this meeting [31].

In the case of self energy type CP violation the amount of lepton asymmetry becomes large for a small mass difference between the two right-handed heavy neutrinos (M_1 and M_2), and is given by [15]:

$$\delta = \frac{1}{8\pi}\mathcal{C}\frac{M_1 M_2}{M_2^2 - M_1^2} \tag{43}$$

where

$$\mathcal{C} = -\mathrm{Im}\left[\sum_\alpha (h^*_{\alpha 1} h_{\alpha 2}) \sum_\beta h^*_{\beta 1} h_{\beta 2})\right]\left(\frac{1}{\sum_\alpha |h_{\alpha 1}|^2} + \frac{1}{\sum_\alpha |h_{\alpha 2}|^2}\right) \tag{44}$$

This contribution becomes significant when the two mass eigenvalues are close to each other. It indicates a resonance like behaviour of the asymmetry if the two mass eigenvalues are nearly degenerate. For very large values of the mass difference this contribution becomes similar to the vertex correction. These two contributions add up to produce the final lepton asymmetry of the universe.

In the triplet higgs scenario lepton number violation comes from the decays of the triplet higgs ξ_a. Consider the decays of ξ_a^{++},

$$\xi_a^{++} \to \left\{ \begin{array}{ll} l_i^+ l_j^+ & (L = -2) \\ \phi^+ \phi^+ & (L = 0) \end{array} \right. \tag{45}$$

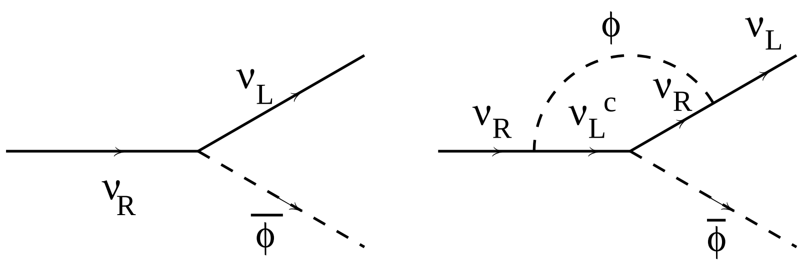

Figure 2. Tree and one loop vertex correction diagrams contributing to the generation of lepton asymmetry in models with right handed neutrinos

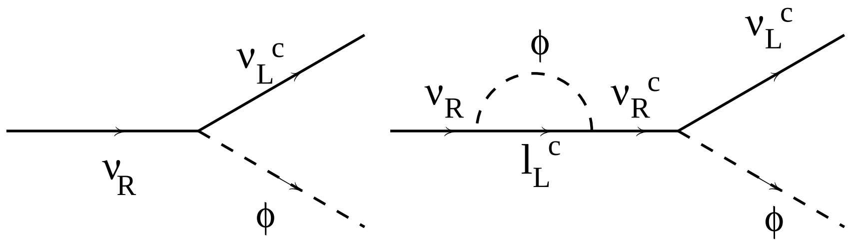

Figure 3. Tree and one loop self energy diagrams contributing to the generation of lepton asymmetry in models with right handed neutrinos

The coexistence of the above two types of final states indicates the non-conservation of lepton number. On the other hand, any lepton asymmetry generated by ξ_a^{++} would be neutralized by the decays of ξ_a^{--}, unless CP conservation is also violated and the decays are out of thermal equilibrium in the early universe. In this case there are no vertex corrections which can introduce CP violation. The only source of CP violation is the self energy diagrams of figure 4.

If there is only one ξ, then the relative phase between any f_{ij} and μ can be chosen real. Hence a lepton asymmetry cannot be generated. With two ξ's, even if there is only one lepton family, one relative phase must remain. As for the possible relative phases among the f_{ij}'s, they cannot generate a lepton asymmetry because they all refer to final states of the same lepton number.

In the presence of the one loop diagram, the mass matrix ${M_a}^2$ and M_a^{*2} becomes different. This implies that the rate of $\xi_b \to \xi_a$ no longer remains to be same as $\xi_b^* \to \xi_a^*$. Since by CPT theorem $\xi_b^* \to \xi_a^* \equiv \xi_a \to \xi_b$, what it means is that now

$$\Gamma[\xi_a \to \xi_b] \neq \Gamma[\xi_b \to \xi_a].$$

This is a different kind of CP violation compared to the CP violation in models with right handed neutrinos. If we consider that the ξ_2 is heavier than ξ_1, then after ξ_2 decayed out the decay of ξ_1 will generate an lepton

