Fermion masses and gauge mediated supersymmetry breaking from a single U(1)

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We present a supersymmetric model of flavor. A single U(1) gauge group is responsible for both generating the flavor spectrum and communicating supersymmetry breaking to the visible sector. The problem of flavor changing neutral currents is overcome, in part using an ''effective supersymmetry'' spectrum among the squarks, with the first two generations very heavy. All masses are generated dynamically and the theory is completely renormalizable. The model contains a simple Froggatt-Nielsen sector and communicates supersymmetry breaking via gauge mediation without requiring a separate messenger sector. By forcing the theory to be consistent with SU(5) grand unification, the model predicts a large $\tan\beta$ and a massless up quark. While respecting the experimental bounds on *CP* violation in the *K* system, the model leads to a large enhancement of *CP* violation in B-$\overline{B}$ mixing as well as in *B* decay amplitudes. [S0556-2821(99)06713-2]

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I. INTRODUCTION

Small dimensionless numbers in physics should have a known dynamical origin [1]. However, nature contains a number of unexplained, seemingly fundamental small quantities, such as the ratio between the weak scale and the Planck scale (M_w/M_p) and the ratios of known fermion masses to the weak scale (M_f/M_w). The former is subject to large radiative corrections in the standard model (SM). But the hierarchy $M_w \ll M_p$ could be explained by dynamically broken supersymmetry with superpartner masses near the weak scale and a superpartner spectrum which satisfies experimental constraints on flavor changing neutral currents (FCNC) and *CP* violation [2]. By contrast, the fermion masses are protected by an approximate chiral symmetry. However, the SM requires tiny dimensionless parameters to reproduce the measured spectrum. These parameters could be produced dynamically by the spontaneous breaking of a flavor symmetry. A complete model would successfully predict the entire spectrum of scalars and fermions with a Lagrangian that only contained coupling constants of order unity. In this article, we present a model of supersymmetry and flavor which is renormalizable and natural, and avoids excessive FCNC. All mass scales are generated dynamically from the fundamental scale of supersymmetry breaking.

One way of mediating supersymmetry breaking to the observable sector is through gauge interactions [3]. In some of the first complete models of gauge mediated supersymmetry breaking (GMSB), a new gauge group, $U(1)_{mess}$, couples to both a dynamical supersymmetry breaking (DSB) sector and a ''messenger'' sector to which supersymmetry breaking is communicated via loop effects [4,5]. The messenger sector consists of superfields that are vector-like with respect to the SM gauge group (G_{sm}) and other superfields that are G_{sm} singlets. At least one G_{sm} singlet has a non-zero vacuum expectation value (VEV) with both scalar and auxiliary components, which in turn give supersymmetric and non-supersymmetric masses respectively to the vector-like fields. Squarks, sleptons and gauginos receive supersymmetry breaking masses from loop corrections involving the messenger sector and SM gauge fields. The mass contributions come from gauge interactions and are therefore flavor independent. Hence, the three generations of scalars are very nearly degenerate, naturally suppressing unwanted contributions to FCNC. Efforts to improve this scenario have been made in the last few years, including attempts to remove the messenger sector and allow the DSB sector to carry G_{sm} quantum numbers [6].

The most successful models of flavor are based on a mechanism developed by Froggatt and Nielsen in the late 1970's [7]. In their original models, the small Yukawa couplings of the SM are forbidden by an additional (gauged)

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$U(1)_F$ symmetry. Quarks and leptons instead couple to Froggatt-Nielsen (FN) fields (heavy fermions in vector-like representations of G_{sm}) and scalar flavons, ϕ (G_{sm} singlets). Non-zero flavon VEVs, $\langle \phi \rangle \ll M_F$ (where M_F is the mass of the FN fields), break the $U(1)_F$ and cause mixing between the heavy and light fermions. This produces Yukawa couplings in the low energy effective theory proportional to the small ratio $\epsilon \sim \langle \phi \rangle / M_F$ to some power n. Here, n depends on the charges of the relevant fermions. A clever choice of charges can produce the correct quark and lepton masses and quark mixing angles, all with couplings of order unity.

These models of fermion masses and GMSB share a number of significant features. Both make use of an additional gauged U(1) symmetry which is spontaneously broken, both contain heavy vector-like quarks and leptons and both contain fields that are singlets under G_{sm}. These similarities are striking and compel one to ask if these two mechanisms can be incorporated efficiently into the same model.[1] There are, however, major differences between the two mechanisms. The biggest difference comes from the fact that in the FN mechanism the vector-like fields and some of the SM fields are charged under the U(1) symmetry. If the same were true in GMSB, the squarks would not, in general, be degenerate. However, large contributions to FCNC and *CP* violation can be suppressed if the first two generations of squarks are very heavy, as in ''effective supersymmetry'' [10]. If the first two generations carry U(1) charges, their scalar components would be heavy due to loop effects, while their fermion masses would be suppressed. Models of this kind have been built with the U(1) anomalies canceled at a high scale by the Green-Schwarz mechanism [9,11].

In this article, we present a model that dynamically generates both fermion and scalar masses using a single gauged U(1) which is non-anomalous. In doing so, we employ a modified version of the FN mechanism. We produce the small ratio $\epsilon \sim \langle \phi \rangle / M_F$ in a similar fashion. However, the range of small parameters comes predominantly from the use of flavons with different VEVs producing different ratios as opposed to different powers of the same ratio. This method requires fewer FN fields (at the cost of requiring more flavons), allowing us to avoid a Landau pole in α_s below $M_{\rm GUT}$. While requiring U(1) charge assignments to be consistent with SU(5), we are able to cancel all gauge anomalies, and we are able to find reasonable fermion mass matrices with fundamental coupling constants of order unity. The spectrum includes a massless up quark, a viable solution to the strong *CP* problem.

The paper is laid out as follows: Sec. II describes the overall design of the model, the mass spectrum of the scalars and the restrictions on the U(1) charges required for this spectrum. Section III describes the fermion mass matrices allowed within these restrictions. Section IV describes the contributions to FCNC and shows that they fall within experimental bounds. Section V describes some interesting cosmological effects of the model, and Sec. VI concludes the paper. The Appendix shows why squarks cannot be degenerate in this approach.

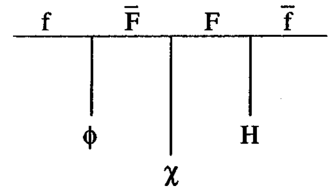


FIG. 1. Source of f-$\overline{f}$ mixing.

II. OVERVIEW

In this section, we describe the overall structure of the model.

A. Supersymmetry breaking

The highest scale defined in our model is the one at which supersymmetry breaks. This breaking occurs in the DSB sector at

$$\Lambda_{DSB} \sim 10^3 - 10^4 \ \ \text{TeV}. \tag{1}$$

This scale is generated dynamically via non-perturbative effects. Because there are currently many types of models in which supersymmetry is known or believed to be broken dynamically [12,13], and because we have very few requirements of this sector, we will leave it largely unspecified. However, the sector must contain a global U(1) symmetry which can be identified with a $U(1)_{mess}$ gauge symmetry that communicates supersymmetry breaking to the rest of the model. Once the DSB sector is integrated out, all lower scales will be generated dynamically through radiative effects.

B. Flavor and the messengers of supersymmetry breaking

In order to naturally produce the small fermion masses of the SM, our model contains Froggatt-Nielsen fields which are in vector-like representations of G_{sm}. A $U(1)_F$ gauge symmetry forbids most of the SM Yukawa couplings. The SM fields[2] f instead couple to the FN fields $(F,\overline{F})$ and flavons (χ,ϕ) in the superpotential (heuristically) as

$$W \sim \chi F \overline{F} + \phi f \overline{F} + H F f, \tag{2}$$

where H is a Higgs superfield. The scalar VEV of χ produces a mass term for the FN fields. If ϕ has a scalar component with a VEV such that $\langle \phi \rangle \ll \langle \chi \rangle$, then the low energy description of this theory will contain the superpotential term $\sim (\langle \phi \rangle / \langle \chi \rangle) H f \overline{f}$ (see Fig. 1). Thus a small coupling is pro-

[1]These similarities were first noted by Arkani-Hamed *et al.* [8]. In their article, they indicate some of the problems with identifying the two sectors. These and other problems are addressed in this paper.

[2]When referring to ''SM fields'' we mean superfields which contain the standard model fields and their superpartners.

duced dynamically from coupling constants of order unity. Different small Yukawa couplings can be produced by flavons with different VEVs. The $U(1)_F$ charges are chosen so as to produce fermion masses and mixing angles that mimic those experimentally measured.[3]

The DSB sector will also have fields charged under $U(1)_F$. All other matter is assumed to couple to the DSB sector only via the $U(1)_F$. Fields carrying this charge will receive contributions to their scalar masses at two loops. By giving the first two generations non-zero flavor charge, we can produce the effective supersymmetry spectrum [10]. The uncharged fields will be lighter and receive their masses at one or two loops below Λ_{DSB} (see Sec. II D). The large masses of the first two generations adequately suppress unwanted contributions to FCNC and *CP* violation (Sec. IV).

