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MESON EXCHANGE CURRENTS IN NUCLEAR WEAK AND ELECTROMAGNETIC INTERACTIONS

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ABSTRACT

Meson theory is applied to the study of the exchange magnetic moment and Gamow-Teller operators. We consider contributions due to the exchange of one pion and of vector mesons and give their detailed expressions in terms of a general classification of the operators. The one-pion-exchange process is separated into Born and non-Born parts corresponding respectively to the Born and non-Born parts in the process of weak or photo-production of a pion by the nucleon. The results of low-energy theorems for this process enable us to reach a model-independent description of the non-Born parts. Corrections to the soft-pion limit are studied by means of simple dynamical models: phenomenological Lagrangian for the weak current and the Chew-Low model for the e.m. current. The exchange operators due to the exchange of ρ and ω mesons are evaluated in the vector dominance model. The applications are concerned with the exchange effect in the isovector and isoscalar magnetic moments of ^{3}He and ^{3}H and in the Gamow-Teller matrix element for ^{3}H beta-decay. The results are obtained with the dominant S-state with a Gaussian radial function. We also discuss their dependence on short-range correlations described in terms of Jastrow factors. The exchange corrections are found to be too small for the current wave functions of the trinucleons. We discuss briefly other agencies that may account for the remaining discrepancies.

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I. INTRODUCTION

In a previous paper[1], we made use of the low-energy theorem based on the PCAC hypothesis to obtain the complete one-pion-exchange (OPE) two-body contribution to the Gamow-Teller matrix element in beta decay and applied it to the process $^3H \rightarrow {}^3H_e + e^- + \bar{\nu}_e$. We have suggested the approach as a successful means of tackling the difficult problem of meson exchange effects in nuclei. In this article, we would like to expand that theme to both weak and electromagnetic interactions by systematically discussing the low-energy theorems, defining as clearly as possible what we mean by the meson exchange corrections, pointing out which pieces of information are model-independent, and emphasizing the basic differences in the meson exchange currents of weak and electromagnetic processes.

The problem of exchange currents in the weak and electromagnetic processes is an old one[2]. However, it still remains unclarified because of the difficulties in defining interactions in a composite system starting from the field theoretic description. We do not pretend to resolve these difficulties. The solution to the problem would involve a fully relativistic equation for many-body systems, which is at present beyond our capability.

Our approach in obtaining the equivalent 2-body current (we neglect many-body currents) relies on the conventional S-matrix method which has the advantage of being both simple and consistent. This has been used before. A renewal of interest in it stems from two sources: one, a recent development in deriving the two-nucleon potential, notably the one-boson-exchange (OBE) models[3], and the other, the success of the low-energy theorems associated with the PCAC and current algebra in describing the π-N interaction[4]. Within the conventional S-matrix framework, the former tells us how to split the totality of the relevant Feynman graphs, and the latter allows us to treat the OPE term correctly to all orders in the strong interaction coupling constant.

In our studies, we encounter both the soft-current and soft-pion theorems. The former is applicable to the Gamow-Teller matrix element where the four momentum transfer k_λ is small. The soft-pion theorem supplements the former when we deal with the OPE graph. For the magnetic moment, however, it is more convenient to take the soft-pion limit as will become clear in Chapters II and III. Let us point out here that the soft-pion theorem has proven to be an extremely powerful technique in understanding both the nuclear force and π-nucleus interaction, thus being applicable to the virtual as well as the real pions. The pion exchanged in nuclei is not really soft; the pion four momentum q_λ does not vanish. But the extrapolation to the relevant value of q^2 has been studied, and shows that the effect is not significant at least for the OPE term[5)]. From these considerations emerges an important and hitherto unappreciated fact that the exchange current in the weak process is quite different from that in the electromagnetic process, that is, the sources of the corrections are different.

In treating the exchanges of mesons other than a single pion, the situation is not simple. The exchange of two soft-pions may be treated, but it cannot represent the true situation. Because of the tensor nature, one or two of the pions can carry considerable momentum. There are also the practical difficulties in the choice of graphs. So we shall not consider the multi-pion exchanges here. We include however the vector meson (ρ and ω) exchanges in an approximate way with the hope that they simulate, at least partially, the effect of multi-pion exchanges.

We apply our theory to the bound three-nucleon systems. Aside from the deuteron, the wave functions of these nuclei are the best known in nuclear physics. There have been innumerable studies, both theoretical and experimental, on the ground state properties of these nuclei: the Coulomb energy difference, the electromagnetic and weak properties such as electric and magnetic form factors, n-d radiative capture, photo - and electro - disintegration, β-decay and muon capture. A lucid discussion on the status of the three-body problem can be found in the review article of Delves and Phillips[6)]. Despite such a long history and considerable efforts, the understanding of the three-nucleon system is far short of the extent to which one knows of the deuteron. The reason is, of course, the extra degrees of freedom an additional nucleon brings in. A reliable calculation of meson exchange corrections in triton beta decay, and in the magnetic moments of ^3H and ^3He would be valuable since once we know the corrections, a set of linear equations describing the beta decay and the magnetic moments can be used to draw conclusions about the structure of the bound three-nucleon systems.

Our calculation is limited in that only the dominant S-state component of the trinucleon wave functions is used. This is only a first step towards more accurate calculations using a more realistic wave function. Furthermore, since our exchange current operators (except for the isocalar magnetic moment) should be reliable, an extended application to other nuclei would be just an exercise in nuclear structure calculations.

In view of the length of the paper, we summarize the essential points here. The two-body meson exchange operator is divided into a one-pion exchange (OPE) term and a heavy-meson exchange (HME) term. We apply the low-energy theorems to the OPE term. The Chapter II deals with this question. We give the theorems relevant to the Gamow-Teller transition, and the isovector and isoscalar magnetic moments. The principal conclusions are that (1) for the Gamow-Teller operator, the theorem specifies the exchange current in terms of two off-shell πN amplitudes, available from Adler's works, (2) for the isovector magnetic moment, it leads to a reliable expression given in terms of generalized Born graphs, and (3) for the isoscalar magnetic moment, the theorem has no predictive power and hence one needs to go beyond it.

The Chapter III is devoted to the classification of two-body meson exchange operators based on general invariance considerations. Explicit expressions are given for the OPE operators when certain assumptions are made. We also indicate how the operators are extracted from Feynman graphs.

In Chapter IV, the one-boson-exchange contributions are explicitly calculated. For both the beta decay and the magnetic moments, models are used to get the HME contribution. For the magnetic moments, the vertex correction coming from N* is calculated by means of the Chew-Low model. The short comings in our treatment of the HME graphs are emphasized, and should be kept in mind in assessing the results of Chapter V.