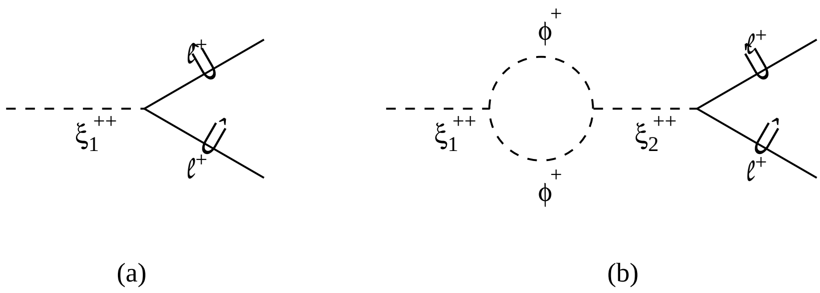

Figure 4. The decay of $\xi_1^{++} \to l^+ l^+$ at tree level (a) and in one-loop order (b). A lepton asymmetry is generated by their interference in the triplet higgs model for neutrino masses.

asymmetry given by,

$$\delta \simeq \frac{Im\left[\mu_1 \mu_2^* \sum_{k,l} f_{1kl} f_{2kl}^*\right]}{8\pi^2 (M_1^2 - M_2^2)} \left[\frac{M_1}{\Gamma_1}\right]. \tag{46}$$

In any of the above two scenarios with right handed neutrinos or with triplet higgs, the lepton number evolves with time following the Boltzmann equation in similar way. The lepton asymmetry n_L thus generated through the CP violation η (where η is the amount of CP violation in the model under consideration), would evolve with time following the Boltzmann equation,

$$\frac{\mathrm{d}n_l}{\mathrm{d}t} + 3Hn_l = \eta \Gamma_h [n_h - n_h^{eq}] - \left(\frac{n_l}{n_\gamma}\right) n_h^{eq} \Gamma_h - 2n_\gamma n_l \langle \sigma |v| \rangle \tag{47}$$

where Γ_h is the thermally averaged decay rate of the heavy particle (whose number density is n_h). The second term on the left side comes from the expansion of the universe, where $H = 1.66 g_*^{1/2} (T^2/M_{Pl})$ is the Hubble constant. n_γ is the photon density and the term $\langle \sigma |v| \rangle$ describes the thermally averaged lepton number violating scattering cross section. The density of the heavy particle satisfies the Boltzmann equation,

$$\frac{\mathrm{d}n_h}{\mathrm{d}t} + 3Hn_h = -\Gamma_h (n_h - n_h^{eq}) \tag{48}$$

It is now convenient to use the dimensionless variable $x = M_h/T$ as well as the particle density per entropy density $Y_i = n_i/s$, and the relation $t = x^2/2H(x=1)$. We also define the parameter $K \equiv \Gamma_h(x=1)/H(x=1)$ as a measure of the deviation from equilibrium. For $K << 1$ at $T \sim M_h$, the system is far from equilibrium; hence the last two terms responsible for the depletion of n_l would be negligible. With these simplifications and the above redefinitions, the Boltzmann equations effectively read:

$$\frac{\mathrm{d}Y_l}{\mathrm{d}x} = (Y_h - Y_h^{eq})\eta K x, \quad \frac{\mathrm{d}Y_h}{\mathrm{d}x} = -(Y_h - Y_h^{eq})Kx. \tag{49}$$

In this limit $K << 1$, it is not difficult to obtain an asymptotic solution for n_l. Although the decay rate of ψ_h is not fast enough to bring the number density n_h to its equilibrium density, it is a good approximation to assume that the universe never goes far away from equilibrium. In other words, we can assume $\mathrm{d}(Y_h - Y_h^{eq})/\mathrm{d}x = 0$ to get an asymptotic value for Y_l, given by $Y_l = n_l/s = \eta/g_*$. However, if $K > 1$, the terms which deplete n_l dominate for some time and the lepton number density reaches its new asymptotic value, which is lower than the value it reaches in the out-of-equilibrium case. In this case although it is difficult to get an analytic solution of the Boltzmann equations, it is possible to get an approximate suppression factor, which is proportional to K.

In our earlier discussions we have considered the out-of-equilibrium condition to be

$$K < 1,$$

which gave us all the bounds on the Majorana masses of the left-handed neutrinos. But as we can see from the above discussions, in models of leptogenesis, where a lepton number violation is associated with a CP violation, the lepton number is not washed out too fast. While in other models fast lepton number wash out any preexisting asymmetry exponentially, in this case the depletion is only linearly. As a result, in models of leptogenesis fast lepton number violation may not wash out the primordial lepton asymmetry of the universe completely and the baryon asymmetry of the universe may still be present after the lepton number violating interaction goes out of equilibrium.

