Archives

Ref.TH.1153-CERN

SOME NOVEL ASPECTS OF MESON EXCHANGES IN NUCLEI *)

M. Rho **)

CERN - Geneva

---

*) Lectures given at the Colloquium on Interaction of Elementary Particles with Nuclei, Valencia, Spain (March 1970).

**) On leave from SPT, CEN Saclay, Gif-sur-Yvette, France.

Ref.TH.1153-CERN
24 April 1970

1. INTRODUCTION

In this series of lectures, I would like to discuss a subject which is rather novel in nuclear physics. It is not a review of what other people have obtained, but rather what I have learned from discussions with Marc Chemtob and A.M. Green [1]. In order not to overburden the audience and readers I shall limit the referencing and formulas to a bare minimum.

The problem we would like to pose is: what is the role of mesons in nuclear structure? This is a question raised many times in the history of nuclear physics, notably in the context of nuclear force and exchange currents, but being an extremely difficult question, has never been answered (to my mind) in a satisfactory way. Even at this moment, there is not any clear-cut answer. But there is a hope that with a new generation of experimental and theoretical tools, one can begin to understand the problem. My aim here is to present some old problems and more modern approaches to tackle them and some novel information one gains from such attempts.

It seems to me that experimentalists can provide us information on the role of mesons in essentially two ways. One, experiments with high energy probes, and the other, precision experiments with low energy probes. I see here an analogy to the quantum electrodynamics (QED), the test of which is made at both low (Lamb shift) and high (short distance) energies. In high energy domain, one hopes to separate the role of baryon resonances (denoted hereon as N^*) from meson exchange phenomena by looking at large angle scattering (say, p-d or π-d scattering). Such scattering process may be particularly sensitive to excitation of one or two nucleons to excited baryon states at some large momentum transfer, while being indifferent to meson clouds. In the low energy domain, for example in magnetic moments, beta decay, μ^- capture, etc., the N^* or meson exchange effects are small in general, and would require very accurate experimental results and very reliable theoretical treatment. Furthermore, such processes may not be selective of N^*'s or mesons, all of which contributing with equal importance.

In this connection, I shall try to convince you that contrary to hopes cherished by several people [2], the two domains, high energy and low energy, discussed above cannot be trivially joined. A lot more work needs to be done for such a feat.

In this lecture, I shall confine the discussion to the low energy domain only. Life is quite complicated here for reasons mentioned above, a lot more so than in the high energy domain. However, we have a powerful tool at our disposal, the low energy theorems based on the PCAC. These theorems, used judiciously, turn out to be extremely useful in clarifying much of the complicated matter. A novelty of the approach taken here as compared with the multitudes of other methods and philosophies is precisely the use of those theorems.

This lecture is divided into three sections. The first section discusses how the meson exchange phenomena occur in nuclear properties, the second section deals with how one can calculate things, the third section on N^* "shell model" and how it compares with the low energy theorems. The metric I use will be that of Pauli; $p = (\underset{\sim}{p}, ip_0)$, and Dirac γ's are Hermitian.

2. HOW AND WHERE DO MESON EXCHANGE EFFECTS APPEAR?

A. Exclusion Principle

One way of looking at the source of meson exchanges is an exclusion principle correction. It is obviously not an entire story; an absence of measurable meson effects is sometimes traced back to the gauge invariance, the well-known example being the electric charge (and also the vector coupling constant G_V in β decay). However, it is one nice way of seeing how they appear and I discuss here the way one can see it for the nuclear force, the pseudoscalar coupling constant in μ^- capture and exchange currents in magnetic moments and β decay (in that order).

Let us first consider a two-particle system, the deuteron. In order to calculate something for this object, it suffices to have a reliable two-nucleon potential V_{12}. There are two schools in getting the V_{12} - a phenomenological way by adjusting parameters to fit all available two-body data (i.e., scattering and bound state), and a theoretic approach which attempts to derive V_{12} from meson theories which fits as well as the phenomenological one does. Although everybody seems to be pessimistic about the status of the meson theories, one seems to agree that the method taken by those who do the meson theoretic calculations has something sound in it. There are many varieties of calculations which I do not aim to review here. But I have in mind the one-boson-exchange potentials (OBEP) which seem to converge gradually towards the phenomenological potentials.

Let me represent the ingredients of a meson theoretic approach by a set of Feynman diagrams as given in Fig. 1, and put in a word of caution that in practice, the matter of whether to include a particular graph or not can be very tricky, because of double counting. Nevertheless OBEP (Fig. 1a) and baryon resonance graphs (Figs. 1d, e) seem to become particularly successful. I have drawn Fig. 1c on the same footing as Fig. 1d, since both represent the vertex correction of the two-pion exchange graph. The double-counting danger lies for example in the graph 1a with ρ exchange and Figs. 1b, c.

We now suppose that a potential obtained in this way correctly describes the deuteron and fits NN scattering data when used in a Schrödinger equation. The question we raise now is whether the same potential is suitable for other heavier nuclei - in particular for bound state properties like binding energy. In principle, the answer is no as there are corrections, although in practice, the corrections may be small. In order to see this, we note that when there are more particles in the system, filling up the available states, the integration over the intermediate states (momentum k) as indicated in Figs. 1b, c which, in two-particle system can run freely over all values is restricted to the range $k_F \lesssim k \lesssim \infty$, where k_F is the maximum momentum up to which the other particles occupy. Therefore, when using the potential for heavier nuclei, it is necessary to subtract away that portion of the term which violates the exclusion principle. Such a subtraction is equivalent to an addition of the three-body force [3] given by Figs. 2a, b, c. In the middle leg, each corresponds to a different contribution to a virtual pion-nucleon scattering, so the sum of it plus any other graphs are lumped into a blob as indicated in Fig. 2d.

It is clear that by putting in more meson exchanges in the three-body force in the same way as the two-body force, one can again find that there is an exclusion principle violation in the three-body force, the correction of which leads to a four-body force. This can go on up to an A-body force for a nucleus with A nucleons.

A quite analogous situation occurs in the pseudoscalar form factor in muon capture [4]. In muon capture in nuclei

$$\mu^- + A_i \rightarrow \nu_\mu + A_f \tag{1}$$

where A_i and A_f represent respectively the initial and final nuclei, the pseudoscalar form factor $F_p(q^2)$ can be different from the free space proton value because of the presence of other nucleons. To see this, first recall how $F_p(q^2)$ is determined for proton. For the reaction, $\mu^- + p \rightarrow n + \nu_\mu$, the axial vector matrix element has two terms:

$$\langle n | J_\lambda^A | p \rangle = i\bar{u}(n)\left[g_A(q^2)\gamma_\lambda\gamma_5 + iF_p(q^2)q_\lambda\gamma_5\right]u(p) \tag{2}$$

One gets the Goldberger-Treiman result

$$F_p(q^2) = \frac{2M g_A(q^2)}{q^2 + m_\pi^2} \tag{3}$$

by saying that F_p is dominated by one-pion exchange as depicted in Fig. 3a. Now, what happens when a muon is captured by a proton inside the nucleus? Clearly $F_p(q^2)$ cannot be the same as the free proton for two (perhaps related) reasons. Firstly, the exchanged pion must feel the presence of other nucleons (that is, the pion wave must be distorted), secondly, there must be an exclusion principle correction. The latter is seen in the following way. The left vertex in Fig. 3a is usually calculated with the renormalized π NN coupling constant g_r(≅13.4). Presumably this g_r represents the sum of all Feynman diagrams like those given in Fig. 4, where again the intermediate states are integrated over all momenta. Making the exclusion principle correction one obtains the correction to the Goldberger-Treiman value in the form of a two-body correction as given by Fig. 3b. One would have drawn this graph even in the language of the distorted pion wave. As we shall see later this can be also viewed as an exchange current correction; more specifically an exchange current to the pseudoscalar part of the axial current in μ capture. All these different languages are in part equivalent and I shall not try to make a distinction as to which is which.

Now note the similarity between Fig. 2d and Fig. 3b. As it stands Fig 3b is obtained from Fig. 2d by replacing an outer nucleon line by the lepton line ($\mu^- \nu_\mu$). The blob is the same. This analogy will enable us to evaluate the correction to F_p from the calculations available for the three-body graph.

Let us finally examine the old exchange current. A more honest definition is given in the next subsection. Here we simply note the analogy of the exchange currents to what we talked about above. Let us denote the current (a four-vector) as J_λ where λ is a Lorentz index (1,2,3,4). The current can be either the electromagnetic current J_λ^{EM} or the axial current J_λ^{A}. A single particle matrix element will be denoted by a wiggly line attached to a nucleon line as in Fig. 5. At the vertex, we stick in a renormalized coupling constant, which again in principle must be calculable in terms of many graphs like the ones of Fig. 5 (for the axial current).

The exchange currents appear as the exclusion principle correction in the same way as we have obtained the correction to the PS coupling constant. Figures 5c and 5d plus many others then lead to a graph depicted in Fig. 6a, where the blob indicates all kinds of complicated stuffs appearing in the matrix element of the current between a pion and nucleons. This graph then stands for one of two-body corrections to a one-particle current operator. Since it is a current attached to the blob it can act only as a correction to the current, whereas Fig. 3b may also be interpreted as a distortion of the pion wave. Despite the PCAC, the way one can go about computing them is somewhat different. We will come back to this point later.

B. A Meson Theoretic View [5)]

Up to now I have been discussing the meson effect as arising from obeying a well-known principle in which every physicist believes. Translating those Feynman graphs into numbers to compare with experiments is easy as the next section will show. But the question of double counting, normalizations, etc., is not obvious from the graphs alone. It is in this context that the following discussion helps in clarifying the situation. I shall restrict to the exchange current, although a similar discussion can perhaps be given for other meson exchange effects.

