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SECOND CLASS CURRENTS AND MESON EXCHANGE EFFECT IN NUCLEAR BETA DECAY

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A B S T R A C T

It is pointed out that two forms of the induced pseudotensor current, equivalent on the mass shell, lead to quite different results in nuclei for $\delta = (ft)_{\beta^+}/(ft)_{\beta^-} - 1$ which was used by Wilkinson as evidences for a possible presence of the second class current, and therefore to an ambiguity in determining the coupling constant g_T. We show that the resolution of this ambiguity requires a consideration of meson-exchange current. The two-body current thus appearing is found to give in fact a dominant contribution to δ.

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Recently Wilkinson [1] revived the possibility that the G parity irregular currents [2] may be present in weak interaction by analyzing mirror beta transitions for a wide range of nuclei. He demonstrated that the presently available data tend to show for the quantity

$$\delta = \frac{(ft)_{\beta^+}}{(ft)_{\beta^-}} - 1 \qquad (1)$$

a linear dependence on $(W_0^+ + W_0^-)$, the sum of the energy release in $\beta^\pm$ decays considered. Since for typical transitions, $W_0^+ + W_0^-$ is of order of 60 m_e (where m_e is the electron mass), the test of the presence of the second class (or G parity irregular) currents in nuclei seems quite feasible. If one ignores the corrections of electromagnetic origin on the first class currents, an assumption which may be justified in some nuclei because of considerable cancellations among various effects [3], a conventional form of the current [i.e., Eq. (3a) defined below] leads in impulse approximation to [4]

$$\delta_a \simeq -\frac{4}{3}\left(\frac{g_T}{g_A}\right)\left(W_0^+ + W_0^-\right) \qquad (2)$$

where g_A is the axial coupling constant, and g_T the induced pseudo-tensor (PT) coupling constant (in unit of electron mass m_e) to be defined below.

The purpose of this paper is to point out that Eq. (2) is interesting not only for the fundamental question of the existence of the second class current in nature, but also for its relevance to meson exchange phenomena in nuclei. We first illustrate that within the framework of impulse approximation, Eq. (2) is ambiguous and then show that the resolution of this ambiguity requires a consideration of meson exchange effects in beta decay.

The vector current J_λ^V is free of second class currents if the CVC is taken. Therefore we consider only the axial current $J_\lambda^{A\pm}$ where $\pm$ stands for $\beta^\mp$ decays, and λ is the Lorentz index. One can write the matrix element of $J_\lambda^{A\pm}$ between two nucleon states in several forms allowed by Lorentz invariance, all equivalent on the mass-shell. We shall consider two of them here. One conventionally used form is [2),5)]

$$\langle p'|J_\lambda^{A\pm}|p\rangle = i\,\bar{u}(p')\,\tau_\pm\,[g_A\gamma_\lambda + i g_P q_\lambda \pm g_T \sigma_{\lambda\mu} q_\mu]\gamma_5\, u(p) \tag{3a}$$

and another form which is sometimes used is

$$\langle p'|J_\lambda^{A\pm}|p\rangle = i\,\bar{u}(p')\,\tau_\pm\,[g_A'\gamma_\lambda + i g_P' q_\lambda \pm i g_T' P_\lambda]\gamma_5\, u(p) \tag{3b}$$

where $q_\lambda = (p - p')_\lambda$, $P_\lambda = (p + p')_\lambda$ and g_A, g_P, g_T are respectively axial, induced pseudoscalar, and induced pseudotensor (PT) form factors, all of which are assumed to be real functions of q^2. The first two terms are called first class currents, the last one second class current. If one ignores the proton-neutron mass difference, then one can identify $g_A = g_A'$, $g_P = g_P'$ and $g_T = g_T'$. Now the usual (and the only workable) procedure in application to nuclear system is to reduce the matrix element (3) non-relativistically and then use it as an effective single-particle operator. This is the usual impulse approximation.

Since the first class current matrix elements approximately factor out in δ (except for some small terms coming from the electromagnetic corrections, etc., which of course should be included in comparing with experiment), it suffices to consider explicitly only the pseudotensor term alone. From now on, we shall refer to Eq. (3) only for that term. Also we shall denote by $\mathcal{J}_\lambda^\pm$ the <u>nuclear axial current</u> relevant to the pseudotensor current alone.

