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THE σ-COMMUTATOR TERM FROM π-NUCLEAR SCATTERING

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ABSTRACT

The value of the σ-commutator term is derived from the physical π-nuclear scattering amplitude with a mass extrapolation technique. Our value is σ_{NN} = 34 MeV in disagreement with the large value recently derived by Cheng and Dashen σ_{NN} = 110 MeV.

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Recently, in connection with the chiral symmetry-breaking in the strong interaction Hamiltonian, there has been considerable interest[1-3] in calculating the charge symmetric π-N amplitude in the soft-pion limit in which both incoming and scattered pions have $\underset{\sim}{q}_{in} = \underset{\sim}{q}_{out} = \underset{\sim}{q} = \underline{0}$ and $q_0 \to 0$. In this limit the charge antisymmetric π-N amplitude a_N^- vanishes linearly in q_0, whereas the charge symmetric amplitude a_N^+ is non-vanishing and can be written as an expectation of the equal-time commutator of an axial charge $Q_A^\pm$ with the divergence of the axial current[4] $D^\pm(x) = \partial_\lambda A_\lambda^\pm(x)$

$$a_N^+(q_0=0) = -\frac{i}{8\pi f_\pi^2(m_N+m_\pi)} \langle N|[Q_A^+, D^-(0)]|N\rangle \tag{1}$$

Rewritten in terms of the SU(2) × SU(2) symmetry-breaking Hamiltonian density $H_{SB}(0)$, this reads as

$$a_N^+(q_0=0) = -\frac{1}{2\pi f_\pi^2\left(1+\frac{m_\pi}{m_N}\right)}\,\sigma_{NN} \tag{2}$$

$$\sigma_{NN} = -\frac{1}{4m_N}\langle N|[Q_A^+,[H_{SB}(0), A_0^-(0)]]|N\rangle \tag{3}$$

Here we use the invariant normalization $\langle p/p'\rangle = 2p_0(2\pi)^3\delta^3(p-p')$ for both nucleons and pions, f_π is the charged pion decay constant (= 0.94 m_π), σ_{NN} is the usual σ-commutator term and m_π and m_N are, respectively, the pion and nucleon masses. Thus the knowledge of σ_{NN} brings important information on the part of the Hamiltonian which breaks the axial charge conservation.

The soft-pion limit (2) cannot be reached directly from experiment. Therefore, one has to perform a mass extrapolation from the physical pion mass (140 MeV) to the zero mass required. Such an extrapolation was done by Von Hippel and Kim[1] for π-N scattering as well as K-N and π-Σ processes. By fitting the elastic and inelastic processes, they obtained

$$\begin{aligned} a_N^+(q_0=0) &\simeq -0.03\, m_\pi^{-1} \\ \sigma_{NN} &\simeq 26 \text{ MeV}. \end{aligned} \tag{4}$$

This result was, however, challenged by Cheng and Dashen[2], who obtained by a completely different method a considerably larger value for the same quantity, $a_N^+(q_0) \simeq -0.12\ m_\pi^{-1}$, corresponding to $\sigma_{NN} \simeq 110$ MeV. A more recent calculation of Höhler et al.[3] in turn disagrees with the latter, but is in closer agreement with the value of Von Hippel and Kim; i.e. σ_{NN} obtained by Höhler et al. is about 39 MeV. Due to these differences, the situation with the symmetry-breaking scheme is at present confusing and we shall not dwell on various theoretical interpretations accommodating those different numbers[5].

Recognizing the desirability of settling these differences we present in this letter a different approach to obtain the σ term. What differentiates our method from others is that here the σ-term will be deduced from π-nuclear interaction*). There are several reasons why working with nuclei can be more advantageous than π-N scattering, despite the many-body complications of nuclei.

1) In contrast to the very poorly known charge symmetric π-N amplitude [different analyses[6] give values ranging from $-0.014\ m_\pi^{-1}$ to $+0.02\ m_\pi^{-1}$], the low-energy π-nuclear amplitudes are very well known. Although there exist no scattering data, they can be extracted from the energy shifts in π-mesic atoms, which have been measured with a high precision for a number of elements[9].