C. Flavor symmetry breaking

We choose Froggatt-Nielsen fields that are vector-like under G_{sm} and chiral under $U(1)_F$. Their masses at the tree level will be proportional to flavon VEVs which break the flavor symmetry. This symmetry breaking is due in part to a Fayet-Iliopoulos (FI) term [15], ξ^2, which appears in the $U(1)_F$ D-term:

$$\frac{g_F^2}{2}\left[\xi^2+\sum_i q_i|\psi_i|^2\right]^2 \tag{3}$$

where g_F is the gauge coupling and q_i are the $U(1)_F$ charges. The fields ψ_i represent all charged fields, including both trivial and non-trivial representations of G_{sm}. Provided that $\Sigma_i q_i$ vanishes, which is necessary for anomaly cancellation, the FI term only receives finite renormalization proportional to supersymmetry breaking effects. We assume that the fundamental FI term vanishes. Then the effective ξ depends on the DSB spectrum, and is generally an order of magnitude below Λ_{DSB}.

At two loops, every scalar with a non-zero q_i receives a supersymmetry breaking mass squared proportional to its charge squared [4,5].[4] Specifically, the contribution to the effective potential is $\tilde{m}^2\Sigma_i q_i^2|\psi_i|^2$, where the DSB sector again determines the exact value of $\tilde{m}^2$. Its magnitude will generally be two orders of magnitude below ξ^2. Thus, after integrating out the DSB sector, the full effective potential looks like

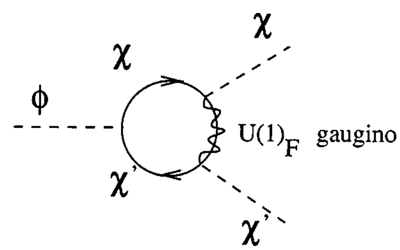
$$V_{eff}=\left|\frac{\partial W}{\partial\psi_i}\right|^2+\{G_{sm}\text{D-terms}\}+\frac{g_F^2}{2}\left[\xi^2+\sum_i q_i|\psi_i|^2\right]^2 + \tilde{m}^2\sum_i q_i^2|\psi_i|^2+\cdots \tag{4}$$

[3]Our model is "notationally" similar but significantly different from another old and interesting approach to flavor by Dimopoulos [14].

[4]We assume there are no direct contact interactions between the DSB sector and the visible sector.

where the ellipsis represents higher dimension supersymmetry breaking terms. The $U(1)_F$ D-term has a large number of flat directions. The parameter $\tilde{m}^2$ comes from the DSB and may have either sign. As we will see in Sec. II E, the squared masses of the third generation and Higgs scalars come from loop corrections which depend on $\tilde{m}^2$. We find we must have $\tilde{m}^2<0$ to keep the squark masses positive. This choice of sign introduces runaway flat directions into Eq. (4). These are curbed by the higher dimensional supersymmetry breaking terms that we have ignored and by superpotential interactions. We will choose a superpotential and a local minimum that allows us to neglect the higher dimension terms.

How can we generate the appropriate flavon VEV hierarchy? One approach is to give VEVs only to χ fields at the tree level. The ϕ flavons receive VEVs at one or more loops. Assume for instance that the two flavons χ and χ' have VEVs. The superpotential interaction $\chi\chi'\phi$ gives a VEV to the flavon ϕ via the diagram (solid and dashed lines represent fermion and scalar fields respectively)



Once ϕ has a VEV, some other flavon ϕ' may receive a VEV by means of a similar diagram if it appears in the superpotential interaction $\phi\phi\phi'$. Such a technique produces a hierarchy of VEVs. In the above case, for instance, $\langle\phi\rangle$ and $\langle\phi'\rangle$ are respectively one loop and three loop factors smaller than $\langle\chi\rangle$.

Generating the hierarchy of VEVs requires that we assign charges to the flavons that allow the required superpotential interactions. It is also important to prevent any field that transforms non-trivially under SU(5) from acquiring a VEV. Finally, additional flavons must be added to the model in order to cancel the $U(1)_F$ and $U(1)_F^3$ anomalies. Preliminary calculations have shown that the above approach should yield a viable scalar potential.

D. Mass generation: Scalars

As we have seen, all $U(1)_F$ charged scalars have masses of at least order $\tilde{m}$. Uncharged scalars receive supersymmetry breaking contributions from a number of different sources. Fields that transform non-trivially under G_{sm} receive contributions from two loop diagrams in the low energy theory (below Λ_{DSB}). Drawing from the results of Poppitz and Trivedi [16], we find that the leading contribution to the mass of an uncharged scalar at two loops is (up to a group theory factor)

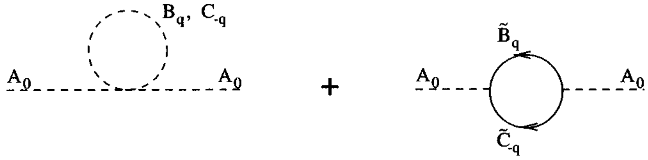


FIG. 2. One loop contribution to the mass of an uncharged scalar, A, appearing in the superpotential term $W \sim ABC$. The fields B and C have $U(1)_F$ charges q and $-q$ respectively.

$$m^2_{unchg} \sim N \frac{\alpha_i^2}{2\pi^2} \Delta m^2 \log\left(\frac{\Lambda^2_{DSB}}{m_f^2} \right)$$

where N is the number of charged Froggatt-Nielsen (FN) pairs, i denotes the relevant gauge group, m_f is the fermionic mass of the Froggatt-Nielsen fields and Δm^2 is of the order of the non-holomorphic contribution to the scalar masses (i.e. $\Delta m^2 \sim \tilde{m}^2$).

The gaugino masses arise at one loop. Using again the results of [16], we find

$$M_i \simeq N \frac{\alpha_i}{4\pi} \frac{F}{m_f}$$

where we have assumed that F is significantly larger than Δm^2. Here $\langle \chi \rangle = M + \theta\theta F$, where χ is a flavon whose VEV gives a mass to FN fields. Thus, $m_f = M$. These results assume $F < M^2$, which is the case for our model. In order for the gauginos (and in particular the W-inos) not to be too light compared to the lightest Higgs boson, we require that F be within an order of magnitude of M^2 (i.e. $F/M^2 > \frac{1}{10}$). By choosing $\tilde{m}$ to be about 20 TeV, we find that the lightest Higgs boson has a mass near the weak scale.

Uncharged fields with direct superpotential couplings to charged fields receive scalar mass contributions from one-loop graphs containing charged fields (Fig. 2) of order

$$\sim \frac{\lambda}{16\pi^2} \tilde{m}^2 \log \frac{M^2_{IR}}{\Lambda^2_{DSB}}, \tag{5}$$

where λ is the superpotential coupling and M_{IR} is of order the mass of the heaviest particle in the loops. This contribution is approximately an order of magnitude larger than the two-loop contribution above.

The mass of an uncharged field may also receive a contribution from a charged field due to $U(1)_F$ breaking if the charged and uncharged fields both appear in the same F-term. For example, let us assume that the superpotential contains $ABC + C\chi D$ where A, B and C are uncharged and χ and D are charged. If χ has a non-zero vacuum expectation value (VEV), the squared masses of the scalar components of A and B receive a contribution proportional to the supersymmetry breaking mass of the scalar component of the charged D field,

$$\sim \frac{\lambda}{16\pi^2} \tilde{m}^2. \tag{6}$$

Moreover, if an uncharged field appears with a charged field in the same F-term, they may mix due to $U(1)_F$ breaking. For example, the F-term contribution to the scalar potential from the field C above is $|AB + D\chi|^2$. If both χ and B have non-zero VEVs, then A and D would mix.

The contributions described in the preceding two paragraphs are not flavor independent. Thus, degenerate squarks are not a feasible method of avoiding FCNC.

E. Constraints on charge assignments and couplings

In choosing a Froggatt-Nielsen sector, our desire is to leave intact perhaps the most compelling feature of the minimal supersymmetric standard model (MSSM), i.e. the unification of gauge coupling constants. To preserve this result, our vector-like FN fields should come in complete SU(5) representations. In addition, $U(1)_F$ charges should be assigned to full multiplets. In addition to maintaining unification, this allows us to satisfy easily the standard anomaly conditions as well as

$$\mathrm{Tr}[Y m_i^2] \simeq 0, \tag{7}$$

where m_i are scalar particle masses and Y is ordinary hypercharge. If this equation were not satisfied, the $U(1)_Y$ D-term would receive an unwanted Fayet-Iliopoulos term at one loop.