The Chapter V deals with the details of experimental situations and theoretical calculations of the Gamow-Teller matrix element, and the isovector and isoscalar magnetic moments in the three-nucleon system. We use for simplicity the dominant S-component of the wave functions. For this reason, the exchange Gamow-Teller matrix element is small as the N^*_{33} cannot contribute. The importance of the D-state is pointed out. For the isovector magnetic moment where the N^*_{33} is expected to play a minor role, our calculated value should be reliable, but there still remains some sizeable discrepancy. Although the exchange isoscalar moment fills up the existing discrepancy, this may be fortuitous because of its model dependence.

Appendix A deals with the problem of defining the meson exchange current from a field theory, and Appendix B deals with the model dependence in the nuclear wave function.

II. LOW-ENERGY THEOREMS

The study of the two-body exchange currents amounts to the description of the process given in Fig. 1. We follow the usual procedure used in the nuclear force problem, and split the contributions into a one-pion exchange (OPE) graph and a heavy-meson exchange (HME) graph as given by Figs. 2a and 2b. As was pointed out by Blin-Stoyle and Tint[7] for the triton β-decay and by Ohtsubo et al.[7] for the axial current in general, a model-independent way would be to look at the two-body exchange contribution by means of the PCAC hypothesis applied directly to the two-body processes

$$\begin{aligned} &N_1 + N_2 \to N_3 + N_4 + \gamma \\ &N_1 + N_2 \to N_3 + N_4 + e^- + \bar{\nu}_e \end{aligned} \tag{II.1}$$

which may be given in the low-energy limit (that is, when the current momentum k approaches zero) by a non-radiative amplitude $T(p_3,p_4;\ p_2,p_1)$ describing the two-particle reaction in the absence of a current. We shall not consider this approach here.

In this Chapter, we discuss the low-energy theorems relevant to the OPE graph of Fig. 2a, although by now these are folk-lores in particle physics. Our philosophy is that this is the dominant contribution in most of the exchange current contributions and therefore should be treated as accurately as possible. The HME terms are treated as corrections, and hence are more model-dependent.

Let us consider the process given by Fig. 2a. The matrix element for such a process is

$$\begin{aligned} &\frac{1}{(2\pi)^3}\,\delta(\vec{p}_1 + \vec{p}_2 + \vec{k} - \vec{p}_1{}' - \vec{p}_2{}') \times \\ &\times \left\langle \pi^n(q) N(p_1') \,|\, J^j_\lambda \,|\, N(p_1) \right\rangle \frac{d_\pi(q^2)}{q^2 + m_\pi{}^2} \left\langle N(p_2') \,|\, J^n_\pi \,|\, N(p_2) \right\rangle + \\ &+ (1 \rightleftarrows 2)\ , \end{aligned} \tag{II.2}$$

where superscripts n and j are the isospin indices for pions and current, $d_\pi(q^2)$ is the pion propagator form factor, and J_π^n is the pion source current. The πNN vertex $d_\pi(q^2)\langle N|J_\pi|N\rangle$ will be taken to be known from dispersion theoretic analysis (see Chapter III, Section 2 for more details). Therefore we need only to know the left vertex which is essentially a matrix element for the production of a pion by a current J_λ^j, which is either $J_\lambda^{e.m.}$ or J_λ^{Aj}. In allowed β-transition we are only interested in the soft-current limit ($k \to 0$). In all cases, however, the pion is assumed to be soft, since it carries a mass ($-q^2$) which is small in the process where a nucleon makes transitions with little momentum transfer, as in the case of β-decay or magnetic moment. On the average, $q^2 \approx m_\pi^2$. Thus the well-known soft-pion theorem is ideally suited to this kind of process occurring in nuclei, in contradistinction to elementary particle physics where the soft-pion limit is only a mathematical limit. We shall argue later that $q^2 = 0$ is a reliable limit, the extrapolation to $q^2 \approx m_\pi^2$ modifying the results unappreciably.

1. DEFINITIONS

We shall follow the notations*) of Adler[8] with minor modifications. The process given by Fig. 3 which represents the reaction

$$J_\lambda^j(k) + N(p_1) \to \pi^n(q) + N(p_1') \qquad \text{(II.3)}$$

is described by the set of kinematic invariants

$$k^2, q^2, \nu \equiv -(p_1 + p_1') \cdot k/2M \; , \quad \nu_B \equiv q \cdot k/2M \qquad \text{(II.4)}$$

in which M is the nucleon mass. We denote the axial current by J_λ^{Aj} and the e.m. current by $J_\lambda^{e.m.} = J_\lambda^{Vj} + J_\lambda^S$, where J_λ^{Vj} and J_λ^S are the isovector and isoscalar components, respectively. When we write simply J_λ^j, it means

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*) We use the natural units $\hbar = c = 1$, $e^2/4\pi = 1/137$, and the Pauli metric where $a\cdot b = a_\mu b_\mu = \vec{a}\cdot\vec{b} + a_4 b_4 = \vec{a}\cdot\vec{b} - a_0 b_0$. The Dirac matrices are all Hermitian and satisfy $\{\gamma_\mu,\gamma_\nu\} = 2\delta_{\mu\nu}$. We also use $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$ and $\sigma_{\mu\nu} = (1/2i)(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$.

both J_λ^{Aj} and $J_\lambda^{e.m.}$ unless otherwise specified. The transition amplitude for the process (II.3) is described in terms of

$$M_\lambda^{nj} \equiv i(q^2 + m_\pi^2) \int d^4x\, e^{-iq\cdot x} \langle p_1' | \{T[\phi_\pi^n(x) J_\lambda^j(0)]\} | \phi_1 \rangle , \qquad \text{(II.5)}$$

where T is the usual time-ordering operator. Let us specify the isospin structure of M_λ by the decompositions:

$$\begin{aligned} M_\lambda^{Anj} &= a_{nj}^{(+)} A_\lambda^{(+)} + a_{nj}^{(-)} A_\lambda^{(-)} \\ M_\lambda^{e.m.} &\equiv M_\lambda^{Vnj} + M_\lambda^{Sn} = \\ &= a_{nj}^{(+)} V_\lambda^{(+)} + a_{nj}^{(-)} V_\lambda^{(-)} + a_n^{(0)} V_\lambda^{(0)} , \end{aligned} \qquad \text{(II.6)}$$

where

$$\begin{aligned} a_{nj}^{(\pm)} &= \tfrac{1}{4} [\tau_n, \tau_j]_\pm \\ a_n^{(0)} &= \tfrac{1}{2} \tau_n , \end{aligned} \qquad \text{(II.7)}$$

and the space-spin structure by

$$\begin{aligned} A_\lambda^{(\pm)} &= \sum_{\ell=1}^{8} A_\ell^{(\pm)} (\nu, \nu_B, k^2, q^2) \bar{u}(p_1') O_\lambda(A_\ell) u(p_1) \\ V_\lambda^{(\pm,0)} &= \sum_{\ell=1}^{6} V_\ell^{(\pm,0)} (\nu, \nu_B, k^2, q^2) \bar{u}(p_1') O_\lambda(V_\ell) u(p_1) . \end{aligned} \qquad \text{(II.8)}$$