2) For the π-mesic atoms, the separation of the electromagnetic and strong interaction effects is done by representing the two by potentials. The reliability of this procedure has been checked by the consistency of the (strong interaction) optical potential found for the elements from $Z = 3$ to $Z = 11$ [10]. Therefore, it is possible to obtain a π-nuclear strong interaction amplitude with the electromagnetic interaction between the pion and nucleus "switched off".

3) The problem of mass extrapolation in nuclei is not any more difficult or more uncertain than in the nucleon case[11]. Moreover, as will be shown below, some of the important correction terms involving off-shell contributions are largely suppressed in nuclei because of the Pauli exclusion principle.

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*) Related ideas have been independently pursued by Gensini[6] and Huang[7].

Our analysis of the charge symmetric amplitude will be made in parallel with the charge antisymmetric one. The reason is that the mass extrapolations are quite similar and the latter case, where both the soft-pion and the physical amplitudes are known, can serve as a test of the reliability of the approximations made. In both cases, the mass extrapolation is best carried out in the framework of non-relativistic potential theory[11]. Then the soft-pion amplitude in the mass-dispersion relation corresponds essentially to the Born amplitude in potential scattering. There are some corrections to this picture and these are evaluated approximately. The close agreement between the values predicted by this method and the experimental ones in the charge exchange process[11] is an encouragement for us to extend the method to the charge symmetric case.

In the Fubini-Furlan technique[12], which we use below, the physical amplitude denoted by $a^+(m_\pi)$ is related to the soft-pion amplitude $a^+(0)$ via a sum rule. This sum rule is truncated and the intermediate states are restricted to nuclear states with zero and one pion. Other intermediate states are ignored consistently with the π-nucleon case[12]. Working in the frame where the nucleus B is at rest ($\underset{\sim}{p}_B = 0$) and the pion has zero three-momentum, we can write

$$a^+(m_\pi) = a^+(0) + \frac{i}{8\pi f_\pi^2(M_B+m_\pi)} \sum_n \left[1 - \left(\frac{E_n - M_B}{m_\pi}\right)^2\right] \langle B|Q_A^+|n\rangle\langle n|D^-(0)|B\rangle + \text{c.t.} \tag{5}$$

$$- \frac{m_\pi^2}{8\pi(M_B+m_\pi)} \sum_{n=B'+\pi} \int d^3x \, \frac{\langle B|j_\pi^+(x)|n\rangle\langle n|j_\pi^-(0)|B\rangle}{(E_n - M_B)\left[m_\pi^2 - (E_n - M_B)^2\right]} + \text{c.t.}$$

where B represents the nuclear ground state, M_B the nuclear mass, E_n the energy of the intermediate state, and $n = B' + \pi$ restricts the summation to intermediate states consisting of a nucleus in a state B' plus a pion. The summation over n in the last term goes over also the isospin of the pion. As one does with the charge antisymmetric amplitude, it is advantageous to separate the coherent rescattering where the nucleus remains in its ground state (i.e. the term corresponding to $B' = B$), which can be identified with the effect of distortion of the pion wave in the potential scattering picture. The remaining terms would correspond to the Born term

$$a^{+}_{born} = a^{+}(0) + \frac{i}{8\pi f_{\pi}^{2}(M_{B}+m_{\pi})} \sum_{n} \left[1 - \left(\frac{E_{n}-M_{B}}{m_{\pi}}\right)^{2}\right] \langle B | Q_{A}^{+} | n \rangle \langle n | D^{-}(0) | B \rangle + c.t.$$

$$- \frac{m_{\pi}^{2}}{8\pi(M_{B}+m_{\pi})} \sum_{\substack{n=B'+\pi \\ B' \neq B}} \int d^{3}x \, \frac{\langle B | j_{\pi}^{+}(x) | n \rangle \langle n | j_{\pi}^{-}(0) | B \rangle}{(E_{n}-M)\left[m_{\pi}^{2} - (E_{n}-M)^{2}\right]} + c.t. \tag{6}$$

The crucial point of our calculation is that the largest term in the mass-dispersion correction -- the coherent rescattering -- need not be considered, in contradistinction to the nucleon case where its evaluation is necessary. What brings out such a fortunate situation here is that a^{+}_{Born} is directly determined from the analysis of the π-mesic data. Note that the soft-pion amplitude $a^{+}(0)$ is closer to a^{+}_{Born} than to $a^{+}(m_{\pi})$. Thus, in order to obtain $a^{+}(0)$ from a^{+}_{Born} one has to evaluate much smaller corrections than in the nucleon case.

