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Exclusive Semileptonic and Rare B Meson Decays in QCD

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Abstract:

We present first complete results for the semileptonic and rare radiative form factors of B mesons weak decay into a light vector meson $(\rho, \omega, K^*, \phi)$ in the light-cone sum rule approach. The calculation includes radiative corrections, higher twist corrections and SU(3) breaking effects. The theoretical uncertainty is investigated in detail. A simple parametrization of the form factors is given in terms of three parameters each. We find that the form factors observe several relations inspired by heavy quark symmetry.

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1 Introduction

The challenge to understand the physics of CP violation related to the structure of the CKM mixing matrix in (and beyond) the Standard Model is fuelling an impressive experimental program for the study of B decays, both exclusive and inclusive. Abundant data in various exclusive channels are expected to arrive within the next few years from the dedicated B factories BaBar and Belle, and their potential impact on our understanding of CP violation at the electroweak scale will depend crucially on our possibility to control the effects of strong interaction. For exclusive decays with only one hadron in the final state the task is to calculate various transition form factors; it has already attracted significant attention in the literature.

In this paper we present the first complete results for the exclusive semileptonic and rare radiative B decays to light vector mesons in the light-cone sum rule approach. Exclusive decays which are the principal concern of this work can be grouped as semileptonic decays:

- $B_{u,d} \to \rho e \nu$,
- $B_s \to K^* e \nu$,

rare decays corresponding to $b \to s$ transitions which we term CKM-allowed:

- $B_{u,d} \to K^* + \gamma, \quad B_{u,d} \to K^* + l^+ l^-$,
- $B_s \to \phi + \gamma, \quad B_s \to \phi + l^+ l^-$,

and $b \to d$ transitions which we call CKM-suppressed:

- $B_d \to (\rho, \omega) + \gamma, \quad B_d \to (\rho, \omega) + l^+ l^-$,
- $B_u \to \rho + \gamma, \quad B_u \to \rho + l^+ l^-$,
- $B_s \to K^* + \gamma, \quad B_s \to K^* + l^+ l^-$.

Let V be a vector meson, i.e. ρ, ω, K^* or ϕ and let p_μ, ϵ^*_μ and m_V be its momentum, polarization vector and mass, respectively. Let p_B (m_B) be the momentum (mass) of the B meson. We define *semileptonic* form factors by ($q = p_B - p$)

$$\langle V(p)|(V-A)_\mu|B(p_B)\rangle = -i\epsilon^*_\mu(m_B+m_V)A_1^V(q^2) + i(p_B+p)_\mu(\epsilon^* p_B)\,\frac{A_2^V(q^2)}{m_B+m_V}$$

$$+iq_\mu(\epsilon^* p_B)\,\frac{2m_V}{q^2}\,\left(A_3^V(q^2)-A_0^V(q^2)\right) + \epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p_B^\rho p^\sigma\,\frac{2V^V(q^2)}{m_B+m_V}\,. \qquad (1.1)$$

Note the exact relations

$$A_3^V(q^2) = \frac{m_B+m_V}{2m_V}\,A_1^V(q^2) - \frac{m_B-m_V}{2m_V}\,A_2^V(q^2),$$

$$A_0^V(0) = A_3^V(0),$$

$$\langle V|\partial_\mu A^\mu|B\rangle = 2m_V(\epsilon^* p_B)A_0^V(q^2). \qquad (1.2)$$

The second relation in (1.2) ensures that there is no kinematical singularity in the matrix element at $q^2 = 0$.

Rare decays are described by the above semileptonic form factors and the following *penguin* form factors

$$\begin{aligned}\langle V|\bar{\psi}\sigma_{\mu\nu}q^{\nu}(1+\gamma_5)b|B(p_B)\rangle = {} & i\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p_B^{\rho}p^{\sigma}\, 2T_1(q^2) \\ & + T_2(q^2)\left\{\epsilon_{\mu}^{*}(m_B^2-m_V^2) - (\epsilon^{*}p_B)\,(p_B+p)_{\mu}\right\} \\ & + T_3(q^2)(\epsilon^{*}p_B)\left\{q_{\mu} - \frac{q^2}{m_B^2-m_V^2}\,(p_B+p)_{\mu}\right\} \qquad (1.3)\end{aligned}$$

with

$$T_1(0) = T_2(0). \qquad (1.4)$$

Here $\psi = s, d$. All signs are defined in such a way as to render the form factors positive.

The physical range of q^2 extends from $q^2_{\rm min} = 0$ to $q^2_{\rm max} = (m_B - m_V)^2$ for three-body decays and $q^2 \equiv 0$ for two-body decays.

The method of light-cone sum rules was first suggested for the study of weak baryon decays in [1] and later extended to heavy meson decays in [2]. It is a nonperturbative approach which combines ideas of QCD sum rules [3] with the twist expansion characteristic for hard exclusive processes in QCD [4] and makes explicit use of the large energy of the final state vector meson at small values of the momentum transfer to leptons, q^2. In this respect, the light-cone sum rule approach is complementary to lattice calculations [5] which are mainly restricted to form factors at small recoil (large values of q^2). Of course, the light-cone sum rules lack the rigour of the lattice approach. Nevertheless, they prove to provide a powerful nonperturbative model which is explicitly consistent with perturbative QCD and the heavy quark limit.

Early studies of exclusive B decays in the light cone sum rule approach were restricted to contributions of leading twist and did not take into account radiative corrections, see Refs. [6, 7] for a review and references to original publications. Very recently, these corrections have been calculated for the semileptonic $B \to \pi, Ke\nu$ decays [8]. In this work we calculate radiative and higher twist corrections to all form factors involving vector mesons, see above, making use of new results on distribution amplitudes of vector mesons, reported in [9, 10, 11]. We find that the corrections in question are fairly small in all cases.

The presentation is organized as follows: In Sec. 2 we remind basic ideas of the light-cone sum rule approach and derive radiative and higher-twist corrections to the form factors in question in a compact form. Section 3 presents our main results and includes discussion of input parameters as well as error estimates. In Sec. 4 we discuss relations between semileptonic and penguin form factors in the heavy quark limit. Section 5 is reserved to a summary and conclusions. The paper has two appendices: In App. A we collect the relevant loop-integrals for the calculation of radiative corrections. App. B contains a summary of the results of [9, 10, 11] on vector meson distribution amplitudes.

2 Method and Calculation

2.1 General Framework

Consider semileptonic $B_d \to \rho e \nu$ and rare $B_d \to K^* \ell^+ \ell^-$ decays as representative examples. We choose a B meson "interpolating current" $j_B = \bar{d} i \gamma_5 b$, so that

$$\langle 0 | j_B | B(p_B) \rangle = \frac{f_B m_B^2}{m_b}\,, \tag{2.1}$$

where f_B is the usual B decay constant and m_b the b quark mass. In order to obtain information on the form factors, we study the set of suitable correlation functions:[1]

$$\begin{aligned} i \int d^4y e^{-ip_B y} \langle \rho(p) | T(V-A)_\mu(0) j_B^\dagger(y) | 0 \rangle \; &= \; -i\Gamma^0(p_B^2, q^2)\epsilon_\mu^* \\ + i\Gamma^+(p_B^2, q^2)\, \frac{\epsilon^* q}{pq}\, (q+2p)_\mu &+ i\Gamma^-(p_B^2, q^2)\, \frac{\epsilon^* q}{pq}\, q_\mu + \Gamma^V(p_B^2, q^2) \epsilon_\mu{}^{\alpha\beta\gamma} \epsilon^*_\alpha q_\beta p_\gamma, \end{aligned} \tag{2.2}$$

$$\begin{aligned} i \int d^4y e^{-ip_B y} \langle K^*(p) | T[\bar{s}\sigma_{\mu\nu}\gamma_5 b](0) j_B^\dagger(y) | 0 \rangle \; &= \; \mathcal{A}(p_B^2, q^2)\{\epsilon^*_\mu (2p+q)_\nu - \epsilon^*_\nu (2p+q)_\mu\} \\ - \mathcal{B}(p_B^2, q^2)\{\epsilon^*_\mu q_\nu - \epsilon^*_\nu q_\mu)\} &- 2\mathcal{C}(p_B^2, q^2)\, \frac{\epsilon^* q}{pq}\, \{p_\mu q_\nu - q_\mu p_\nu\}. \end{aligned} \tag{2.3}$$

The Lorentz-invariant functions $\Gamma^{0,\pm,V}, \mathcal{A}, \mathcal{B}, \mathcal{C}$ can be calculated in QCD for large Euclidian p_B^2. More precisely, if $m_b^2 - p_B^2 \ll 0$, then the correlation functions in (2.2), (2.3) are dominated by the region of small y^2 and can systematically be expanded in powers of the deviation from the light-cone $y^2 = 0$. The light-cone expansion presents a modification of the usual Wilson operator product expansion, such that relevant operators are nonlocal and are classified in terms of twist rather than dimension. Matrix elements of nonlocal light-cone operators between the vacuum and the vector meson state define meson *distribution amplitudes* [4] which describe the partition of the meson momentum between the constituents in the infinite momentum frame. In particular, there exist two leading twist distribution amplitudes for vector mesons, see App. B, corresponding to longitudinal and transverse polarizations, respectively:

$$\langle \rho | \bar{u}(0) \gamma_\mu d(z) | 0 \rangle \; = \; f_\rho m_\rho p_\mu\, \frac{\epsilon^* z}{pz} \int_0^1 du\, e^{i\bar{u}pz}\, \phi_\parallel(u,\mu), \tag{2.4}$$

$$\langle \rho | \bar{u}(0) \sigma_{\mu\nu} d(z) | 0 \rangle \; = \; -i f_\rho^T(\mu) (\epsilon^*_\mu p_\nu - p_\mu \epsilon^*_\nu) \int_0^1 du\, e^{i\bar{u}pz}\, \phi_\perp(u,\mu), \tag{2.5}$$

[1] In this work we define invariant functions with respect to the Lorentz-structure $\frac{\epsilon^* q}{pq}$ instead of $\epsilon^* q$ [12] in order to remove a kinematical singularity for $p \to 0$.

and similarly for K^* and ϕ. Here z is an auxiliary light-like vector, u is the momentum fraction carried by the valence quark and the decay constants f_ρ, f_ρ^T are defined in App. B. μ specifies the scale: Extraction of the leading asymptotic behaviour in field theories invariably produces singularities which reflect themselves in the scale-dependence of distribution amplitudes. As always, this scale-dependence cancels in physical quantities by a corresponding dependence of coefficient functions.

The invariant amplitudes in (2.2), (2.3) can be calculated in terms of meson distribution amplitudes in complete analogy with the calculation of structure functions in deep inelastic lepton-nucleon scattering in terms of nucleon parton distributions: The off-shellness $m_b^2 - p_B^2$ plays the role of photon virtuality Q^2. As an illustration, consider the tree-level leading-twist result for Γ^0, adapted from Ref. [12]:

$$\Gamma^0(p_B^2, q^2) = \int_0^1 du \, \frac{1}{m_b^2 - up_B^2 - \bar{u}q^2} \, f_V^T \phi_\perp(u) \, \frac{m_b^2 - q^2}{2u}. \tag{2.6}$$

We want to emphasize that the procedure is rigorous at this point: all corrections can (in principle) be included in a systematic way and their evaluation is precisely what makes the subject of this work.

The subtle part concerns the extraction of the B meson contribution to the invariant amplitudes. The exact amplitude Γ^0 (in nature) has a pole at $p_B^2 = m_B^2$ corresponding to the intermediate B meson state, and this contribution can be written in terms of the form factor $A_1^{B\to\rho}$ defined in (1.1):

$$\Gamma^0_{B \text{ meson}} = (m_B + m_\rho) A_1^{B\to\rho}(q^2) \cdot \frac{1}{m_B^2 - p_B^2} \cdot \frac{m_B^2 f_B}{m_b} \tag{2.7}$$

On the other hand, the QCD calculation at $p_B^2 \ll m_b^2$ is only approximate and, continued analytically to "Minkowskian" $p_B^2 > m_b^2$, produces a smooth imaginary part with no sign of a pole-behaviour. To proceed, we invoke the concept of *duality*, assuming that exact spectral density and the one calculated in QCD coincide *on the average*, that is integrated over a sufficient region of energies. In particular, we assume that the B meson contribution is obtained by the integral of the QCD spectral density over the *duality region*:

$$\Gamma^0_{\text{B meson}} = \frac{1}{2\pi i} \int_{m_b^2}^{s_0} \frac{ds}{s - p_B^2} \, \text{Disc}_{\,p_B^2} \Gamma^0_{\text{QCD}}(s, q^2) \tag{2.8}$$

The parameter $s_0 \approx (34 - 35)$ GeV^2 is called "continuum threshold" and is fixed from QCD sum rules for f_B, see e.g. [13]. Equating the two above representations, one obtains a *light-cone sum rule* for the form factor A_1. Sum rules for the other form factors are constructed in precisely the same manner.

While the accuracy of the QCD calculation can be controlled (and improved), the duality approximation introduces an irreducible uncertainty in predictions for the form factors, which is usually believed to be of order (10–15)%. Practical calculations in the sum rule framework

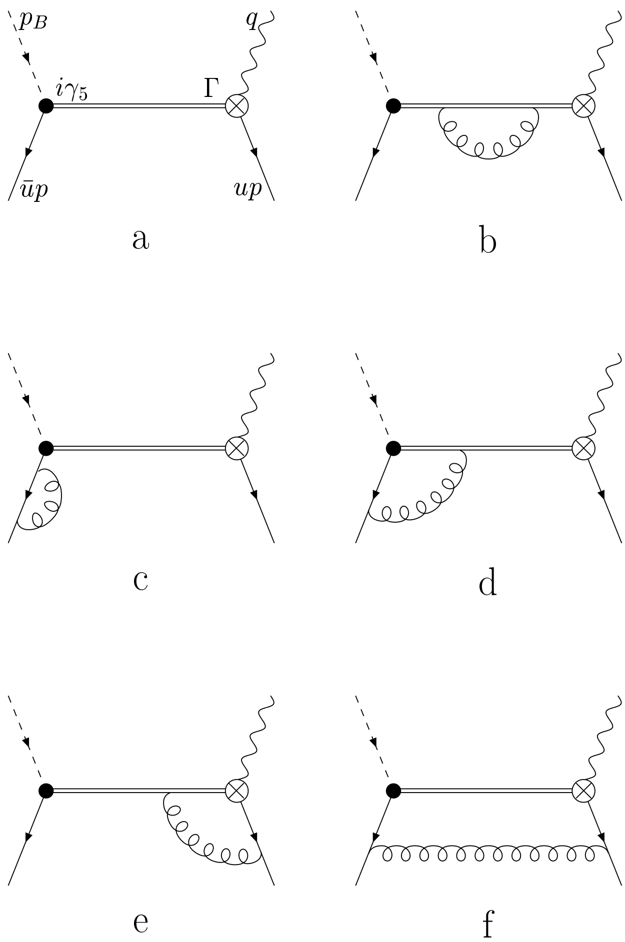

Figure 1: The leading order diagram (a) and one-loop radiative corrections (b-f).

involve some technical tricks to reduce this uncertainty, e.g. Borel transformation which we will not discuss here. These techniques are well established and their detailed description in the particular context of light-cone sum rules can be found e.g. in Refs. [7, 12]. The work [12] also contains a detailed comparison of the light-cone sum rule approach to traditional QCD sum rules and can serve as introduction for the more theoretically-minded reader.