In nuclear physics, we are given a Hamiltonian which we suppose is good for deuteron. Let us denote it as the sum of the kinetic energy part H_0 and a potential V_{12}:

$$H = H_0 + V_{12} \tag{4}$$

The eigenstates and eigenvalues of H will be denoted by $|\varphi_m\rangle$ and E_m respectively with a complete set of quantum numbers m:

$$H|\varphi_m\rangle = E_m|\varphi_m\rangle \tag{5}$$

Suppose that in the presence of some external disturbance, say, the e.m. field, there is an operator H_{int} such that

$$\langle \varphi_n | H_{int} | \varphi_m \rangle \tag{6}$$

describes correctly the transition matrix element between an initial state with quantum number m and final state with n. We would like now to find this H_{int} from the way the potential V_{12} is defined. To do so, we consider a field theoretic Hamiltonian including the e.m. interaction (we take this for definiteness; any interaction will do),

$$h_T = h + v_\gamma \tag{7}$$

where

$$h = h_0 + v_\pi$$

$$h_0 = \sum_{i=1,2} h_N(i) + h_\pi$$

$$v_\pi = \sum_{i=1,2} h_{\pi NN}(i)$$

$$v_\gamma = h_{\gamma\pi\pi} + \sum_{i=1,2} h_{\gamma NN}(i) .$$

In this equation the subscript N stands for nucleon, π for pions, $h_{\pi NN}$ for bare π N interaction; h_0 is the unperturbed portion, and the coupling of π to N occurs through v_π. In the following, we shall treat the problem to first order in v_γ but hopefully to all orders in v_π [6].

We denote a two-nucleon eigenstate of h by a round ket $|\Psi_m)$;

$$h|\Psi_m) = E_m |\Psi_m) \tag{8}$$

Note that H [Eq. (4)] is defined so that the eigenvalue is the same as that of Eq. (8). The wave functions differ of course. The non-interacting two-nucleon state without pions will be denoted by $|0_m)$. Then the formal solution for $|\Psi_m)$ is

$$|\Psi_m) = |0_m) + \frac{1}{e} t \, |0_m) \tag{9a}$$

with

$$t = v_\pi + v_\pi \frac{1}{e} t$$
$$e = E_m - h \, . \tag{9b}$$

Now, an electromagnetic transition between states with quantum numbers n and m is

$$(\Psi_n | v_\gamma | \Psi_m) \tag{10}$$

from which we would like to extract H_γ as

$$\langle \varphi_n | H_\gamma | \varphi_m \rangle = (\Psi_n | v_\gamma | \Psi_m) \, , \quad H_\gamma \equiv H_{int}^{EM} \, . \tag{11}$$

Note that the pion degrees of freedom must appear in the operator H_γ on the left side, since $|\varphi\rangle$ has no explicit dependence on them, while they appear principally in the wave function $|\Psi)$ and to a lesser degree in the operator v_γ on the right-hand side.

What we need to do is to express $|\Psi)$ in terms of $|\varphi\rangle$. The latter in turn can be written in terms of $|0)$ as

$$|\varphi_m\rangle = |0_m) + \frac{Q_0}{e} t \, |0_m) \tag{12}$$

where Q_0 is a projection operator which picks out a two-nucleon state with no pions, and vanishes when it operates on a state with pions. The potential V_{12} of Eq. (4) is <u>defined</u> so that

$$|\varphi_m\rangle = |0_m) + \frac{1}{e} T \, |0_m) , \quad T = V_{12} + V_{12} \frac{1}{e} T \tag{13}$$

If we define an operator t_π which introduces pions explicitly,

$$t_\pi = v_\pi + v_\pi \frac{Q_\pi}{e} t_\pi \tag{14}$$

in which $Q_\pi \equiv 1 - Q_0$ is a projection operator which picks out states with pions, then from Eq. (9b) and (14), one has

$$t = t_\pi + t_\pi \frac{Q_0}{e} t \tag{15}$$

and therefore Eq. (9a) becomes

$$|\Psi_m) = \left(1 + \frac{Q_\pi}{e} t_\pi\right) |\varphi_m\rangle \tag{16}$$

which is what we wanted. Equation (11) leads to

$$H_\gamma = \left(1 + \frac{Q_\pi}{e} t_\pi\right)^\dagger v_\gamma \left(1 + \frac{Q_\pi}{e} t_\pi\right) . \tag{17}$$

We are considering a bound two-particle system and hence $|\Psi)$ should be normalized accordingly

$$(\Psi_m | \Psi_m) \equiv N_m^{-2} = 1 + \langle \varphi_m | t_\pi^\dagger \left(\frac{Q_\pi}{e}\right)^2 t_\pi | \varphi_m \rangle \tag{18}$$

so that a properly normalized operator is

$$(H_\gamma)_{nm} = N_n N_m \left(1 + \frac{Q_\pi}{e} t_\pi\right)^\dagger v_\gamma \left(1 + \frac{Q_\pi}{e} t_\pi\right) \tag{19}$$

where the subscripts (nm) apply only to the normalization factors. For two particles in scattering state, N^2 turns out to be equivalent to the wave function renormalization Z_2 in field theory, in which case N^2 does not appear in a calculation where renormalized quantities are used. When the two particles spend some of the time near each other exchanging pions between them, however, N^2 is certainly more than just Z_2. When t_π acts on the same nucleon in Eq. (18) (that is, pions are emitted from nucleon 1 and reabsorbed by the same nucleon), then it would just give the Z_2 factor. When pions are emitted by a nucleon and reabsorbed by another, then it is an extra contribution. Thus, we can write

$$N_{m}^{2} = Z_2 \left[1 + Z_2 \langle \varphi_m | t_\pi^+ \left(\frac{Q_\pi}{e}\right)^2 t_\pi | \varphi_m \rangle_{ND} \right]^{-1} \tag{20}$$

where ND means non-diagonal in nucleon co-ordinates for t_π [7].
Considering the case $n = m$, Eq. (19) would read as

$$(H_\gamma)_{mm} = \left[1 + Z_2 \langle \varphi_m | t_\pi^+ \left(\frac{Q_\pi}{e}\right)^2 t_\pi | \varphi_m \rangle_{ND} \right]^{-1} Z_2 \left(1 + \frac{Q_\pi}{e} t_\pi\right)^+ v_\gamma \left(1 + \frac{Q_\pi}{e} t_\pi\right). \tag{21}$$

The renormalization game to put the second factor on the right in terms of renormalized quantities is beyond my scope here (even if such a game exists). We shall simply understand that all the single-particle process (including pions emitted and reabsorbed) multiplied by z_2 is equivalent to v_γ with renormalized coupling constants, etc. Next, terms non-diagonal in nucleon co-ordinates (for t_π) multiplied by Z_2 will be assumed to contribute to two-body operator with only renormalized quantities appearing.

The lowest order graphs (now in g_r) are given in Fig. 8. These graphs have the old-fashioned propagators and should not be confused with Feynman graphs. Notice that because of the operator Q_π, the graphs of Fig. 9 should not be included. They are already taken into account in the wave function $|\varphi\rangle$, that is in $\langle \varphi_n | v_\gamma | \varphi_m \rangle$. Equation (21) contains further two-pion exchanges, three-pion exchanges, etc. In practice, it would be impossible to calculate the multi-pion exchange graphs. Therefore, we shall in practice do as is done in nuclear force. We shall take the full one-pion exchange term as given in Fig. 6 with the blob indicating all possible vertex corrections, and then for the multi-pion exchanges, we shall introduce heavier mesons.

We treat the normalization correction

$$Z_2 \langle \varphi_m | t_\pi^+ \left(\frac{Q_\pi}{e}\right)^2 t_\pi | \varphi_m \rangle_{ND}$$

in the same spirit, although it would be even tougher to give a rigorous renormalization scheme in this case. We shall take only the OPE term as given by Fig. 7b and ignore all the rest. Heavy meson exchange terms are numerically very small, so that this would be a good procedure provided my interpretation of the correction is correct.

The point of this subsection is that for the case where two nucleons can spend some time near each other (at a distance of m_π^{-1}), there is a non-trivial normalization correction, and that if one were to use nuclear wave functions $|\varphi\rangle$, one needs to exclude a certain set of graphs to avoid double counting. The prescriptions for them are clear, provided one can do the renormalization.

3. LESSONS FROM LOW ENERGY THEOREMS

We now proceed to do some calculations getting hints from the low-energy theorems (involving pions) which have been discussed extensively in the literature. We need only the most unsophisticated version. I shall treat the material in the same order as I did in the previous section.

A. Three-body Force [3)]

The relevant diagram we are interested in is given by Fig. 2d. Heavy meson exchanges can also be considered, but there are good reasons to believe that they contribute insignificantly. To simplify the problem, let us consider nuclear matter, and calculate the correction from the three-body force to the nuclear matter binding energy. I do not think that anybody has derived a three-body force in co-ordinate space corresponding to Fig. 2d. But it is not necessary to do so. One quick way of evaluating the contribution of Fig. 2d as used by Ref. 3) is to consider the three-body graph as a propagator correction to the two-body graph of Fig. 1a (with pion exchange). To do that, we need to consider the direct term where the second nucleon N_2 suffers no momentum transfer, therefore both pions carrying the same momentum q. Of course, an exchange term would not have such kinematics, but apparently it is small. We neglect it. I have drawn in Fig. 10 that portion of the graph corresponding to a forward scattering of a virtual pion off a nucleon. Now, if the pion in Fig. 1a has a propagator (note that I am using the metric $q^2 = \underset{\sim}{q}^2 - q_0^2$)

$$\frac{1}{q^2 + m_\pi^2} \tag{22}$$

a direct term of Fig. 2d may be viewed as effectively modifying the pion mass in nuclear matter to

$$\frac{1}{q^2 + m_\pi^2 + \delta m_\pi^2} \tag{23}$$

which when expanded gives (22) as the lowest term, and $(q^2+m_\pi^2)^{-1}$ $\delta m_\pi^2 \, (q^2+m_\pi^2)^{-1}$ as the next term which is just the graph (2d). If we denote the forward scattering amplitude depending upon two available invariants $-p \cdot q/M$ and q^2 corresponding to Fig. 10 as

$$T^{\pi N}\left(-\frac{p \cdot q}{M}, q^2\right)$$

and the density of nuclear matter as ρ , then

$$\delta m_\pi^2 = \rho \, T^{\pi N}\left(-\frac{p \cdot q}{M}, q^2\right) . \tag{24}$$

In nuclear matter, the isospin antisymmetric combination of the amplitude $T^{\pi N(-)}$ does not contribute; hence only the symmetric one $T^{\pi N(+)}$ is needed in Eq. (24). In order to compute the three-body force effect, it suffices to consider the difference

$$\Delta = (q^2 + m_\pi^2 + \delta m_\pi^2)^{-1} - (q^2 + m_\pi^2)^{-1} \tag{25}$$

and, therefore, δm_π^2 or $T^{\pi N(+)}$. However, not the entire scattering amplitude contributes. A positive energy nucleon intermediate state (Fig. 11) should not be included, since it is already taken into account in the Schrödinger equation with a two-body potential. Denote the amplitude with the harmful term properly subtracted by $T^{\pi N(+)\prime}$. Then,

$$(\delta m_\pi^2)_1 = \rho\, T^{\pi N(+)\prime}\left(-\frac{p.q}{M}, q^2\right) \tag{26}$$

where the subscript 1 is to distinguish this formula from a later one. If one chooses the frame $p = (0,0,0,iM)$, then

$$-\frac{p.q}{M} = q_0$$

which is small in nuclear matter. We may set it to zero. Then the amplitude depends only on $q^2 \approx \vec{q}^{\,2}$, which, however, differs depending upon which graphs one looks at. In Fig. 12, I have drawn a three-body force by putting a cross on the propagator 12a; Fig. 12b represents a contribution from the three-body force when it is iterated once. Now, Brown and Green [3] find that for Fig. 12a, $q^2 \approx m_\pi^2$, and for Fig. 12b, $q^2 \approx 6.5 m_\pi^2$. Thus, the pion exchanged in the second graph is not so soft. The reason is of course the tensor nature of the force.