In Eq. (II.8), $A_\ell^{(\pm)}(\ldots)$ and $V_\ell^{(\pm,0)}(\ldots)$ are amplitudes which depend on the invariants Eq. (II.4), and $O_\lambda(A_\ell)$ and $O_\lambda(V_\ell)$ are covariants given in Table 1. The aim of any theory is to determine the amplitudes $A_\ell^{(\pm)}$ and $V_\ell^{(\pm,0)}$.

We shall often use the matrix elements of the currents between nucleon states:

$$<p_1'|J_\lambda^{Aj}|p_1> = i\bar{u}(p_1')\left[g_A(Q^2)\gamma_\lambda\gamma_5 - ih_A(Q^2)Q_\lambda\gamma_5\right]\frac{\tau_j}{2}u(p_1) \qquad \text{(II.9a)}$$

$$<p_1'|J_\lambda^{e.m.}|p_1> = i\bar{u}(p_1')\left\{\gamma_\lambda\left[\tfrac{1}{2}F_1^S(Q^2) + \frac{\tau_3}{2}F_1^V(Q^2)\right] - \right.$$

$$\left. - \sigma_{\lambda\eta}Q_\eta\left[\tfrac{1}{2}F_2^S(Q^2) + \frac{\tau_3}{2}F_2^V(Q^2)\right]\right\}u(p_1) , \qquad \text{(II.9b)}$$

where $Q \equiv p_1' - p_1$, and the form factors at $Q^2 = 0$ have the values

$$g_A(0) = 1.23 \pm 0.01$$

$$F_1^S(0) = F_1^V(0) = 1$$

$$F_2^S(0) = \frac{\kappa_S}{2M} = -\frac{0.12}{2M} \qquad \text{(II.10)}$$

$$F_2^V(0) = \frac{\kappa_V}{2M} = \frac{3.70}{2M} .$$

2. AXIAL CURRENT

In this section, we discuss in a concise manner what we mean by the low-energy theorem relevant to the β-decay. In the limit as $k \to 0$ which is valid in β-decay where both energy and three-momentum transfers are small, the Adler-Dothan theorem[8,9] provides an expression for M_λ^A valid up to order k. The derivation is given in the papers of Adler and Dothan[9], and Adler[8]; so we shall simply give the results. Let us separate M_λ^A into a "Born term" $(M_\lambda^A)^B$ and a non-Born term $\bar{M}_\lambda^A$,

$$M_\lambda^A = (M_\lambda^A)^B + \bar{M}_\lambda^A . \qquad \text{(II.11)}$$

Then the Born term is defined by

$$(M_\lambda^A)^B = \bar{u}(p_1') \left[i g_r \tau_n \gamma_5 \; S(p_1 + k) \; i g_A(k^2) \frac{\tau_j}{2} \gamma_\lambda \gamma_5 + \right.$$

$$\left. + \; i g_A(k^2) \; \frac{\tau_j}{2} \; \gamma_\lambda \gamma_5 \; S(p_1 - q) \; i g_r \tau_n \gamma_5 \right] u(p_1), \quad \text{(II.12)}$$

where

$$S(P) = (i\gamma \cdot P + M)^{-1} .$$

This corresponds to Fig. 4. Rearranging the non-Born term a little from the original expression of Refs. 8 and 9, we may write

$$\bar{M}_\lambda^A = i\bar{u}(p_1') \left\{ a_{nj}^{(+)} \gamma(q^2) q_\lambda + a_{nj}^{(-)} \left[-2\beta(q^2) P_\lambda - 2i\alpha(q^2) M \gamma_\lambda \right] \right\} u(p_1) \quad ,$$

where

$$\alpha(q^2) = \left[\bar{A}_1^{(-)}(q^2) - \tfrac{1}{2}\bar{A}_4^{(-)}(q^2) \right]_{\nu=\nu_B=k^2=0}$$

$$\beta(q^2) = - \left[\bar{A}_1^{(-)}(q^2) + \bar{A}_2^{(-)}(q^2) \right]_{\nu=\nu_B=k^2=0} \quad \text{(II.13)}$$

$$\gamma(q^2) = \bar{A}_3^{(+)}(q^2) \Big|_{\nu=\nu_B=k^2=0} \quad .$$

Thanks to the PCAC on which the low-energy theorem is based:

$$\partial_\lambda J_\lambda^{An} = \frac{M \, m_\pi^2 g_A}{g_r(0)} \, \phi_\pi^n \quad , \quad \text{(II.14)}$$

α, β, and γ can be given in terms of the invariant πN scattering amplitudes $A^{\pi N}$, $B^{\pi N}$:

$$\bar{u}(p_1') \; i T^{\pi N(\pm)} \; u(p_1) =$$

$$= -i\bar{u}(p_1') \left[A^{\pi N(\pm)} - i\gamma \cdot k \; B^{\pi N(\pm)} \right] u(p_1)$$

by

$$\alpha(q^2) = \frac{g_A}{g_r(0)} \bar{B}^{\pi N(-)} \Big|_{\nu=\nu_B=k^2=0}$$

$$\beta(q^2) = \frac{g_A}{g_r(0)} \left[\frac{\partial \bar{A}^{\pi N(-)}}{\partial \nu}\right]_{\nu=\nu_B=k^2=0} \qquad \text{(II.15)}$$

$$\gamma(q^2) = \frac{g_A}{g_r(0)} \left[\frac{\partial \bar{A}^{\pi N(+)}}{\partial \nu_B}\right]_{\nu=\nu_B=k^2=0} .$$

The α and β are related at $q^2 = 0$ by the Adler-Weisberger relation[4)]

$$-\frac{2M^2}{g_r(0)g_A} \left[\beta(0) + \alpha(0)\right] = 1 - g_A^{-2} . \qquad \text{(II.16)}$$

It is thus advantageous to work at $q^2 = 0$, in which case only two off-shell πN amplitudes completely specify M_λ^A since the Born term Eq. (II.12) is given unambiguously. One may, however, ask at this point how α, β, and γ extrapolate from their values at $q^2 = 0$ to $q^2 \approx m_\pi^2$. Such an extrapolation necessarily involves models, since it deals with momenta which cannot be reached directly by experiments.