2.2 Radiative Corrections

Radiative corrections to the sum rules correspond to one-loop corrections to the coefficient functions in front of leading twist distribution amplitudes and are given by the diagrams shown in Fig. 1. The calculation is done in dimensional regularization and it is sufficient to consider matrix elements over on-shell massless quark and antiquark carrying momentum fraction up and $\bar{u}p$, respectively. The transversely polarized and longitudinally polarized meson states are projected onto by

$$\langle V_\perp(p)|\bar u_a(0)d_b(x)|0\rangle = -\frac{i}{4}\, f_V^T[\sigma_{\mu\nu}]_{ba}\epsilon^{*\mu}p^\nu \int du\, e^{i\bar u px}\phi_\perp(u) \tag{2.9}$$

$$\equiv -\frac{1}{8}\, f_V^T[\sigma^{\mu\nu}\gamma_5]_{ba}\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\rho}p^\sigma \int du\, e^{i\bar u px}\phi_\perp(u), \tag{2.10}$$

$$\langle V_\parallel(p)|\bar u_a(0)d_b(x)|0\rangle \equiv \frac{1}{4}\, f_V m_V [\not p]_{ba}\,\frac{\epsilon^* x}{px} \int du\, e^{i\bar u px}\phi_\parallel(u)$$

$$\stackrel{m_V^2\to 0}{\longrightarrow} \frac{1}{4}\, f_V[\not p]_{ba} \int du\, e^{i\bar u px}\phi_\parallel(u), \tag{2.11}$$

where a, b are spinor indices, respectively. In the last line in (2.11) we made use of the fact that for ultrarelativistic longitudinal vector mesons $\epsilon_\mu \to p_\mu/m_V$ up to $O(m_V^2/|\vec p|^2)$ corrections. This is a justified approximation for the calculation of radiative corrections to leading twist accuracy to which end the meson mass can be neglected throughout. For further use we introduce notations for the projection operators:

$$\mathcal{P}_\parallel = \frac{1}{4} f_V \not p,$$

$$\mathcal{P}_\perp = -\frac{i}{4} f_V^T \sigma_{\alpha\beta}\epsilon^{*\alpha}p^\beta \quad \text{or} \quad \mathcal{P}_\perp^{(5)} = -\frac{1}{8} f_V^T \sigma_{\alpha\beta}\gamma_5\epsilon^{\alpha\beta\rho\sigma}\epsilon^*_\rho p_\sigma. \tag{2.12}$$

These will be treated as D-dimensional objects in what follows.

The calculation in question is in principle straightforward and similar to the existing calculations of NLO corrections to hard exclusive processes [14, 15, 16, 17]. One has to consider one-loop diagrams with a heavy quark and two different kinematic invariants q^2 and p_B^2, which makes formulas rather cumbersome, however. The specific requirement is to organize the expressions in a form suitable for a dispersion representation in p_B^2, cf. Eq. (2.8), so that continuum subtraction can be made.

Analytic expressions are available since recently for B decays to light pseudoscalar mesons π, K [8]. For vector mesons the number of form factors is so large that working out (relatively) compact analytic expressions is not worth the effort. In this work we prefer to give the formulae in terms of traces and general momentum integrals (see below and App. A) which can be compiled and evaluated numerically using MATHEMATICA programming language[2].

A usual subtlety concerns treatment of γ_5. The results for the form factors given below are obtained using "naive dimensional regularization" (NDR) and the same scheme has to be applied to the calculation of Wilson coefficients for penguin operators.

There are two form factors in whose calculation one encounters an odd number of γ_5 in traces, which could cause ambiguities: V and T_1. Only transverse mesons contribute

[2]The computer code is available from P.B. upon request.

to these form factors. In both cases, a possible ambiguity comes solely from the B vertex correction in Fig. 1d, whereas in all other diagrams contraction of γ matrices over γ_5 can be avoided. There are several ways out: (a) use a 't Hooft-Veltman prescription for γ_5 and apply a finite renormalization to restore the Ward identities as in [18]; (b) instead of the "natural" projection (2.9), use (2.10) which introduces a second γ_5 and thus eliminates the problem; (c) modify the definition of the form factors (1.3) to

$$\langle V|\bar{s}\sigma_{\mu\nu}\gamma_5 b|B\rangle = A(q^2)\left\{\epsilon^*_\mu(p_B+p)_\nu - (p_B+p)_\mu\epsilon^*_\nu\right\} - B(q^2)\left\{\epsilon^*_\mu q_\nu - q_\mu\epsilon^*_\nu\right\}$$
$$- C(q^2)\,\frac{\epsilon^* p_B}{m_B^2-m_V^2}\,\left\{(p_B+p)_\mu q_\nu - q_\mu(p_B+p)_\nu\right\}. \tag{2.13}$$

Using

$$\sigma_{\mu\nu}\gamma_5 = -\frac{i}{2}\,\epsilon_{\mu\nu\rho\sigma}\,\sigma^{\rho\sigma}$$

and contracting with q^ν, one finds

$$A(q^2) = T_1(q^2),$$
$$B(q^2) = \frac{m_B^2-m_V^2}{q^2}\left[T_1(q^2)-T_2(q^2)\right],$$
$$C(q^2) = T_3(q^2) - \frac{m_B^2-m_V^2}{q^2}\left[T_1(q^2)-T_2(q^2)\right], \tag{2.14}$$

from which the relation (1.4) follows. It is thus sufficient to calculate A, B and C instead of T_i with the premium to avoid any γ_5 problem. We have checked that all of the above prescriptions yield identical results.

After these preliminary remarks, we are now in the position to calculate the diagrams in Fig. 1. The tree-level contribution of Fig. 1a equals

$$T^{(0)} = \frac{i}{s}\,\mathrm{Tr}(\Gamma(\not{p}_B - \bar{u}\not{p} + m_b)\gamma_5\mathcal{P}), \tag{2.15}$$

where Γ is the Dirac-structure of the weak vertex, $\mathcal{P}$ is one of the projection operators defined in Eqs. (2.12) and

$$s = m_b^2 - up_B^2 - \bar{u}q^2.$$

It proves convenient to replace in Eq. (2.15) the running $\overline{\mathrm{MS}}$ b quark mass by the one-loop pole mass, which is given by

$$m_{\mathrm{pole}} = m_{\overline{\mathrm{MS}}}\left\{1 + C_F\,\frac{g^2}{4\pi^2}\left(1-\frac{3}{4}\,\ln\,\frac{m^2}{\mu^2}\right)\right\}. \tag{2.16}$$

This replacement induces the radiative correction

$$T^{\rm pole} = 2i\,\frac{m_b^2}{s^2}\,C_F\,\frac{g^2}{4\pi^2}\left(1-\frac{3}{4}\,\ln\,\frac{m_b^2}{\mu^2}\right)Tr(\mathcal{P}\Gamma(\not p_B-\bar u\not p+m_b)\gamma_5)$$
$$-\,\frac{i}{s}\,m_bC_F\,\frac{g^2}{4\pi^2}\left(1-\frac{3}{4}\,\ln\,\frac{m_b^2}{\mu^2}\right)Tr(\mathcal{P}\Gamma(\not p_B-\bar u\not p)\gamma_5). \tag{2.17}$$

The general strategy is to simplify the traces as much as possible, but to keep $\mathcal{P}$ and Γ arbitrary. Also contraction of γ matrices over γ_5 is only allowed in the B vertex correction.

It turns out that all one-loop diagrams can be expressed in terms of the following traces:

$$\begin{aligned}
\mathrm{Tr}_1 &= Tr\,(\mathcal{P}\Gamma\not q\gamma_5) \equiv Tr\,(\mathcal{P}\Gamma\not p_B\gamma_5)\,,\\
\mathrm{Tr}_2 &= Tr\,(\mathcal{P}\Gamma\gamma_5)\,,\\
\mathrm{Tr}_3 &= Tr\,(\mathcal{P}\not q\Gamma\gamma_5) \equiv Tr\,(\mathcal{P}\not p_B\Gamma\gamma_5)\,,\\
\mathrm{Tr}_4 &= Tr(\mathcal{P}\not q\Gamma\not q\gamma_5).
\end{aligned} \tag{2.18}$$

Let us also introduce

$$a_{\mathcal{P}}\mathcal{P} := \gamma_\alpha\mathcal{P}\gamma^\alpha, \qquad a_\Gamma\Gamma := \gamma_\alpha\Gamma\gamma^\alpha. \tag{2.19}$$

The b quark self-energy diagram in Fig. 1b is

$$T_b^{\rm SE} = -\frac{g^2C_F}{s^2}\,\Big\{\big[4m_b^2\,(Y+(1-\epsilon)Z)+2s(1-\epsilon)(Y-Z)\big]\,(\,\mathrm{Tr}_1+m_b\,\mathrm{Tr}_2)$$
$$-2m_bs\,(Y+(1-\epsilon)Z)\;\mathrm{Tr}_2\Big\}\,, \tag{2.20}$$

where $D=4-2\epsilon$ and the expressions for momentum integrals Y, Z are given in App. A. The self-energy insertions in external light quark legs in Fig. 1c do only contribute logarithmic terms in dimensional regularization,

$$T_l^{\rm SE} = \frac{g^2C_F}{s}\,(\mathrm{Tr}_1+m_b\mathrm{Tr}_2)\left(\ln\frac{m_b^2}{\mu_{UV}^2}-\ln\frac{m_b^2}{\mu_{IR}^2}\right), \tag{2.21}$$

where we distinguish between the ultra-violet scale μ_{UV}, which is to be identified with the renormalization scale of the curent j_B and the penguin operators, and the infra-red renormalization-scale μ_{IR} corresponding to the factorization scale in meson distribution amplitudes.

For the B vertex correction in Fig. 1d, one obtains:

$$T^B = 2\,\frac{g^2C_F}{s}\,\Big\{\big(-8\bar C(1-\epsilon)-1-m_b^2\bar B+\bar u(p_B^2-q^2)\bar A\big)\,(\,\mathrm{Tr}_1+m_b\,\mathrm{Tr}_2)-m_bs\bar B\,\mathrm{Tr}_2\Big\}\,. \tag{2.22}$$

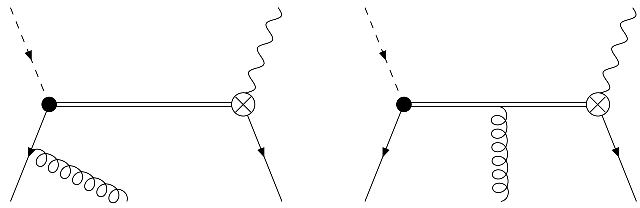

Figure 2: The higher-twist contributions.

For the weak vertex in Fig. 1e we find

$$T^W = -\frac{g^2 C_F}{s}\left\{a_\Gamma^2\, C\,(\,\mathrm{Tr}_1 + m_b\,\mathrm{Tr}_2) + a_\Gamma\, D\,(q^2\,\mathrm{Tr}_3 + m_b\,\mathrm{Tr}_4) + (p_B^2 - q^2)ua_\Gamma\, E\,\mathrm{Tr}_3\right.$$
$$\left. +2(p_B^2 - q^2)uA(\,\mathrm{Tr}_1 + m_b\,\mathrm{Tr}_2) + m_b B\left[2(m_b\,\mathrm{Tr}_1 + q^2\,\mathrm{Tr}_2) - a_\Gamma(m_b\,\mathrm{Tr}_3 + \,\mathrm{Tr}_4)\right]\right\}. \quad (2.23)$$

Finally, the box-diagram in Fig. 1f can be written as

$$T^{\mathrm{box}} = -g^2 C_F a_{\mathcal{P}}\left\{a_{\mathcal{P}} H(\,\mathrm{Tr}_1 + m_b\,\mathrm{Tr}_2) + I(-m_b\,\mathrm{Tr}_4 + (s - m_b^2)\,\mathrm{Tr}_3) + B_{u=1}\,\mathrm{Tr}_3\right\}, \quad (2.24)$$

where $B_{u=1}$ is the limiting value of B for $u \to 1$

Definitions and explicit expressions for the one-loop integrals A, B, C etc. are given in App. A.

2.3 Higher Twist Contributions

Higher twist terms generically refer to contributions to the light-cone expansion of the correlation functions (2.2) and (2.3) which are suppressed by powers of $1/(m_b^2 - p_B^2)$. In the sum rules, such corrections are suppressed by powers of the Borel parameter. Higher twist corrections are usually divided into "kinematical", originating from nonzero mass of the vector meson, and "dynamical", related to contributions of higher Fock states and transverse quark motion. In this paper we take into account both effects to twist 4 accuracy, making use of the new results on distribution amplitudes of vector mesons reported in [10, 11] and summarized in App. B.