Summary

Any fast lepton number violation in the universe can, in principle, wash out baryon asymmetry of the universe. The survival of the baryon asymmetry of the universe thus gives constraints on the left-handed Majorana neutrino masses. There are constraints which are dependent on models of the neutrino masses, but it is also possible to give constraints from the lepton number violating interactions of $W^{\pm}$ due to the Majorana masses of the left-handed neutrinos directly. Although these constraints are independent of any models, these constraints also gets weakened in models of leptogenesis.

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References

[1] Kolb E W and Turner M S 1989, *The Early Universe* (Addison-Wesley, Reading, MA).

[2] Sakharov A D 1967, *Pis'ma Zh. Eksp. Teor. Fiz.* **5** 32.

[3] Yoshimura M 1978, *Phys. Rev. Lett.* **41** 281; E 1979: *Phys. Rev. Lett.* **42** 7461;

[4] Mohapatra R N 1992, *Unification and Supersymmetry* (Springer-Verlag);
Masiero A 1984, in *Grand Unification with and without Supersymmetry and Cosmological Implications*, International School for Advanced Studies Lecture Series No. 2, (World Scientific) 1;
Zee A 1982, (ed.) *Unity of Forces in the Universe* **1** (World Scientific);
Langacker P 1981, Phys. Rep. **72** 185.

[5] 't Hooft G 1976, *Phys. Rev. Lett.* **37** 8.

[6] Kuzmin V A, Rubakov V A and Shaposhnikov M E 1985, *Phys. Lett.* **B 155** 36.

[7] Fukugita M and Yanagida T 1986, *Phys. Lett.* **B 174** 45.

[8] Fukugita M and Yanagida T 1990, *Phys. Rev.* **D 42** 1285;
Barr S M and Nelson A E 1991, *Phys. Lett.* **B 246** 141.

[9] Khlebnikov S Yu and Shaposhnikov M E 1988, *Nucl. Phys.* **B 308** 885;
Harvey J A and Turner M S 1990, *Phys. Rev.* **D 42** 3344.

[10] Shaposhnikov M E 1987, *Nucl. Phys.* **B 287** 757;
Shaposhnikov M E 1988, *Nucl. Phys.* **B 299** 797;
Dolgov A D 1992, *Phys. Rep.* **222** 309;
Rubakov V A and M.E. Shaposhnikov 1996, *Usp. Fiz. Nauk* **166** 493; 1996 *Phys. Usp.* **39** 461.

[11] Shaposnikov M E 1987, *Nucl. Phys.* **B 287** 757;
Shaposhnikov M E 1988, **B 299** 797;
Turok N and Zadrozny J 1990, *Phys. Rev. Lett.* **65** 2331;
Kazarian A, Kuzmin S and Shaposnikov M E 1992, *Phys. Lett.* **B 276** 131.
McLerran L, Shaposnikov M E, Turok N and Voloshin M 1991, *Phys. Lett.* **B 256** 451.

[12] Bochkarev A I, Kuzmin S V and Shaposnikov M E 1990, *Phys. Lett.* **244 B** 275;
Dine M, Huet P and Singleton Jr R 1992, *Nucl. Phys.* **B 375** 625;
Shaposnikov M E 1993, *Phys. Lett.* **316 B** 112.

[13] Langacker P, Peccei R D and Yanagida T 1986, *Mod. Phys. Lett.* **A 1** 541;
Luty M A 1992, *Phys. Rev.* **D 45** 445;
Mohapatra R N and Zhang X 1992, *Phys. Rev.* **D 45** 5331;
Enqvist K and Vilja I 1993, *Phys. Lett.* **B 299** 281;
Murayama H, Suzuki H, Yanagida T and Yokoyama J 1993, *Phys. Rev. Lett.* **70** 1912;
Acker A, Kikuchi H, Ma E and Sarkar U 1993, *Phys. Rev.* **D 48** 5006;
Murayama H, Suzuki H, Yanagida T and Yokoyama J 1993, *Phys. Rev. Lett.* **70** 1912;
O'Donnell P J and Sarkar U 1994, *Phys. Rev.* **D 49** 2118;
Buchmüller W and Plümacher M 1996, *Phys. Lett.* **B 389** 73;
Ganguly A, Parikh J C and Sarkar U 1996, *Phys. Lett.* **B 385** 175;
Plümacher M 1997, *Z. Phys.* **C 74** 549;
Faridani J, Lola S, O'Donnell P J and Sarkar U 1998, hep-ph/9804261.