It has been shown in particular by Brown and Green that numerically the different models yield similar results up to $q^2 \approx 10\ m_\pi^2$ [5)]. Thus from a practical point of view, the model-dependence is not an obstacle.

We have applied a method similar to the one used by Brown and Green for three-body force, and found that the extrapolation in q^2 changes insignificantly from the values at $q^2 = 0$. More specifically, using the same arguments and procedures [i.e. saturation of the dispersion integral by (3,3) resonance, zero-width approximation, etc.], we found that

$$\alpha(q^2) = a(q^2)\alpha(0)$$

$$\beta(q^2) = b(q^2)\beta(0) \qquad \text{(II.17)}$$

$$\gamma(q^2) = c(q^2)\gamma(0) \;,$$

with

$$a(q^2) \approx \left(1 + \frac{q^2}{\lambda_a M^2}\right) \frac{K(q^2)}{K(0)}$$

$$b(q^2) \approx \left(1 + \frac{q^2}{\lambda_b M^2}\right) \frac{K(q^2)}{K(0)}$$

$$c(q^2) \approx \left(1 + \frac{q^2}{\lambda_c M^2}\right) \frac{K(q^2)}{K(0)} \;,$$

where the magnitude of each of λ_a, λ_b, λ_c is greater than unity. It is thus consistent with

$$a(q^2) \approx b(q^2) \approx c(q^2) \approx \frac{K(q^2)}{K(0)}$$

for $q^2 \ll M^2$. If the pionic form factor of nucleon $K(q^2)$ is assumed to extrapolate to $q^2 = m_\pi^2$ as smoothly as it does from $q^2 = m_\pi^2$ to $q^2 = 0$, then we would expect that

$$a(m_\pi^2) \approx b(m_\pi^2) \approx c(m_\pi^2) \approx 0.95 \;,$$

which gives a 5% variation from $q^2 = 0$ value. We shall neglect this variation from now on.

There is a variant of the low-energy theorems that can be obtained by considering directly the soft-pion limit ($q \to 0$). In this limit the only surviving amplitudes are $\bar{A}_2^{(-)}$ and $\bar{A}_4^{(-)}$ for which the PCAC places the constraints

$$\bar{A}_2^{(-)} = \frac{g_r(0)}{M\, g_A} F_2^V(k^2)$$

$$\bar{A}_4^{(-)} = -\frac{g_r(0)}{M^2\, g_A} \left[F_1^V(k^2) - g_A g_A(k^2) + 2M\, F_2^V(k^2)\right] .$$

One can also take advantage of the relations (II.15) valid at $k \to 0$, arbitrary q, and the above relations valid at $q \to 0$, arbitrary k, describing the same quantity $\bar{M}^A_\lambda$, to obtain the combined constraints which would specify $\bar{M}^A_\lambda$ up to $O(qk, q^2, k^2)$. The results are given in Adler-Dothan [9]. The advantage of this method is that it allows $\bar{A}^{(-)}_1$ and $\bar{A}^{(+)}_3$ to be specified separately in addition to $\bar{A}^{(-)}_2$, $\bar{A}^{(-)}_4$.

3. ELECTROMAGNETIC CURRENT

Unlike the axial case where the $k \to 0$ limit is appropriate, we shall instead take the soft-pion limit for the electromagnetic current. The reason is that the $k \to 0$ limit is not quite appropriate for the magnetic moment, and furthermore we want to rely on the experiments of the threshold production of pions for which the soft-pion theorem is known to be reliable for charged pions.

A shortest derivation of the theorem is through the modified PCAC relation in the presence of electromagnetic field A_λ [10]

$$\partial_\lambda J^{A(\pm)}_\lambda \mp ieA_\lambda J^{A(\pm)}_\lambda = c\phi^{(\pm)}_\pi$$

$$\partial_\lambda J^{A(0)}_\lambda = \frac{c}{\sqrt{2}}\,\phi^{(0)}_\pi$$

$$J^{A(\pm)}_\lambda = J^{A1}_\lambda \mp iJ^{A2}_\lambda \qquad \text{(II.18)}$$

$$J^{A(0)}_\lambda = J^{A3}_\lambda$$

$$c = \frac{\sqrt{2}M\, m^2_\pi\, g_A}{g_r(0)} = f_\pi\, m^2_\pi \ ,$$

where $(\pm,0)$ stand for the charge states of the pion. For charged pions, the photoproduction amplitude is

$$\langle p_1' | J^{(\pm)}_\pi | p_1 k\rangle = -\frac{i}{f_\pi}\left\{ \pm\, e\varepsilon_\lambda \langle p_1' | J^{A(\pm)}_\lambda | p_1\rangle + {} \right.$$

$$\left. {} + q_\lambda \langle p_1' | \bar{J}^{A(\pm)}_\lambda | p_1 k\rangle \right\}$$

$$\equiv \varepsilon_\lambda M^{e.m.}_\lambda(\pi^\pm) \ , \qquad \text{(II.19)}$$

where ε_λ is the photon polarization four vector, and the bar in the second term on the r.h.s. means that a pion-pole term

$$- \frac{f_\pi \, q^2}{q_\pi^2 + m_\pi^2} \left\langle p_1' \right| J_\pi^{(\pm)} \left| p_1 k \right\rangle$$

has been separated out. Similar equations can be written for the neutral pion. Now imposing the gauge condition $k_\lambda M_\lambda^{e.m.} = 0$, one finds that $M_\lambda^{e.m.}$ is given to order (m_π/M) by the Born terms of Fig. 5.

The first term in Eq. (II.19) yields the seagull term Fig. (5a) and a piece of Fig. 5d. (Note that the seagull term is equivalent to the Born term with $N\bar{N}$ intermediate state. It contributes only to the charged pion production.) One can see this by examining the matrix element of J_λ^A in Eq. (II.9a) and using the Goldberger-Treiman relation to express h_A in terms of g_A and letting $Q^2 = 0$ in the form factors. The Figs. 5b, 5c, and a part of Fig. 5d come from the second term of Eq. (II.19). Note that we are to take the full electromagnetic vertex as defined in Eq. (II.9b) and $F_\pi(q^2) \to F_\pi(0) = 1$ to ensure the gauge condition. It is important to note that to the order (m_π/M), no non-Born terms contribute, in contradistinction to the axial current where non-trivial contributions come from the non-Born terms.