The calculation is most conveniently done using the background field approach of [19]. The diagrams of the type shown in Fig. 2a are taken into account within this method by the expansion of the nonlocal quark-antiquark operator in powers of the deviation from the light-cone and give rise to contributions of two-particle distribution amplitudes of higher twist, see Eqs. (B.12) and (B.27). The contribution of the gluon emission from heavy quark is calculated using the light-cone expansion of the quark propagator [19, 20]:

$$\langle 0|T\{b(x)\bar{b}(0)\}|0\rangle =$$

$$= iS_b(x) - ig\int\frac{d^4k}{(2\pi)^4}\,e^{-ikx}\int_0^1 dv\left[\frac{1}{2}\,\frac{\not k + m_b}{(m_b^2-k^2)^2}\,G^{\mu\nu}(vx)\sigma_{\mu\nu} + \frac{v}{m_b^2-k^2}\,x_\mu G^{\mu\nu}(vx)\gamma_\nu\right],\tag{2.25}$$

where $S_b(x)$ is the free quark propagator. As in the case of radiative corrections, our strategy in this work is to derive the most general expression for all form factors in question, suitable for implementation in analytic/numerical calculations using MATHEMATICA. We obtain

$$\begin{aligned}
\mathrm{CF} = \frac{1}{4}\int_0^1 du\Big\{ & if_V m_V\Big[\Big(\Phi_\parallel^{(i)}(u)\epsilon^*_\alpha\,\frac{\partial}{\partial Q_\alpha} + \frac{\epsilon^* q}{pq}\,\frac{1}{16}\,m_V^2\mathbb{A}(u)\,\frac{\partial^2}{\partial Q_\rho\partial Q^\rho}\Big)\,Tr(\Gamma S_b(Q)\gamma_5\not p) \\
& - g_\perp^{(v)}(u)Tr(\Gamma S_b(Q)\gamma_5\not\epsilon^*) - \frac{\epsilon^* q}{pq}\,\frac{1}{2}\,m_V^2\mathbb{C}^{(i)}(u)\,\frac{\partial}{\partial Q_\alpha}\,Tr(\Gamma S_b(Q)\gamma_5\gamma_\alpha) \\
& -\frac{i}{4}\,\epsilon_{\alpha\beta\gamma\delta}\epsilon^{*\beta}p^\gamma g_\perp^{(a)}(u)\,\frac{\partial}{\partial Q_\delta}Tr(\Gamma S_b(Q)\gamma_\alpha)\Big] - f_V^T\Big[\Big(\phi_\perp(u) - \frac{1}{16}\,m_V^2\mathbb{A}_T(u)\,\frac{\partial^2}{\partial Q_\rho\partial Q^\rho}\Big) \\
& \times Tr(\Gamma S_b(Q)\gamma_5\sigma_{\alpha\beta})\epsilon^{*\alpha}p^\beta - \frac{\epsilon^* q}{pq}\,m_V^2\mathbb{B}_T^{(i)}(u)p^\alpha\,\frac{\partial}{\partial Q_\beta}Tr(\Gamma S_b(Q)\gamma_5\sigma_{\alpha\beta}) \\
& - \frac{i}{2}\,\Big(1 - \frac{m_q + m_{\bar q}}{m_V}\,\frac{f_V}{f_V^T}\Big)\,m_V^2 h_\parallel^{(s)}(u)\epsilon^{*\alpha}\,\frac{\partial}{\partial Q_\alpha}\,Tr(\Gamma S_b(Q)\gamma_5) \\
& - \frac{1}{2}\,m_V^2\mathbb{C}_T^{(i)}(u)\epsilon^{*\alpha}\,\frac{\partial}{\partial Q_\beta}\,Tr(\Gamma S_b(Q)\gamma_5\sigma_{\alpha\beta})\Big]\Big\} \\
& +\frac{i}{4}f_V m_V\int_0^1 dv\int\mathcal{D}\alpha\,\Big[m_b^2 - (q + (\alpha_1 + v\alpha_3)p)^2\Big]^{-2}\Big[2v(pq)\,(\mathcal{A}(\underline{\alpha}) + \mathcal{V}(\underline{\alpha}))\,Tr(\Gamma\not\epsilon^*\not p\gamma_5) \\
& + m_V^2\,\frac{\epsilon^* q}{pq}\,\Big(2\Phi(\underline{\alpha}) + \Psi(\underline{\alpha}) - 2\widetilde{\Phi}(\underline{\alpha}) - \widetilde{\Psi}(\underline{\alpha})\Big)\,Tr(\Gamma(\not q + m_b)\not p\gamma_5) \\
& +4m_V^2 v(\epsilon^* q)\,\Big(\widetilde{\Phi}(\underline{\alpha}) - \Phi(\underline{\alpha})\Big)\,Tr(\Gamma\gamma_5) - m_V^2 v\,\frac{\epsilon^* q}{pq}\,\Psi(\underline{\alpha})Tr(\Gamma\not q\not p\gamma_5)\Big] \\
& +\frac{i}{4}f_V^T m_V^2\int_0^1 dv\int\mathcal{D}\alpha\,\Big[m_b^2 - (q + (\alpha_1 + v\alpha_3)p)^2\Big]^{-2}\Big[-2v(\epsilon^* q)\mathcal{T}(\underline{\alpha})Tr(\Gamma\not p\gamma_5) \\
& + \Big(\mathcal{S}(\underline{\alpha}) - \widetilde{\mathcal{S}}(\underline{\alpha}) + T_1^{(4)}(\underline{\alpha}) - T_2^{(4)}(\underline{\alpha}) + T_3^{(4)}(\underline{\alpha}) - T_4^{(4)}(\underline{\alpha})\Big)\,Tr(\Gamma(\not q + m_b)\not\epsilon^*\not p\gamma_5) \\
& + 2v\,\Big(T_2^{(4)}(\underline{\alpha}) - T_4^{(4)}(\underline{\alpha}) - \mathcal{S}(\underline{\alpha}) - \widetilde{\mathcal{S}}(\underline{\alpha})\Big)\,\Big[(\epsilon^* q)Tr(\Gamma\not p\gamma_5) - (pq)Tr(\Gamma\not\epsilon^*\gamma_5)\Big] \\
& + 2v\,\Big(T_3^{(4)}(\underline{\alpha}) - T_4^{(4)}(\underline{\alpha}) - \widetilde{\mathcal{S}}(\underline{\alpha})\Big)\,Tr(\Gamma\not q\not p\not\epsilon^*\gamma_5)\Big],
\end{aligned}\tag{2.26}$$

where $Q = q + \bar{u}p$ and CF $\in \{\Gamma^{0,\pm,V}, \mathcal{A}, \mathcal{B}, \mathcal{C}\}$. Definitions and explicit expressions for the numerous distribution amplitudes are collected in App. B[3]. In addition, we use the notation

$$\Phi_{\parallel}^{(i)}(u) = \int_0^u dv\left(\phi_{\parallel}(v) - g_{\perp}^{(v)}(v)\right). \tag{2.27}$$

To leading twist accuracy, our result agrees with the expressions available in the literature, see [21, 12, 22][4].

3 Results

In this section we present results of the numerical analysis of the light-cone sum rules for the form factors defined in (1.1) and (1.3) for B and B_s decays. The sum rules depend on several parameters, those needed to describe the B meson on the one hand and those describing the vector meson on the other hand. The former ones are essentially f_B (f_{B_s}), the leptonic decay constant defined in (2.1), the b quark mass m_b, the continuum threshold s_0 introduced in (2.8) and the Borel parameter M^2 mentioned in Sec. 2.1. Lacking experimental determination of f_B and f_{B_s}, we determine their values from two-point QCD sum rules to $O(\alpha_s)$ accuracy (see e.g. [13]), which fixes s_0 depending on m_b and also the "window" in M^2, in which the sum rules are evaluated. We then use the same values for m_b, s_0 and M^2 in both the QCD sum rule for f_B and the light-cone sum rules for the form factors,[5] which helps to reduce the systematic uncertainty of the approach. The corresponding parameter sets and results for the decay constants are given in Tab. 1. The question of the value of the b quark mass has attracted considerable attention recently; following these developments [23], we use the value $m_b = (4.8 \pm 0.1)$ GeV. Our results for f_B agree well with new lattice determinations [24].

The parameters of light mesons are collected in App. B, Tabs. A and B. These parameters are evaluated at the factorization scale $\mu_{IR}^2 = m_B^2 - m_b^2 = 4.8\,\text{GeV}^2$, which is the typical virtuality of the virtual b quark in the process. The penguin form factors also depend on the ultra-violet renormalization scale of the effective weak Hamiltonian, for which we choose $\mu_{UV} = m_b$. Using the central values of all parameters, we obtain the form factors plotted in Figs. 3 and 4. For their representation in algebraic form, a parametrization in terms of three parameters proves convenient:

$$F(q^2) = \frac{F(0)}{1 - a_F\,\frac{q^2}{m_B^2} + b_F\left(\frac{q^2}{m_B^2}\right)^2}, \tag{3.1}$$

[3]Despite appearance, the number of nonperturbative parameters in the description of higher-twist distributions is small since they are related by exact equations of motion, see [10, 11] and App. B.

[4]The sum rule for T_1 given in [21, 22] misses a contribution of $\Phi_{\parallel}$; this term can formally be viewed as part of the kinematic higher twist correction which is included in [21, 22] only partially.

[5]To be precise, the expansion parameter of the light-cone correlation function is uM^2 rather than M^2. Because of this, in the light-cone sum rules we use an "effective" Borel parameter M^2_{eff} defined by $\langle u\rangle M^2_{\text{eff}} \equiv M^2_{2pt}$, M^2_{2pt} being the Borel parameter used in the QCD sum rules for f_B.

m_b [GeV]	4.7	4.8	4.9
s_0 [GeV^2]	34.5 ± 0.5	33.5 ± 0.5	32.5 ± 0.5
f_B [MeV]	177 ± 5	150 ± 5	123 ± 5
s_0 [GeV^2]	35.5 ± 0.5	34.5 ± 0.5	33.5 ± 0.5
f_{B_s} [MeV]	191 ± 5	162 ± 5	135 ± 5

Table 1: Values for f_B and f_{B_s} from QCD sum rules in dependence on the b quark mass. The Borel parameter window is $M^2 = (4-8)\,\text{GeV}^2$.

with the fit parameters $F(0)$, a_F and b_F. Here m_B is the mass of the relevant B meson, i.e. $m_{B_{u,d}}$ for $B_{u,d}$ decays and m_{B_s} for B_s decays. This parametrization describes all 28 form factors to an accuracy of 1.8% or better for $0 \leq q^2 \leq 17\,\text{GeV}^2$.

Let us now discuss the dependence of the results on the input parameters and approximations involved. First we note that the net impact of radiative corrections is very small at small q^2 and at most 5% at $q^2 = 0$. Their effect increases, however, at large q^2 and leads to a decrease of the form factors A_2 and T_3 at $q^2 = 17\,\text{GeV}^2$ by 20% with respect to their tree-level values; the impact on the other form factors stays in the 5% range. The small effect of radiative corrections was anticipated in the tree-level analysis of Ref. [12] and also observed in the calculation of $O(\alpha_s)$ corrections for $B \to$ pseudoscalar decays [8]. It is due to the fact that the biggest contribution to radiative corrections (in Feynman gauge) comes from the B vertex correction diagram, which enters both the calculation of f_B and the light-cone correlation functions and cancels in the ratio that gives the form factors. Although literally we only calculated radiative corrections to the leading twist contribution to the light-cone expansion, it is unlikely that yet unknown corrections to the higher-twist terms could change this pattern dramatically. We thus believe that radiative corrections are under good control.

The next question concerns the convergence of the light-cone expansion. The higher-twist terms have several sources: some depend on the intrinsic properties of the multiparticle Fock-states of the vector meson and some appear as meson mass corrections to the two-particle valence state. The latter ones, described in terms of the same parameters as the leading twist distribution amplitudes, turn out to be numerically dominant, which is very welcome as the matrix elements describing the multiparticle states are only poorly known. To be specific, putting all intrinsic higher-twist parameters ζ of Tab. B to zero, the form factors change by at most 3%. Hence, we conclude that the light-cone expansion is under good control as well.

The dependence of form factors on the sum rule parameters is small, too. Changing m_b by $\pm 100\,\text{MeV}$ makes a 5% effect at most and is most pronounced at large q^2; at $q^2 = 0$ it is a 0.8% effect. This result means that, like for radiative corrections, there is a strong cancellation of m_b dependence in the ratio of the light-cone correlation function and f_B. The same statement holds for the dependence on the continuum threshold within the limits specified in Tab. 1. For the dependence on the Borel parameter we find an $\sim 7\%$ effect,

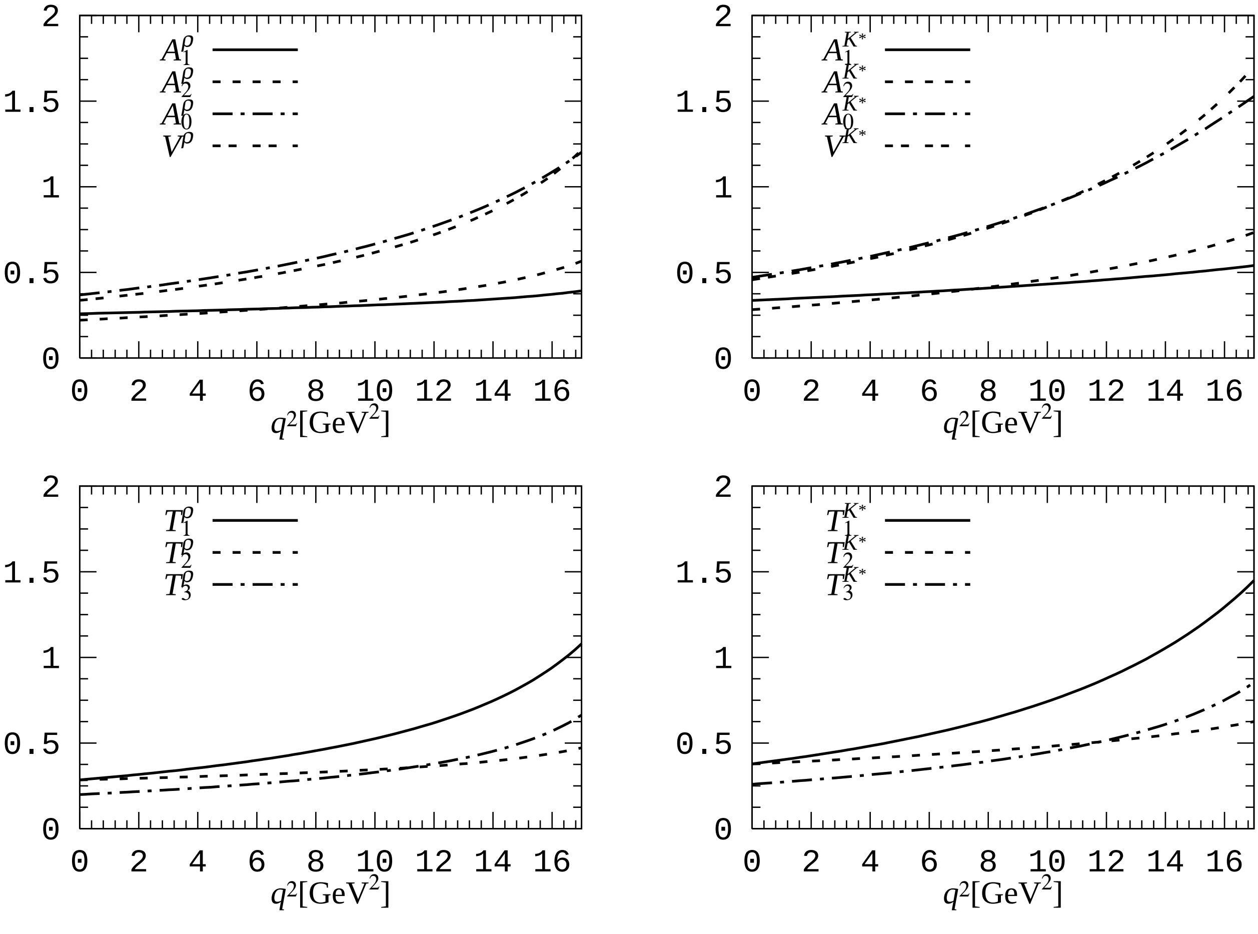

Figure 3: Light-cone sum rule results for $B_{u,d} \to$ vector meson form factors. Renormalization scale for T_i is $\mu = m_b = 4.8\,\mathrm{GeV}$. Further parameters: $m_b = 4.8\,\mathrm{GeV}$, $s_0 = 33.5\,\mathrm{GeV}^2$, $M^2 = 6\,\mathrm{GeV}^2$.