Now reducing non-relativistically, we find that Eq. (3a) yields [6]

$$\mathcal{J}_0^{\pm}(\vec{x}) = \pm g_T \sum_{i=1}^{A} \xi_{\pm}(i)\, \vec{\sigma}(i) \cdot \vec{k}\;\; \delta(\vec{x}-\vec{x}_i)$$

$$\vec{\mathcal{J}}^{\pm}(\vec{x}) = \pm W_0^{\mp} g_T \sum_{i=1}^{A} \xi_{\pm}(i) \vec{\sigma}(i)\, \delta(\vec{x}-\vec{x}_i) \tag{4a}$$

whereas the PT current of Eq. (3b) yields

$$\mathcal{J}_0^{\pm}(\vec{x}) = \pm g_T \sum_i \xi_{\pm}(i) \vec{\sigma}(i) \cdot \vec{k}\;\; \delta(\vec{x}-\vec{x}_i)$$

$$\vec{\mathcal{J}}^{\pm}(\vec{x}) = O\left(\frac{1}{M}\right) \tag{4b}$$

where we have made the identification of $(\vec{q}, iq_0)$ by $(\vec{k}, iW_0)$, the momentum and energy actually carried away by the leptons : $\vec{k} = \vec{k}_e + \vec{k}_\nu$, $W_0 = E_e + E_\nu$. Thus the two forms of the current, equivalent on the mass-shell, give the same time-component, but quite different spatial component $\vec{\mathcal{J}}$ when applied to a many-body system. Clearly this is an ambiguity due to the impulse approximation manifesting the many-body nature of a nucleus. Does such an ambiguity occur in any other nuclear processes ? The answer is yes with a major difference : in all electromagnetic processes and also in weak interactions via first class currents, the ambiguity arises in small correction terms, and hence does not play a significant role. Only in the present case does it appear in the major term. To see that the ambiguity is a serious one we calculate δ using both (4a) and (4b). If one uses (4a), one finds Eq. (2). But with Eq. (4b), one gets instead

$$\delta_b \simeq \frac{2}{3}\left(\frac{g_T}{g_A}\right)(W_0^+ + W_0^-) \tag{5}$$

which differs from Eq. (2) <u>not only in magnitude but also in sign</u>. It is perhaps possible to combine the Wilkinson analysis [1] which can provide the magnitude of the coefficient of $(W_0^+ + W_0^-)$, and the muon capture experiments of the Louvain group [7] which effectively determine the induced coupling constants to discriminate the alternative in sign. But a priori, there is no sacred criterion as to which of the two equivalent forms of the current should be more suited to nuclei than other.

We now show that the ambiguity can be resolved completely if we take into account "correlations" due to meson exchanges. This we do with one assumption : that the nuclear PT current is divergenceless

$$\partial_\lambda \mathcal{J}^{\pm}_\lambda(x) \simeq 0 \quad . \tag{6}$$

As can be verified directly from both (3a) and (3b), for the nucleons, the divergence of the PT current is trivially zero. It becomes, however, an assumption, though plausible, when the current is a nuclear current. We can make the assumption somewhat weaker by introducing the nuclear Coulomb interaction via the minimal coupling

$$\partial_\lambda \mathcal{J}^{\pm}_\lambda(x) \mp ie\, a_0(x)\, \mathcal{J}^{\pm}_0(x) \simeq 0 \tag{7}$$

where $a_0(x)$ is the Coulomb field. Let us for the moment drop the Coulomb interaction and in order to obtain an equivalent two-body operator, apply the Adler-Dothan theorem [8),9)] to the two-nucleon process

$$N(p_1) + N(p_2) \longrightarrow N(p_1') + N(p_2') + e(k_e) + \nu(k_\nu) \tag{8}$$

where $k_e + k_\nu = k$ is the momentum carried away by the leptons, and $N(p)$ stands for a nucleon with momentum p. Let us write the amplitude for (8) as

$$\begin{aligned} M_\lambda &= M_\lambda^{ext'} + \Delta M_\lambda + O(k) \; , \\ M_\lambda^{ext'} &= \bar{u}(p_1')\bar{u}(p_2') \, [\, \Gamma_\lambda(1')\, S_F(p_1'+k)\, T^P(p_2', p_1'+k\, ; p_2, p_1) \; + \\ & T^P(p_2', p_1'\, ; p_2, p_1-k)\, S_F(p_1-k)\, \Gamma_\lambda(1) \; + \; (1 \rightleftarrows 2)\,]\, u(p_1) u(p_2) + O(k) \; . \end{aligned} \tag{9}$$