3.1 Threshold production of pions

In order to have an idea how good Eq. (II.19) (and a similar equation for the neutral pion) described by Fig. 5 can be for the isovector $V^{(\pm)}$ and isoscalar $V^{(0)}$ amplitudes for our purpose, it is instructive to look at the experiments on the photoproduction of pions at threshold. We shall assume that the features observed at $q^2 = -m_\pi^2$ (physical pions) can give us some information about the $q^2 \approx m_\pi^2$ point.

It is now well established that Fig. 5 describes very reliably the charged pion ($\pi^\pm$) production, whereas it appears to be insufficient for π^0 production where the main terms Figs. 5a and 5d vanish. Also theoretically, the corrections to the soft-pion theorem are found to be very small for $\pi^\pm$, while important for π^0 [11]. Now to translate these observations in terms of the isovector and isoscalar amplitudes, we note from Eqs. (II.6) and (II.7) that:

$$M_\lambda^{e.m.}\ (\gamma p \to \pi^+ n) = \frac{1}{\sqrt{2}}\left[V_\lambda^{(-)} + V_\lambda^{(0)}\right]$$

$$M^{e.m.}\ (\gamma n \to \pi^- p) = -\frac{1}{\sqrt{2}}\left[V_\lambda^{(-)} - V_\lambda^{(0)}\right]$$

$$M_\lambda^{e.m.}\ (\gamma p \to \pi^0 p) = \left[V_\lambda^{(+)} + V_\lambda^{(0)}\right]/2 \qquad \text{(II.20)}$$

$$M^{e.m.}\ (\gamma n \to \pi^0 n) = \left[V_\lambda^{(+)} - V_\lambda^{(0)}\right]/2\ .$$

Or inversely

$$V_\lambda^{(-)} = \frac{1}{\sqrt{2}}\left[M_\lambda^{e.m.}\ (\gamma p \to \pi^+ n) - M_\lambda^{e.m.}\ (\gamma n \to \pi^- p)\right]$$

$$V_\lambda^{(+)} = \left[M_\lambda^{e.m.}\ (\gamma p \to \pi^0 p) + M_\lambda^{e.m.}\ (\gamma n \to \pi^0 p)\right], \qquad \text{(II.21)}$$

$$V_\lambda^{(0)} = \frac{1}{\sqrt{2}}\left[M_\lambda^{e.m.}\ (\gamma p \to \pi^+ n) + M_\lambda^{e.m.}\ (\gamma n \to \pi^- p)\right],$$

where $M_\lambda^{e.m.}\ (\gamma p \to \pi^+ n)$ stands for the amplitude for $\gamma + p \to \pi^+ + n$. Substituting into Eq. (II.21) the experimental values for $\pi^\pm$ production and the theoretical estimate of π^0 production by Furlan et al.[11], we have found that

$$\left|\frac{V_\lambda^{(+)}}{V_\lambda^{(-)}}\right| \approx 6 \times 10^{-3}$$

$$\left|\frac{V_\lambda^{(0)}}{V_\lambda^{(-)}}\right| \approx 7 \times 10^{-2}\ . \qquad \text{(II.22)}$$

Thus in the isovector part we may safely ignore $V_\lambda^{(+)}$. The amplitude $V_\lambda^{(-)}$ being the sum of numerically reliable quantities is thus correctly given by the low-energy theorem. The same would hold for the amplitudes we are interested in if the extrapolation from $q^2 = -m_\pi^2$ to $q^2 \approx m_\pi^2$ is as smooth as is assumed.

On the other hand, $V_\lambda^{(0)}$ so obtained would not be good, since it is a difference between two large numbers, and any further corrections to it can become significant.

III. TWO-BODY EXCHANGE OPERATORS

1. GENERAL CLASSIFICATION

In this section, we discuss how the non-relativistic two-body operators for the magnetic moment (MM) and the Gamow-Teller (GT) β-decay transitions are extracted, given the appropriate meson theories (next section) to calculate the Feynman graphs. Let us first classify the functional forms for these operators based on general invariance requirements.

We assume that GT and MM exchange operators have the same transformation properties as the corresponding one-body operators. They are both axial vectors. The GT operator is an isovector, whereas the MM has an isovector as well as an isoscalar component. For their construction we have at our disposal position coordinates of the two nucleons $\vec{x}_1, \vec{x}_2$, their conjugate momenta $\vec{p}_1, \vec{p}_2$, the Pauli spin matrices $\vec{\sigma}_1, \vec{\sigma}_2$, and isospin matrices $\vec{\tau}(1), \vec{\tau}(2)$.

For the isospin parts of the operators, all possible two-body terms formed with $\vec{\tau}(1)$ and $\vec{\tau}(2)$ must reduce to three isoscalars:

$$\vec{\tau}(1) \cdot \vec{\tau}(2) \ , \qquad [1 \pm \tau_3(1)\tau_3(2)] \ , \tag{III.1}$$

and three isovectors:

$$[\vec{\tau}(1) \times \vec{\tau}(2)]_j \ , \qquad [\tau_j(1) \pm \tau_j(2)] \ , \tag{III.2}$$

where the MM operators involve the isovector component j = 3 and the GT operators j = (1) ± i(2).

For the space-spin parts it is convenient to work with the relative and C.M. coordinates:

$$\vec{r} = (\vec{x}_1 - \vec{x}_2) \ , \qquad \vec{R} = (\vec{x}_1 + \vec{x}_2)/2 \ .$$

Operators depending on $\vec{R}$ (i.e. translationally non-invariant terms) or on the momentum operators $\vec{p}_1, \vec{p}_2$ (non-local terms) are, in principle, allowed, but we shall not consider them in our classification. The space-spin parts must then be axial-vectors formed with $\vec{\sigma}_1, \vec{\sigma}_2$, and $\vec{r}$. They must be of the following seven forms ($\hat{r} \equiv \vec{r}/|\vec{r}|$):

$$
\begin{aligned}
&(\vec{\sigma}_1 \times \vec{\sigma}_2) \ , \\
&(\vec{\sigma}_1 \pm \vec{\sigma}_2) \ , \\
&(\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \hat{r}\, \hat{r} \\
&(\vec{\sigma}_1 \pm \vec{\sigma}_2) \cdot \hat{r}\, \hat{r} \ , \\
&\left[(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \times \hat{r}) + (\vec{\sigma}_1 \times \hat{r})(\vec{\sigma}_2 \cdot \hat{r})\right] .
\end{aligned}
\tag{III.3}
$$