It is well known that the addition of complete SU(5) multiplets to the standard model does not ruin coupling constant unification. In order for the gauge couplings to remain perturbative from one-loop running to the grand unified theory (GUT) scale, the following inequality must be satisfied:

$$3n_{10} + n_5 \lesssim 5, \tag{8}$$

where n_{10} is the number of $\{\mathbf{10}, \overline{\mathbf{10}}\}$ pairs in addition to the standard model fields, and n_5 is the number of additional $\{\overline{\mathbf{5}}, \mathbf{5}\}$ pairs. Two loop contributions to the beta functions will modify this condition, with two loop gauge contributions generally reducing slightly the number of additional fields allowed and superpotential couplings increasing this number—we will assume that the net two-loop effects are not too important. A realistic model of fermion masses that satisfies this condition will have $n_{10} = 1$ and $n_5 = 1$ or 2. Thus, the particle content of our model includes

(i) three generations of matter in SU(5) multiplets, $\{\mathbf{10}^g_q, \overline{\mathbf{5}}^g_r\}$, where $g(=1,2,3)$ is the generation index, and q and r denote $U(1)_F$ charges,

(ii) two Higgs superfields, H^u and H^d,[5]

(iii) Froggatt-Nielsen fields in vector representations of SU(5), $\{\mathbf{10}^V_d, \overline{\mathbf{10}}^V_e\}$, $\{\overline{\mathbf{5}}^V_l, \mathbf{5}^V_m\}$, and possibly $\{\overline{\mathbf{5}}^{V'}_n, \mathbf{5}^{V'}_p\}$,

[5]The SU(5) representations of the Higgs fields are intentionally left unspecified. We do not intend here to build a complete grand unified theory, but we wish to allow unification to be possible in the context of our model. We only require that H^u and H^d contain the standard Higgs doublets.

(iv) flavons [SU(5) singlets] which have non-zero VEVs — some at the tree level (χ), and others at one or more loops (ϕ), and

(v) additional fields ($A,B,C, \dots$) which help produce a ''cascade'' of flavon VEVs.

Another major constraint on the charge assignments of these fields comes from the experimental limits on FCNC [2]. There are different ways to constrain squark (and slepton) masses in order to limit supersymmetric contributions to FCNC. One way is to make their masses degenerate, thus suppressing their contribution through a supersymmetric Glashow-Iliopoulos-Maiani (GIM) mechanism. Degeneracy is a natural result and thus a virtue of the original GMSB models [3–5]. In those models, squark and slepton masses are dominated by loop corrections involving flavor-blind G_{sm} couplings. However, the additional structure in our model produces significant flavor dependent contributions to sparticle masses, destroying this degeneracy. Therefore, to suppress FCNC, we instead decouple the problem by making the first two generations heavy [9–11]. This can be achieved naturally by simply requiring the particles in the first two generations $(\mathbf{10}_a^1, \mathbf{10}_b^2, \overline{\mathbf{5}}_i^1, \overline{\mathbf{5}}_j^2)$, to have non-zero $\mathrm{U}(1)_\mathrm{F}$ charges. We do find, however, that some level of degeneracy must still exist between the first two generations.

The following observations impose additional constraints on our model:

To avoid fine tuning, at least one Higgs boson must have a mass at the weak scale. Therefore, one Higgs superfield must be uncharged [under $\mathrm{U}(1)_\mathrm{F}$] and must not have any contact interactions with charged fields.

The Higgsino mass will come from a μ-type term in the superpotential,

$$W \supset X H^u H^d. \tag{9}$$

Thus, to satisfy the previous condition, both Higgs fields must be uncharged.

The top quark's Yukawa coupling is of order unity and therefore does not come from the Froggatt-Nielsen mechanism, but from a direct coupling to the Higgs boson:

$$W \supset H^u \mathbf{10}_c^3 \mathbf{10}_c^3 \tag{10}$$

where c is the flavor charge and the 3 indicates the generation. We conclude that $c=0$ by $\mathrm{U}(1)_\mathrm{F}$ invariance. Note also that $c=0$ guarantees that the H^u mass contribution is not much larger than the weak scale.

Figure 3 summarizes the resulting spectrum.

III. FERMION MASSES

We want Yukawa coupling matrices in the low energy effective theory that reproduce the known experimental values of fermion masses and mixing angles. In order to have a model from which the fermion masses of the SM appear naturally, we must produce the small parameters in the Yukawa matrices dynamically. We accomplish this with a modified FN mechanism and a hierarchy of flavon VEVs. This section describes the allowed fermion mass matrices.

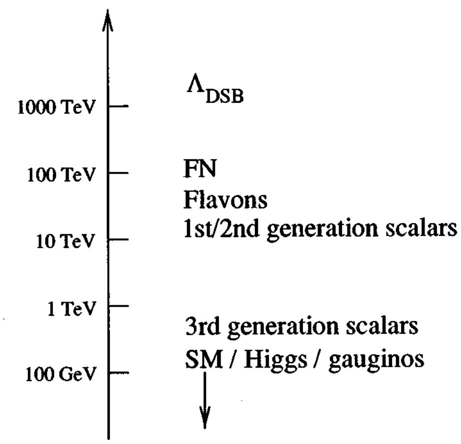


FIG. 3. Spectral structure of the model.

A. Framework

The masses of the fermions are generated by superpotential terms like $M_{ij}\psi_i\psi_j$, where M_{ij} are the scalar VEVs of Higgs bosons or flavon superfields. To construct these superpotential terms, we apply the following guidelines:

(1) We work in the context of SU(5). This means our $\mathrm{U}(1)_\mathrm{F}$ charge assignments are consistent with SU(5).

(2) We want the model to be natural. Any superpotential interaction should appear with a coupling constant of order unity.

(3) The Higgs fields are uncharged. The up-type Higgs field cannot couple directly to charged fields and the fields it couples to have restricted interactions with charged fields. The third generation **10** is also uncharged.

(4) The first two generations must be charged in order to avoid large FCNC (this will be shown explicitly).

From the following arguments, we will conclude that the FN sector must include one $\{\mathbf{10},\overline{\mathbf{10}}\}$ pair and *two* $\{\mathbf{5},\overline{\mathbf{5}}\}$ pairs. The model predicts a massless up quark and a large value of $\tan\beta$.

The masses of up-type quarks come from the superpotential terms

$$H^u \mathbf{10}\,\mathbf{10} \quad \text{and} \quad \varphi \mathbf{10}\,\overline{\mathbf{10}}, \tag{11}$$

while those of down-type quarks and leptons come from the terms

$$H^d \overline{\mathbf{5}}\,\mathbf{10}, \quad \varphi \mathbf{10}\,\overline{\mathbf{10}} \quad \text{and} \quad \varphi \overline{\mathbf{5}}\,\mathbf{5}. \tag{12}$$

Because of the SU(5) symmetry, the charged lepton mass matrix will be proportional to the down quark mass matrix. Deviations will derive from SU(5) breaking and will depend on the Higgs sector of the (grand unified) model. We will assume that this can be done such that the correct lepton masses are predicted, and thus for convenience, we shall speak only in terms of quark masses.

The quark content of the SU(5) multiplets are

$$\mathbf{10}_q^g \supset \{\overline{u_q^g}, u_q^g, d_q^g\}$$

$$\overline{\mathbf{10}}_q^{\bar V}\supset\{u_q^{\bar V},\overline{u}_q^{\bar V},\overline{d}_q^{\bar V}\}$$

$$\overline{\mathbf{5}}_q^{g}\supset\{\overline{d}_q^{g}\}$$

$$\mathbf{5}_q^{\bar V}\supset\{d_q^{\bar V}\},$$

where $g(=1,2,3,V)$ is a generation index, and q is the $U(1)_F$ charge of the multiplet. Schematically, the tree-level mass matrices look like

	$\overline{u}_a^1$	$\overline{u}_b^2$	$\overline{u}_0^3$	$\overline{u}_d^V$	$\overline{u}_e^{\bar V}$
u_a^1, u_b^2, u_0^3, u_d^V			$\langle$up-Higgs$\rangle$		$\langle$flavons$\rangle$
$u_e^{\bar V}$			$\langle$flavons$\rangle$		$\langle H^d\rangle$

and

	$\overline{d}_i^1$	$\overline{d}_j^2$	$\overline{d}_k^3$	$\overline{d}_l^V$	$(\overline{d}_n^{V'})$	$\overline{d}_e^{\bar V}$
d_a^1, d_b^2, d_0^3, d_d^V			$\langle$down-Higgs$\rangle$			$\langle$flavons$\rangle$
$d_m^{\bar V}$, $(d_p^{\bar V'})$			$\langle$flavons$\rangle$			$\langle H^u\rangle$

where the 6th row and 5th column of the down quark mass matrix represent the optional $(\overline{\mathbf{5}},\mathbf{5})$ pair. Now, following the above mentioned guidelines on charge constraints, we can fill in these matrices.