Now, what does a low-energy theorem tell us? The Adler consistency condition [8] gives us valuable informations at the symmetry point $q_0 = q^2 = 0$;

$$T^{\pi N(+)}(0,0) = T^{\pi N(+)'}(0,0) = 0 .$$

The remaining art of the game is to extrapolate from $q^2 = 0$ to $q^2 \approx 6.5 m_\pi^2$. There are several ways of doing this, but let me just report on two of them. The results seem to be more or less the same for q^2 relevant to us. One method is to separate $T^{\pi N(+)'}$ into a Born term evaluated with the renormalized coupling constant g_r [which in our case is just the pair term, (Fig. 13) and the "recoil" term (Fig. 14) [9]] and a non-Born (NB) term. Since the Born term extrapolates by itself, what remains to be done is to extrapolate the NB term. This is done by a forward dispersion relation, where an over-all constant is fixed so that the NB term cancels the Born term at the symmetry point. The other method is to take more advantage of the Adler condition in assuming that whatever object it is that cancels the pair term at $q^2 = 0$ (σ meson?) continues cancelling as q^2 moves away from zero, and some other object which vanishes exactly at $q^2 = 0$, shows up as q^2 becomes non-zero. Dispersion theoretic calculations indicate that such an object is reasonably represented by $N^*_{3,3}(1236)$. So one may extrapolate by calculating a graph with N^* intermediate state as given by Fig. 15. I shall not go into discussions as to how one does the calculations. For those interested in the details, Ref. 3) is recommended.

The results of extrapolation are given in Table 1. The dispersion method labelled as D, and the N^* method labelled as N^* give about the same extrapolation up to $q^2 \sim 10 m_\pi^2$. There are two quantities given there, one $(\delta m_\pi^2)_1$ as defined by Eq. (26), and the other $(\delta m_\pi^2)_2$ defined by

$$(\delta m_\pi^2)_2 = \rho K^2(q^2)\, T^{\pi N(+)'}(q_0, q^2) \tag{27}$$

where $K(q^2)$ is the pionic form factor of nucleon, which decreases for larger q^2. This definition is what should be used in calculating the three-body contribution [Eq. (25)]. The reason for this is as follows. A three-body force contribution to Figs. 12a and 12b carries the pionic form factor $K(q^2)$ at each πNN vertex, whereas a OPE force of Fig. 1a is usually calculated with the form factor on the mass shell $K(-m_\pi^2) = 1$. Therefore, to interpret the three-body force as being a correction to a two-body force via a propagator modification as was done here, $K^2(q^2)$ has to be multiplied into $(\delta m_\pi^2)_1$ to have a correct form factor dependence.

What Brown and Green obtain is that the three-body force contributions from Figs. 12a and 12b are respectively about -1 and $+4$ MeV/particle with a total of $\sim+3$ MeV/particle. This is not a small number compared with the two-body force contribution $\sim+10$ MeV/particle.

B. Pseudoscalar Coupling Constant [4)]

The blob appearing in the two-body contribution to the pseudoscalar form factor is quite similar to that of the three-body force. There are differences, however. Firstly, the pion (π_1 in Fig. 3b) coupled to the leptons can carry only a definite momentum which turns out to be $q_1^2 \approx m_\mu^2 \approx \frac{1}{2}m_\pi^2$ whereas the pion (π_2 in Fig. 3b) exchanged between the nucleons has on the average $q_2^2 \approx m_\pi^2$, or larger if the tensor nature of the force comes in. In three-body force, we have considered the case where both pions carry the same momentum, hence a forward scattering amplitude. Now, if we are to use correlated wave functions to evaluate the nuclear matrix elements, then only the non-iterated graph (Fig. 3b) is relevant for which π_2 may be taken to have $\sim m_\pi^2$. The second difference is that while π_1 must be a charged pion, π_2 can be $\pi^\pm$ or π^0. This means that both $T^{\pi N(+)}$ and $T^{\pi N(-)^2}$ can, in principle, contribute.

For these two reasons, it is not quite correct to consider Fig. 3b as a propagator correction to Fig. 3a. A correct way is to evaluate Fig. 3b as a genuine two-body operator as is done for the exchange current. It is of course quite complicated. For our purpose of making a rough estimate, we shall ignore the difference between q_1^2 and q_2^2, and consider Fig. 3b as a propagator correction, that is, we ignore π^0 exchange which is equivalent to ignoring $T^{\pi N(-)}$ 10).

Then, the Goldberger-Treiman relation [Eq. (3)] is modified in nuclear matter to

$$F_p^{\text{Nuclear}}(q^2) = \frac{2M\, g_A(q^2)}{q^2 + m_\pi^2 + (\delta m_\pi^2)_1} = \eta\, F_p^{\text{Free}}(q^2) \tag{28}$$

where

$$\eta = \frac{q^2 + m_\pi^2}{q^2 + m_\pi^2 + (\delta m_\pi^2)_1} \tag{29}$$

and F_p^{free} is that given by Eq. (3) for free nucleon. Note that here we need $(\delta m_\pi^2)_1$ instead of $(\delta m_\pi^2)_2$ [see Eqs. (26) and (27)]. This is because each vertex in Fig. 3a requires the form factor, unlike in nuclear force. To obtain a number, let us put $q^2 = \frac{1}{2} m_\pi^2$ which is about the right momentum transfer for μ capture in nuclei, and read off from Table 1,

$$\delta m_\pi^2 \approx -0.26\, m_\pi^2 \tag{30}$$

and so that

$$\eta \approx 1.2. \tag{31}$$

a 20% enhancement over the G-T value. If one takes

$$C_P \equiv m_\mu F_P(q^2)/g_A(q^2) \simeq 7$$

usually quoted as the G-T value, one would have

$$C_P^{\text{Nuclear}} \lesssim 8.5 \tag{32}$$

I consider this as an upper limit, because among other reasons, the nuclear matter density is perhaps an overestimate if capture occurs near nuclear surface. I heard from the Louvain group that their experiment in B^{11} sets an upper limit of about 9.

Before closing this part, let me point out a few things which are important to bear in mind. We recall that in calculating δm_π^2, we took away the positive energy nucleon intermediate state for the reason that it is already in the nuclear wave function (in the language developed in Section 3.B, it is the wave function $|\varphi_m\rangle$). One may work instead with the uncorrelated state $|0_m)$, in which case we must include that term also. This, in principle, could be a systematic way of doing the calculation, but, in practice, quite difficult. If one takes only the nuclear intermediate states, one may compute the entire pseudoscalar nuclear matrix element by putting in particle-hole bubbles to modify the pion propagator. What I mean is depicted in Fig. 16 which is a Goldstone diagram rather than a usual Feynman graph. The particles and holes interact through the effective interaction G (usually called Brueckner G or K matrix), not just through the exchange of a pion. There are a couple of disadvantages in looking at the problem in this way, however. From the practical point of view, the extraction of $F_p^{\text{Nucl.}}$ is not straightforward. One has to divide it by a nuclear matrix element of the operator without the coupling constant, and this has to be done in the same way as for what we have above. The other point can be more serious. Consider for example a graph with only one particle-one hole bubble. One cannot use the pseudoscalar coupling for the πNN vertex, for then we will have the well-known disaster coming from the $N\bar{N}$ (pair) term.

Let us assume that that pair term is somehow suppressed as it must be. Then, a simple calculation with $\vec{\sigma}\cdot\vec{p}$ coupling shows that $\delta m_\pi^2\,(q_o,q^2)$ has a pole at $q_o \approx q^2/2M$ which, for $q^2 \approx \frac{1}{2}m_\pi^2$ is about 5 MeV. This is not too far from the actual value $q_o \sim 10$ MeV. The consequence of this phenomenon is that δm_π^2 can be very sensitive to what transition energy one is dealing with. Therefore, it is important to treat the p-h interaction very carefully. If this is not done properly, one will have a spurious enhancement which does not exist in non-perturbative approach. On the other hand, if this Born term is taken into account in the wave function $|\varphi_m\rangle$, we have no such problem. The enhancement expected near $q_o \approx 5$ MeV in a finite order of perturbation theory like Fig. 16 is merely a disease in the approach, not a real effect. Therefore, the large enhancement ($>50\%$) found by Wycech [4] based on this method is most probably subject to this criticism.

C. Exchange Currents [11]

I have discussed in Section 2.B a meson theoretic way of looking at the exchange current operator. One knows that it is impossible to calculate anything in that way. Instead we shall take the attitude of nuclear force people in that multi-pion exchanges are simulated by heavy meson exchanges. Let us split the graph into a OPE contribution (Fig. 6a) and heavier meson exchanges (abbreviated as HME), (Fig. 6b). The OPE term we try to calculate as accurately as possible, and the rest somewhat roughly, considered as a correction. This is not always justified, especially if the OPE turns out to be small. In that case, the numbers can serve only as an estimate of size.