That no new terms can occur upon introducing the isospin and spin exchange operators P^{τ}_{12} and P^{σ}_{12} can be seen by using the following identities among the isospin matrices:

$$
\begin{aligned}
\left[\vec{\tau}(1) \times \vec{\tau}(2)\right] P^{\tau}_{12} &= i\left[\vec{\tau}(1) - \vec{\tau}(2)\right] , \\
\left[1 - \tau_3(1)\, \tau_3(2)\right] P^{\tau}_{12} &= \vec{\tau}(1) \cdot \vec{\tau}(2) - \tau_3(1)\tau_3(2) \ ,
\end{aligned}
\tag{III.4}
$$

and likewise for the spin matrices. Because of the antisymmetry of nuclear wave functions $P^{M}_{12}\, P^{\sigma}_{12}\, P^{\tau}_{12} = -1$, the Majorana operator (P^{M}_{12}) would therefore bring nothing new. Thus the operators (III.3) give the full space-spin dependence within multiplicative scalar functions of the relative coordinate. The complete operators are formed by combining these with the isospin parts (III.1) and (III.2) in all possible ways under the restriction that they are fully symmetric under the interchange of all particle coordinates. We thus arrive at the general classifications of the isovector and isoscalar components:

$$H_j = \frac{G}{2} \left\{ (\vec{\tau}(1) \times \vec{\tau}(2))_j \left[(\vec{\sigma}_1 \times \vec{\sigma}_2) g_I + \vec{T}_{12}^{(\times)} g_{II} \right] \right.$$

$$+ \left[\vec{\tau}(1) - \vec{\tau}(2) \right]_j \left[(\vec{\sigma}_1 - \vec{\sigma}_2) \; (h_I + h_I^\tau P_{12}^\tau + h_I^\sigma P_{12}^\sigma) + \right.$$

$$\left. + \vec{T}_{12}^{(-)} (h_{II} + h_{II}^\tau P_{12}^\tau + h_{II}^\sigma P_{12}^\sigma) \right] +$$

$$+ \left[\vec{\tau}(1) + \vec{\tau}(2) \right]_j \left[(\vec{\sigma}_1 + \vec{\sigma}_2) \; j_I + \right.$$

$$\left. + \vec{T}_{12}^{(+)} \; j_{II} + \vec{\Sigma}_{12} \; j_{III} \right] \quad , \qquad \text{(III.5)}$$

$$H_s = \frac{G}{2} \left\{ \tau_3(1)\tau_3(2) \left[(\vec{\sigma}_1 + \vec{\sigma}_2) k_I + \vec{T}_{12}^{(+)} \; k_{II} + \vec{\Sigma}_{12} \; k_{III} \right] \right.$$

$$+ \left[(\vec{\sigma}_1 + \vec{\sigma}_2) \; \ell_I + \vec{T}_{12}^{(+)} \; \ell_{II} + \vec{\Sigma}_{12} \; \ell_{III} \right]$$

$$\left. + \left[\vec{\tau}(1) \cdot \vec{\tau}(2) \right] \left[(\vec{\sigma}_1 + \vec{\sigma}_2) \; m_I + \vec{T}_{12}^{(+)} \; m_{II} + \vec{\Sigma}_{12} \; m_{III} \right] \right\}, \qquad \text{(III.6)}$$

where we have used the definitions:

$$\vec{\Sigma}_{12} = \frac{i}{3} \left[(\vec{\sigma}_1 \cdot \hat{r}) \; (\vec{\sigma}_2 \times \hat{r}) + (\vec{\sigma}_1 \times \hat{r}) \; (\vec{\sigma}_2 \cdot \hat{r}) \right] ,$$

$$\vec{T}_{12}^{\odot} = \left[(\vec{\sigma}_1 \odot \vec{\sigma}_2) \cdot \hat{r} \; \hat{r} - \frac{1}{3} \; (\vec{\sigma}_1 \odot \vec{\sigma}_2) \right] , \qquad \odot = \pm, \times \; . \qquad \text{(III.6a)}$$

The operators $\vec{T}_{12}^{\odot}$ are the D-wave parts of $(\vec{\sigma}_1 \odot \vec{\sigma}_2) \cdot \hat{r} \; \hat{r}$, obtained by subtracting out their average over orientations of $\hat{r}$. The functions g_I, g_{II}, h_I, h_I^τ, etc., are arbitrary scalar functions of $|\vec{r}| = r$, which must all be real by virtue of the transformation properties of the operators under time reversal (see Appendix 3 of Ref. 2). The constant G is defined by

$$G = \begin{cases} e/2M & (MM) \\ -g_A & (GT) \end{cases} \tag{III.7}$$

We note that of the 20 terms entering our classification, only 12 of them would be allowed in the phenomenological theory of Osborne and Foldy[12]. The additional 8 terms are those involving the functions $h^{\tau}_{I,II}$, $h^{\sigma}_{I,II}$, and also those involving functions with subscript III, namely j_{III}, h_{III}, etc. They had to be rejected in the phenomenological theory of Osborne-Foldy as a consequence of the assumptions used in its construction and the restrictions imposed on the two-body operators by time reversal[13]. To be more specific, their assumption that the exchange operators are proportional to the NN potential imposes reality conditions on the radial functions, which for the above-mentioned terms are incompatible with conditions imposed by time reversal. For example, a term such as

$$\left\{ (\vec{\sigma}_1 \times \vec{\sigma}_2) \left[\vec{\tau}_j(1) - \vec{\tau}_j(2) \right] G(r) \right\}$$

transforms properly under time reversal only if we allow G(r) to become pure imaginary. On general grounds there is, however, no reason to discard these terms, which in fact will appear in our explicit meson-theory descriptions.

The classification of operators non-invariant under translation is a rather fastidious task which we do not wish to undertake here. We simply write down some typical terms we shall encounter in the calculations:

$$\begin{aligned} H' = G \Big\{ & \left[(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \hat{R}\, \hat{r} - (\vec{\sigma}_1 + \vec{\sigma}_2)\, (\hat{r} \cdot \hat{R}) \right] F_I \\ & + \left[(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \hat{R}\, \hat{r} - (\vec{\sigma}_1 - \vec{\sigma}_2)\, (\hat{r} \cdot \hat{R}) \right] F_{II} \\ & + \left[(\vec{\sigma}_1 \cdot \hat{r})\, (\vec{\sigma}_2 \times \hat{R}) + (\vec{\sigma}_1 \times \hat{R})\, (\vec{\sigma}_2 \cdot \hat{r}) \right] F_{III} \\ & + i(\hat{r} \times \hat{R}) F_{IV} \Big\} , \end{aligned} \tag{III.8}$$

where F_I, ..., F_{IV} are real scalar functions of r which may depend on spin and isospin operators, and have appropriate symmetries so that H is fully symmetric under interchange of two coordinates.