where $\Gamma_\lambda(1')$ is the PT current vertex inserted to the nucleon line 1', $S_F(p)$ the Feynman propagator $(i\gamma\cdot p + M)^{-1}$, T^p the positive energy on-shell nucleon-nucleon T matrix and $O(k)$ implies that terms explicitly of order k have been neglected [10]. One can easily show that $M_\lambda^{ext'}$ leads to the impulse approximation results of $\mathcal{J}_\lambda$ of Eq. (4), when T^p is interpreted in terms of a nuclear potential V. Now in order to determine ΔM_λ, we use the condition (6) which in this case is

$$k_\lambda\left(M_\lambda^{ext'} + \Delta M_\lambda\right) = O(k^2) \quad .$$

If one takes the PT current of (3a), then since $\sigma_{\lambda\mu} k_\lambda k_\mu = 0$, one gets $k_\lambda M_\lambda^{ext'} = 0$ which means that

$$\Delta M_\lambda^{\pm} = O(k) \quad . \tag{10}$$

Therefore to the order we are considering there is no correction of two-body origin to Eq. (4a). On the other hand, for the current (3b), we have with $T_o^P \equiv T^P(p_2', p_1' \,;\, p_2, p_1)$,

$$\Delta M_\lambda^{\pm} = \mp i\,\bar{u}(p_1')\,\bar{u}(p_2')\left\{g_T\left[(\tau_\pm\gamma_\lambda\gamma_5)_1 , T_o^P\right]_- + (1 \rightleftarrows 2)\right\} u(p_1)\,u(p_2) + O(k) \,. \tag{11}$$

Translated into an equivalent nuclear current [9], this just gives

$$\begin{aligned} \mathcal{J}_0^{(2)\pm}(\vec{x}) &= O\left(\frac{1}{M}\right) , \\ \vec{\mathcal{J}}^{(2)\pm}(\vec{x}) &= \mp g_T\left[V, \sum_i \tau_\pm(i)\,\vec{\sigma}(i)\,\delta(\vec{x}-\vec{x}_i)\right]_- , \end{aligned} \tag{12}$$

where the superscript 2 stands for a non-relativistic two-body operator.

In allowed approximation which is used in obtaining δ , what we need is $\vec{\mathcal{J}}^{(2)\pm} = \int d^3x\, \vec{\mathcal{J}}^{(2)\pm}(\vec{x})$ so that adding $[H_0, \sum_i \tau_\pm(i)\vec{\sigma}_i]_- = 0$ where H_0 is the kinetic energy term, we get (as a relation for matrix elements)

$$\vec{\mathcal{J}}^{(2)\pm} = \pm g_T W_0^{\mp} \sum_i \tau_\pm(i)\vec{\sigma}(i) \quad . \tag{13}$$

Thus the two-body current restores nicely the spatial current which was missing in Eq. (4b). We shall see below that Eq. (13) is actually valid without the allowed approximation.

In order to incorporate the Coulomb effect and to avoid the allowed approximation, we can use an argument analogous to Siegert theorem [9]. This is possible since Eq. (12) tells us that to the order considered there is no two-body exchange correction to the $\mathcal{J}_0^\pm(\vec{x})$. We then rewrite Eq. (7) as

$$\vec{\nabla}\cdot\vec{\mathcal{J}}^\pm(\vec{x}) = -i\,[H - H_c, \mathcal{J}_0^\pm(\vec{x})]_- \tag{14}$$

which follows from

$$\pm e a_0 \mathcal{J}_0^\pm(x) = [H_c, \mathcal{J}_0^\pm(x)]_-$$

where H_c is the two-body Coulomb Hamiltonian. One may include the p - n mass difference in H_c, but it can be verified that it is cancelled exactly in δ . Substituting into Eq. (14) $\mathcal{J}_0^\pm(x)$ as given by Eq. (4), it follows that [11]

$$\vec{\mathcal{J}}^\pm(\vec{x}) = \mp g_T\,[H - H_c, \sum_i \tau_\pm(i)\vec{\sigma}(i)\delta(\vec{x}-\vec{x}_i)]_- + \vec{R}(\vec{x}) \tag{15}$$