Let us first consider the second term on the right-hand side of Eq. (4). This represents the effect of pion absorption. A part of it comes from the absorption by a single nucleon, leading mainly to low-excited states, with the excitation energy much less than the pion mass. This effect has been estimated by d'Auria et al.[13] by assuming that only one excited state saturates the transition matrix element. Although the approximations made here are rough, the value calculated by them (which is less than 0.001 m_{π}^{-1}) can be safely ignored. The remaining contribution arises from absorption by two or more nucleons. Unfortunately, there is no reliable quantitative estimate of this effect. Nevertheless, the magnitude is probably not larger than the imaginary part of the amplitude (i.e. 0.009 m_{π}^{-1}). Our neglect of this contribution constitutes perhaps the largest uncertainty in the calculation, although its negligible contribution in the charge antisymmetric amplitude may be taken as evidence against very large multi-nucleon absorption effect.

To evaluate the last term of Eq. (6) [the incoherent scattering], we make the static approximation for the nucleus, further neglecting the energy difference between the excited states B′ and the ground state B. Then the closure may be used. This approximation clearly overestimates the correction*). Next the matrix element $\int d^3x \langle B|j_\pi^\alpha(\underset{\sim}{x})|B'\pi^B\rangle$ is taken as a sum of individual nucleon contributions. Ignoring off-shell effects and momentum dependence of the pion-nucleon amplitude, we can write

$$(8\pi M_B)^{-1}\int d^3x \langle B_0|j_\pi^+(x)|B'_{q'}\pi^-_q\rangle = \left(1+\frac{m_\pi}{m_\pi}\right)(2\pi)^3\delta(q+q')$$
$$\int d^3r\, e^{i\underset{\sim}{q}\cdot\underset{\sim}{r}}\langle B|\sum_{i=1}^{A}\delta(\underset{\sim}{r}-\underset{\sim}{r}_i)[a_N^+ + a_N^-\tau_i^3]|B\rangle \tag{7}$$

and similarly for the charge exchange process in which case $\sqrt{2}a_N^-\tau_i^+$ replaces the factor $(a_N^+ + a_N^-\tau_i^3)$. From now on we restrict to isospin-zero nuclei (generalization to non-zero isospin case is straightforward). Then the incoherent scattering can be reduced to the following

$$a^+_{Born} - a^+(0) = A\,\frac{m_\pi^2}{\pi^2\left(1+\frac{m_\pi}{M_B}\right)}\left(1+\frac{m_\pi}{m_N}\right)^2\left[a_N^{-2}+\frac{1}{2}a_N^{+2}\right]\int d^3q\,\frac{C(q)}{q^2\omega_q^2} \tag{8}$$

where A is the number of nucleons, $w_q = \sqrt{q^2 + m_\pi^2}$, and $C(q) - 1$ is the correlation function

$$C(q)-1 = \frac{1}{A}\int d^3r\, d^3r'\, e^{iq(\underset{\sim}{r}-\underset{\sim}{r}')}\Big\{\langle B|\sum_{i\neq j}\tau_i^3\tau_j^3\,\delta(\underset{\sim}{r}-\underset{\sim}{r}_i)\,\delta(\underset{\sim}{r}'-\underset{\sim}{r}_j)|B\rangle$$
$$-\langle B|\sum_i\tau_i^3\,\delta(\underset{\sim}{r}-\underset{\sim}{r}_i)|B\rangle\langle B|\sum_j\tau_j^3\,\delta(\underset{\sim}{r}'-\underset{\sim}{r}_j)|B\rangle \tag{9}$$

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*) For an average excitation energy of 20 MeV, the difference is $a^+_{Born} - a^+(0) \simeq -0.0078\ A\ m_\pi^{-1}$ instead of the value $-0.0095\ A\ m_\pi^{-1}$ of Eq. (11) that we use.