The low energy theorem is applicable to the OPE graph in the following way. If one looks at the blob in Fig. 6a, one notes that it is just a production of pion off a nucleon through the current J_λ. The pion is virtual, and would have a small positive value for q^2, if the initial and final nucleons are within and near the Fermi sea. We already know that it is $q^2 \approx m_\pi^2$. The soft pion theorem is then applied to the matrix element [for reference, see Dashen and Adler, in Ref. 12)]

$$(q^2+m_\pi^2)\int d^4x \; e^{-iq\cdot x}\langle p'|T(\phi_\pi(x)J_\lambda(0))|p\rangle \tag{33}$$

in which T is the time-ordering operator. As is usually done, we shall decompose it into a Born term and a NB term. The former is a definition from which the NB term is determined by the low energy theorems. The soft pion result would be exact if its conditions are met. The pions in nuclei are very nearly soft in a process like Fig. 6a and in fact a rough calculation correcting for the non-softness of the pions shows a rather negligible correction [Ref. 11) for details]. Let me emphasize that this is an important point - that we can treat the pion term rather accurately.

Now, given the blob, and πNN vertex

$$i g_r K(q^2)\,\bar{u}(p_2')\gamma_5\tau_j\,u(p_2)$$

we can obtain a two-body current $j_\lambda(\vec{x}_1,\vec{x}_2)$ from the Feynman graph (Fig. 6a) by Fourier transforming it,

$$\delta(\vec{x}_1'-\vec{x}_1)\,\delta(\vec{x}_2'-\vec{x}_2)\,j_\lambda(\vec{x}_1,\vec{x}_2) = (2\pi)^{-6}\int d\vec{p}_1\cdots d\vec{p}_2{}'\,M_\lambda(\vec{p}_1,\cdots,\vec{p}_2{}')\,e^{-i(\vec{p}_1\cdot\vec{x}_1+\vec{p}_2\cdot\vec{x}_2-\vec{p}_1{}'\cdot\vec{x}_1{}'-\vec{p}_2{}'\cdot\vec{x}_2{}')} \tag{34}$$

where M_λ is what the Feynman rule gives for Fig. 6a. In general, $j_\lambda(\vec{x}_1,\vec{x}_2)$ depends upon derivative operators acting on the δ functions. These operators which are non-local will not be included in our discussion. Including them would make life terribly complicated. Besides, they are numerically small when taken between wave functions we are dealing with. In the limit that the current carries small momentum, we only need the space component of j_k; i.e., $k = 1,2,3$. For the Gamow-Teller matrix element in β decay, the time component contributes only in higher order of (p/M) and thus need not be included.

For the magnetic moment, the operator is

$$\hat{\mu} = -\left(\frac{i}{2}\vec{\nabla}_{\vec{k}} \times \vec{J}\right)_{k=0}$$

so that only $\vec{j}$ would be needed.

The formulas one obtains in this way are unfortunately very lengthy and unilluminating. I shall not reproduce them here. Those who can amuse themselves with complicated formulas are referred to Ref. 11). What I would like to do here is simply to summarize what we learn from it.

a. Magnetic moments

The low energy theorem teaches us that the isovector E.M. production of pions at small four-momentum (physically at threshold) is well described by the Born graphs [to the accuracy of (m_{π}/M) where M is the nucleon mass] whereas for the isoscalar production the Born terms are small, and hence the theorem is unable to say anything [12]. This means that the exchange current $\vec{j}^{v}$ (v for isovector) can be unambiguously calculated by the Born graphs, but not $\vec{j}^{s}$ (s for isoscalar). It also means that we can at best do a model dependent calculation for the latter. To be complete, we shall do all we can do - calculate Feynman graphs of the type given in Fig. 17 by using phenomenological Lagrangians. For $\vec{j}^{v}$ this is merely a correction.

As for the HME graphs (Fig. 6b), the vector dominance model is used whenever possible. It turns out that only ρ and ω make significant contributions, so that we have ignored any other mesons. There is an inherent uncertainty due to the uncertainty in the coupling constants. For the isoscalar moment this is a serious drawback; however, for the isovector moment it amounts to a small number.

Let me now go into a little detail on some subtle points. Recall that given a two-body operator, the matrix element is to be obtained by sandwiching it between the wave functions $|\varphi_m\rangle$ containing nuclear correlations. This $|\varphi_m\rangle$ contains for example the short-range correlation reflecting the particular short-range behaviour of potentials. Therefore, in principle, there should be no ambiguity for short-ranged operators if the short-range part of the wave function were correctly accounted for. But this short-range behaviour of wave function is a major open question in nuclear physics; hence, we have to rely on models. Now, which are the short-ranged operators? Firstly, some of the Born terms are. In particular, the "recoil terms" (as we shall call them) of Figs. 8c and 8d can be shown to have a radial dependence (K's are Bessel functions of 2nd kind)

$$K_0(m_\pi r) - K_1(m_\pi r)/m_\pi r \tag{35}$$

which is fairly sensitive to the wave function at short distances. Another example is the vertex correction graph (Fig. 17b) which, if one evaluates blindly has the radial function

$$\frac{e^{-m_\pi r}}{m_\pi r} - \left(\frac{m_\rho}{m_\pi}\right)^3 \frac{e^{-m_\rho r}}{m_\rho r} \tag{36}$$

whose matrix element can be very wrong if one ignores the (correlation) hole in the wave function.

Another delicate point is the normalization, that is, how to compute it in practice. In the discussion of a previous section, I mentioned that we would calculate it to the lowest order in the renormalized coupling constant (see Fig. 7b) while "higher order" terms interpreted in terms of heavy meson exchanges were hand-waved away (the reason being due to the hole in the wave function). Frankly, I think that this procedure is quite questionable. And it is serious especially when one would like to understand a small discrepancy like in deuteron. It does not however affect the <u>ratio</u>

between a single-particle operator and the exchange current, the normalization factor being an over-all factor.

I have examined how much change this normalization amounts to in the tri-nucleon systems ^3H and ^3He. It is found to be

$$N^2 \approx \frac{Z_2}{0{,}98}$$

for the ground state. Let us now confront what I asserted above with some numbers calculated for ^3H and ^3He. They are given in Table 2. μ_2^v and μ_2^s are calculated values for exchange moments (isovector and isoscalar respectively) for ^3H. For ^3He they are $-\mu_2^v$ and μ_2^s. One can summarize the results as follows:

(1) μ_2^v: the Born terms are dominant as the low-energy theorem says. The NB contribution is negligible. The HM contribution amounts to 10-20% of the Kroll-Ruderman term.

(2) μ_2^s: the Born, NB, and HM terms are about equal. Thus, the results are extremely model dependent.

b. Gamow-Teller matrix element

Here the momenta of both the current (k) and the pion (q) are soft. A low energy theorem for such situation was worked out some years ago by Adler and Dothan [13]. What it says is that once the Born terms are defined properly, the NB term can be computed uniquely in terms of off-shell π N scattering amplitudes (in fact, derivatives of the amplitude with respect to k) [14]. Due to the latter which require models for extrapolation, the calculation of the exchange current for β decay is somewhat model-dependent. The error is hard to assess, but it must be of the order of the error made in testing the Adler consistency condition which is 10-20%.

For allowed transitions, the axial vector current gives the one-body operator

$$H_1^A = g_A \sum_i \tau_i^{(+)} \vec{\sigma}_i \tag{37}$$

In general the two-body operator for the pion exchange is very complicated, but if the nuclear wave function has the symmetry

$$P_{12}^r |\varphi\rangle = |\varphi\rangle \quad \left(P_{12}^r f(r_1, r_2) = f(r_2, r_1) \right)$$

then the matter simplifies enormously. This is the case of the Gamow-Teller transition in

$$H^3 \rightarrow He^3 + e^- + \bar{\nu}_e \tag{38}$$

on which I shall talk mostly. An attempt to make some order out of the complications for other nuclei is being made at this moment and perhaps we will have a chance to talk about it on some other occasion. Now, for the process (38) and for the predominant component of the wave function (see next subsection for symmetries of $|\varphi\rangle$), we get a simple operator for the NB term

$$H_2^A = g_A \sum_{i<j} h_{ij}^{NB}$$

$$h_{ij}^{NB} = \text{Constant} \times (\alpha - \beta) \frac{e^{-m_\pi r}}{m_\pi r} (\vec{\tau}_i \times \vec{\tau}_j)_+ (\vec{\sigma}_i \times \vec{\sigma}_j) \tag{39}$$

where α and β are different combinations of π N amplitudes (off shell), and the constant is irrelevant for the discussion. $-\beta$ arises from the identity for <u>symmetric state</u> φ_s [15]

$$\beta (\vec{\tau}_i - \vec{\tau}_j)_+ (\vec{\sigma}_i - \vec{\sigma}_j) \varphi_s = + \beta (\vec{\tau}_i \times \vec{\tau}_j)_+ (\vec{\sigma}_i \times \vec{\sigma}_j) \varphi_s \tag{40}$$

Let me just mention that in other nuclei, and also for the D state of ^3He and ^3H, it will be quite different and furthermore because of the tensor force, tensor operators do not vanish as they do above.

The predictions based on the low energy (Adler-Dothan) theorem for α and β are given in Table 3. The numerical values (labelled as Adler) are obtained from Adler's estimates of the off-shell amplitudes. Note that α and β have about the same magnitude and the same sign, and therefore in the process (38) there is a large cancellation. Due to this, the contribution turns out to be minute. Denote the matrix elements of H_1^A and H_2^A as M_1^A and M_2^A respectively. Then, Table 4 gives for the triton β decay

$$\delta^A = M_2^A / M_1^A \tag{41}$$

in terms of r_c, the hard core radius. Indeed, the column "NB" shows that the cancellation is almost complete. One may wonder why the cancellation is so close. To see this we recalculate α and β by sticking in relevant Feynman graphs. We assume that N^* graphs (Fig. 17a) play the dominant role for the NB terms. We note in Table 3 that indeed the $N^*(\frac{3}{2},\frac{3}{2})$ practically saturates α and β while other N^*'s are negligible. [There is of course the ρ term (Fig. 17b), but as we mentioned before, this needs a careful handling because of the short range behaviour. It is negligible if one calculates the matrix element with Eq. (36) and with correlated wave functions.] It is well known that $N^*(\frac{3}{2},\frac{3}{2})$ cannot contribute in tri-nucleon ground state because of the isospin; thus it is the <u>selection rule</u> that makes the NB minute. Such a selection rules does not necessarily apply to heavier nuclei. It is plausible that the NB can make a sizeable contribution in some nuclei which would also point to a role of $N^*(\frac{3}{2},\frac{3}{2})$ in nuclei.

Unlike the NB term no cancellation occurs within the Born term and also within the HM term. They are small individually. The near cancellation between Born and HM terms is probably fortuitous. Whatever the case, Born + HM is small. This is probably the case also for other nuclei. Thus the search of $N^*(\frac{3}{2},\frac{3}{2})$ role in β decay of nuclei other than ^3H is an interesting open problem.