Our strategy for avoiding large FCNC requires $a,b\neq 0$. Therefore, any field that appears in one of the first two rows of either matrix has a contact interaction with a charged field. However, the up-type Higgs field must not interact with $U(1)_F$-charged particles, so the first two rows of the up matrix will be devoid of Higgs (VEVs). That matrix will have a zero eigenvalue, thus predicting a massless up quark. A vanishing up quark mass is a possible solution to the strong *CP* problem, as the strong phase is no longer physical and can be rotated into the up quark field via an axial rotation. For complete details on the viability of a massless up quark, see [17].

To complete the up matrix, we note that if $d\neq 0$, this matrix would have two zero eigenvalues. Since we are confident that the charm mass is not zero, we set $d=0$.

Also, the Froggatt-Nielsen field $\overline{u}_0^V$ must interact with $\overline{u}_e^{\bar V}$ through a flavon χ_{-e} to receive a mass $\langle\chi_{-e}\rangle$ much greater than the weak scale. But $\overline{u}_0^V$ must interact with H^u as well if the up matrix has only one zero eigenvalue. To avoid corrections to the up Higgs mass of order $\tilde{m}/4\pi$ the fields interacting with $\overline{u}_e^V$ must be uncharged. That is, $e=0$.

Assuming that all allowed couplings exist, we find that the up matrix is completely determined and takes the form

$$M^u=\begin{array}{c} \\ u_a^1\\ u_b^2\\ u_0^3\\ u_0^V\\ u_0^{\bar V}\end{array}\begin{array}{c}\begin{array}{ccccc}\overline{u}_a^1 & \overline{u}_b^2 & \overline{u}_0^3 & \overline{u}_0^V & \overline{u}_0^{\bar V}\end{array}\\ \begin{pmatrix} 0 & 0 & 0 & 0 & \langle\phi_{-a}\rangle\\ 0 & 0 & 0 & 0 & \langle\phi_{-b}\rangle\\ 0 & 0 & \langle H^u\rangle & \langle H^u\rangle & 0\\ 0 & 0 & \langle H^u\rangle & \langle H^u\rangle & \langle\chi_0\rangle\\ \langle\phi_{-a}\rangle & \langle\phi_{-b}\rangle & 0 & \langle\chi_0\rangle & \langle H^d\rangle\end{pmatrix}\end{array}, \tag{13}$$

where the fields $\mathbf{10}_0^3$ and $\mathbf{10}_0^V$ have been rotated to remove the (3,5) and (5,3) entries. For generic couplings, the (4,4) and (5,5) entries have little effect on the final results. For convenience, we henceforth set them to zero.[6] This matrix produces the following up-type Yukawa couplings in the low energy theory:

$$\overline{u}\langle H^u\rangle\begin{pmatrix}0 & 0 & \sim\epsilon_1\\ 0 & 0 & \sim\epsilon_2\\ \sim\epsilon_1 & \sim\epsilon_2 & \sim 1\end{pmatrix}u, \tag{14}$$

where

$$\epsilon_1=\frac{\langle\phi_{-a}\rangle}{\langle\chi_0\rangle}$$

$$\epsilon_2=\frac{\langle\phi_{-b}\rangle}{\langle\chi_0\rangle}.$$

The tildes represent the (order 1) couplings that have not yet been included.

Now we shall attempt to design a down mass matrix with only one additional $\{\overline{\mathbf{5}},\mathbf{5}\}$ pair. First, to prevent a zero eigenvalue, there must be at least one $\langle H^d\rangle$ entry in one of the first two rows. However, since we wish to produce the small Yukawa couplings of the first two generations dynamically, we place the entry in the 4th column. To do this, we let $l=-b$ (choosing $-a$ would lead to the same conclusions). Examining the first three columns, we see that in order to avoid a zero eigenvalue, at least two of i, j and k must be zero. This is in contradiction with our decoupling strategy for avoiding FCNC, hence ruling out this scenario. One could ask if by setting all $i=j=k=0$, these squarks would be degenerate. However (see the Appendix), the degeneracy is broken by large flavor-dependent contributions.

We must include two $\{\overline{\mathbf{5}},\mathbf{5}\}$ pairs in the FN sector. Making similar arguments as those above, we see our matrix is limited to

[6]These couplings are relevant when dealing with the ''μ-term problem.'' For details, see our Conclusion.

$$M^d = \begin{array}{c} \\ d_a^1 \\ d_b^2 \\ d_0^3 \\ d_0^V \\ d_m^{\overline{V}} \\ d_p^{\overline{V}'} \end{array} \begin{array}{c} \begin{array}{cccccc} \overline{d_i^1} & \overline{d_j^2} & \overline{d_0^3} & \overline{d_l^V} & \overline{d_{-b}^{V'}} & \overline{d_0^{\overline{V}}} \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & ? & 0 & \langle \phi_{-a} \rangle \\ 0 & 0 & 0 & ? & \langle H^d \rangle & \langle \phi_{-b} \rangle \\ 0 & 0 & \langle H^d \rangle & ? & 0 & 0 \\ 0 & 0 & \langle H^d \rangle & ? & 0 & \langle \chi_0 \rangle \\ \langle \phi_{-i-m} \rangle & \langle \phi_{-j-m} \rangle & \langle \phi_{-m} \rangle & \langle \chi_{-l-m} \rangle & \langle \phi_{b-m} \rangle & 0 \\ \langle \phi_{-i-p} \rangle & \langle \phi_{-j-p} \rangle & \langle \phi_{-p} \rangle & \langle \phi_{-l-p} \rangle & \langle \chi_{b-p} \rangle & 0 \end{pmatrix} \end{array}, \tag{15}$$

where the question marks label undetermined entries. We see that l can be either $(-a)$ or zero, and any of the flavons in the last two rows can be removed.

B. A Model

We now present a specific example of the above framework that yields the correct quark mass ratios and Cabibbo-Kobayashi-Maskawa (CKM) angles.

If the up matrix is fixed, the down mass matrix would still allow many choices. We start by choosing $l=-a$ ($l=0$ would work as well). The first four entries of the fourth column of M^d are then $\langle H^d \rangle$, 0, 0 and 0. We want, for simplicity, to limit the number of flavons appearing in the matrices. Again, we can use the freedom offered by the down mass matrix. We can remove one flavon from each of the first two columns — the entries (6,1) and (5,2) are taken to be 0. We also can take the entries (5,5) and (6,4) to be 0. As for the third column one may ask if one could remove the two flavons in the entries (5,3) and (6,3) since it would not generate a zero eigenvalue. However, in such a scenario the value of V_{ub} comes out too small as is explained farther down. We take only entry (6,3) to be 0. The resultant matrices are

$$M^u = \begin{pmatrix} 0 & 0 & 0 & 0 & \langle \phi_{-a} \rangle \\ 0 & 0 & 0 & 0 & \langle \phi_{-b} \rangle \\ 0 & 0 & \langle H^u \rangle & \lambda_1 \langle H^u \rangle & 0 \\ 0 & 0 & \lambda_1 \langle H^u \rangle & 0 & \langle \chi_0 \rangle \\ \langle \phi_{-a} \rangle & \langle \phi_{-b} \rangle & 0 & \langle \chi_0 \rangle & 0 \end{pmatrix} \tag{16}$$

and

$$M^d = \begin{pmatrix} 0 & 0 & 0 & \mu_3 \langle H^d \rangle & 0 & \langle \phi_{-a} \rangle \\ 0 & 0 & 0 & 0 & \mu_2 \langle H^d \rangle & \langle \phi_{-b} \rangle \\ 0 & 0 & \langle H^d \rangle & 0 & 0 & 0 \\ 0 & 0 & \mu_1 \langle H^d \rangle & 0 & 0 & \langle \chi_0 \rangle \\ \langle \phi_{-m-i} \rangle & 0 & \langle \phi_{-m} \rangle & \langle \chi_{a-m} \rangle & 0 & 0 \\ 0 & \langle \phi_{-p-j} \rangle & 0 & 0 & \langle \chi_{b-p} \rangle & 0 \end{pmatrix}, \tag{17}$$

where the λ's and μ's are coupling constants of order 1 that cannot be absorbed by redefining the VEVs. Assuming $\langle \chi \rangle \gg \langle \phi \rangle$ and $\langle \chi \rangle \gg \langle H^{u,d} \rangle$, we can integrate out the Froggatt-Nielsen fields, yielding the 3×3 fermion mass matrices

$$M_3^u = \langle H^u \rangle \begin{pmatrix} 0 & 0 & \lambda_1 \epsilon_1 \\ 0 & 0 & \lambda_1 \epsilon_2 \\ \lambda_1 \epsilon_1 & \lambda_1 \epsilon_2 & 1 \end{pmatrix}$$