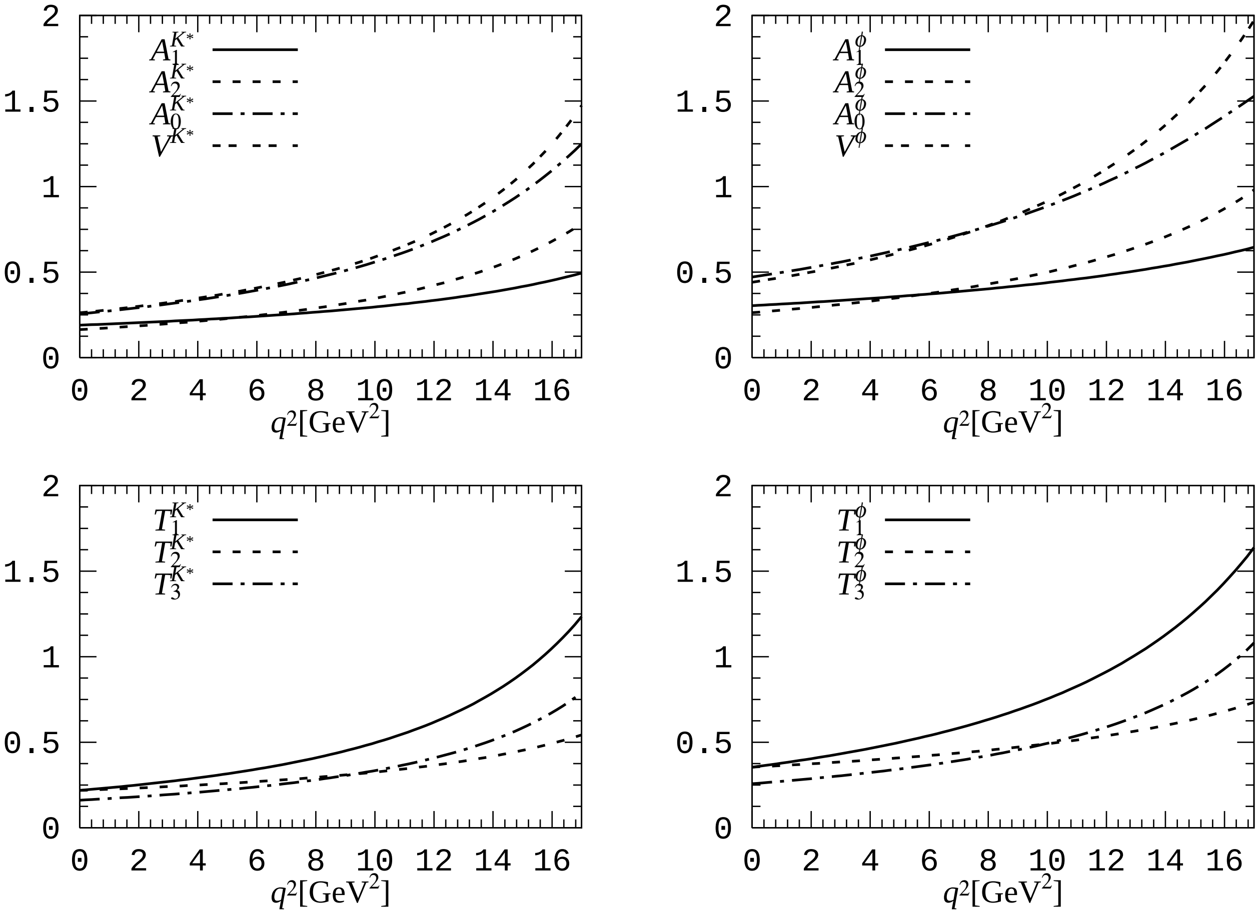

Figure 4: Light-cone sum rule results for $B_s \to$ vector meson form factors. Renormalization scale for T_i is $\mu = m_b = 4.8\,\mathrm{GeV}$. Further parameters: $m_b = 4.8\,\mathrm{GeV}$, $s_0 = 34.5\,\mathrm{GeV}^2$, $M^2 = 6\,\mathrm{GeV}^2$.

	$F(0)$	a_F	b_F	$F(0)$	a_F	b_F	
A_1^ρ	0.261	0.29	-0.415	0.337	0.60	-0.023	$A_1^{K^*}$
A_2^ρ	0.223	0.93	-0.092	0.283	1.18	0.281	$A_2^{K^*}$
A_0^ρ	0.372	1.40	0.437	0.470	1.55	0.680	$A_0^{K^*}$
V^ρ	0.338	1.37	0.315	0.458	1.55	0.575	V^{K^*}
T_1^ρ	0.285	1.41	0.361	0.379	1.59	0.615	$T_1^{K^*}$
T_2^ρ	0.285	0.28	-0.500	0.379	0.49	-0.241	$T_2^{K^*}$
T_3^ρ	0.202	1.06	-0.076	0.261	1.20	0.098	$T_3^{K^*}$

Table 2: $B_{u,d}$ decay form factors in a three parameter fit. Renormalization scale for T_i is $\mu = m_b = 4.8\,\text{GeV}$. The theoretical uncertainty is estimated as 15%.

	$F(0)$	a_F	b_F	$F(0)$	a_F	b_F	
$A_1^{K^*}$	0.190	1.02	-0.037	0.296	0.87	-0.061	A_1^ϕ
$A_2^{K^*}$	0.164	1.77	0.729	0.255	1.55	0.513	A_2^ϕ
$A_0^{K^*}$	0.254	1.87	0.887	0.382	1.77	0.856	A_0^ϕ
V^{K^*}	0.262	1.89	0.846	0.433	1.75	0.736	V^ϕ
$T_1^{K^*}$	0.219	1.93	0.904	0.348	1.82	0.825	T_1^ϕ
$T_2^{K^*}$	0.219	0.85	-0.271	0.348	0.70	-0.315	T_2^ϕ
$T_3^{K^*}$	0.161	1.69	0.579	0.254	1.52	0.377	T_3^ϕ

Table 3: B_s decay form factors in a three parameter fit. Renormalization scale for T_i is $\mu = m_b = 4.8\,\text{GeV}$. The theoretical uncertainty is estimated as 15%.

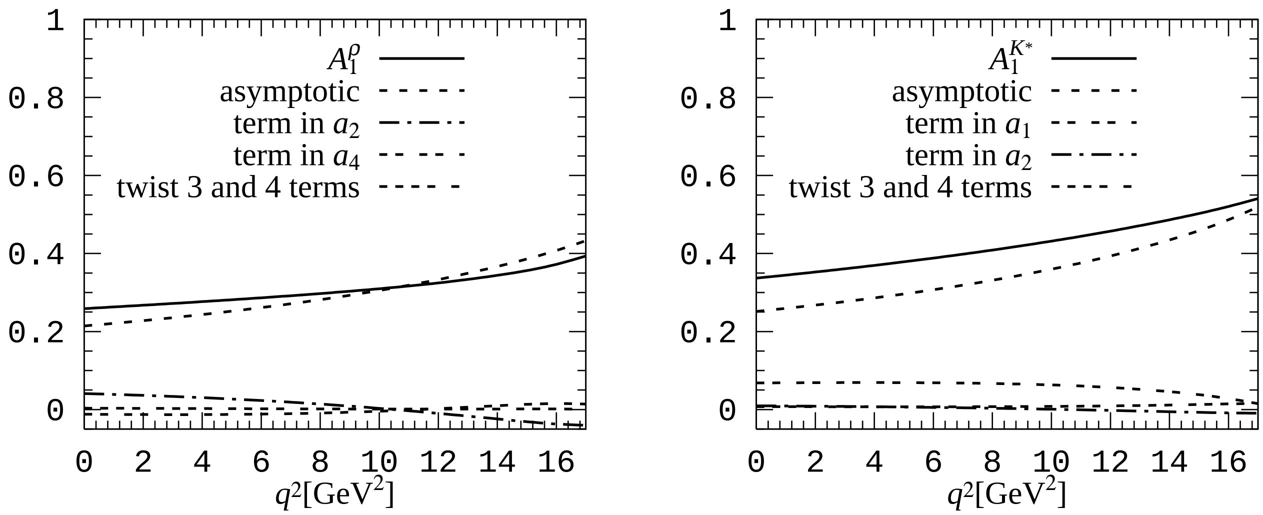

Figure 5: Separate contributions to the form factors A_1^ρ and $A_1^{K^*}$.

increasing with q^2, which again reminds us of the fact that the light-cone sum rules become less reliable for large q^2.

Overall normalization of the form factors depends on the vector meson decay constants f_V and f_V^T, the former one determined experimentally, the latter one calculated from QCD sum rules (see Tab. A). The corresponding uncertainty is at most 3%.

Adding up all the errors in quadrature, we obtain an uncertainty of the form factors of ca. 11%.

The shape of leading twist distribution amplitudes, characterized by the the Gegenbauer moments $a_{2,\rho}^{\parallel,\perp}$ for the ρ and $a_{1,K^*}^{\parallel,\perp}$, $a_{2,K^*}^{\parallel,\perp}$ for the K^*, affects most significanly the slope of the form factors and is illustrated in Fig. 5 on two examples: A_1^ρ and $A_1^{K^*}$. The curves labeled "asymptotic" designate the form factors as obtained by putting the a_i to zero in Eqs. (B.15) and (B.30); the corresponding meson distribution amplitudes are completely model-independent and dictated by perturbative QCD. The curves labeled "a_i" show corrections to this limit which take into account nonperturbative corrections to the distribution amplitudes; for illustration we assumed in this figure the value $a_{4,\rho}^\perp = a_{4,\rho}^\parallel = 0.1$ at $\mu = 1\,\text{GeV}$ as a ball-park estimate for potential higher-order terms; this contribution is not included in the final results. The curves labeled "twist 3 and 4 terms" show the contribution induced by the ζs in Tab. B and for the K^* also contain terms explicitly proportional to the strange quark mass. It is obvious that the "asymptotic" contribution grossly dominates, and the remaining terms only add marginal corrections. It is also obvious that the twist 3 and 4 terms do not have much overall influence, whereas the contribution in a_2 (for A_1^ρ) and a_1 (for $A_1^{K^*}$) tend to slow down the increase of the form factors as functions of q^2. All involved parameters (except for the couplings f_V and f_V^T) come with considerable theoretical uncertainty. However, the only important error is that in $a_{2,\rho}$ and a_{1,K^*}. Taken together, they contribute of order 10% to the uncertainty in our predictions. Adding this number (in quadrature) to

F	A_1	A_2	V	T_1	T_3
$F^{K^*}(0)/F^\rho(0)$	1.30 ± 0.13	1.28 ± 0.13	1.36 ± 0.14	1.33 ± 0.13	1.29 ± 0.13

Table 4: Size of SU(3) breaking for $B_{u,d}$ decays into ρ or K^*.

the $\sim 11\%$ error from other sources, we end up with a total uncertainty of light-cone sum rule preductions of order $\sim 15\%$, which is our final error estimate. An improvement is to be expected if future lattice calculations achieve an accuracy better than that quoted in Tab. A.

A few remarks are in order on the pattern of SU(3) symmetry breaking. It enters our calculation at the following places:

- difference in decay constants: $f_{K^*}/f_\rho \approx f_{K^*}^T/f_\rho^T = 1.14$, $f_{B_s}/f_B = 1.08$;
- different meson masses and continuum thresholds s_0 (Tab. 1);
- different vector meson distribution amplitudes (Tab. A).

Fig. 5 also illustrates the relative size of these effects: the difference between the "asymptotic" curves is almost exclusively due to the difference in f_ρ and f_{K^*} and makes a 17% effect. For K^*, the a_2 are small, whereas the a_1 are large and thus increase the form factor. For $B_s \to \bar{K}^*$ decays, the sign in a_1 is negative and f_{B_s} is larger than f_B, so that we observe considerably smaller form factors, see Tab. 3. The total SU(3) breaking corrections amount to $\approx 35\%$, half of which comes from the decay constants and half from the bigger momentum carried by the s quark in the strange hadron. Specifically, for $B_{u,d}$ decay form factors at $q^2 = 0$ we obtain the values given in Tab. 4.

In Fig. 6 we present a comparison of our results for $B \to \rho$ semileptonic and rare radiative form factors with the lattice calculations by the UKQCD collaboration [25, 26]. The agreement is very good. We wish to emphasize that the light-cone sum rule approach is theoretically more sound at small values of q^2, and in this sense is complementary to lattice techniques which work best in the large q^2 region. A similar comparison for $B \to K^*$ decays is presented in Fig. 7. The agreement is somewhat worse in this case; the lattice data favor smaller SU(3) breaking effects. This question deserves further study.

Finally, in Tab. 5 we present a comparison of the results of this work for the form factor values at $q^2 = 0$ with earlier sum rule calculations and the lattice results obtained using the light-cone sum rule constraints.

4 The Heavy Quark Limit

The behaviour of B decay form factors in the limit $m_b \to \infty$ is interesting for various reasons. This limit was already discussed in some detail in Refs. [21, 12, 8] so that in this paper we only summarize the main points.