To summarize: in triton β decay, the exchange current contribution is very small ($\lesssim 1\%$), due to the suppression of the NB by selection rules. This is quite different from the magnetic moment where the Born term plays the major role, and gives about 10% correction.

c. Consequences of exchange currents in ^3H and ^3He

What is the significance (if any) of the results we have obtained? Let me now try to answer this question by examining the structure of the wave function of the tri-nucleon system or indirectly the nuclear force. For the three bound nucleons and in the ground state, ^3H (^3He) has $J = \frac{1}{2}$ $(\frac{1}{2})$, $T = \frac{1}{2}$ $(\frac{1}{2})$, and $T_z = -\frac{1}{2}$ $(+\frac{1}{2})$. An eigenwave function of the nuclear Hamiltonian H would be a linear combination of different space symmetries while keeping over-all antisymmetry required by statistics. The symmetries are S (fully symmetric), S' (mixed-symmetric), D (due to tensor force), and so on;

$$\varphi = \alpha_S \psi_S + \alpha_D \psi_D + \alpha_{S'} \psi_{S'} + \cdots \tag{42}$$

Other symmetries other than S, S', D turn out to be insignificant and so we shall not talk about them here. Both α's and ψ's will reflect the nature of nuclear force. By this I mean that a stronger tensor force will induce larger α_D; a hard core in the potential will punch a hole in ψ's, the size of the hole depending upon the nature of the repulsion.

One cannot say anything about ψ's from the static quantities like magnetic moment and Gamow-Teller matrix elements, but one can say something about α's. This is because the single-particle operators have matrix elements depending only on the $P_i \equiv |\alpha_i|^2$,

$$\mu_1^S = \frac{\gamma_p+\gamma_n}{2}(P_S+P_{S'}-P_D)+\frac{1}{2}P_D$$

$$\mu_1^V = \frac{\gamma_n-\gamma_p}{2}(P_S-\frac{1}{3}P_{S'}+\frac{1}{3}P_D)+\frac{1}{6}P_D$$

$$M_1^A = \sqrt{3}\,(P_S-\frac{1}{3}P_{S'}+\frac{1}{3}P_D) \tag{43}$$

where $(\gamma_p \mp \gamma_n)/2$ is the $\begin{pmatrix}\mu^V\\ \mu^S\end{pmatrix}$ for nucleon, equal to $\begin{pmatrix}-2.353\\ 0.426\end{pmatrix}$ in nuclear magneton.

I list in Table 5 the discrepancy between experiments and single-particle values $\mu_1^{V,S}(P_S,P_{S'},P_D)$ and $M_1^A(P_S,P_{S'},P_D)$ which clearly depend upon the probabilities P. Allowing large errors for the HM and NB (for magnetic moments), our results are consistent with only the upper two rows. On the other hand, the Hamada-Johnston potential which fits scattering and deuteron data implies when used in the tri-nucleon system 9% D, and 2% S' which are in definite disagreement with our results. Supposing our calculations are reliable, this would imply that there is a sizable additional effect in many-nucleon system which makes it invalid to use blindly a two-body potential in many body system. We have noted this in the binding energy of nuclear matter. If what we find is true, then there would be a serious gap in our understanding of nuclear structure. But then our calculation may be erroneous! It would be very important to do this calculation in more sophisticated and accurate way. Our calculation reported here as the exchange current correction is done with only the S state. As we have seen, the β decay in particular can be sensitive to $N^*(\frac{3}{2},\frac{3}{2})$ whenever allowed and hence sizable D state (9%) would bring in the N^* effect which would invalidate the above argument. This is another important point we hope to look into soon. I would like to urge others to do so also.

d. Quasi-elastic neutrino scattering

Although I have no results to show, the process

$$\nu + A_i \rightarrow \mu^- + A_f \tag{44}$$

at low momentum transfer q^2 (small angle Θ) where $q = p_\nu - p_\mu$, $\hat{p}_\nu \cdot \hat{p}_\mu = \cos\Theta$ has an interesting feature which if reconfirmed experimentally might be an evidence of meson exchange phenomenon. What is observed up to now is that for high energy neutrino scattering off nucleus A_i to μ^- and all final states of A_f, the suppression of cross-section as $\Theta \rightarrow 0$ is much less than what the Pauli exclusion principle allows for a one-particle operator. If one neglects the muon mass and if $N = Z$, the cross-section would be zero for $\Theta = 0$ in the impulse approximation. Of course, $m_\mu \neq 0$; hence, the Pauli suppression is not complete even at $\Theta = 0$, unless $E_\nu \gg m_\mu$. In any case, when cats are away, even mice can take over. The hope is that when the main terms are suppressed, the exchange current would show up better [17].

In reality, when $\cos\Theta \rightarrow 1$, $|\vec{q}|$ approaches minimum nuclear excitation energy ~10-20 MeV. So even at that limit the situation is not exactly the same as the β decay. But the CVC and the Siegert theorem say that at $|\vec{q}| = 0$, there is no exchange current for the vector current. I shall not consider it. For the axial current, it turns out that for $|\vec{q}| \ll \frac{1}{R}$, where R is an inter-nucleon distance, a good approximation for the exchange current is

$$H_2^{\nu} = g_A \sum_{ij} \exp(i\vec{q}\cdot\vec{R}_{ij})\, h_{ij}^{\beta}$$

$$\vec{R}_{ij} = (\vec{r}_i + \vec{r}_j)/2 \tag{45}$$

where h_{ij}^{β} is that for the β decay, $|\vec{q}| \rightarrow 0$. Now, if we define the difference

$$\Delta(\cos\theta) = \frac{d\sigma}{d(\cos\theta)} - \left[\frac{d\sigma}{d(\cos\theta)}\right]_{\text{Impulse}} \tag{46}$$

"impulse" meaning the single-particle operator case, then Δ which is the exchange correction to the cross-section is as $\theta \to 0$.

$$\Delta(\cos\theta) \to \frac{G^2}{2\pi}\left[\frac{2}{3}g_A^2\,(1+\sin^2\frac{\theta}{2})\right]\sum_f (E_\mu)_{fi}^2 \left\{ |\int\sigma_1 + \int\sigma_2|^2 - |\int\sigma_1|^2 \right\} \tag{47}$$

where

$$\int\sigma_1 = \langle f|\sum_i \tau_i^{(+)}\vec{\sigma}_i\, e^{i\vec{q}\cdot\vec{r}_i}|i\rangle$$

$$\int\sigma_2 = \langle f|\sum_{i<j} e^{i\vec{q}\cdot\vec{R}_{ij}}\, R_{ij}^{\beta}|i\rangle$$

and

$$(E_\mu)_{fi} = E_\nu - (E_{A_f} - E_{A_i})$$

is the energy carried away by the outgoing muon.

The same discussion about the possible role of $N^*(\frac{3}{2},\frac{3}{2})$ applies here, perhaps more so because the transition here involves kicking a particle from below to above the Fermi sea. Let us wait and see.

4. WHAT ABOUT PUTTING N*'s INTO THE WAVE FUNCTION?

There is an interesting idea [2)] floating around in its infancy that, on the one hand, we may be able to determine the N* components in the nuclear wave function by some high energy experiment, and on the other hand perhaps what we call exchange current effects may be explained away in terms of these N* components. If this were true it would be nice, because one would then know how to correlate N^*'s, short range correlations, and meson exchange effects all at the same time, all of which are open problems in modern nuclear physics.

The lowest N^* lies ~300 MeV above the nucleon state and hence the excitation involved is much greater than any low energy nuclear physics experience. Any nuclear physicist will agree that the excitation energy of ~ 50 MeV is already too complicated. Then, how can one hope to see any effect coming from a 300 MeV excitation? I think the answer is found in what Danos has been emphasizing: that N^* comes in at very short range which pops up in a manner quite different from what happens at 0-50 MeV. Thus the back angle scattering of protons off deuteron involves momentum transfer just where N^*'s become relevant. The evidence from such experiments is that a 1-2.5% N^* probability is likely.

Now, we would like to ask the following question: can the N^* probability of that amount account for the discrepancy usually ascribed to the exchange current? The deuteron is not a good place to look, because only isoscalar moment contributes and an explanation of a small number by N^*'s would not mean much. The next candidate is the tri-nucleon system which has a sizable discrepancy in the isovector moment. Unfortunately an $N^*(\frac{3}{2},\frac{3}{2})$ cannot be mixed into the wave function, which makes the testing somewhat difficult. (Other nuclei are too complicated at least for quantitative studies.) Danos suggests [2], however, that $N^*(\frac{1}{2},\frac{1}{2})$ at 1400 MeV could account for the isovector moment discrepancy (see Table 5) if the probability is about 1%.

Now, suppose that $N^*(\frac{1}{2},\frac{1}{2})$ is the sole agent for the isovector moment discrepancy. Next we ask: is this picture consistent with other exchange currents, say, the β decay? <u>The answer is no, that $N^*(\frac{1}{2},\frac{1}{2})$ plays a very little role and that meson exchange current is really a meson exchange current as the low energy theorem says.</u> To see this, let us write a good bona-fide tri-nucleon wave function including the N^*'s as

$$\Psi = \alpha_S \varphi_S + \alpha_D \varphi_D + \alpha_{S'} \varphi_{S'} + \cdots + \sum_R \alpha_R \varphi_R \tag{48}$$

where φ_R is a wave function wherein one nucleon is in a resonance R. In our case, we take only the Roper resonance $N^*(\frac{1}{2},\frac{1}{2})$. Now, evaluate the matrix element of the magnetic moment operator $\hat{\mu}$, and the β decay operator $\hat{\beta} \equiv \sum_i \tau_i^{(+)} \vec{\sigma}_i$. The presence of φ_R leads to

$$\delta\mu = \alpha_S^* \alpha_R \langle \varphi_S | \hat{\mu} | \varphi_R \rangle + \text{c.c.}$$

$$\delta M^A = \alpha_S^* \alpha_R \langle \varphi_S(He^3) | \hat{\beta} | \varphi_R(H^3) \rangle + \alpha_S \alpha_R^* \langle \varphi_R(He^3) | \hat{\beta} | \varphi_S(H^3) \rangle \tag{49}$$