and

$$M_3^d = \langle H^d \rangle \begin{pmatrix} \mu_3 \epsilon_5 & 0 & \mu_3 \epsilon_3 + \mu_1 \epsilon_1 \\ 0 & \mu_2 \epsilon_4 & \mu_1 \epsilon_2 \\ 0 & 0 & 1 \end{pmatrix}$$

with

$$\epsilon_1 = -\frac{\langle \phi_{-a} \rangle}{\langle \chi_0 \rangle}, \quad \epsilon_2 = -\frac{\langle \phi_{-b} \rangle}{\langle \chi_0 \rangle}$$

$$\epsilon_3 = -\frac{\langle \phi_{-m} \rangle}{\langle \chi_{a-m} \rangle}, \quad \epsilon_4 = -\frac{\langle \phi_{-p-j} \rangle}{\langle \chi_{b-p} \rangle}$$

$$\epsilon_5 = -\frac{\langle \phi_{-m-i}\rangle}{\langle \chi_{a-m}\rangle}.$$

We can now see why the (5,3) entry of M^d cannot vanish. The angle V_{ub} is equal to the inner product $v_u^\dagger v_b$. The vector v_u is the eigenvector of $M_3^u M_3^{u\dagger}$ corresponding to the eigenvalue equal to the squared mass of the up quark (which is 0) and v_b is the eigenvector of $M_3^d M_3^{d\dagger}$ corresponding to the eigenvalue equal to the squared mass of the bottom quark. We have (up to some normalization factors of order 1)

$$v_u = \begin{pmatrix} 1 \\ -\frac{\epsilon_1}{\epsilon_2} \\ 0 \end{pmatrix}$$

$$v_b = \begin{pmatrix} \mu_1\epsilon_1 + \mu_3\epsilon_3 + O(\epsilon^3) \\ \mu_1\epsilon_2 + O(\epsilon^3) \\ \sim 1 \end{pmatrix}$$

where ϵ is of the order of the ϵ_i in the matrices. Typically, ϵ is less than 0.05. It follows that

$$V_{ub} = \mu_3\epsilon_3 + O(\epsilon^3).$$

If ϵ_3 were 0, that is if there were no entry (5,3) in the 6×6 down matrix, V_{ub} would be of order 10^{-4}, an order of magnitude too small to meet the experimental range. This short computation also applies to the general form of the down mass matrix. The $\overline{\mathbf{5}}^3$ field must always interact with one of the $\mathbf{5}^V$ fields. The mass of the right-handed (RH) bottom squark depends on this interaction. If $\mathbf{5}^V$ is charged, the mass of the RH bottom squark would be of order $\tilde{m}/4\pi$. If $\mathbf{5}^V$ is uncharged, the mass of the RH bottom squark would be at the weak scale.[7]

It remains to evaluate the orders of magnitude of the different ϵ_i. We find

$$\epsilon_2 \simeq \sqrt{\frac{m_{charm}}{m_{top}}}$$

$$\epsilon_1 \simeq V_{us}\epsilon_2$$

$$\epsilon_4 \simeq \frac{m_{strange}}{m_{bottom}}$$

$$\epsilon_5 \simeq \frac{m_{down}}{m_{strange}}\epsilon_4$$

and

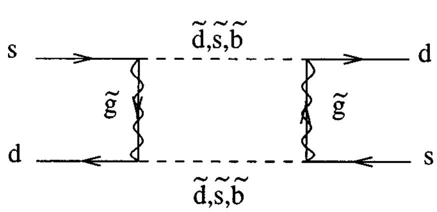
$$\epsilon_3 \simeq V_{ub}\ (\mu_1 - \lambda_1)\epsilon_2 \simeq V_{cb} \tag{18}$$

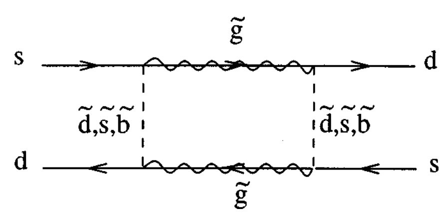
which implies

$$\mu_1 - \lambda_1 \simeq \frac{1}{2}.$$

IV. FCNC AND *CP* VIOLATION

Several new interactions may contribute to K^0-$\overline{K}^0$ mixing and ϵ_K, beyond the usual weak interaction contributions. These usually provide the most stringent constraints on supersymmetric models. The potentially largest contribution is from a gluino exchange box diagram





We wish to compute in the framework of our model the two main contributions to K^0-$\overline{K}^0$ mixing, namely the contribution of the quarks via the usual weak interactions and the contribution from the squarks due to the strong interactions. We work with the specific model described in Sec. III B. In this example, we shall find that the squark contribution to the K_S/K_L mass difference is small, both because the first two squark generations are heavy, and because the squark mixing angles among the first two generations of the down sector are very small [18]. The largest supersymmetric contribution comes from the left handed bottom squark. The phase of this contribution is naturally almost real, and so the contribution to ϵ_K is sufficiently small.

To generate *CP* violation, there must exist at least one complex parameter in the interaction Lagrangian that cannot be made real by redefining fields. The relevant interactions

[7]The superpotential may contain couplings which contribute to a right-handed bottom squark mass above the weak scale.

are listed in the up and down mass matrices (16) and (17). Assuming that all coupling constants and VEVs are complex, suitable redefinitions of the fields that do not receive VEVs allow us to make some of the entries in the mass matrices real. However, two entries cannot be made real. We can choose these to be the (3,3) and (5,3) entries of the down matrix. The up and down matrices are then (ignoring real coupling constants of order 1)

$$M^{up}=\begin{pmatrix} 0 & 0 & 0 & 0 & \langle\phi_{-a}\rangle \\ 0 & 0 & 0 & 0 & \langle\phi_{-b}\rangle \\ 0 & 0 & \langle H^u\rangle & \langle H^u\rangle & 0 \\ 0 & 0 & \langle H^u\rangle & 0 & \langle\chi_0\rangle \\ \langle\phi_{-a}\rangle & \langle\phi_{-b}\rangle & 0 & \langle\chi_0\rangle & 0 \end{pmatrix}$$

and

$$M^{down}=\begin{pmatrix} 0 & 0 & 0 & \langle H^d\rangle & 0 & \langle\phi_{-a}\rangle \\ 0 & 0 & 0 & 0 & \langle H^d\rangle & \langle\phi_{-b}\rangle \\ 0 & 0 & \eta_{33}\langle H^d\rangle & 0 & 0 & 0 \\ 0 & 0 & \langle H^d\rangle & 0 & 0 & \langle\chi_0\rangle \\ \langle\phi_{-m-i}\rangle & 0 & \eta_{53}\langle\phi_{-m}\rangle & \langle\chi_{a-m}\rangle & 0 & 0 \\ 0 & \langle\phi_{-p-j}\rangle & 0 & 0 & \langle\chi_{b-p}\rangle & 0 \end{pmatrix},$$

where η_{33} and η_{53} are two complex numbers of modulus 1.

All of the other variables in the matrices are real. The corresponding superpotential reads

$$\begin{aligned} W_{FN}=&H^d 10^1_a \overline{5}^{V}_{-a}+\phi_{-a}10^1_a\overline{10}^{\overline{V}}_0+\phi_{-b}10^2_b\overline{10}^{\overline{V}}_0+H^d 10^2_b\overline{5}^{V'}_{-b}\\ &+\eta_{33}H^d 10^3_0\overline{5}^3_0+H^d 10^V_0\overline{5}^3_0+\chi_0\overline{10}^{\overline{V}}_0 10^V_0+\phi_{-m-i}\overline{5}^1_i 5^{\overline{V}}_m\\ &+\eta_{53}\phi_{-m}\overline{5}^3_0 5^{\overline{V}}_m+\chi_{a-m}5^{\overline{V}}_m\overline{5}^V_{-a}+\phi_{-p-j}\overline{5}^2_j 5^{\overline{V}'}_p\\ &+\chi_{b-p}5^{\overline{V}'}_p\overline{5}^{V'}_{-b}+\frac{1}{2}H^u 10^3_0 10^3_0+H^u 10^3_0 10^V_0. \end{aligned} \tag{19}$$

Integrating out the heavy fields, we find the following up and down matrices for the light fermions:

$$M^u_3=\langle H^u\rangle\begin{pmatrix} 0 & 0 & \epsilon_1 \\ 0 & 0 & \epsilon_2 \\ \epsilon_1 & \epsilon_2 & 1 \end{pmatrix}$$

and

$$M^d_3=\langle H^d\rangle\begin{pmatrix} \epsilon_5 & 0 & \eta_{53}\epsilon_3+\epsilon_1 \\ 0 & \epsilon_4 & \epsilon_2 \\ 0 & 0 & \eta_{33} \end{pmatrix}$$

from which we get the CKM matrix

$$V^{CKM}=\begin{pmatrix} 1 & -\frac{\epsilon_1}{\epsilon_2} & -\eta^*_{33}\eta_{53}\epsilon_3 \\ \frac{\epsilon_1}{\epsilon_2} & 1 & (\eta^*_{33}-1)\epsilon_2 \\ (1-\eta_{33})\epsilon_1 & (1-\eta_{33})\epsilon_2 & 1 \end{pmatrix}.$$

Only the significant phases have been retained. The phases of η_{33} and η_{53} are assumed to be of order 1. The remaining entries have phases of order 10^{-2} or less. Such a CKM matrix yields reasonable values of Δm_K and ϵ_K from the weak interactions.