where $\vec{R}$ is a possible correction term satisfying $\vec{\nabla}\cdot\vec{R} = 0$. It is, however, easy to convince oneself that $\vec{R} = O(\vec{k}^2)$; therefore we may drop it safely. Equation (15) modifies δ to

$$\delta \simeq -\frac{4}{3}\left(\frac{g_T}{g_A}\right)\left(W_0^+ + W_0^- + \frac{3}{2}(\Delta E_c^+ + \Delta E_c^-)\right) \tag{16}$$

where

$$\Delta E_c^{\pm} = \langle f | H_c | f \rangle - \langle i | H_c | i \rangle$$

is the Coulomb energy difference for the $\beta^{\pm}$ decay from a state i to a state f. We emphasize that this correction, pertaining only to the second class current, has nothing to do with the electromagnetic effects (associated with the first class currents) neglected above. It is interesting to note that the Coulomb correction is really quite small because of the cancellation of the large isovector Coulomb energy in the sum $\Delta E_c^+ + \Delta E_c^-$. For instance for the $A = 12$ triplets, the correction amounts to less than 1 MeV reduction to $W_0^+ + W_0^- \approx 30$ MeV. The place where the correction might perhaps have influence on the energy dependence is in the positon decays

$$^{18}Ne \xrightarrow{\beta_1^+} {}^{18}F \xrightarrow{\beta_2^+} {}^{18}O .$$

In this case, the energies subtract, so that the Coulomb correction $\frac{3}{2}(\Delta E_c(\beta_1^+) - \Delta E_c(\beta_2^+)) \approx -0.8$ MeV is not negligible compared with $W_0(\beta_1^+) - W_0(\beta_2^+) \approx 2.8$ MeV. A quantitative assessment of the correction would be, however, harder in combinations of this sort, since other finer corrections than the second class would have to be reliably estimated. In general the correction would become smaller for heavier nuclei (roughly as $A^{-1/3}$) and would be negligible in most of the cases.

Let us now enquire whether one can represent the two-body current Eq. (12) by a set of Feynman graphs. In an analogous case of meson exchange electromagnetic currents, one knows [12),13)] that one-body current plus the two-body currents obtained from the Figure are sufficient to satisfy the continuity equation $\partial_\lambda J_\lambda^{e.m.}(x) = 0$. Let us see whether one can make a similar correspondence for the PT current. On invariance ground, the pionic current (graph a) cannot contribute to the axial current, be it first or second class. The graph b does contribute, however, and its contribution to the spatial component of the current can be calculated in a standard way [14)] ;

$$\vec{\mathcal{J}}^{(2)\pm} = \int d^3x\, \vec{\mathcal{J}}^{(2)\pm}(\vec{x}) = \mp\frac{1}{6}\left(\frac{g_r^2}{4\pi}\right)\frac{g_T m_\pi^3}{M^2}\sum_{i<j}\left[\tau(i)-\tau(j)\right]_\pm P_{ij}^{\tau}$$
$$\times\left\{ (\vec{\sigma}(i)-\vec{\sigma}(j))\, Y_0(\rho) + 3\vec{T}_{ij}^{(-)}\, Y_2(\rho)\right\} \quad + \text{N.L.} \tag{17}$$

where $P_{ij}^{\tau,\sigma}$ is the exchange operator in τ, σ space, g_r the renormalized πN coupling constant (≈ 13.5), $\vec{T}_{ij}^{(-)} = (\vec{\sigma}_i - \vec{\sigma}_j)\cdot\hat{r}\hat{r} - \frac{1}{3}(\vec{\sigma}_i - \vec{\sigma}_j)$, $Y_0(\rho) = e^{-\rho}/\rho$, $Y_2(\rho) = (1 + 3/\rho + 3/\rho^2)\, Y_0(\rho)$, $\rho = m_\pi r$, $\vec{r} = \vec{r}_i - \vec{r}_j$, and N.L. stands for non-local terms which may be ignored. Equation (17) should be compared to Eq. (12) obtained in replacing V by a one-pion-exchange potential V_π ; i,e.,