Taking α's to be real and evaluating the matrix elements explicitly one gets

$$\delta\mu^V = 2\alpha_S\alpha_R \frac{2}{3\sqrt{5}} \left\{ \frac{\mu^*_{-\tau} - \mu^*_{\tau}}{2} \right\}$$

$$\delta M^A = 2\alpha_S\alpha_R \frac{2}{3\sqrt{5}} M^A(N^*_{-\frac{1}{2}} \to N_{\frac{1}{2}}) \tag{50}$$

where

$$\mu^*_\tau = \langle N_\tau \uparrow | \hat{\mu}_3 | N^*_\tau \uparrow \rangle, \quad \tau = \pm\tfrac{1}{2} \text{ for } \binom{p}{n},$$

thus $N^*_{\frac{1}{2}} \equiv N^{*+}$ and $N^*_{-\frac{1}{2}} \equiv N^{*0}$, the superscript + and 0 standing for the charge state. The arrow indicates the spin. M^A is defined to be

$$M^A \equiv \langle N_{\frac{1}{2}} \| \hat{\beta} \| N^*_{-\frac{1}{2}} \rangle \tag{51}$$

where the reduced matrix element is taken with respect to spin. We now appeal to experiments for information on μ^* and M^A. In the literature, μ^* is quoted to be [18)]

$$\mu^*_\tau = 0.42\,\mu_\tau, \qquad \frac{\mu^*_{-\tau} - \mu^*_\tau}{2} = 0.42\,\mu^V \tag{52}$$

where μ_τ is the nucleon moment. From a generalized Goldberger-Treiman relation, one obtains from the width of $N^*(\frac{1}{2},\frac{1}{2})$ [19)]

$$M^A(N^*_{\frac{1}{2}} \to N_{\frac{1}{2}}) \approx 0.36\, M^A(n \to p) \tag{53}$$

Now, $\mu^V \approx \mu^V_1$, and $M^A(n \to p) \approx M^A_1$ where subscript 1 indicates the one-body matrix element in the tri-nucleon system. Hence, we have

$$\frac{\delta\mu^V/\mu^V_1}{\delta M^A/M^A_1} \approx 1 \tag{54}$$

which says that if the N^* contributes 10% isovector moment, then it is expected to contribute the same amount to the Gamow-Teller matrix element. I think this is in violent disagreement with the experiments and the low energy theorem. Clearly, if one were to pursue this idea, one would have to find another mechanism in β decay which would kill off the large N^* contribution. I have tried this, but I did not find one yet.

Alternatively, one may argue that the isovector moment discrepancy does not come from N^* at all (if at all it must be tiny) and is due entirely to something else. This would mean that either α_R or μ_τ^* or both are small. In fact, an order of magnitude calculation comparing coupling constants gives [19)]

$$\alpha_R^2 \approx 0.3 \tag{55}$$

Also, a large class of models predicts $\mu_{\gamma}^{*} = 0$, and the failure up to now to photoproduce $N^{*}(\frac{1}{2},\frac{1}{2})$ again seems to support much smaller μ_{γ}^{*} than used above. <u>But then this is what the low energy theorem has been telling us</u>. Just look back at the "derivation" of the exchange current by means of wave functions (Section 2.B) - whether one puts the blame on the operator or on the wave function is a matter of taste. Clearly the absence of the N^{*} strength in the exchange current for magnetic moment speaks also for its absence in the wave function!

Then, how can one reconcile the large N^{*} probability needed for high energy phenomena (whichever N^{*} it is), and the exchange moments and β decay? This question can probably be answered in the same line as the "effective charges" used in electromagnetic transitions in nuclei. Such a scheme requires an unambiguous way of classifying mesons and baryons so as not to double count things. One way is the quark model, but then there are other problems. This whole thing is an open problem for future.

ACKNOWLEDGMENT

I wish to thank Dr. P. Pascual for congenial atmosphere in which to talk about this subject.

NOTE ADDED AFTER LECTURES

After having written this note I learned that Fujita et al. [20] claim to have found an extra term in the exchange current operator in beta decay which increases δ^{A} by a few percent. I think that that term is included correctly if one evaluates Fig. 17b in the phenomenological Lagrangian method, and amounts to a negligible correction if the PCAC values (Adler) are used. This tricky "yes we have" and "no you have not" argument is described more in Ref. 11).

Table 1 [a)]

	q^2/m_π^2	0.5	1	4	8	12	16
c)	b)						
$(\delta m_\pi^2)_1$	D	-0.15	-0.25	-0.61	-0.86	-0.75	-0.57
	N*		-0.29	-0.66	-0.91	-1.00	-1.07
$(\delta m_\pi^2)_2$	D	-0.13	-0.20	-0.38	-0.33	-0.18	-0.08
	N*		-0.23	-0.41	-0.35	-0.24	-0.15

δm_π^2 in unit of f^{-2} vs. q^2/m_π^2. The density ρ is taken to be that of nuclear matter; i.e., $k_F = 1.35f^{-1}$.

a) Numbers from Ref. 3).

b) D = dispersion method, N* = N* method

c) $(\delta m_\pi^2)_1 = \rho T^{\pi N'(+)}$; $(\delta m_\pi^2)_2 = \rho K^2(q^2) T^{\pi N'(+)}$

Table 2

		Born			NB	HM	Total
	r_c	Feynman graphs	Normalization	Sum			
μ_2^v	0 f	0.139	0.127	0.266	-0.0008	-0.046	0.219
	0.2	0.150	0.093	0.243	-0.0008	-0.036	0.206
	0.4	0.161	0.058	0.219	-0.0007	-0.026	0.193
	0.6	0.165	0.037	0.202	-0.0006	-0.017	0.185
	0.8	0.164	0.024	0.188	-0.0005	-0.012	0.176
μ_2^s	0	-0.018	0.024	0.006	0.0038	0.0053	0.015
	0.2	-0.013	0.017	0.004	0.0036	0.0047	0.013
	0.4	-0.0082	0.011	0.003	0.0033	0.0035	0.0093
	0.6	-0.0053	0.0070	0.002	0.0029	0.0025	0.0071
	0.8	-0.0034	0.0045	0.002	0.0025	0.0019	0.0055

Exchange isovector moment μ_2^v and isoscalar moment μ_2^s calculated as a function of hard core radius r_c for the Kroll-Ruderman (Born), non-Born (NB), and heavy meson (HM) contributions. Experimental values are $\mu_{exp}^v = 2.553$, $\mu_{exp}^s = 0.426$ in nuclear magneton. Gaussian radial wave function fitting Coulomb energy is used throughout.

Table 3

	α	β
Adler	0.70	0.50
$N^*(\frac{3}{2},\frac{3}{2})$	0.55	0.55
$N^*(\frac{1}{2},\frac{1}{2})$	0.09	0
$N^*(\frac{3}{2},\frac{1}{2})$	-0.07	-0.02

Amplitudes α and β calculated from low energy theorem (Adler) and from N^* graphs of Fig. 17a.

$$\left.\begin{aligned} \alpha &\equiv m_\pi^2 \left(\frac{g_A}{g_r}\right) \frac{1}{K(0)} \left(\bar{B}^{\pi N(-)}\right)_0 \\ \beta &\equiv -m_\pi^2 \left(\frac{g_A}{g_r}\right) \frac{1}{K(0)} \left(\frac{\partial \bar{A}^{\pi N(-)}}{\partial \nu}\right)_0 \end{aligned}\right\} \text{Refs. 11,12)}$$

Table 4

r_c	Born			NB	HM	Total
	Feynman graphs	Normalization	Sum			
0	-0.028	0.054	0.026	-0.0035	-0.020	0.0019
0.2	-0.019	0.039	0.020	-0.0034	-0.016	0.0013
0.4	-0.0098	0.025	0.015	-0.0030	-0.011	0.00053
0.6	-0.0048	0.016	0.011	-0.0027	-0.0076	0.00088
0.8	-0.0018	0.010	0.008	-0.0023	-0.0054	0.00083

$\delta^A = M_2^A/M_1^A$ for the triton beta decay. Calculation for NB is based on the Adler values. The same radial function as Table 2 is used.

Table 5

P_S	P_D	$P_{S'}$	$\delta\mu^V$ a)	$\delta\mu^s$	$\delta M^A/M^A_1$
96	4	0	0.269 (10.5%)	0.003 (0.7%)	-0.06%
94	6	0	0.304 (11.9%)	0.009 (2.1%)	1.31%
92.8	6	1.2	0.342 (13.4%)	0.009 (2.1%)	2.97%
92	6	2 b)	0.368 (14.4%)	0.009 (2.1%)	4.09%
91	8	1 c)	0.370 (14.5%)	0.013 (3.1%)	4.04%
89	9	2 d)	0.419 (16.4%)	0.020 (4.7%)	6.11%

Discrepancies between experiments and single particle values in isovector, isoscalar moments and Gamow-Teller matrix element. P_S, P_D, $P_{S'}$ are respectively S, S' and D state probabilities. Experimental values used are $\mu^V(\mu^S) = 2.553\ (0.426)$, $M^A_{exp} = 1.685$.

a) $\mu^{v,s}_{exp} - \mu^{v,s}_1(P_S, P_{S'}, P_D)$. The numbers in parentheses are % defined by $\delta\mu/\mu_1$.

b) Known in the trade as Gibson wave function [see Ref. 16)].

c) Old Blatt-Delves wave function [Ref. 16)].

d) New Blatt-Delves wave function [Ref. 16)].

REFERENCES AND FOOTNOTES

1) None of them, however, is responsible for possible erroneous statements in the lecture.

2) L.S. Kisslinger, Phys.Letters 29B, 211 (1969);
H. Arenhövel and M. Danos, Phys.Letters 28B, 299 (1969).

3) For an extensive discussion on the three-body force, see
G.E. Brown and A.M. Green, Nuclear Phys. A137, 1 (1969).

4) The discussions on the pseudoscalar form factor are based on an unpublished work (or a thought?) of
A.M. Green and M. Rho;
see also
S. Wycech, Nuclear Phys. B14, 133 (1969).

5) This part is entirely based on the thesis of
M. Chemtob, "Les courants d'interaction nucléaires à deux corps", Université de Paris (1969). Although most of the contents may be found in many old articles, I give this reference for its thorough references into the matter.

6) This may or may not make sense, but what we do finally in the next section seems to be meaningful.