The contribution of the gluino box to K^0-$\overline{K}^0$ mixing remains to be computed. To compute this requires the squark mass matrix. We consider tree level and one loop mass terms generated by the effective scalar potential.

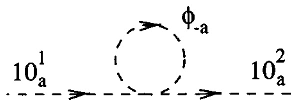
We assume that all of the flavons appearing in one line or column of the mass matrices are distinct (this is automatically satisfied if all the standard model fields have different charges). This implies that there are no off-diagonal one-loop corrections to the squark mass matrix of order

$$\frac{\tilde{m}^2}{16\pi^2}\log\left(\frac{\Lambda^2_{DSB}}{\langle\chi\rangle^2}\right).$$

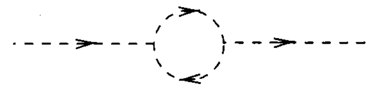
Indeed, if for example $a=b$, we could have $\phi_{-a}=\phi_{-b}$. The F-term of $\overline{10}^{\overline{V}}_0$ would yield the interaction

$$\mathbf{10}_a^1\mathbf{10}_a^{2\,*}\phi_{-a}\phi_{-a}^*$$

from which we could get the one-loop scalar graph



whose supersymmetry breaking part is of the order of the above correction. The only one loop corrections to off diagonal terms come from the supersymmetry breaking part of scalar graphs such as



Both LH squarks and RH squarks will contribute to these processes.

We first consider the case of the LH down squarks, since (as we shall see) their contribution is the largest. At the tree level, the masses of the LH down type squarks come from the Hermitian matrix (omitting the VEV symbol $\langle\ \ \rangle$ for clarity)

$$\begin{array}{c} \\ d_a^1 \\ d_b^2 \\ d_0^3 \\ d_0^V \\ d_m^{\overline{V}} \\ d_p^{\overline{V}'} \end{array}
\begin{array}{c} \begin{array}{cccccc} d_a^{1\,*} & d_b^{2\,*} & d_0^{3\,*} & d_0^{V\,*} & d_m^{\overline{V}\,*} & d_p^{\overline{V}'\,*} \end{array} \\
\begin{pmatrix} \tilde{m}^2 & \phi_{-a}\phi_{-b} & 0 & \phi_{-a}\chi_0 & \chi_{a-m}H^d & 0 \\ & \tilde{m}^2 & 0 & \phi_{-b}\chi_0 & 0 & \chi_{b-p}H^d \\ & & m_{\rm weak}^2 & \eta_{33}H^{d\,2} & \eta_{33}\eta_{53}^*\phi_{-m}H^d & 0 \\ & & & \chi_0^2 & \eta_{53}^*\phi_{-m}H^d & 0 \\ & & & & \chi_{a-m}^2 & 0 \\ & & & & & \chi_{b-p}^2 \end{pmatrix} \end{array}$$

where we have written only the dominant contribution to each matrix element.

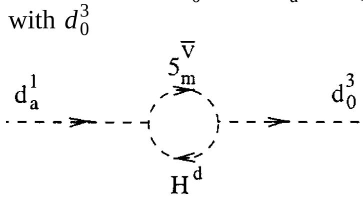
There are other tree level contributions to the masses of the LH down squarks since some mixing occurs with the RH down squarks. However, as we note at the end of the present section, these terms are small and can be ignored for an order of magnitude computation.

As mentioned above, any off-diagonal entry of the mass matrix may receive a one-loop contribution. The correction is of the form

$$\frac{1}{16\pi^2}\langle A\rangle\langle B\rangle\log\left(\frac{m_{fermion}^2}{m_{scalar}^2}\right).$$

The masses appearing in the logarithm are the masses of the heavier scalar particle running in the loop and of its fermionic partner.

Some loop corrections come from known superpotential interactions between SU(5) and flavon fields in Eq. (19). For instance, from the F-terms of $\overline{5}_0^3$ and $\overline{5}_{-a}^V$, we get a diagram that mixes d_a^1 with d_0^3



The resulting term is

$$\eta_{53}\eta_{33}^*\frac{1}{16\pi^2}\langle\phi_{-m}\rangle\langle\chi_{a-m}\rangle\frac{\tilde{m}^2}{\langle\chi_{a-m}\rangle^2},$$

where $\tilde{m}^2/\langle\chi\rangle^2$ comes from expanding the logarithm.

Other loop corrections may arise from terms in the flavon superpotential. Without knowing explicitly the flavon superpotential, we cannot tell if one specific entry receives a correction and if so what the VEVs $\langle A\rangle$ and $\langle B\rangle$ are. We will assume that any off-diagonal term in the matrix receives such a correction with a phase of order 1 and that the VEV product is of the order of $\langle\phi\rangle\langle\chi\rangle$ with $\langle\phi\rangle/\langle\chi\rangle\simeq 10^{-2}$. This last value is an overestimate (most likely the VEV product is of the order of the product of two ϕ VEVs). For example, assuming a flavon superpotential containing the terms $CD\phi_{-a}$ and $CD'\phi_{-b}$, we obtain an off-diagonal term mixing d_a^1 and d_b^2. The loop correction is equal to $\sim(1/16\pi^2)\langle D\rangle^*\langle D'\rangle\tilde{m}^2/\langle\chi_{a-m}\rangle^2$. We assume that $\langle D\rangle^*\langle D'\rangle$ is of the same order as $\langle\phi\rangle\langle\chi\rangle$.

We may now estimate the angles at the squark-quark-gluino vertex with the quarks and squarks taken as mass eigenstates. For the LH quarks, the angles are given by the matrix that diagonalizes $M_3^d M_3^{d\,\dagger}$:

$$\begin{pmatrix} 1 & -(\eta_{53}^*\epsilon_3+\epsilon_1)\epsilon_2 & \eta_{33}\epsilon_1 \\ (\eta_{53}\epsilon_3+\epsilon_1)\epsilon_2 & 1 & -\eta_{33}\epsilon_2 \\ -\eta_{33}^*\epsilon_1 & \eta_{33}^*\epsilon_2 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 10^{-3} & 10^{-2} \\ & 1 & 10^{-1} \\ & & 1 \end{pmatrix}.$$

Off-diagonal elements in the third line and column have the same order one phase. The elements (1,2) and (2,1) have a smaller phase of order 10^{-1}.

For the squarks, we first integrate out the heavy fields and then rotate the light squark mass matrix. Doing so, we find the following symmetric matrix as an estimate for the rotation matrix:

$$\begin{pmatrix} 1 & \epsilon_1\epsilon_2\frac{\langle\chi\rangle^2}{\tilde{m}^2} & \frac{1}{16\pi^2}\frac{\langle\phi\rangle}{\langle\chi\rangle} \\ & 1 & \frac{1}{16\pi^2}\frac{\langle\phi\rangle}{\langle\chi\rangle} \\ & & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 10^{-2} & 10^{-4} \\ & 1 & 10^{-4} \\ & & 1 \end{pmatrix}$$

where the off-diagonal elements of the third line and column have phases of order 1 and the entries (1,2) and (2,1) have a phase of order 10^{-2}. The VEV $\langle\chi\rangle$ is a generic value for the VEVs of χ_0, χ_{a-m} and χ_{b-p}, which we assume to be all of the same order. The value of $\langle\chi\rangle/\tilde{m}$ depends on the DSB. A typical value is $\langle\chi\rangle^2/\tilde{m}^2=10$. As before, $\langle\phi\rangle/\langle\chi\rangle$ is taken to be 10^{-2}.

The product of the above matrices yields the matrix Z_{LL} at the squark-quark-gluino vertices:

$$Z_{LL}\sim\begin{pmatrix} 1 & 10^{-2} & 10^{-2} \\ & 1 & 10^{-1} \\ & & 1 \end{pmatrix},$$

where off-diagonal elements (1,3) and (2,3) have phases which are of order 1 but differ by a term of order 10^{-2}. The (1,2) angle has a phase of 10^{-2}.