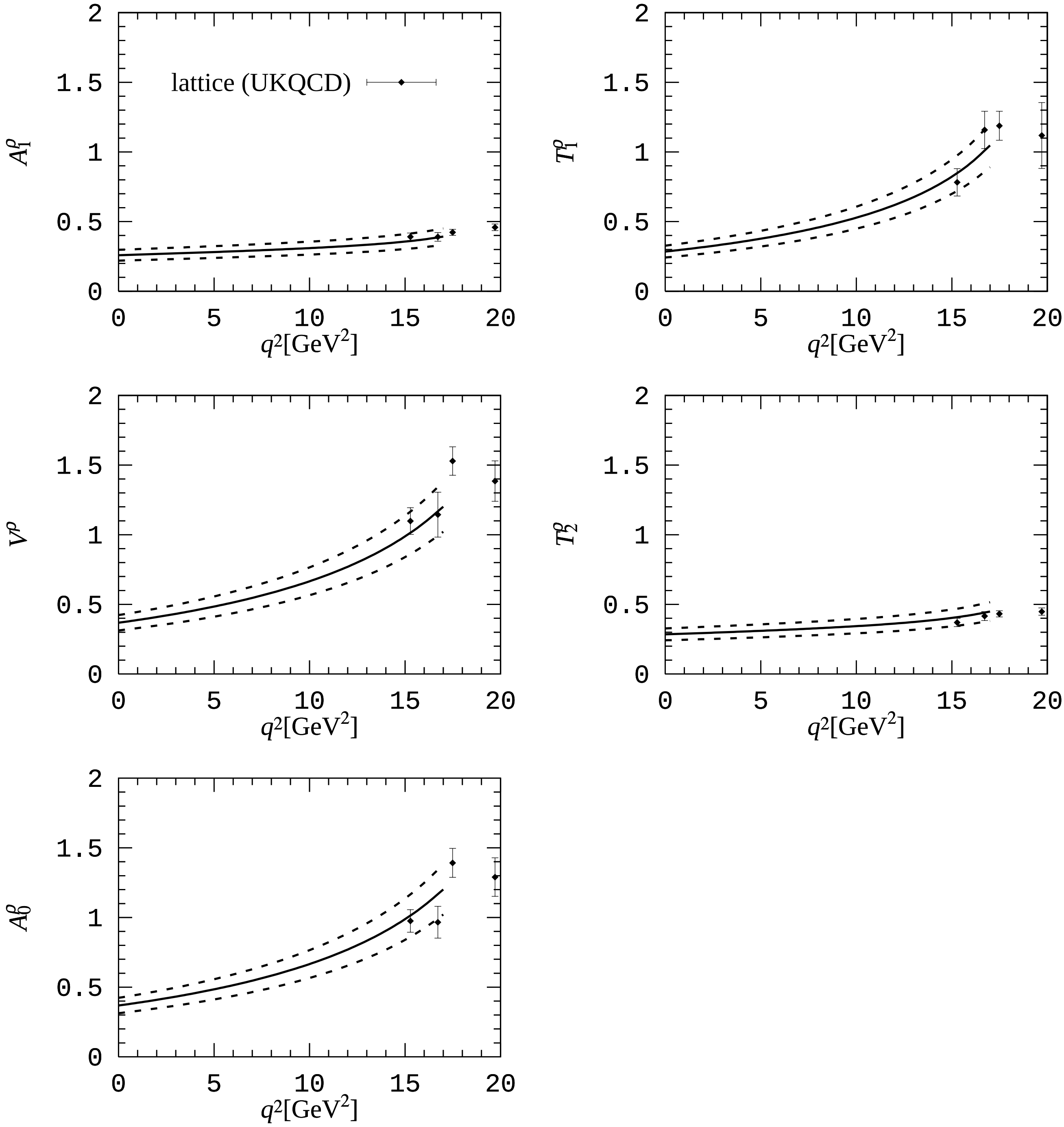

Figure 6: Comparison of the light-cone sum rule predictions for the $B \to \rho$ form factors with lattice calculations [25, 26]. Lattice errors are statistical only. The dashed curves show the 15% uncertainty range.

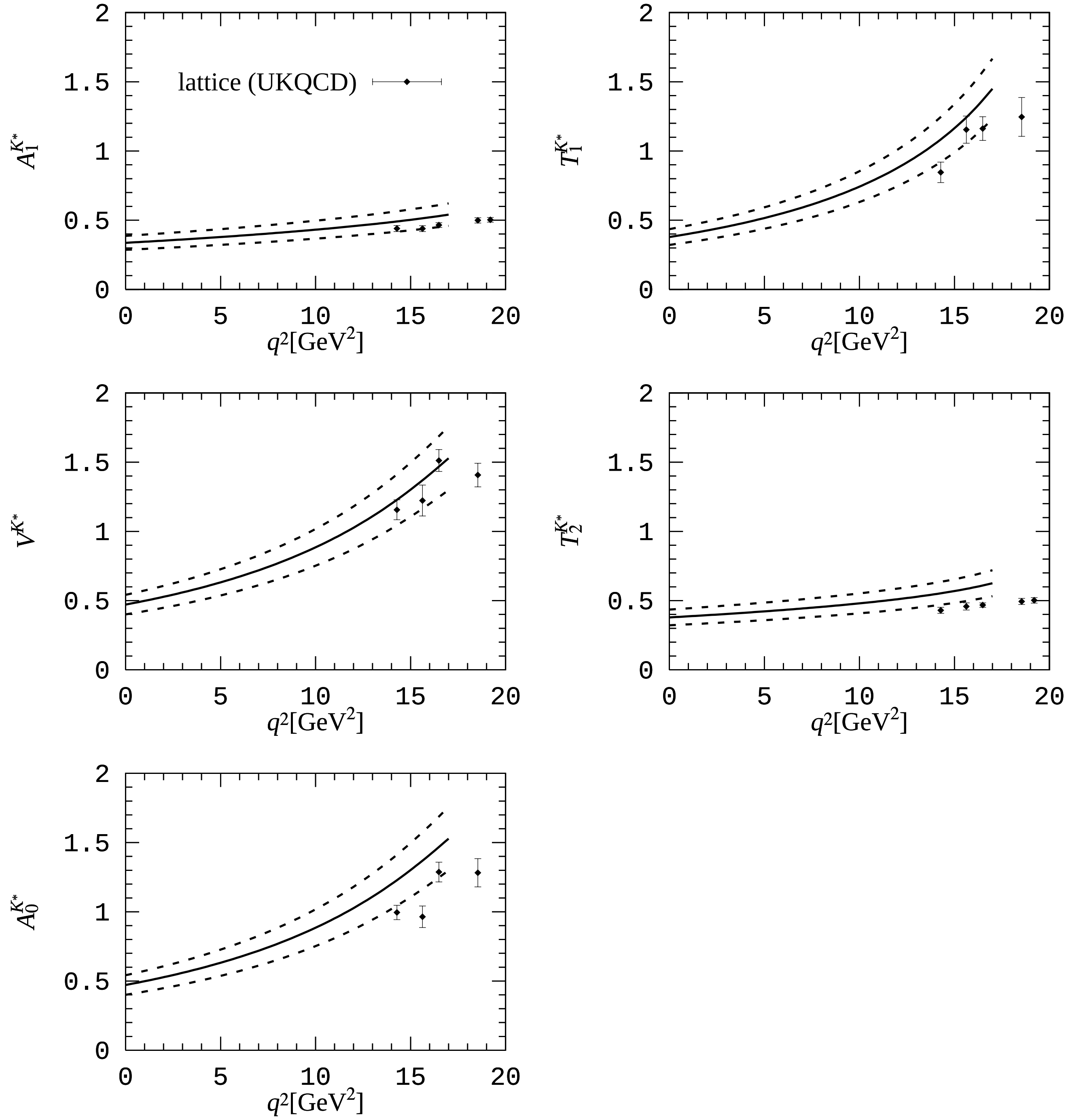

Figure 7: Comparison of the light-cone sum rule predictions for the $B \to K^*$ form factors with lattice calculations [25, 26]. Lattice errors are statistical only. The dashed curves show the 15% uncertainty range.

	this work (LCSR)	[21, 12] (LCSR)	[22] (LCSR)	[26](lattice +LCSR)	[27] (3ptSR)
$A_1^\rho(0)$	0.26 ± 0.04	0.27 ± 0.05	0.30 ± 0.05	$0.27^{+0.05}_{-0.04}$	0.5 ± 0.1
$A_2^\rho(0)$	0.22 ± 0.03	0.28 ± 0.05	0.33 ± 0.05	$0.26^{+0.05}_{-0.03}$	0.4 ± 0.2
$V(0)^\rho$	0.34 ± 0.05	0.35 ± 0.07	0.37 ± 0.07	$0.35^{+0.06}_{-0.05}$	0.6 ± 0.2
$T_1^\rho(0)$	0.29 ± 0.04	0.24 ± 0.07	0.30 ± 0.10	–	–
$T_3^\rho(0)$	0.20 ± 0.03	–	0.20 ± 0.10	–	–
$A_1^{K^*}(0)$	0.34 ± 0.05	0.32 ± 0.06	0.36 ± 0.05	$0.29^{+0.04}_{-0.03}$	0.37 ± 0.03
$A_2^{K^*}(0)$	0.28 ± 0.04	–	0.40 ± 0.05	–	0.40 ± 0.03
$V^{K^*}(0)$	0.46 ± 0.07	0.38 ± 0.08	0.45 ± 0.08	–	0.47 ± 0.03
$T_1^{K^*}(0)$	0.38 ± 0.06	0.32 ± 0.05	0.34 ± 0.10	$0.32^{+0.04}_{-0.02}$	0.38 ± 0.06
$T_3^{K^*}(0)$	0.26 ± 0.04	–	0.26 ± 0.10	–	0.6

Table 5: Comparison of results from different works on form factors at $q^2 = 0$.

The first question concerns the scaling behavior of form factors as functions of the b quark mass. The behavior depends on the momentum transfer and is different for small and large recoil. For $q^2 \to 0$, or, more precisely, for $m_b^2 - q^2 \sim O(m_b^2)$, all form factors in question scale as $\sim 1/m_b^{3/2}$. This behavior can be proven in perturbative QCD taking into account Sudakov suppression of large transverse distances, but is not restricted to this regime and extends to "soft" terms as well [12, 6]. For $m_b^2 - q^2 \sim O(m_b)$, on the other hand, the form factors obtained from light-cone sum rules satisfy the scaling behavior predicted by Heavy Quark Effective Theory (HQET) [28]. For realistic values of the b quark mass, these two regimes are not well separated one from another; therefore large corrections to asymptotic scaling are to be expected. Some estimates of preasymptotic corrections are presented in Refs. [21, 12]. They have to be considered as indicative only. We do not attempt to further quantify preasymptotic corrections in this work.

The second question concerns possible relations between different form factors in the heavy quark limit. Heavy quark symmetry implies exact relations between semileptonic and penguin form factors at small recoil and renormalization scale $\mu = m_b$ [28], which can conveniently be written using the penguin form factor definitions in Eq. (2.13):

$$A(q^2) + B(q^2) = \frac{2m_B}{m_B + m_V}\, V(q^2), \tag{4.1}$$

$$A(q^2) - B(q^2) = \frac{(m_B + m_V)}{m_B}\, A_1(q^2) - \frac{m_B^2 - q^2 + m_V^2}{m_B} \frac{V(q^2)}{m_B + m_V}, \tag{4.2}$$

$$C(q^2) = -\frac{m_B - m_V}{m_B}\,V(q^2) + \frac{m_V(m_B^2 - m_V^2)}{m_B q^2}\left[A_0(q^2) - A_3(q^2)\right]$$

$$+\frac{m_B - m_V}{2m_B}\,A_2(q^2). \qquad (4.3)$$

Writing the relations in this form emphasizes their different behavior in the heavy quark limit: At small recoil both sides of Eqs. (4.1) and (4.3) are of order $\sqrt{m_b}$, while Eq. (4.2) relates combinations of form factors, which are of order $1/\sqrt{m_b}$. The numerical comparison for $B \to \rho$ transitions is presented in Fig. 8. We note that (a) Eq. (4.1) is satisfied with high accuracy, (b) the relation (4.2) is violated. However, both sides are numerically small compared to Eq. (4.1), in agreement with the expected $1/m_b$ suppression. (c) The relation (4.3) is satisfied very well at $q^2 \to 0$ and holds with 20% accuracy at large q^2; both sides turn out to be small at large recoil, which implies significant cancellations between the terms on the right-hand side.

For phenomenological applications it is more appropriate to rewrite the Isgur-Wise relations (4.1)–(4.3) in terms of the form factors defined in (1.3):

$$T_1(q^2) = \frac{m_B^2 + q^2 - m_V^2}{2m_B}\frac{V(q^2)}{m_B + m_V} + \frac{m_B + m_V}{2m_B}A_1(q^2), \qquad (4.4)$$

$$\frac{m_B^2 - m_V^2}{q^2}\Big[T_1(q^2) - T_2(q^2)\Big] = \frac{3m_B^2 - q^2 + m_V^2}{2m_B}\frac{V(q^2)}{m_B + m_V} - \frac{m_B + m_V}{2m_B}A_1(q^2), \qquad (4.5)$$

$$T_3(q^2) = \frac{m_B^2 - q^2 + 3m_V^2}{2m_B}\frac{V(q^2)}{m_B + m_V} + \frac{m_B^2 - m_V^2}{m_B q^2}m_V A_0(q^2)$$

$$-\frac{m_B^2 + q^2 - m_V^2}{2m_B q^2}\Big[(m_B + m_V)A_1(q^2) - (m_B - m_V)A_2(q^2)\Big]. \qquad (4.6)$$

Note that such a rewriting mixes terms of different order in $1/m_b$ in the small recoil region, and in this sense is not fully consistent with the derivation in [28]. It can be justified, however, by observing that the hierarchy of contributions is different at large recoil and all the terms become formally of the same order. The numerical comparison for $B \to \rho$ transitions is presented in Fig. 9. The accuracy proves to be excellent for the relation (4.4), which is observed to within 3% accuracy, and good for (4.5) with deviations of at most 8%. Relation (4.6), however, is violated by 20% for $q^2 > 15\,\mathrm{GeV}^2$. Since fidelity of the sum rules worsens in the high-q^2 region, it is not clear whether this disagreement indicates a genuine $1/m_b$ correction or is an artifact. Our results reinforce an earlier observation in [21] that the relation in (4.4) is satisfied within $\approx (5-7)\%$ in the whole region of q^2 to leading-twist accuracy in the light-cone sum rule approach, and strongly support the conjecture of [29] about the validity of heavy quark symmetry relations in the region of small q^2 in heavy-to-light transitions.