7) Perhaps a graphical explanation is easier to see. First we note that

$$\langle \varphi_m | t_\pi^+ \left(\frac{Q_\pi}{e}\right)^2 t_\pi | \varphi_m \rangle = -\frac{\partial}{\partial E_m} \langle \varphi_m | t_\pi^+ \frac{Q_\pi}{e} t_\pi | \varphi_m \rangle$$

and the matrix element

$$\langle \varphi_m | t_\pi^+ \frac{Q_\pi}{e} t_\pi | \varphi_m \rangle$$

is just the second order (in t_π) energy. The diagonal term looks like Fig. 7a, and the non-diagonal term like Fig. 7b.

8) The content of the Adler condition needed here is thoroughly discussed in Ref. 3).

9) The recoil term is found to be small by A.M. Green (private communication).

10) There is a reason to believe that $T^{\pi N(-)}$ is small for $q_0 \approx 0$, $q^2 \neq 0$. See
J. Hamilton, Nuclear Phys. B1, 449 (1967).

11) Details can be found in a paper being written by Marc Chemtob and myself.

12) This is a variant of the well-known Kroll-Ruderman theorem translated into an isospin language. The relevant K-R theorem is best studied in the monograph "Current Algebras" by S. Adler and R. Dashen.

13) See the monograph quoted in Ref. 12).

14) Refer to Ref. 11) for more details relevant to the problem.

15) This would be true independently of the wave function if the radial part of the operator were zero-ranged; i.e., if $m_\pi \to \infty$.

16) L.M. Delves and A.C. Phillips, Reviews of Modern Phys. 41, No. 3, 497 (1969).

17) The sentiment among some physicists is that one has to wait for better experiments. I am basing my observation on the paper by
R.L. Kustom et al., Phys.Rev. Letters 22, 1014 (1969).

18) See Danos et al., in Ref. 2)

19) H. Primakoff, Weak interactions in Nuclear Physics, Karlsruhe Lecture (1969).

20) J. Fujita, H. Ohtsubo and G. Takeda, submitted to Phys.Letters (early 1970).

FIGURE CAPTIONS

Figure 1 : Feynman diagrams considered in two-body force. The symbol M stands for all mesons exchanged between two nucleons. Fig. c is not usually considered explicitly when phenomenological coupling constants are used.

Figure 2 : Feynman graphs for three-body force. Fig. a contains a part which is contained in an iterated two-body force, which is subtracted away to construct the three-body force. Fig. d describes lumping all the vertex correction into the blob.

Figure 3 : Two-body correction to the pseudoscalar form factor in μ capture (b). Fig. a is the usual graph to obtain the Goldberger-Treiman relation for a free nucleon.

Figure 4 : Possible graphs contributing to the renormalized π N coupling constant g_r.

Figure 5 : Possible graphs contributing to the renormalized axial vector coupling constant g_A. The wiggle here denotes the axial current.

Figure 6 : Graphs representing the exchange currents for J_λ^{EM} or J_λ^A. Fig. a is the "dominant" pion exchange, Fig. b the heavy meson exchange.

Figure 7 : Graphs for normalization N_m^2 described in Section 2.B. Fig. a leads to Z_2 (presumably), Fig. b to the ND (non-diagonal) term.

Figure 8 : Old-fashioned perturbation graphs (not Feynman graphs) for pion exchange current. Figs. c and d are usually referred to as recoil term. The wiggly lines represent the current (either J_λ^{EM} or J_λ^A).

Figure 9 : Old-fashioned perturbation graphs which should not be included in the exchange current.

Figure 10 : Graph representing pion-nucleon scattering.

Figure 11 : Positive energy nucleon Born term which should be subtracted away from Fig. 10. Otherwise double counting!

Figure 12 : Two-body graphs with modified propagator used to calculate three-body effects. The cross means that a third nucleon interacts with the pion, thus causing δm_π^2 in the propagator. Fig. b corresponds to a once iterated graph where only one pion propagator is modified. This gives rise to the tensor contribution.

Figure 13 : Pair term ($N\bar{N}$ Born term) in πN scattering.

Figure 14 : Recoil term in πN scattering.

Figure 15 : N* graph used to extrapolate $T^{(+)}(q_0,q^2)$ in q^2.

Figure 16 : Change to the pseudoscalar form factor in μ capture due to the modification of virtual pion field by particle-hole excitation in nuclei. The dot represents full nuclear interaction given by the matrix elements of Brueckner G matrix.

Figure 17 : Non-Born contributions to pion exchange currents to be calculated with phenomenological Lagrangians.