The contribution of the LH squarks alone to the box diagram is [2]

$$\langle\bar{K}|H_{LL}|K\rangle=\frac{1}{3}\alpha_s^2 Z_{LL}^{1i*}Z_{LL}^{2i}Z_{LL}^{1j*}Z_{LL}^{2j} \times\left(-\frac{11}{36}I_1+\frac{x_g}{9}I_2\right)\frac{1}{\tilde{m}^2}m_K f_K^2,$$

where

$$I_1=\int_0^{+\infty}dy\,\frac{y^2}{(y+x_i)(y+x_j)(y+x_g)^2}$$

$$I_2=\int_0^{+\infty}dy\,\frac{y}{(y+x_i)(y+x_j)(y+x_g)^2}$$

and

$$x_i=\frac{m_i^2}{\tilde{m}^2}.$$

The indices i and j refer to the squarks and run from 1 to 3. The index g stands for the gluino, whose mass is at the weak scale ($x_g\simeq 10^{-4}$). Parameters m_K, f_K and α_s are the K mass, K decay constant and strong coupling constant. We use the values $m_K=490$ MeV, $f_K=160$ MeV and $\alpha_s(M_W)=0.12$.

Given the above Z_{LL} matrix and taking $\tilde{m}\simeq 20$ TeV, we find that all contributions to Δm_K and ϵ_K are within the experimental values. For instance, a LH bottom squark of mass m_{weak} gives a contribution to Δm_K of 10^{-13} MeV and to ϵ_K of 10^{-3}. Other possibilities involving first or second generation squarks give smaller contributions.

The same computation done with the RH down quarks and squarks gives the following matrix Z_{RR} (again neglecting left-right mixing):

$$Z_{RR}\sim\begin{pmatrix} 1 & 10^{-4} & 10^{-4} \\ & 1 & 10^{-3} \\ & & 1 \end{pmatrix},$$

where all off diagonal entries have a phase of order 1. This matrix gives contributions to Δm_K and ϵ_K well below the experimental bounds.

We now consider the mixing between LH and RH squarks and confirm that it can be neglected.

At the tree level, a mu term $\mu H^u H^d$ generates mixing between d_0^3 and $\bar{d}_0^3$ and also between the light squarks, $\bar{d}_0^3$, d_a^1, and d_b^2, and heavy squarks, namely d_0^V, $\bar{d}_{-a}^V$ and $\bar{d}_{-b}^{V'}$. These terms come with a coefficient $\mu\langle H^u\rangle\simeq m_{\text{weak}}^2$ and possibly a phase of order 1. They are of the same order as the loop corrections previously considered and would not change the order of magnitude of Z_{LL} and Z_{RR}.

Additional mixing may occur due to the flavon superpotential. For instance, the flavon ϕ_{-b} could appear in the flavon superpotential in a term $\phi_{-b}DD'$. Assuming that D and D' receive a VEV, a mixing term between d_b^2 and $\bar{d}_0^V$ is generated. Its coefficient is $c_b=\langle D\rangle^*\langle D'\rangle^*$. Similarly, via ϕ_{-a}, we discover a mixing term between d_a^1 and $\bar{d}_0^V$ could also be generated (with coefficient c_a equal to a product of flavon VEVs). This would mean that when integrating out $\bar{d}_0^V$, the entry (1,2) of Z_{RR} would receive a contribution $c_a c_b/\langle\chi\rangle^2\tilde{m}^2$. The previous analysis could be invalidated if c_a and c_b were large, causing contributions to FCNC and CP violation beyond experimental bounds. We must constrain the choice of the flavon superpotential. We assume that the VEV product $\langle D\rangle\langle D'\rangle$ is of the order of $\langle\chi\rangle\langle\phi\rangle$ with a small phase ($\simeq 10^{-2}$) or of the order of $\langle\phi\rangle^2$ with a phase of order 1. If so, the orders of magnitude of Z_{LL} and Z_{RR} are unchanged. These restrictions are reasonable since most ϕ fields do not interact directly with a χ field.

With these restrictions, left-right angles remain small (of the order of 10^{-3} or less) except for the mixing angle between d_0^3 and $\overline{d_0^3}$ which is about 10^{-2}. The contribution to the gluino box of left-right effects is then well below the experimental values.

We may conclude that the framework of our model accommodates the current experimental bounds on FCNC and *CP* violation for the *K* system. The important point in this analysis was to assume that there were no loop corrections to off-diagonal entries given by

$$\frac{\tilde{m}^2}{16\pi^2}\log\left(\frac{\Lambda_{DSB}^2}{\langle\chi\rangle^2}\right)$$

which would generate angles of order 10^{-1}. With such a correction, FCNC is still within the experimental values. However, *CP* violation would be much larger than the experimental bound if the correction came with a phase of order 1.

This analysis is applicable to the *B* system. The entries in the third column of Z_{LL} are large and with a phase of order 1 and the mass of the left-handed bottom squark is at the weak scale. Therefore, the supersymmetric contribution to *CP* violation in B-$\overline{B}$ mixing can be as large as the weak interaction contribution [19]. Also new contributions to *CP* violating decay amplitudes may arise with significant departures from the SM predictions. As for FCNC phenomena in *B* physics, the model provides sizable new contributions to the mixing and the *B* radiative decays, but always keeping below the experimental results.

Another possible constraint on new sources of *CP* violation comes from electric dipole moment (EDM) bounds on the neutron and on atoms. Our model contains a massless up quark and thus there is no strong *CP* violation. Though there are several new sources of *CP* violation, supersymmetric contributions to EDM's are sufficiently suppressed due to the large mass of the first two superpartner generations.

V. SOME COSMOLOGICAL CONSIDERATIONS

Supersymmetric models, where the messenger sector is identified with the Froggatt-Nielsen sector and a single U(1) symmetry is used both to give large masses to the first two generations of sfermions and to generate the flavor spectrum, are of considerable interest [20] from the cosmological point of view. Indeed, this class of low energy supersymmetry breaking models naturally predicts (superconducting) cosmic strings [21]. The presence of a Fayet-Iliopoulos D-term ξ^2 induces the spontaneous breakdown of the U(1) gauge symmetry along some field direction in the messenger sector. Let us denote this field direction generically by φ and its U(1) charge by q_φ. In this case local cosmic strings are formed whose mass per unit length is given by $\mu\sim\xi^2$ [21]. Since ξ is a few orders of magnitude larger than the weak scale, cosmic strings are not very heavy. The crucial point is that some quark and/or the lepton superfields are charged under the U(1) group. Let us focus on one of the sfermion fields, $\tilde{f}$ with generic U(1) charge q_f, such that sign $q_f=\mathrm{sgn}q_\varphi$. The potential for the fields φ and $\tilde{f}$ is written as

$$V(\tilde{f},\varphi)=q_\varphi^2\tilde{m}^2|\varphi|^2+q_f^2\tilde{m}^2|\tilde{f}|^2+\frac{g^2}{2}(q_\varphi|\varphi|^2+q_f|\tilde{f}|^2+\xi^2)^2+\lambda|\tilde{f}|^4, \tag{20}$$

where we have assumed, for simplicity, that $\tilde{f}$ is F-flat. The parameter λ is generated from the standard model gauge group D-terms and vanishes if we take $\tilde{f}$ to denote a family of fields parameterizing a D-flat direction.

At the global minimum $\langle\tilde{f}\rangle=0$ and the electric charge, baryon and/or the lepton numbers are conserved. The soft breaking mass term for the sfermion reads

$$\Delta m_{\tilde{f}}^2=q_f(q_f-q_\varphi)\tilde{m}^2, \tag{21}$$

and is positive by virtue of the hierarchy $q_\varphi<q_f<0$ (recalling $\tilde{m}^2<0$). Consistency with experimental bounds requires $\Delta m_{\tilde{f}}^2$ to be of the order of $(20\ \text{TeV})^2$ or so, which in turn requires $\xi^2\sim(4\pi/g^2)\tilde{m}^2\sim(10^2\,\text{TeV})^2$. Notice that $\Delta m_{\tilde{f}}^2$ does not depend upon ξ^2.

Let us analyze what happens in the core of the string. In this region of space, the vacuum expectation value of the field vanishes, $\langle|\varphi|\rangle=0$, and non-zero values of $\langle|\tilde{f}|\rangle$ are energetically preferred in the string core:

$$\langle|\tilde{f}|^2\rangle=\frac{-\tilde{m}^2q_f^2-g^2\xi^2q_f}{g^2q_f^2+2\lambda}. \tag{22}$$

Since the vortex is cylindrically symmetric around the *z* axis, the condensate will be of the form $\tilde{f}=\tilde{f}_0(r,\theta)\,e^{i\eta_f(z,t)}$, where r and θ are the polar coordinate in the (x,y) plane. One can check easily that the kinetic term for $\tilde{f}$ also allows a non-zero value of $\tilde{f}$ in the string and therefore one expects the existence of bosonic charge carriers inside the strings. The latter are, therefore, superconducting.