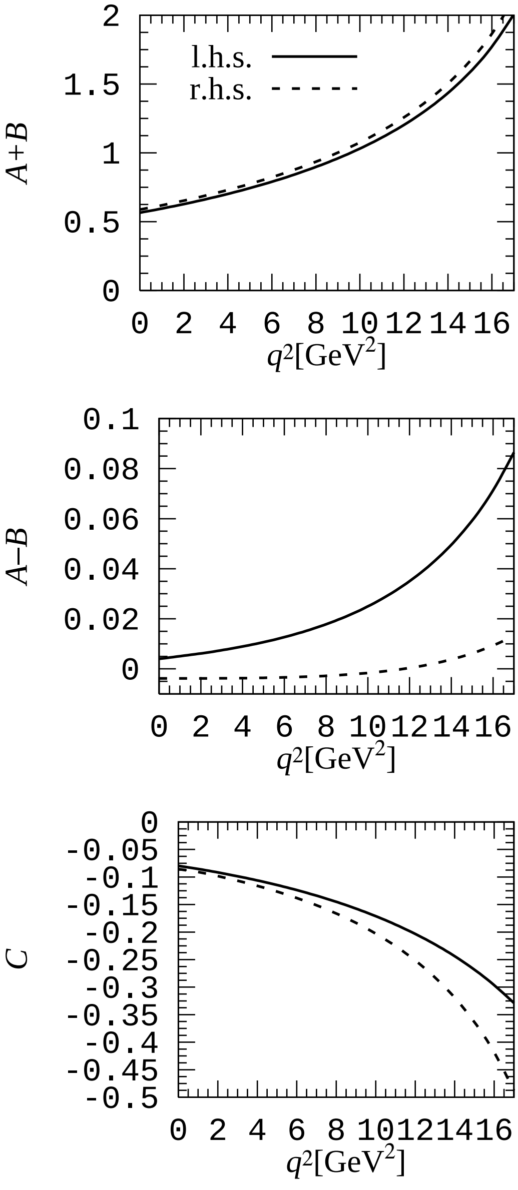

Figure 8: Isgur-Wise relations (4.1)–(4.3) for $B \to \rho$ transitions. Renormalization scale is $\mu = m_b$. Solid and dashed curves correspond to expressions appearing on the l.h.s. and r.h.s., respectively.

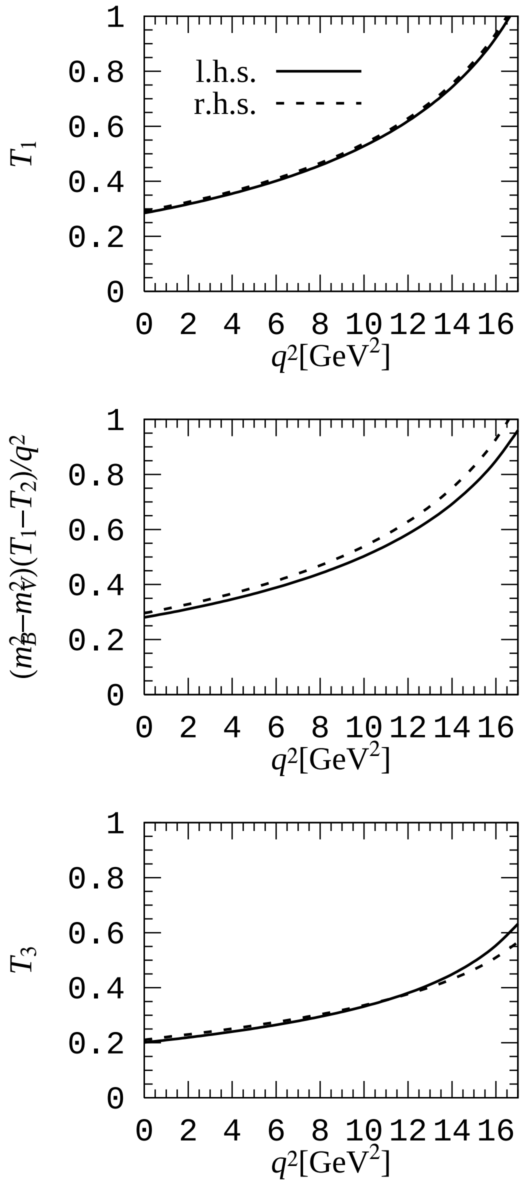

Figure 9: Isgur-Wise relations (4.4)–(4.6) for $B \to \rho$ transitions. Renormalization scale is $\mu = m_b$. Solid and dashed curves correspond to expressions appearing on the l.h.s. and r.h.s., respectively.

5 Conclusions

We have given a complete analysis of B decay form factors to light vector mesons in the light-cone sum rule approach. The principal new contribution of this work are radiative corrections and higher-twist corrections to the sum rules, which are calculated for the first time. We observe that the light-cone sum rules turn out to be very robust against corrections in the light-cone expansion, which numerical impact proves to be minimal. Radiative corrections seem to be well under control. In cases that higher-twist corrections are important, they are dominated by meson mass effects which do not involve free parameters. The theoretical accuracy of the approach is thus restricted entirely by the duality approximation for the extraction of the B meson state from the continuum and contributions of higher resonances. The usual "educated guess" is that accuracy of such an extraction is of order 10% which provides an irreducible error. Effects of yet higher radiative corrections and yet higher twists are likely to be much less; therefore, the sum rules derived in this work cannot be improved significantly. The numerical analysis, however, can and should eventually be updated, once estimates for the meson distributions amplitudes, b quark mass and f_B become more precise. In particular, lattice calculations of the tensor couplings f_V^T and the parameters $a_{1,2}^{\parallel}$, $a_{1,2}^{\perp}$ for meson distribution amplitudes would be most welcome.

Acknowledgements

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A One-loop Integrals

For the calculation of radiative corrections, we need the following integrals:

$$\int \frac{d^Dk}{(2\pi)^D} \frac{k_\alpha}{(k+up)^2k^2[(q-k)^2-m^2]} = Aup_\alpha + Bq_\alpha, \tag{A.1}$$

$$\int \frac{d^Dk}{(2\pi)^D} \frac{k_\alpha(q-k)_\beta}{(k+up)^2k^2[(q-k)^2-m^2]} = Cg_{\alpha\beta} + Dq_\alpha q_\beta + Eq_\alpha up_\beta + Fup_\alpha q_\beta + \dots, \tag{A.2}$$

$$\int \frac{d^Dk}{(2\pi)^D} \frac{k_\alpha k_\beta}{k^2(k-up)^2(k+\bar{u}p)^2[(up+q-k)^2-m^2]} = Hg_{\alpha\beta} + Iq_\alpha q_\beta + \dots, \tag{A.3}$$

$$\int \frac{d^Dk}{(2\pi)^D} \frac{1}{k^2[(up+q-k)^2-m^2]} = Y, \tag{A.4}$$

$$\int \frac{d^D k}{(2\pi)^D} \frac{k_\alpha}{k^2[(up+q-k)^2-m^2]} = Z\,(q+up)_\alpha, \tag{A.5}$$

where the dots stand for terms which are irrelevant for the present calculation. The functions $\bar{A}$, $\bar{B}$, $\bar{C}$ are obtained from A, B and C by the replacement

$$u \to \bar{u}; \qquad q \to -p_B. \tag{A.6}$$

We shall use the notations

$$s \equiv m^2 - up_B^2 - \bar{u}q^2, \qquad \frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} - \gamma_E + \ln 4\pi. \tag{A.7}$$

with $D = 4-2\epsilon$. In order to perform Borel transformation and continuum subtraction, the following spectral representations for the above integrals prove the most convenient:

$$A = \frac{i}{(4\pi)^2} \int\limits_{m^2}^{\infty} \frac{\mathrm{d}t}{t-\xi} \frac{1}{(t-q^2)^2} \left\{ (m^2-q^2)\left[-\frac{1}{\hat{\epsilon}} - 1 + \log\frac{(t-m^2)^2}{\mu^2 t}\right] + t - q^2 - \frac{q^2(m^2-t)}{t} \right\}$$

$$\begin{aligned}
u(p_B^2-q^2)A &= \frac{\mathrm{i}}{(4\pi)^2}\left\{\frac{1}{\hat{\epsilon}} + 2 - \log\frac{s}{\mu^2} + \int_{m^2}^{\infty}\frac{\mathrm{d}t}{t-\xi}\right.\\
&\quad\left.\times\frac{1}{t-q^2}\left[(m^2-q^2)\left(-\frac{1}{\hat{\epsilon}} - 1 + \log\frac{(m^2-t)^2}{\mu^2 t}\right) - \frac{q^2}{t}(m^2-t)\right]\right\}
\end{aligned}$$

$$\begin{aligned}
\bar{u}(q^2-p_B^2)\bar{A} &= \frac{\mathrm{i}}{(4\pi)^2}\left\{\frac{1}{\hat{\epsilon}} + 2 - \log\frac{s}{\mu^2} + \int_{m^2}^{\infty}\frac{\mathrm{d}t}{t-\xi}\right.\\
&\quad\left.\times\left[\left(1+\frac{m^2-t}{t-p_B^2}\right)\left(-\frac{1}{\hat{\epsilon}} - 1 + \log\frac{(m^2-t)^2}{\mu^2 t}\right) + \left(\frac{1}{t} - \frac{1}{t-p_B^2}\right)(m^2-t)\right)\right]\right\}
\end{aligned}$$

$$B = \frac{i}{(4\pi)^2}\int_{m^2}^{\infty}\frac{\mathrm{d}t}{t-\xi}\frac{m^2-t}{t(t-q^2)}$$

$$\bar{B} = \frac{i}{(4\pi)^2}\frac{1}{\bar{u}}\int_{m^2}^{\infty}\frac{\mathrm{d}t\,(m^2-t)}{t(t-q^2)}\left(\frac{1}{t-p_B^2} - \frac{u}{t-\xi}\right)$$

$$C = \frac{i}{(4\pi)^2}\frac{1}{4}\left\{-\frac{1}{\hat{\epsilon}} - 3 + \log\frac{m^2-\xi}{\mu^2} + \int_{m^2}^{\infty}\frac{\mathrm{d}t}{t-\xi}\frac{(2m^2-q^2)t-m^4}{t(t-q^2)}\right\}$$

$$\bar{C} = \frac{i}{(4\pi)^2}\frac{1}{4}\left\{-\frac{1}{\hat{\epsilon}} - 3 + \log\frac{s}{\mu^2} + \int_{m^2}^{\infty}\mathrm{d}t\left[\frac{1}{t-\xi} - \frac{1}{\bar{u}}\frac{(m^2-t)^2}{t(t-q^2)}\left(\frac{1}{t-p_B^2} - \frac{u}{t-\xi}\right)\right]\right\}$$

$$D = \frac{i}{(4\pi)^2}\frac{1}{2}\int_{m^2}^{\infty}\frac{\mathrm{d}t}{t-\xi}\frac{m^4-t^2}{t^2(t-q^2)}$$

$$E = \frac{i}{(4\pi)^2}\,\frac{1}{2u(p_B^2-q^2)} + \frac{1}{u(p_B^2-q^2)}\,(m^2B - q^2D)$$

$$F = A + E$$

$$H = \frac{i}{(4\pi)^2}\frac{1}{2}\int_{m^2}^{\infty} \mathrm{d}t\, \left\{\frac{1}{t-\xi}\left[\frac{1}{\hat{\epsilon}} + 2 - \log\frac{(t-m^2)^2}{t\mu^2}\right]\left[\frac{u(m^2-q^2)}{(t-q^2)^2} + \frac{\bar{u}(m^2-t)}{(t-p_B^2)^2} + \frac{\bar{u}}{t-p_B^2}\right]\right.$$
$$\left. - \frac{1}{(t-p_B^2)(t-q^2)}\frac{t+m^2}{t}\right\}$$

$$I = \frac{i}{(4\pi)^2}\int_{m^2}^{\infty} \mathrm{d}t\, \left\{\frac{m^2-t}{(t-\xi)t}\left[\frac{u}{(t-q^2)^2} + \frac{\bar{u}}{(t-p_B^2)^2}\right] + \frac{m^2}{t^2(t-p_B^2)(t-q^2)}\right\}$$

$$Y = \frac{i}{(4\pi)^2}\left(\frac{1}{\hat{\epsilon}} - \ln\frac{s}{\mu^2} + 2 - \frac{m^2}{m^2-s}\ln\frac{m^2}{s}\right)$$

$$Z = \frac{1}{2}\,\frac{i}{(4\pi)^2}\left(\frac{1}{\hat{\epsilon}} - \ln\frac{m^2}{\mu^2} - \frac{m^2}{m^2-s} + 2 - \frac{s^2}{(m^2-s)^2}\ln\frac{s}{m^2}\right). \tag{A.8}$$

B Summary of Meson Distribution Amplitudes

The expressions collected in this appendix are principally the result of recent studies reported in Refs. [9, 10, 11]. We use a simplified version of the set of twist 4 distributions [11] taking into account contributions of the lowest conformal partial waves only, and for consistency discard contributions of higher partial waves in twist 3 distributions in cases that they enter physical amplitudes multiplied by additional powers of m_ρ. The SU(3) breaking effects are taken into account in leading twist distributions and partially for twist 3, but neglected for twist 4. Explicit expressions are given below for a (charged) ρ meson. Distribution amplitudes for other vector mesons are obtained by trivial substitutions.

Throughout this appendix we denote the meson momentum P_μ and introduce the light-like vectors p and z such that

$$p_\mu = P_\mu - \frac{1}{2}z_\mu\frac{m_\rho^2}{pz}. \tag{B.1}$$

The meson polarization vector $e_\mu^{(\lambda)}$ is decomposed in projections onto the two light-like vectors and the orthogonal plane as

$$e_\mu^{(\lambda)} = \frac{(e^{(\lambda)}\cdot z)}{pz}\left(p_\mu - \frac{m_\rho^2}{2pz}z_\mu\right) + e_{\perp\mu}^{(\lambda)}. \tag{B.2}$$

B.1 Chiral-even distributions

Two-particle quark-antiquark distribution amplitudes are defined as matrix elements of non-local operators on the light-cone [10]

$$\langle 0|\bar{u}(z)\gamma_\mu d(-z)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho \left[p_\mu \frac{e^{(\lambda)}\cdot z}{p\cdot z} \int_0^1 du\, e^{i\xi p\cdot z}\phi_\parallel(u,\mu^2) + e^{(\lambda)}_{\perp\mu} \int_0^1 du\, e^{i\xi p\cdot z} g_\perp^{(v)}(u,\mu^2) - \frac{1}{2} z_\mu \frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2} m_\rho^2 \int_0^1 du\, e^{i\xi p\cdot z} g_3(u,\mu^2)\right] \tag{B.3}$$

and

$$\langle 0|\bar{u}(z)\gamma_\mu\gamma_5 d(-z)|\rho^-(P,\lambda)\rangle = \frac{1}{2}\left(f_\rho - f_\rho^T \frac{m_u+m_d}{m_\rho}\right) m_\rho \epsilon_\mu{}^{\nu\alpha\beta} e^{(\lambda)}_{\perp\nu} p_\alpha z_\beta \int_0^1 du\, e^{i\xi p\cdot z} g_\perp^{(a)}(u,\mu^2). \tag{B.4}$$

For brevity, here and below we do not show the gauge factors in between the quark and the antiquark field and use the shorthand notation

$$\xi = u-(1-u) = 2u-1.$$

The vector and tensor decay constants f_ρ and f_ρ^T are defined as usually as

$$\langle 0|\bar{u}(0)\gamma_\mu d(0)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho e^{(\lambda)}_\mu, \tag{B.5}$$

$$\langle 0|\bar{u}(0)\sigma_{\mu\nu} d(0)|\rho^-(P,\lambda)\rangle = i f_\rho^T (e^{(\lambda)}_\mu P_\nu - e^{(\lambda)}_\nu P_\mu). \tag{B.6}$$

The distribution amplitude $\phi_\parallel$ is of twist 2, $g_\perp^{(v)}$ and $g_\perp^{(a)}$ are twist 3 and g_3 is twist 4. All four functions $\phi = \{\phi_\parallel, g_\perp^{(v)}, g_\perp^{(a)}, g_3\}$ are normalized as

$$\int_0^1 du\, \phi(u) = 1, \tag{B.7}$$

which can be checked by comparing both sides of the defining equations in the limit $z_\mu \to 0$ and using the equations of motion. We keep the (tiny) corrections proportional to the u and d quark masses m_u and m_d to indicate the SU(3) breaking corrections for K^* and ϕ mesons.