$$\vec{\mathcal{J}}_\pi^{(2)\pm} = \mp g_T\left[V_\pi, \sum_i \tau_\pm(i)\vec{\sigma}(i)\right]_-$$
$$= \mp\frac{1}{6}\left(\frac{g_r^2}{4\pi}\right)\frac{g_T m_\pi^2}{M^2}\sum_{i<j}\left[\tau(i)-\tau(j)\right]_\pm \left\{(\vec{\sigma}(i)-\vec{\sigma}(j))(P_{ij}^{\tau}+P_{ij}^{\sigma})\right.$$
$$\left.\times\, Y_0(\rho) + 3\vec{T}_{ij}^{(-)}(P_{ij}^{\tau} - P_{ij}^{\sigma}/2)\, Y_2(\rho)\right\} \tag{18}$$

Thus the graph b is seen to reproduce all the terms proportional to P^{τ}_{ij}, but fails to give terms proportional to P^{σ}_{ij}. In contrast to the electromagnetic current, additional graphs are clearly necessary to saturate Eq. (12) ; i.e., to obtain the P^{σ}_{ij} terms. Unfortunately search for such graphs will require a knowledge of how the second class current couples to other hadrons than nucleon, an information which is of course not available at present. Therefore this intriguing question of the origin of the additional terms must wait until the presence of the second class current itself is more convincingly established.

To summarize, we have obtained an equivalent non-relativistic G parity irregular current $\mathcal{J}_\lambda$ applicable to nuclear β decay going beyond the usual impulse approximation. The major contribution to $\mathcal{J}$ comes from the spatial part $\vec{\mathcal{J}}$ which is entirely of many-body origin. It is seen that although the conventionally used PT current of (3a) gives the entire nuclear current $\vec{\mathcal{J}}$ (4a) in the form of impulse approximation, it, however, is not a <u>genuine</u> one-body operator, but an <u>effective</u> one, whose meaning becomes clearer if one uses the form of (3b). It is tempting to suggest that a clear demonstration of a linear dependence on $W_o^+ + W_o^-$ in mirror β decay and a determination of the sign consistent with Eq. (2) (perhaps through muon capture) could be an indication not only of a second class current but also of a meson exchange phenomenon.

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5) We use the Pauli metric throughout : γ_λ (λ = 1,2,3,4,5) are all Hermitian,

$$\sigma_{\lambda\mu} = \frac{1}{2i}[\gamma_\lambda, \gamma_\mu]_-$$

and

$$p \cdot q = (\vec{p}\cdot\vec{q} + p_4 q_4) = (\vec{p}\cdot\vec{q} - p_0 q_0)$$

6) We shall always work to the lowest order in (1/M) where M is the nucleon mass. Also terms of order ($\vec{k}^2$, W_o^2) will be consistently ignored in the non-relativistic expressions. This is justified for the energy-momentum transfer involved.

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9) H. Ohtsubo, J.I. Fujita and G. Takeda - "Low Energy Theorems for Nuclear Weak and Electromagnetic Currents", I.N.S. Report 148, Tokyo (1970).

10) It must be emphasized that we do, however, keep the terms of order $\vec{k}$, W_o in the non-relativistic expression for the nuclear current. For instance $2M\bar{u}(p')\gamma_5 u(p)$ is explicitly zeroth order in $Q = p - p'$, but the non-relativistic limit is $-\vec{\sigma}\cdot\vec{Q}$, linear in $\vec{Q}$.

11) The easiest way to obtain this is in momentum space. Or see Refs. 9) and 12).

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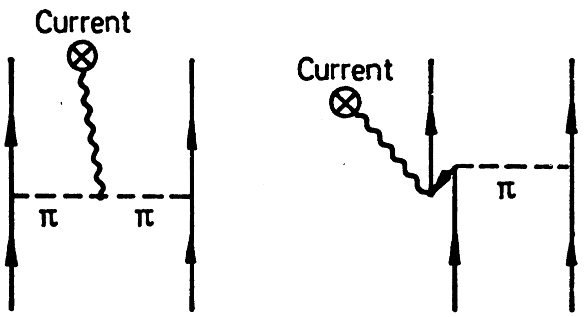
FIGURE CAPTION

Pion exchange graphs needed to satisfy the continuity equation for the electromagnetic current.
Graph a vanishes for the axial current, and
Graph b fails to saturate the two-body current needed for the approximate continuity equation assumed for the induced tensor current.



a) b)

FIG.1