These superconducting cosmic strings formed at temperatures within a few orders of magnitude of the weak scale may generate primordial magnetic fields [21] and even give rise to the observed baryon asymmetry [22]. Indeed, during their evolution, the superconducting cosmic strings carry some baryon charge. The latter is efficiently preserved from the sphaleron erasure and may be released in the thermal bath at low temperatures. In such a case, the charge carriers inside the strings are provided by the scalar superpartner of the fermions that carry baryon (lepton) number. Since these scalar condensates are charged under $SU(2)_L$, baryon number violating processes are frozen in the core of the strings and the baryon charge number cannot be wiped out at temperatures larger than $T_{EW}\sim100$ GeV. In other words, the superconducting strings act like ''bags'' containing the baryon charge and protect it from sphaleron wash-out throughout the evolution of the Universe, until baryon number violating processes become harmless. This mechanism is efficient even if the electroweak phase transition in the MSSM is of the second order and therefore does not impose any upper bound on the mass of the Higgs boson [22].

VI. CONCLUSION

We have presented a renormalizable model of low energy flavor and supersymmetry breaking in which all mass scales are produced dynamically. A U(1) gauge group mediates large contributions to the masses of the first two generations of scalars, of order 20 TeV, while suppressing the masses of their fermionic partners. Excessive FCNC is successfully avoided, in part, by decoupling the scalars of the first two families. *CP* violation in the kaon system is also predicted to be within experimental bounds. We are able to produce the observed fermion masses and mixing angles while maintaining perturbative unification of gauge coupling constants at M_{GUT}. However, we did not explicitly construct a complete model of flavon interactions having the correct vacuum, though we have made it plausible that one could be produced.

Our goal was to produce a model in which the sectors responsible for scalar masses and fermion masses could be identified. The resulting model, as an unintended consequence, potentially solves at least two major problems of fundamental physics. First, the model predicts a massless up quark. This is the simplest viable solution to the strong *CP* problem. Second, the model predicts the existence of light superconducting cosmic strings, which could be the source of the magnetic fields that are observed on the cosmological scale. These strings may also be responsible for the baryon asymmetry of the universe.

Our model suffers from the same ''μ-term'' problem that exists in most gauge mediated models [23]. We can naturally generate a μ-term via loop corrections if we include, for example, the terms $H_u\mathbf{10}^V\mathbf{10}^V$, $H_d\overline{\mathbf{10}}^V\overline{\mathbf{10}}^V$ and $\chi\mathbf{10}^V\overline{\mathbf{10}}^V$. At one loop, a μ-term appears with coupling constant $\mu\sim(1/16\pi^2)F/M$, where $\langle\chi\rangle=M+\theta\theta F$. As pointed out by Dvali, Pomarol and Giudice [23], the scalar coupling, $B_\mu H_u H_d$, also appears at one loop with $B_\mu\sim(1/16\pi^2)F^2/M^2\sim(4\pi\mu)^2$, which may be too large for natural electroweak symmetry breaking. However, because this model naturally contains a large mass for the scalar H_d, the weak scale Higgs VEV may still be produced naturally. Otherwise, it may be possible to adopt the mechanisms of Ref. [23] to suppress this B_μ term, or to produce acceptable μ and B_μ terms via the mechanism of Ref. [25].

As the first renormalizable and explicit example of the effective supersymmetry [10] approach to flavor and supersymmetry breaking, this model reproduces the success of the standard model in explaining the observed size of FCNC and absence of lepton flavor violation (LFV). In fact this model is surprisingly successful, as the supersymmetric contributions to *CP* violating effects in K-$\bar{K}$ mixing, which even with 20 TeV squarks are potentially 100 times too large, are sufficiently small. The *CP* violating phases in B_d-$\bar{B}_d$ and B_s-$\bar{B}_s$ mixing receive a large nonstandard contribution from left-handed bottom squark exchange. It remains to be calculated whether any other nonstandard FCNC, *CP* violating, and LFV effects are large enough to be revealed by new, more stringent experiments [24].

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APPENDIX: FLAVOR DEPENDENT CONTRIBUTIONS TO UNCHARGED SQUARKS

In this appendix, we show explicitly that leaving all three generations of right-handed down squarks uncharged will not produce a degenerate spectrum. To see this, let us look at the matrix explicitly. Filling in the remaining entries and rotating fields to simplify the matrix we have

$$M^d=\begin{array}{c}\\ d_a^1\\ d_b^2\\ d_0^3\\ d_0^V\\ d_m^{\bar V}\end{array}\begin{array}{c}\begin{array}{ccccc}\overline{d_0^1} & \overline{d_0^2} & \overline{d_0^3} & \overline{d_{-b}^V} & \overline{d_0^{\bar V}}\end{array}\\ \begin{pmatrix} 0 & 0 & 0 & 0 & \langle\phi_{-a}\rangle\\ 0 & 0 & 0 & \langle H^d\rangle & \langle\phi_{-b}\rangle\\ 0 & 0 & \langle H^d\rangle & 0 & 0\\ 0 & \langle H^d\rangle & \langle H^d\rangle & 0 & \langle\chi_0\rangle\\ \langle\phi_{-m}\rangle & \langle\phi_{-m}\rangle & \langle\phi_{-m}\rangle & \langle\chi_{b-m}\rangle & 0\end{pmatrix}\end{array} \tag{A1}$$

This matrix produces the following down-type Yukawa couplings in the low energy theory:

$$\overline{d^g}\langle H^d\rangle\begin{pmatrix} 0 & \sim\epsilon_1 & \sim\epsilon_1\\ \sim\epsilon_3 & \sim\epsilon_2+\sim\epsilon_3 & \sim\epsilon_2+\sim\epsilon_3\\ 0 & 0 & \sim 1\end{pmatrix}_{gh} d^h, \tag{A2}$$

where

$$\epsilon_3=\frac{\langle\phi_{-m}\rangle}{\langle\chi_{b-m}\rangle}.$$

To see that, for example, the RH down squarks are not degenerate, we examine the squark mass (squared) matrix. For our purposes, we can ignore terms proportional to $\langle H^d\rangle$ (which would be of the same order as the bottom quark mass). In this approximation, the relevant superpotential couplings are

$$W \supset \sum_{i=1}^{3} \lambda_i \overline{\mathbf{5}}_0^i \mathbf{5}_m^{\overline{V}} \phi_{-m} + \lambda_V \overline{\mathbf{5}}_{-b}^V \mathbf{5}_m^{\overline{V}} \chi_{b-m} + \lambda_{\overline{V}} \overline{\mathbf{10}}_e^{\overline{V}} \mathbf{10}_0^V \chi_0, \tag{A3}$$

and the mass squared matrix is

$$\begin{pmatrix} |\lambda_1|^2|\phi|^2 & \lambda_1^*\lambda_2|\phi|^2 & \lambda_1^*\lambda_3|\phi|^2 & \lambda_1^*\lambda_V\phi^*\chi & 0 \\ \lambda_2^*\lambda_1|\phi|^2 & |\lambda_2|^2|\phi|^2 & \lambda_2^*\lambda_3|\phi|^2 & \lambda_2^*\lambda_V\phi^*\chi & 0 \\ \lambda_3^*\lambda_1|\phi|^2 & \lambda_3^*\lambda_2|\phi|^2 & |\lambda_3|^2|\phi|^2 & \lambda_3^*\lambda_V\phi^*\chi & 0 \\ \lambda_V^*\lambda_1\chi^*\phi & \lambda_V^*\lambda_2\chi^*\phi & \lambda_V^*\lambda_3\chi^*\phi & |\lambda_V|^2|\chi|^2 & 0 \\ 0 & 0 & 0 & 0 & |\lambda_{\overline{V}}|^2|\chi_0|^2 \end{pmatrix} \tag{A4}$$

where, for simplicity, $\phi = \phi_{-m}$ and $\chi = \chi_{b-m}$. This matrix has three zero eigenvalues and two eigenvalues of order $\langle \chi \rangle^2$. The three generations of squarks receive degenerate weak-scale contributions to their masses from the two loop diagrams in Fig. 1 of [4]. However, the FN fields receive large supersymmetry breaking contributions to their masses (of order $\sqrt{\tilde{m}^2}$). When $\tilde{m}^2$ is added to the (4,4) component of the matrix, there is one less zero eigenvalue. For $\tilde{m}^2 \lesssim \xi^2 \sim \langle \chi \rangle^2$ and $\langle \phi \rangle \ll \langle \chi \rangle$, this matrix has two eigenvalues of order $\langle \chi \rangle^2$, and one of order $(\langle \phi \rangle / \langle \chi \rangle)^2 \tilde{m}^2 = \epsilon_3^2 \tilde{m}^2$. In order to produce the correct mass ratios and mixing angles without significant fine-tuning, it turns out that ϵ_3 must be of order 10^{-2}. For $\tilde{m}^2 \sim (20\,\mathrm{TeV})^2$, the third eigenvalue is of order $(200\,\mathrm{GeV})^2$, thereby destroying the weak-scale degeneracy. A more careful analysis reveals additional flavor-dependent contributions at one loop. In fact, the only way to protect this degeneracy is to require all of the SU(5) multiplets (and hence, all flavons) to be uncharged under $U(1)_F$, clearly a useless choice.

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