As for non-local operators, there exists a phenomenological classification of them by Sachs and Austern[14]. In our calculation where the static approximation is made for nucleons, non-local operators are met only infrequently, so we shall not go into them; they do not contribute in our application to the three-nucleon systems.

2. DEFINITION OF EXCHANGE OPERATORS FROM FEYNMAN DIAGRAMS

The most convenient way of looking at the meson exchanges is to divide them into an OPE term and a multipion exchange contribution which is to be simulated partly by heavy meson exchanges (HME). We have given the corresponding Feynman graphs in Fig. 2. They represent two-nucleon scattering from initial momenta p_1, p_2 to final momenta p_1', p_2' while undergoing an interaction with the external field (electromagnetic or weak). The relevant currents are the e.m. or weak currents. The deduction of exchange operators from the Feynman amplitudes associated with those graphs will now be discussed.

Let us first consider the OPE graph, Fig. 2a. The matrix element associated with it is denoted by $\langle p_1' p_2' | J_\lambda^j | p_1 p_2 \rangle$. If the vertex corresponding to the pion production by a current J_λ is written as $\langle \pi^n(q) N(p_1') | J_\lambda^j | N(p_1) \rangle$ where the superscript n stands for the isospin index for pion, then using the conventional form of the πNN vertex

$$\left\langle N(p_2') \middle| J_\pi^n \middle| N(p_2) \right\rangle = i\, g_r\, K_{\pi NN}(q^2)\; \bar{u}(p_2') \gamma_5 \tau_n u(p_2) \ , \qquad \text{(III.9)}$$

we may write

$$\tilde{J}_\lambda^j \equiv (2\pi)^3\, \delta(\vec{p}_1 + \vec{p}_2 + \vec{k} - \vec{p}_1{}' - \vec{p}_2{}') \left\langle p_1' p_2' \middle| J_\lambda^j \middle| p_1 p_2 \right\rangle$$

$$= (2\pi)^{-3}\, \delta(\vec{p}_1 + \vec{p}_2 + \vec{k} - \vec{p}_1{}' - \vec{p}_2{}') \times$$

$$\times \left\{ \left\langle \pi^n(q) N(p_1') \middle| J_\lambda^j \middle| N(p_1) \right\rangle \frac{d_\pi(q^2)}{q^2 + m_\pi{}^2} \left\langle N(p_2') \middle| J_\pi^n \middle| N(p_2) \right\rangle + \right.$$

$$\left. + (1 \rightleftarrows 2) \right\} \qquad \text{(III.10)}$$

in which $(1 \rightleftarrows 2)$ stands for the term obtained by interchange of nucleons 1 and 2 in initial and final states. The $\tilde{J}^j_\lambda$ can be considered as a matrix element in momentum space of the two-body current operator for two-nucleon systems. Equation (III.10) represents an amplitude expressed in renormalized perturbation theory, thus containing a pionic form factor of nucleon $K_{\pi NN}$ and the pionic propagator form factor d_π. These quantities are normalized at the pion pole $q^2 = -m_\pi^2$. In principle, the off-shell behaviour in these form factors can be very important. However, if we want to pursue the calculation in the line adopted in nuclear force theory, we need to be careful. The reason is that when obtaining two-nucleon potentials by using phenomenological coupling constants and masses for pion and heavier mesons, the form factor effect is assumed to be automatically taken care of. This fact was duly recognized by Brown and Green[5)] in their treatment of three-body force. Since we are going to use correlated nuclear wave functions, and put in also the heavy meson exchanges, it is deemed necessary and consistent to put $K(q^2) \equiv K_{\pi NN}(q^2)\, d_\pi(q^2)$ on the mass shell $K(-m_\pi^2) = 1$. This argument does not apply to the weak or photoproduction vertex $\langle \pi N | J^j_\lambda | N \rangle$, whose off-shell-dependence cannot be ignored. In our case, we work with the amplitude at the soft pion limit $q^2 = 0$ which is sufficiently reliable for small q^2.

As it stands, the representation (III.10) is not suitable for nuclear physics applications. The reason is that conventional description of nuclear structure is usually given in terms of non-relativistic, single-time wave functions depending on the space and intrinsic coordinates of nucleons. The transformations required for such a description are standard ones. Matrix elements with respect to Dirac spinors must, firstly, be expressed in terms of Pauli spinors and, secondly, transformed to position space. In the non-relativistic reduction of Dirac to Pauli spinors, the static limit is taken. This is achieved by expressing matix elements in a particular representation for the Dirac matrices and retaining leading terms in the limit where the nucleon mass $M \to \infty$. The procedure reproduces the lowest-order terms in a Foldy-Wouthuysen transformation. The representation in configuration space is then obtained by doing a Fourier transform on each nucleon momentum.

$$<\vec{x}_1'\vec{x}_2'|J^j_\lambda|\vec{x}_1\vec{x}_2> = \delta(\vec{x}_1 - \vec{x}_2)\delta(\vec{x}_1' - \vec{x}_2')J^j_\lambda(\vec{x}_1,\vec{x}_2)$$

$$= \frac{1}{(2\pi)^6}\int d\vec{p}_1 \ldots d\vec{p}_2' \times$$

$$\times\ e^{-i(\vec{p}_1\cdot\vec{x}_1 + \vec{p}_2\cdot\vec{x}_2 - \vec{p}_1'\cdot\vec{x}_1' - \vec{p}_2'\cdot\vec{x}_2')}\tilde{J}^j_\lambda \qquad \text{(III.11)}$$

where $J^j_\lambda(\vec{x}_1,\vec{x}_2)$ is the diagonal part of the matrix element. It represents the Fourier transform of the current density of the two-nucleon system as the momentum k of the external field is held fixed. Any momentum-dependent term would come from a derivative contained in $J^j_\lambda(\vec{x}_1,\vec{x}_2)$ operating on the delta functions.

In Table 1, non-relativistic expressions in the static limit are given for each covariant $O_\lambda(A)$ and $O_\lambda(V)$. The πNN vertex is, on the other hand,

$$\bar{u}(p_2')\gamma_5 u(p_2) \rightarrow \frac{\vec{\sigma}\cdot\vec{q}}{2M}$$

and in the static limit $q^2 \simeq \vec{q}^{\,2}$.