Various nuclear models give similar values for C(q), so we shall use the Fermi gas model[14]. This includes only the Pauli exclusion correlation, but it is known that other correlations such as short-range correlation are unimportant. The result is well known,

$$C(q) = \frac{3}{2}\frac{q}{2p_F} - \frac{1}{2}\left(\frac{q}{2p_F}\right)^3 \qquad q \leq 2p_F$$
$$= 0 \qquad q > 2p_F \tag{10}$$

where $q = |\underset{\sim}{q}|$, and $p_f \simeq 1.95\ m_\pi$ is the Fermi momentum. An important point here is that at small values of $q \leq 2p_f$, from which the integral (6) gets its major contribution, there is a large suppression due to the exclusion principle. This can be easily understood as an increasing difficulty for a nucleon to find a level to make a transition into as the momentum transfer q decreases. If it were not for the exclusion principle, C(q) would be unity for all q; thus the integral of (6) would be considerably larger.

Evaluating Eq. (6) with Eq. (8), and using the experimental values $(1 + m_\pi/m_N)a_N^- \approx 0.10\ m_\pi^{-1}$, $a_N^+ \approx 0$, we obtain

$$\left(1 + \frac{m_\pi}{M_B}\right)\left[a^+_{Born} - a^+(0)\right] = 0.0095\ A\ m_\pi^{-1} \tag{11}$$

In the absence of the exclusion principle, the value would be $0.02\ A\ m_\pi^{-1}$. The available data from π-mesic atoms lead to an accurate value[10] of a^+_{Born}

$$\left(1 + \frac{m_\pi}{M_B}\right) a^+_{Born} = -0.034\ A\ m_\pi^{-1} \tag{12}$$

Therefore the soft-pion amplitude is

$$\left(1 + \frac{m_\pi}{M_B}\right) a^+(0) = -0.044\ A\ m_\pi^{-1} \tag{13}$$

In order to reach the nucleon value from this, we make the reasonable assumption that $(1 + m_\pi/m_B)a^+(0)$ can be extrapolated to $(1 + m_\pi/m_N)a_N^+(0)$ linearly in nucleon number. Then we finally get

$$a_N^+(0) \simeq -\frac{0.044}{1+\frac{m_\pi}{m_N}} m_\pi^{-1} = -0.038\, m_\pi^{-1} \tag{14}$$

or $\quad \sigma_{NN} \simeq 34$ MeV.

This value is quite close to those obtained by Von Hippel and Kim, and Höhler et al., but definitely smaller than that of Cheng and Dashen.

As in the Von Hippel-Kim calculation, our mass extrapolation would be questionable if there exist large off-shell effects in the π-N amplitude. We have two reasons to think that our approximation of neglecting them should not be blamed for the discrepancy with Cheng and Dashen.

i) Large off-shell effects have not been observed in the mass extrapolation for the antisymmetric amplitude. In this case there appears the combination of the off-shell amplitudes $a^-(a^- + 2a^+)$, which should also display the off-shell effects of the charge symmetric π-N amplitude.

ii) Even if there exist large off-shell effects in the π-N amplitude the error committed in neglecting them would be much less in the π-nuclear process than in the π-nucleon one. The reason is that one is dealing here with a small correction term which is largely suppressed by the exclusion principle. Even if our estimate of the incoherent rescattering contribution would have a large relative error, this still would not influence our conclusion.

Another effect which should be taken into account for a more accurate calculation is the two-nucleon absorption. It can be significant if its influence on the real part of the amplitude turned out to be largely attractive (> 0). However, we consider the last possibility to be very unlikely.

Therefore we conclude that a difference between the soft pion and the Born amplitude as large as 0.08 A m_π^{-1} (as needed to understand the value of Cheng and Dashen) is highly improbable. The smallness of the observed value of the Born amplitude should reflect the smallness of the σ-commutator term. The value of Cheng and Dashen appears thus in the light of this study of the π-nuclear interaction to be implausible.

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