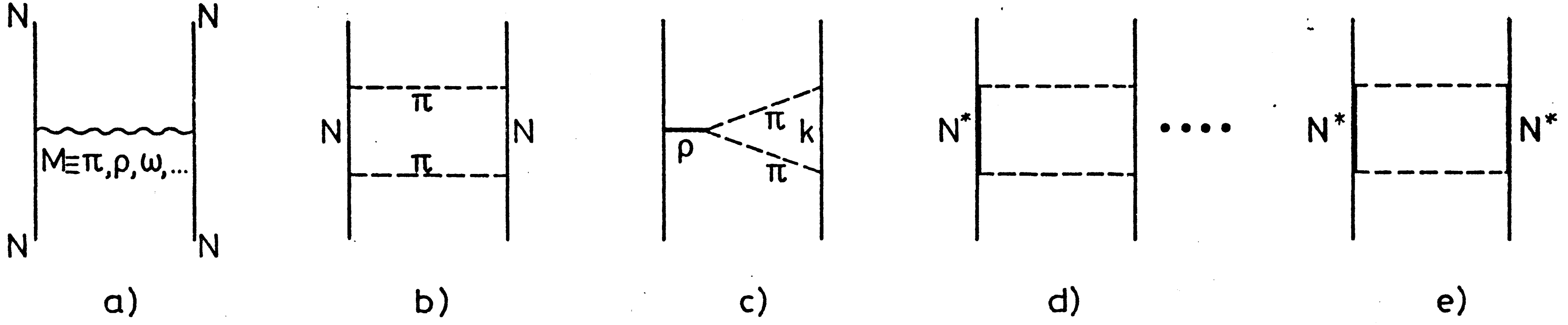

FIG.1

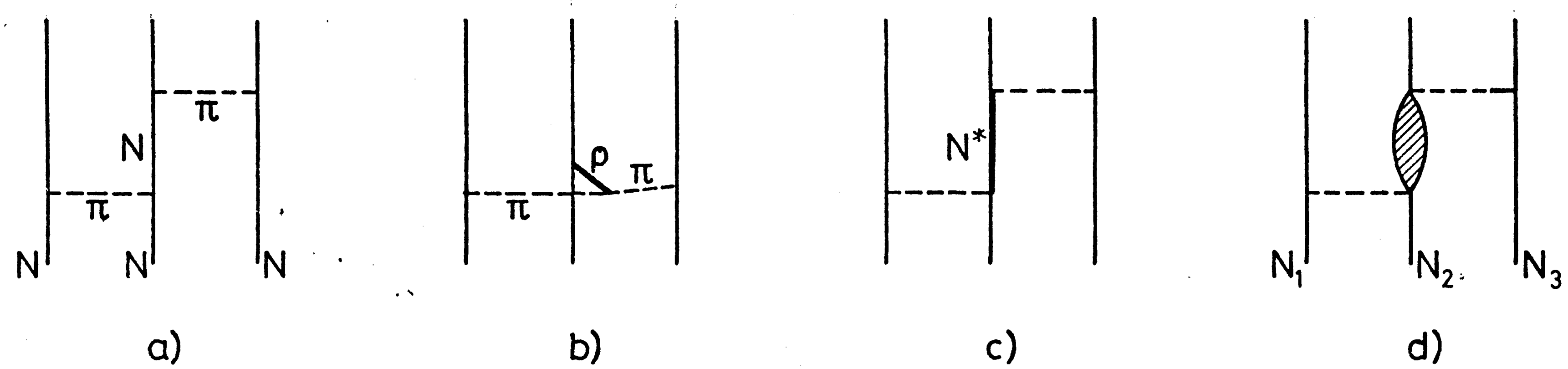

FIG. 2

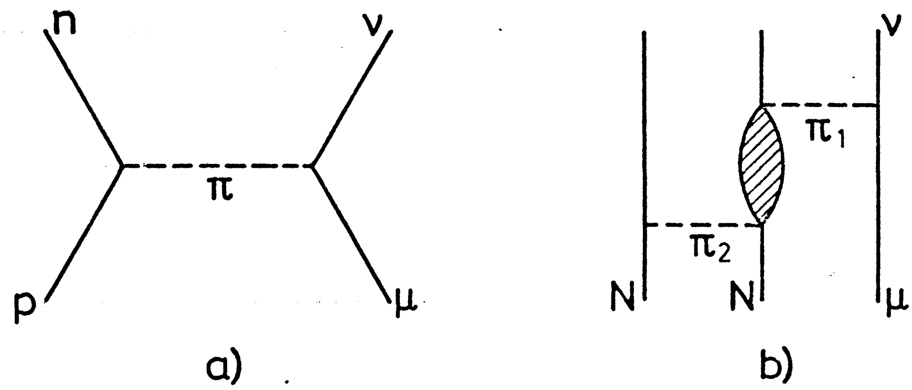

FIG. 3

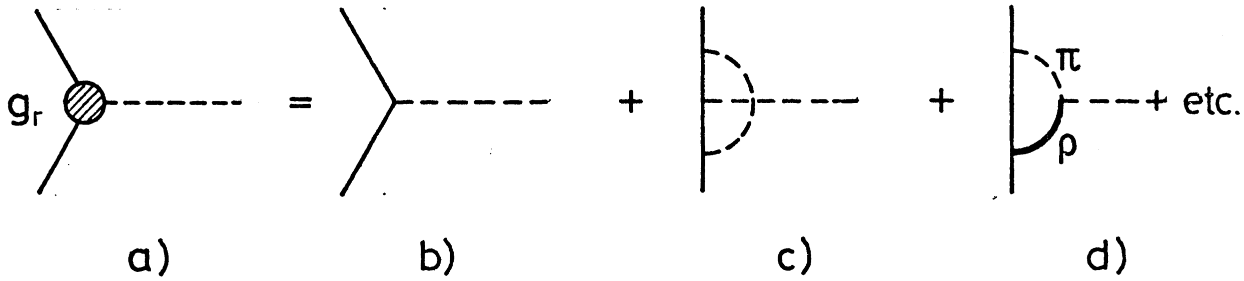

FIG.4

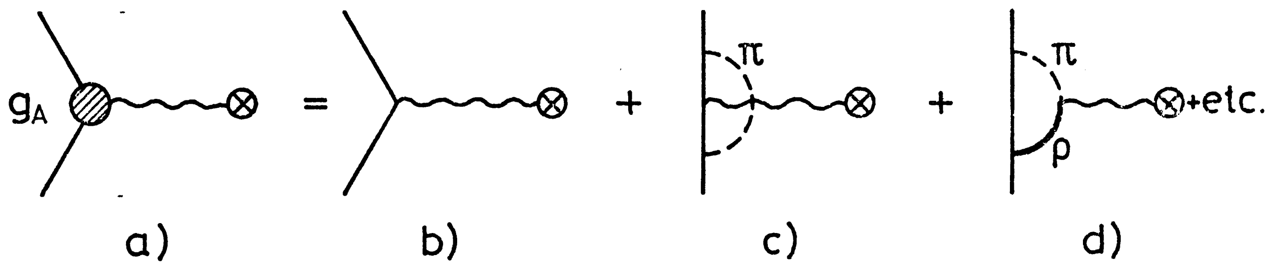

FIG. 5

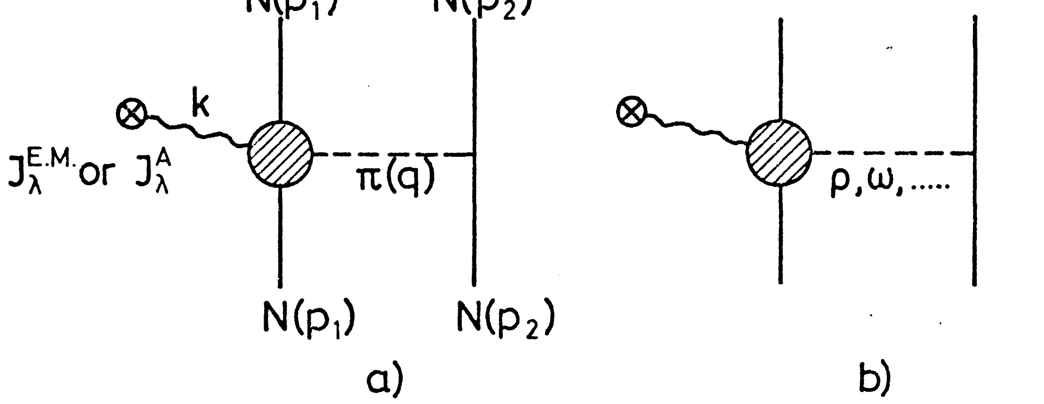

FIG. 6

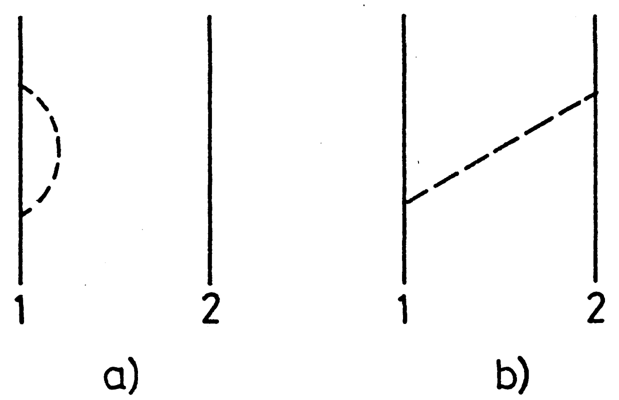

FIG. 7

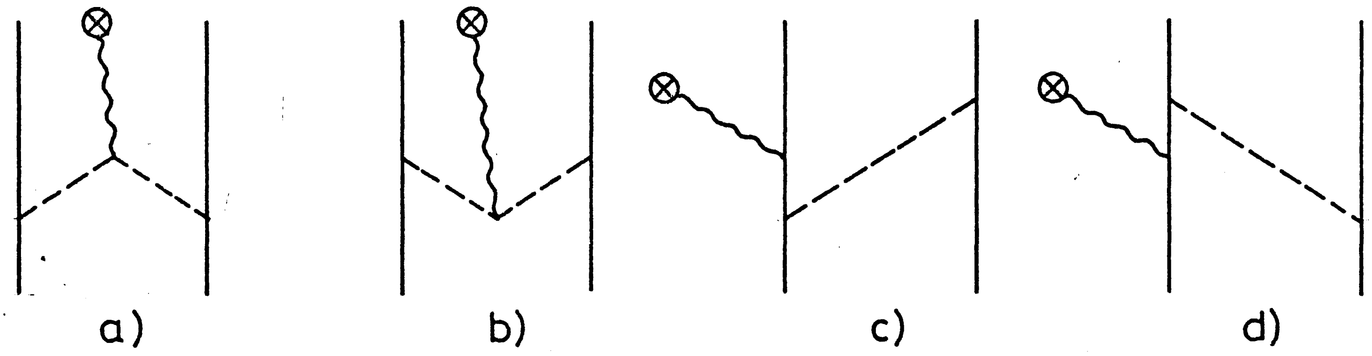

FIG. 8

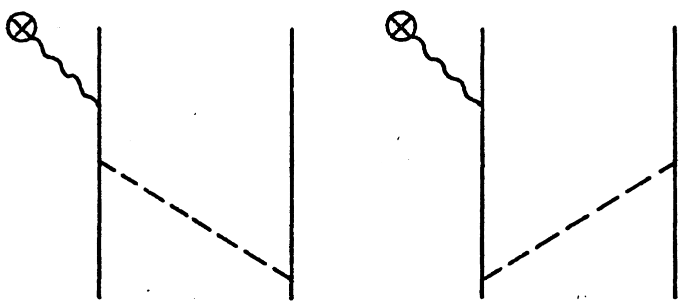

FIG. 9

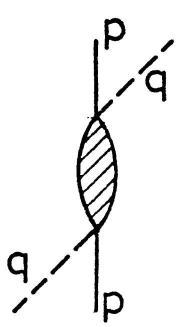

FIG. 10

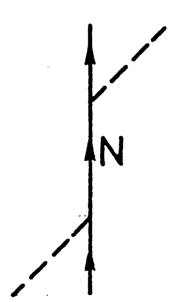

FIG. 11

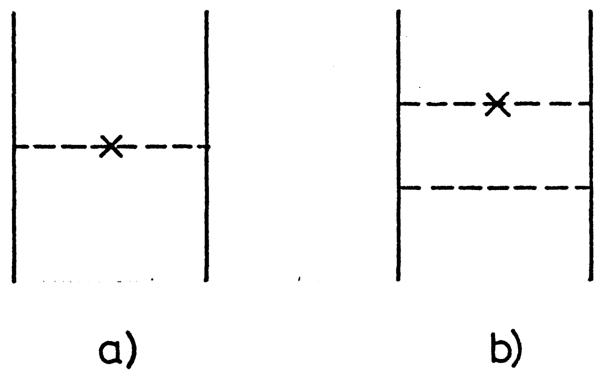

FIG. 12

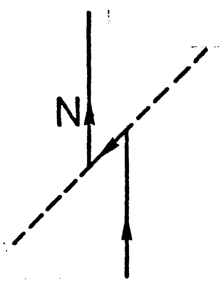

FIG. 13

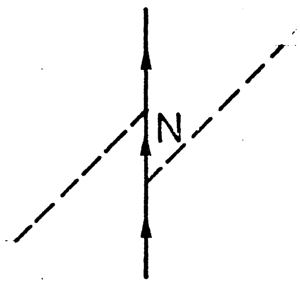

FIG. 14

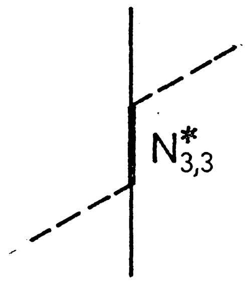

FIG. 15

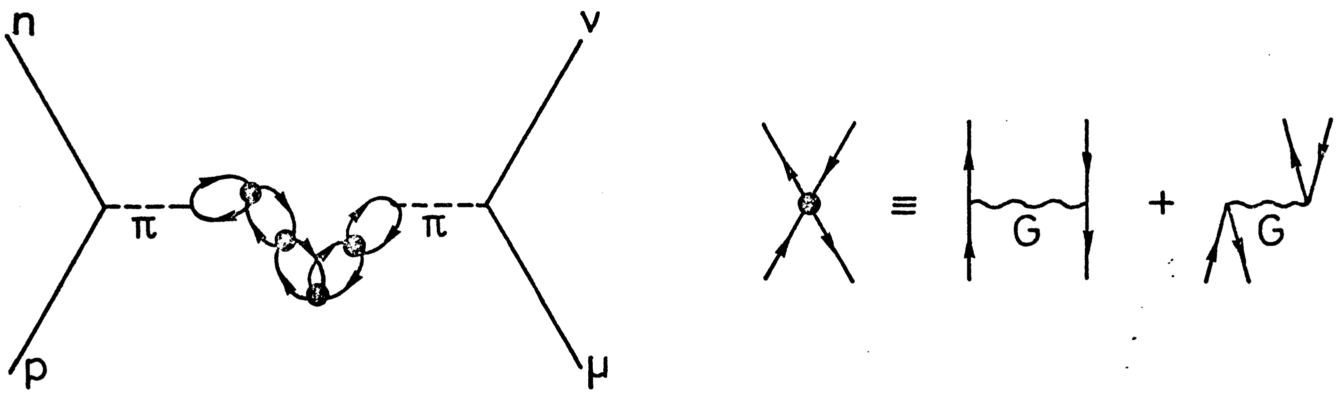

FIG. 16

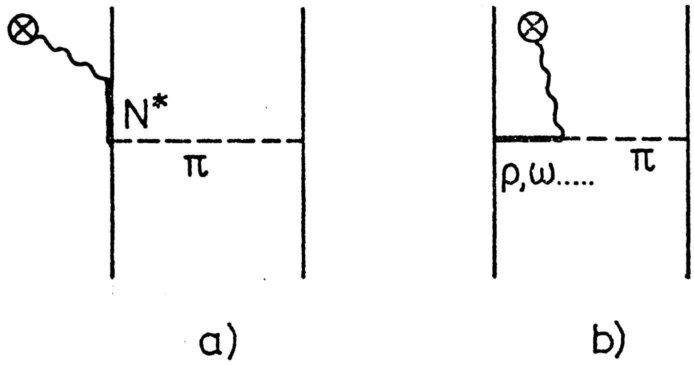

FIG.17