In addition, we have to define three-particle distributions

$$\langle 0|\bar{u}(z) g\widetilde{G}_{\mu\nu}\gamma_\alpha\gamma_5 d(-z)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho p_\alpha [p_\nu e^{(\lambda)}_{\perp\mu} - p_\mu e^{(\lambda)}_{\perp\nu}]\mathcal{A}(v,pz) + f_\rho m_\rho^3 \frac{e^{(\lambda)}\cdot z}{pz}[p_\mu g^\perp_{\alpha\nu} - p_\nu g^\perp_{\alpha\mu}]\widetilde{\Phi}(v,pz) + f_\rho m_\rho^3 \frac{e^{(\lambda)}\cdot z}{(pz)^2} p_\alpha [p_\mu z_\nu - p_\nu z_\mu]\widetilde{\Psi}(v,pz) \tag{B.8}$$

$$\langle 0|\bar{u}(z)gG_{\mu\nu}i\gamma_\alpha d(-z)|\rho^-(P)\rangle = f_\rho m_\rho p_\alpha[p_\nu e^{(\lambda)}_{\perp\mu} - p_\mu e^{(\lambda)}_{\perp\nu}\mathcal{V}(v,pz)$$

$$+ f_\rho m_\rho^3 \frac{e^{(\lambda)}\cdot z}{pz}[p_\mu g^\perp_{\alpha\nu} - p_\nu g^\perp_{\alpha\mu}]\Phi(v,pz)$$

$$+ f_\rho m_\rho^3 \frac{e^{(\lambda)}\cdot z}{(pz)^2} p_\alpha [p_\mu z_\nu - p_\nu z_\mu]\Psi(v,pz) \qquad \text{(B.9)}$$

where

$$\mathcal{A}(v,pz) = \int \mathcal{D}\underline{\alpha} e^{-ipz(\alpha_u - \alpha_d + v\alpha_g)}\mathcal{A}(\underline{\alpha}), \qquad \text{(B.10)}$$

etc., and $\underline{\alpha}$ is the set of three momentum fractions $\underline{\alpha} = \{\alpha_d, \alpha_u, \alpha_g\}$. The integration measure is defined as

$$\int \mathcal{D}\underline{\alpha} \equiv \int_0^1 d\alpha_d \int_0^1 d\alpha_u \int_0^1 d\alpha_g\, \delta(1 - \sum \alpha_i). \qquad \text{(B.11)}$$

The distribution amplitudes $\mathcal{V}$ and $\mathcal{A}$ are of twist 3 while the rest is twist 4 and we have not shown further Lorentz structures corresponding to twist 5 contributions[6].

Calculation of exclusive amplitudes involving a large momentum transfer reduces to evaluation of meson-to-vacuum transition matrix elements of nonlocal operators which can be expanded in powers of the deviation from the light-cone (see text). To twist 4 accuracy one can use the expression for the axial-vector matrix element in (B.4) as it stands, replacing the light-cone vector z_μ by the actual quark-antiquark separation x_μ. For the vector operator, the light-cone expansion to the twist 4 accuracy reads:

$$\langle 0|\bar{u}(x)\gamma_\mu d(-x)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho \Bigg\{ \frac{e^{(\lambda)}x}{Px}\int_0^1 du\, e^{i\xi Px}\Big[\phi_\parallel(u,\mu) + \frac{m_\rho^2 x^2}{4}\mathbb{A}(u,\mu)\Big]$$

$$+ \left(e^{(\lambda)}_\mu - P_\mu \frac{e^{(\lambda)}x}{Px}\right)\int_0^1 du\, e^{i\xi Px}\, g_\perp^{(v)}(u,\mu)$$

$$- \frac{1}{2}x_\mu \frac{e^{(\lambda)}x}{(Px)^2} m_\rho^2 \int_0^1 du\, e^{i\xi Px}\mathbb{C}(u,\mu)\Bigg\} \qquad \text{(B.12)}$$

where

$$\mathbb{C}(u) = g_3(u) + \phi_\parallel(u) - 2g_\perp^{(v)}(u) \qquad \text{(B.13)}$$

and $\mathbb{A}(u)$ can be related to integrals of three-particle distributions using equations of motion. All distribution functions in (B.12) are assumed to be normalized at the scale $\mu^2 \sim x^{-2}$ (to leading logarithmic accuracy). In practical calculations it is sometimes convenient to use integrated distributions

$$\mathbb{C}^{(i)}(u) = -\int_0^u dv\, \mathbb{C}(v), \qquad \mathbb{C}^{(ii)}(u) = -\int_0^u dv\, \mathbb{C}^{(i)}(v). \qquad \text{(B.14)}$$

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[6]We use a different normalization of three-particle twist 3 distributions compared to [10].

For the leading twist 2 distribution amplitude $\phi_\parallel$ we use

$$\phi_\parallel(u) = 6u\bar{u}\left[1 + 3a_1^\parallel\,\xi + a_2^\parallel\,\frac{3}{2}(5\xi^2-1)\right] \tag{B.15}$$

with parameter values as specified in Tab. A. The expressions for higher-twist distributions given below correspond to the simplest self-consistent approximation which satisfies the QCD equations of motion [10, 11]:

- Three-particle distributions of twist 3:

$$\mathcal{V}(\underline{\alpha}) = 540\,\zeta_3\omega_3^V(\alpha_d-\alpha_u)\alpha_d\alpha_u\alpha_g^2, \tag{B.16}$$

$$\mathcal{A}(\underline{\alpha}) = 360\,\zeta_3\alpha_d\alpha_u\alpha_g^2\Big[1+\omega_3^A\frac{1}{2}(7\alpha_g-3)\Big]. \tag{B.17}$$

- Two-particle distributions of twist 3:

$$\begin{aligned} g_\perp^{(a)}(u) &= 6u\bar{u}\left[1+a_1^\parallel\xi+\left\{\frac{1}{4}a_2^\parallel+\frac{5}{3}\,\zeta_3\left(1-\frac{3}{16}\,\omega_3^A+\frac{9}{16}\omega_3^V\right)\right\}(5\xi^2-1)\right] \\ &\quad + 6\,\widetilde{\delta}_+\,(3u\bar{u}+\bar{u}\ln\bar{u}+u\ln u)+6\,\widetilde{\delta}_-\,(\bar{u}\ln\bar{u}-u\ln u), \end{aligned} \tag{B.18}$$

$$\begin{aligned} g_\perp^{(v)}(u) &= \frac{3}{4}(1+\xi^2)+a_1^\parallel\,\frac{3}{2}\,\xi^3+\left(\frac{3}{7}\,a_2^\parallel+5\zeta_3\right)\left(3\xi^2-1\right) \\ &\quad +\left[\frac{9}{112}\,a_2^\parallel+\frac{15}{64}\,\zeta_3\Big(3\,\omega_3^V-\omega_3^A\Big)\right]\left(3-30\xi^2+35\xi^4\right) \\ &\quad +\frac{3}{2}\,\widetilde{\delta}_+\,(2+\ln u+\ln\bar{u})+\frac{3}{2}\,\widetilde{\delta}_-\,(2\xi+\ln\bar{u}-\ln u), \end{aligned} \tag{B.19}$$

- Three-particle distributions of twist 4:

$$\begin{aligned} \widetilde{\Phi}(\underline{\alpha}) &= \Big[-\frac{1}{3}\zeta_3+\frac{1}{3}\zeta_4\Big]30(1-\alpha_g)\alpha_g^2, \\ \Phi(\underline{\alpha}) &= \Big[-\frac{1}{3}\zeta_3+\frac{1}{3}\zeta_4\Big]30(\alpha_u-\alpha_d)\alpha_g^2, \\ \widetilde{\Psi}(\underline{\alpha}) &= \Big[\frac{2}{3}\zeta_3+\frac{1}{3}\zeta_4\Big]120\alpha_u\alpha_d\alpha_g, \\ \Psi(\underline{\alpha}) &= 0. \end{aligned} \tag{B.20}$$

- Two-particle distributions of twist 4:

$$\mathbb{A}(u) = \left[\frac{4}{5} + \frac{20}{9}\zeta_4 + \frac{8}{9}\zeta_3\right] 30u^2(1-u)^2,$$

$$g_3(u) = 6u(1-u) + \left[\frac{10}{3}\zeta_4 - \frac{20}{3}\zeta_3\right](1-3\xi^2),$$

$$\mathbb{C}(u) = \left[\frac{3}{2} + \frac{10}{3}\zeta_4 + \frac{10}{3}\zeta_3\right](1-3\xi^2),$$

$$\mathbb{C}^{(ii)}(u) = \left[\frac{3}{2} + \frac{10}{3}\zeta_4 + \frac{10}{3}\zeta_3\right] u^2(1-u)^2, \tag{B.21}$$

where the dimensionless couplings ζ_3 and ζ_4 are defined as local matrix elements

$$\langle 0|\bar{u} g \tilde{G}_{\mu\nu}\gamma_\alpha\gamma_5 d|\rho^-(P,\lambda)\rangle = f_\rho m_\rho \zeta_3 \left[e^{(\lambda)}_\mu \left(P_\alpha P_\nu - \frac{1}{3}m_\rho^2 g_{\alpha\nu}\right) - e^{(\lambda)}_\nu \left(P_\alpha P_\mu - \frac{1}{3}m_\rho^2 g_{\alpha\mu}\right)\right]$$
$$+ \frac{1}{3} f_\rho m_\rho^3 \zeta_4 \left[e^{(\lambda)}_\mu g_{\alpha\nu} - e^{(\lambda)}_\nu g_{\alpha\mu}\right] \tag{B.22}$$

and have been estimated from QCD sum rules [30, 31]. The terms in $\delta_\pm$ and $\widetilde{\delta}_\pm$ specify quark-mass corrections in twist 3 distributions induced by the equations of motion. The numerical values of these and other coefficients are listed in Tabs. A and B[7]. Note that we neglect SU(3) breaking effects in twist 4 distributions and in gluonic parts of twist 3 distributions.

B.2 Chiral-odd distributions

There exist four different two-particle chiral-odd distributions [10] defined as

$$\langle 0|\bar{u}(z)\sigma_{\mu\nu} d(-z)|\rho^-(P,\lambda)\rangle = i f_\rho^T \left[(e^{(\lambda)}_{\perp\mu} p_\nu - e^{(\lambda)}_{\perp\nu} p_\mu) \int_0^1 du\, e^{i\xi p\cdot z} \phi_\perp(u,\mu^2)\right.$$
$$+ (p_\mu z_\nu - p_\nu z_\mu)\frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2} m_\rho^2 \int_0^1 du\, e^{i\xi p\cdot z} h_\parallel^{(t)}(u,\mu^2)$$
$$\left. + \frac{1}{2}(e^{(\lambda)}_{\perp\mu} z_\nu - e^{(\lambda)}_{\perp\nu} z_\mu)\frac{m_\rho^2}{p\cdot z}\int_0^1 du\, e^{i\xi p\cdot z} h_3(u,\mu^2)\right], \tag{B.23}$$

$$\langle 0|\bar{u}(z) d(-z)|\rho^-(P,\lambda)\rangle = -i\left(f_\rho^T - f_\rho \frac{m_u + m_d}{m_\rho}\right)(e^{(\lambda)}\cdot z) m_\rho^2 \int_0^1 du\, e^{i\xi p\cdot z} h_\parallel^{(s)}(u,\mu^2). \tag{B.24}$$

[7] In the notations of Ref. [10], $\omega^A_{1,0} \equiv \omega^A_3$, $\zeta^A_3 \equiv \zeta_3$ and $\zeta^V_3 \equiv (3/28)\zeta_3\omega^V_3$.

The distribution amplitude $\phi_\perp$ is twist 2, $h_\parallel^{(s)}$ and $h_\parallel^{(t)}$ are twist 3 and h_3 is twist 4. All four functions $\phi = \{\phi_\perp, h_\parallel^{(s)}, h_\parallel^{(t)}, h_3\}$ are normalized to $\int_0^1 du\, \phi(u) = 1$.