[14] Ignatev A Yu, Kuzmin V A and Shaposhnikov M E 1979, *JETP Lett.* **30** 688;
Botella F J and Roldan J 1991, *Phys. Rev.* **D 44** 966.
Liu J and Segre G 1993, *Phys. Rev.* **D 48** 4609;
Flanz M, Paschos E A, and Sarkar U 1995, *Phys. Lett.* **B 345** 248;
Covi L, Roulet E and Vissani F 1996, *Phys. Lett.* **B 384** 169;
Pilaftsis A 1997, *Phys. Rev.* **D 56** 5431;
Covi L and Roulet E 1997, *Phys. Lett.* **B 399** 113;
Covi L, Roulet E and Vissani F 1998, *Phys. Lett.* **B 424** 101;
Buchmüller W and Plümacher M 1997, hep-ph/9710460 (revised);
Flanz M and Paschos E A 1998, hep-ph/9805427.

[15] Flanz M, Paschos E A, Sarkar U and Weiss J 1996, *Phys. Lett.* **B 389** 693.

[16] Fischler W, Giudice G F, Leigh R G and Paban S 1991, *Phys. Lett.* **258 B** 45;
Buchmüller W and Yanagida T 1993, *Phys. Lett.* **302 B** 240.

[17] Dreiner H and Ross G G 1993, *Nucl. Phys.* **B 410** 188;
Ilakovac A and Pilaftsis A 1995, *Nucl. Phys.* **B 437** 491.

[18] Sarkar U 1997, *Phys. Lett.* **B 390** 97.

[19] Campbell B A, Davidson S, Ellis J and Olive K A 1991, *Phys. Lett.* **B 256** 457.

[20] Fry J N, Olive K A and Turner M S 1980, *Phys. Rev. Lett.* **45** 2074; 1980 *Phys. Rev.* **D 22** 2953; 1980 *Phys. Rev.* **D 22** 2977;
Kolb E W and Wolfram S 1980, *Nucl. Phys.* **B 172** 224.

[21] Gell-Mann M, Ramond P and Slansky R 1979, in *Supergravity*, Proceedings of the Workshop, Stony Brook, New York, 1979, ed. by P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam);
Yanagida T 1979, in *Proc of the Workshop on Unified Theories and Baryon*

Number in the Universe, Tsukuba, Japan, edited by A. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba).

[22] Masiero A and Mohapatra R N 1981, *Phys. Lett.* **B 103** 343.

[23] Davidson S 1998, *Proc. Int. Symp. on Lepton and Baryon number Violation, Trento*; 1997 *Phys. Lett.* **B 391** 287.

[24] Nandi S and Sarkar U 1986, *Phys. Rev. Lett.* **56** 564;
Joshipura A S and Sarkar U 1986, *Phys. Rev. Lett.* **57** 33;
Masiero A, Nanopoulos D V and Sanda A I 1986, *Phys. Rev. Lett.* **57** 663;
Mann R B and Sarkar U 1988, *Int. Jour. Mod. Phys.* **A 3** 2165;

[25] Farhi E and Susskind L 1979, *Phys. Rev.* **D20** 3404;
Dimopoulos S 1980, *Nucl. Phys.* **B 168** 69;
Zee A 1980, *Phys. Lett.* **93 B** 389;
Wolfenstein L 1980, *Nucl. Phys.* **B175** 93;
Nussinov S 1985, *Phys. Lett.* **165 B** 55.

[26] Ma E and Sarkar U 1998, *Phys. Rev. Lett.* **80** 5716.

[27] Pati J C and Salam A 1974, *Phys. Rev.* **D10** 275;
Mohapatra R N and Pati J C 1975, *Phys. Rev.* **D11** 566;
Mohapatra R N and Senjanović G 1980, *Phys. Rev. Lett.* **44** 912;
Marshak R E and Mohapatra R N 1980, *Phys. Rev. Lett.* **44** 1316.

[28] Pati J C, Salam A and Sarkar U 1983, *Phys. Lett.* **133 B** 330.

[29] Gelmini G B and Roncadelli M 1981, *Phys. Lett.* **99B** 411.

[30] Renton P B 1997, *Int. J. Mod. Phys.* **A12** 4109.

[31] Covi L 1998, *Proc. Int. Symp. on Lepton and Baryon number Violation, Trento*