Three-particle chiral-odd distributions are defined to twist 4 accuracy as

$$\begin{aligned}
\langle 0|\bar{u}(z)\sigma_{\alpha\beta} g G_{\mu\nu}(vz) d(-z)|\rho^-(P,\lambda)\rangle \; &= \\
&= f_\rho^T m_\rho^3 \frac{e^{(\lambda)}\cdot z}{2(p\cdot z)}[p_\alpha p_\mu g^\perp_{\beta\nu} - p_\beta p_\mu g^\perp_{\alpha\nu} - p_\alpha p_\nu g^\perp_{\beta\mu} + p_\beta p_\nu g^\perp_{\alpha\mu}]\mathcal{T}(v,pz) \\
&+ f_\rho^T m_\rho^2 [p_\alpha e^{(\lambda)}_{\perp\mu} g^\perp_{\beta\nu} - p_\beta e^{(\lambda)}_{\perp\mu} g^\perp_{\alpha\nu} - p_\alpha e^{(\lambda)}_{\perp\nu} g^\perp_{\beta\mu} + p_\beta e^{(\lambda)}_{\perp\nu} g^\perp_{\alpha\mu}] T_1^{(4)}(v,pz) \\
&+ f_\rho^T m_\rho^2 [p_\mu e^{(\lambda)}_{\perp\alpha} g^\perp_{\beta\nu} - p_\mu e^{(\lambda)}_{\perp\beta} g^\perp_{\alpha\nu} - p_\nu e^{(\lambda)}_{\perp\alpha} g^\perp_{\beta\mu} + p_\nu e^{(\lambda)}_{\perp\beta} g^\perp_{\alpha\mu}] T_2^{(4)}(v,pz) \\
&+ \frac{f_\rho^T m_\rho^2}{pz}[p_\alpha p_\mu e^{(\lambda)}_{\perp\beta} z_\nu - p_\beta p_\mu e^{(\lambda)}_{\perp\alpha} z_\nu - p_\alpha p_\nu e^{(\lambda)}_{\perp\beta} z_\mu + p_\beta p_\nu e^{(\lambda)}_{\perp\alpha} z_\mu] T_3^{(4)}(v,pz) \\
&+ \frac{f_\rho^T m_\rho^2}{pz}[p_\alpha p_\mu e^{(\lambda)}_{\perp\nu} z_\beta - p_\beta p_\mu e^{(\lambda)}_{\perp\nu} z_\alpha - p_\alpha p_\nu e^{(\lambda)}_{\perp\mu} z_\beta + p_\beta p_\nu e^{(\lambda)}_{\perp\mu} z_\alpha] T_4^{(4)}(v,pz) \\
&+ \ldots
\end{aligned} \tag{B.25}$$

and

$$\begin{aligned}
\langle 0|\bar{u}(z) g G_{\mu\nu}(vz) d(-z)|\rho^-(P,\lambda)\rangle &= i f_\rho^T m_\rho^2 [e^{(\lambda)}_{\perp\mu} p_\nu - e^{(\lambda)}_{\perp\nu} p_\mu] S(v,pz), \\
\langle 0|\bar{u}(z) i g \widetilde{G}_{\mu\nu}(vz)\gamma_5 d(-z)|\rho^-(P,\lambda)\rangle &= i f_\rho^T m_\rho^2 [e^{(\lambda)}_{\perp\mu} p_\nu - e^{(\lambda)}_{\perp\nu} p_\mu] \widetilde{S}(v,pz).
\end{aligned} \tag{B.26}$$

Of these seven amplitudes, $\mathcal{T}$ is twist 3 and the other six are twist 4.

The light-cone expansion of the nonlocal tensor operator can be written to twist 4 accuracy as

$$\begin{aligned}
\langle 0|\bar{u}(x)\sigma_{\mu\nu} d(-x)|\rho^-(P,\lambda)\rangle &= \\
&= i f_\rho^T \left[(e^{(\lambda)}_\mu P_\nu - e^{(\lambda)}_\nu P_\mu)\int_0^1 du\, e^{i\xi Px}\left[\phi_\perp(u) + \frac{m_\rho^2 x^2}{4}\mathbb{A}_T(u)\right]\right. \\
&\quad + (P_\mu x_\nu - P_\nu x_\mu)\frac{e^{(\lambda)}\cdot x}{(Px)^2} m_\rho^2 \int_0^1 du\, e^{i\xi Px}\mathbb{B}_T(u) \\
&\quad \left. + \frac{1}{2}(e^{(\lambda)}_\mu x_\nu - e^{(\lambda)}_\nu x_\mu)\frac{m_\rho^2}{Px}\int_0^1 du\, e^{i\xi Px}\mathbb{C}_T(u)\right],
\end{aligned} \tag{B.27}$$

where $\mathbb{B}_T$ and $\mathbb{C}_T$ are expressed in terms of the distribution amplitudes defined above as

$$\mathbb{B}_T(u) = h_\parallel^{(t)}(u) - \frac{1}{2}\phi_\perp(u) - \frac{1}{2}h_3(u),$$

$$\mathbb{C}_T(u) = h_3(u) - \phi_\perp(u), \tag{B.28}$$

and $\mathbb{A}_T$ can be related to integrals of three-particle distribution functions using the equations of motion.

We introduce notations, similar to Eq. (B.14):

$$\mathbb{B}_T^{(i)}(u) = -\int_0^u dv\, \mathbb{B}_T(v),$$

$$\mathbb{C}_T^{(i)}(u) = -\int_0^u dv\, \mathbb{C}_T(v). \tag{B.29}$$

For the leading twist 2 distribution amplitude $\phi_\perp$ we use

$$\phi_\perp(u) = 6u\bar{u}\left[1 + 3a_1^\perp\,\xi + a_2^\perp\,\frac{3}{2}(5\xi^2-1)\right] \tag{B.30}$$

with parameter values as specified in Tab. A. The expressions for higher-twist distributions given below correspond to the simplest self-consistent approximation which satisfies all QCD equations of motion [10, 11]:

- Three-particle distribution of twist 3:

$$\mathcal{T}(\underline{\alpha}) = 540\,\zeta_3\omega_3^T(\alpha_d-\alpha_u)\alpha_d\alpha_u\alpha_g^2. \tag{B.31}$$

- Two-particle distributions of twist 3:

$$h_\parallel^{(s)}(u) = 6u\bar{u}\left[1 + a_1^\perp\xi + \left(\frac{1}{4}a_2^\perp + \frac{5}{8}\,\zeta_3\omega_3^T\right)(5\xi^2-1)\right]$$
$$+ 3\,\delta_+\,(3u\bar{u} + \bar{u}\ln\bar{u} + u\ln u) + 3\,\delta_-\,(\bar{u}\ln\bar{u} - u\ln u), \tag{B.32}$$

$$h_\parallel^{(t)}(u) = 3\xi^2 + \frac{3}{2}\,a_1^\perp\,\xi(3\xi^2-1) + \frac{3}{2}a_2^\perp\,\xi^2\,(5\xi^2-3) + \frac{15}{16}\zeta_3\omega_3^T(3-30\xi^2+35\xi^4)$$
$$+ \frac{3}{2}\,\delta_+\,(1+\xi\,\ln\bar{u}/u) + \frac{3}{2}\,\delta_-\,\xi\,(2+\ln u+\ln\bar{u}) \tag{B.33}$$

- Three-particle distributions of twist 4:

$$T_1^{(4)}(\underline{\alpha}) = T_3^{(4)}(\underline{\alpha}) \;=\; 0,$$

$$T_2^{(4)}(\underline{\alpha}) = 30\widetilde{\zeta}_4^T(\alpha_d-\alpha_u)\alpha_g^2,$$

$$T_4^{(4)}(\underline{\alpha}) = -30\zeta_4^T(\alpha_d-\alpha_u)\alpha_g^2,$$

$$S(\underline{\alpha}) = 30\zeta_4^T(1-\alpha_g)\alpha_g^2,$$

$$\widetilde{S}(\underline{\alpha}) = 30\widetilde{\zeta}_4^T(1-\alpha_g)\alpha_g^2. \tag{B.34}$$

V	$\rho^\pm$	$K^*_{u,d}$	$\bar{K}^*_{u,d}$	ϕ
f_V[MeV]	198 ± 7	226 ± 28	226 ± 28	254 ± 3
f_V^T[MeV]	160 ± 10	185 ± 10	185 ± 10	215 ± 15
	152 ± 9	175 ± 9	175 ± 9	204 ± 14
$a_1^\parallel$	0	0.19 ± 0.05	-0.19 ± 0.05	0
		0.17 ± 0.04	-0.17 ± 0.04	
$a_2^\parallel$	0.18 ± 0.10	0.06 ± 0.06	0.06 ± 0.06	0 ± 0.1
	0.16 ± 0.09	0.05 ± 0.05	0.05 ± 0.05	
$a_1^\perp$	0	0.20 ± 0.05	-0.20 ± 0.05	0
		0.18 ± 0.05	-0.18 ± 0.05	
$a_2^\perp$	0.20 ± 0.10	0.04 ± 0.04	0.04 ± 0.04	0 ± 0.1
	0.17 ± 0.09	0.03 ± 0.03	0.03 ± 0.03	
δ_+	0	0.24	0.24	0.46
		0.22	0.22	0.41
δ_-	0	-0.24	0.24	0
		-0.22	0.22	
$\widetilde{\delta}_+$	0	0.16	0.16	0.33
		0.13	0.13	0.27
$\widetilde{\delta}_-$	0	-0.16	0.16	0
		-0.13	0.13	

Table A: Masses and couplings of vector meson distribution amplitudes including SU(3) breaking. In cases that two values are given, the upper one corresponds to the scale $\mu^2 = 1\,\text{GeV}^2$ and the lower one to $\mu^2 = m_B^2 - m_b^2 = 4.8\,\text{GeV}^2$, respectively. We use $m_s(1\,\text{GeV}) = 150\,\text{MeV}$ and put the u and d quark mass zero.

	ζ_3	ω_3^A	ω_3^V	ω_3^T	ζ_4	ζ_4^T	$\tilde{\zeta}_4^T$
V	0.032	-2.1	3.8	7.0	0.15	0.10	-0.10
	0.023	-1.8	3.7	7.5	0.13	0.07	-0.07

Table B: Couplings for twist 3 and 4 distribution amplitudes for which we do not include SU(3) breaking. Renormalization scale as in previous table.

- Two-particle distributions of twist 4:

$$h_3(u) = 6u(1-u) + 5[\zeta_4^T + \widetilde{\zeta}_4^T](1-3\xi^2),$$

$$\mathbb{A}_T(u) = 30u^2(1-u)^2\left[\frac{2}{5} + \frac{4}{3}\zeta_4^T - \frac{8}{3}\widetilde{\zeta}_4^T\right]. \qquad \text{(B.35)}$$

The constants ζ_4^T and $\widetilde{\zeta}_4^T$ are defined as

$$\langle 0|\bar{u}gG_{\mu\nu}d|\rho^-(P,\lambda)\rangle = if_\rho^T m_\rho^3 \zeta_4^T (e_\mu^{(\lambda)}P_\nu - e_\nu^{(\lambda)}P_\mu),$$

$$\langle 0|\bar{u}g\widetilde{G}_{\mu\nu}i\gamma_5 d|\rho^-(P,\lambda)\rangle = if_\rho^T m_\rho^3 \widetilde{\zeta}_4^T (e_\mu^{(\lambda)}P_\nu - e_\nu^{(\lambda)}P_\mu) \qquad \text{(B.36)}$$

and have been estimated in [1] from QCD sum rules:

$$\zeta_4^T \simeq -\widetilde{\zeta}_4^T \simeq 0.10. \qquad \text{(B.37)}$$

Other parameters are given in Tab. A[8]. Like in the chiral-even case, we neglect SU(3) breaking corrections in twist 4 distributions.

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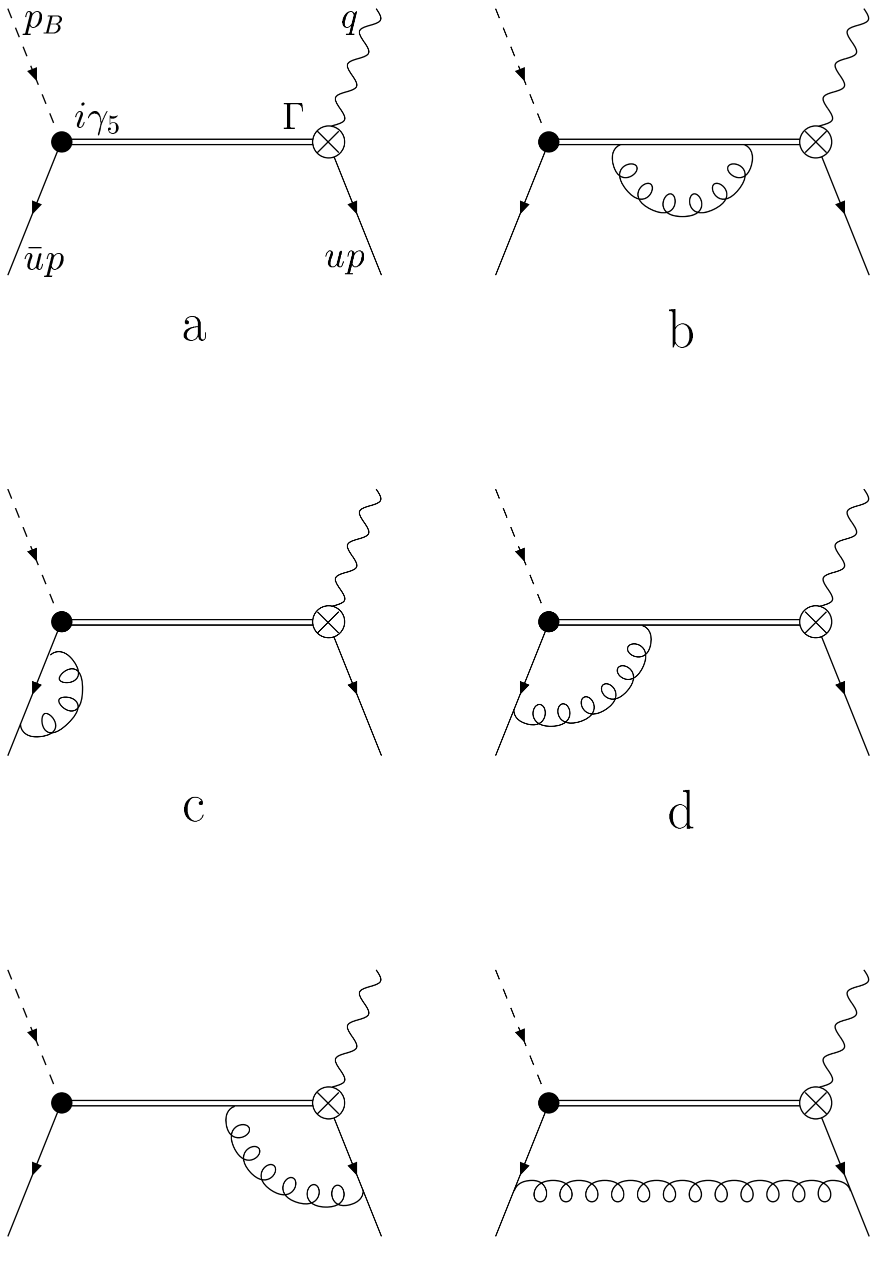
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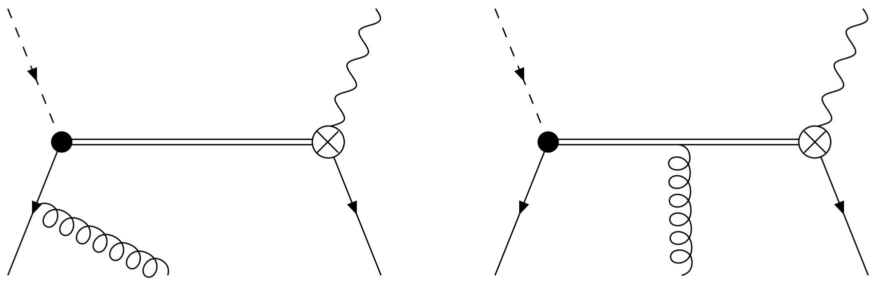

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