As an illustration, let us consider the axial current. To make an unambiguous separation of local and non-local terms, one should be careful in performing the integrals in Eq. (III.11). Of the four integrals over nucleon momenta, one, say $\vec{p}_1'$, is used to eliminate the delta function expressing momentum conservation. The remaining integrals are then expressed in terms of the independent momenta $\vec{p}_1$, $(\vec{p}_2' + \vec{p}_2)/2$ and $(\vec{p}_2' - \vec{p}_2) \equiv \vec{q}$. The argument of the exponent is written as:

$$-i\left[\vec{p}_1\cdot(\vec{x}_1 - \vec{x}_1') + \frac{\vec{p}_2' + \vec{p}_2}{2}\cdot(\vec{x}_2 - \vec{x}_2') + \vec{q}\cdot\left(\vec{x}_1' - \frac{\vec{x}_2 + \vec{x}_2'}{2}\right) - \vec{k}\cdot\vec{x}_1'\right] .$$

The integrals over the first two momenta introduce the delta functions displayed in Eq. (III.11) so that any dependence on them is equivalent to the terms:

$$\vec{p}_1 \rightarrow \vec{p}_1$$

$$\frac{\vec{p}_1' + \vec{p}_2}{2} \rightarrow \vec{p}_2 + \vec{q}$$

where $\vec{p}_1$ and $\vec{p}_2$ are momentum operators acting on the ket wave functions. Putting Eq. (III.10) into (III.11), we can write

$$J_\lambda^{Aj}(\vec{x}_1,\vec{x}_2) = \frac{1}{(2\pi)^3} \left\{ \int d\vec{q}\ \bar{u}(\vec{p}_1')\ O_\lambda(A_s) \left[A_s^{(+)} a_{nj}^{(+)} + A_s^{(-)} a_{nj}^{(-)} \right] u(\vec{p}_1) \times \right.$$

$$\left. \times \frac{e^{-i\vec{q}\cdot\vec{r}}}{q^2 + m_\pi^2}\ g_r \bar{u}(\vec{p}_2') i\gamma_5 \tau_n u(\vec{p}_2) + (1 \rightleftarrows 2) \right\} \qquad \text{(III.12)}$$

where s = 1, ..., 8, and $a_{nj}^{(\pm)} = \frac{1}{4} \left[\tau_n, \tau_j\right]_\pm$ as defined before. After expressing all the dependences on pion momentum by a derivative, i.e. $q_\ell \rightarrow i\partial_\ell$, $\ell = 1,2,3$, the integration over $\vec{q}$ brings in the familiar Yukawa function

$$\int d\vec{q}\ \frac{e^{-i\vec{q}\cdot\vec{r}}}{\vec{q}^2 + m_\pi^2} = 2\pi^2 m_\pi\ Y_0(x_\pi) \ , \qquad \text{(III.13)}$$

where $Y_0(x) = e^{-x}/x$, with $x_\pi = m_\pi r$. If the invariant amplitudes $A_s^{(\pm)}$ have no dependence on q, the integrand in Eq. (III.12) depends through $O_\lambda(A_s)$ and γ_5 on the pion momentum, at most quadratically. As we have seen in the previous section, the amplitudes $A_s^{(\pm)}$ have that feature in low-energy limits. But suppose we consider explicitly the vertex correction due to ρ (Fig. 6c), giving rise to the two-body operator of Fig. 7. In the limit $k \rightarrow 0$, the amplitude relevant to this contains the factor

$$\frac{1}{q^2 + m_\rho^2} \qquad \text{(III.14)}$$

if the Feynman graph is evaluated in the usual way. In such a case, the integration is not Eq. (III.13), but

$$\int d\vec{q}\,\frac{e^{-i\vec{q}\cdot\vec{r}}}{(\vec{q}^2+m_\pi^2)\,(\vec{q}^2+m_\rho^2)} = \frac{2\pi^2}{m_\pi\left[\left(\frac{m_\rho}{m_\pi}\right)^2-1\right]}\left[Y_0(x_\pi)-\left(\frac{m_\rho}{m_\pi}\right)Y_0\left(\frac{m_\rho}{m_\pi}x_\pi\right)\right] \qquad \text{(III.15)}$$

If, furthermore, pions can propagate with either positive or negative frequencies alone as is the case in some of the Born terms (see the next subsection), then Eq. (III.12) does not hold, and radial functions different from the Yukawa types [Eq. (III.13) or Eq. (III.15)] appear.

Now given $\tilde{J}^j_\lambda$, what are the relevant operators for the magnetic moment and the β-decay? To answer this, we recall that only the space part of the current four-vector is needed. The Gamow-Teller operator is simply the monopole part of the axial current, and the magnetic operator is the dipole part of the current, i.e.

$$\begin{aligned} &\vec{J}^{Aj}\Big|_{k=0} && \text{for β-decay} \\ &\vec{M} \equiv -\frac{i}{2}\vec{\nabla}_{\vec{k}}\times\vec{J}^{e.m.}\Big|_{k=0} && \text{for MM}\ . \end{aligned} \qquad \text{(III.16)}$$

We conclude this section by giving the expressions of the radial dependences of the operators in the case where the amplitudes $A_s^{(\pm)}$, $V_s^{(\pm,0)}$ are independent of the invariants ν, ν_B, and q^2. In Table 2, we give the functions g, h, j for the GT operators, and in Table 3 the functions g, h, j, k, ℓ, m for the MM operators. We note that this condition is met for the NB parts of the amplitudes found in the application of the low-energy theorems. It does not, however, apply to the Born parts which have singular terms in the soft-pion or soft-current limits. For this and other reasons to which we will return in the next Chapter, the Born parts must be treated separately.

Note that for the GT operator where $k \to 0$, A_5, ..., A_8 do not contribute, and for the MM operator, V_5 and V_6 vanish since $k^2 = 0$ for real photon. Furthermore, in the non-relativistic (NR) limit, V_1 and V_2 are negligible compared to V_3 and V_4, since the matrix elements of $0_\lambda(V_1)$ and $0_\lambda(V_2)$ are proportional to k_0 or M^{-1}. These are the reasons why Table 2 contains only A_s, $s = 1, \ldots, 4$ and Table 3 only V_s, $s = 3,4$. We also note that at the limit $\nu = \nu_B = k^2 = q^2 = 0$ which comes in the low-energy theorem, some amplitudes vanish due to crossing symmetry, and these are indicated therein.

IV. ONE-BOSON EXCHANGES

In this Chapter we calculate the two-body exchange operators due to OPE and heavy-meson exchange (HME) contributions. For the OPE term, we use the low-energy theorem information, and supplement the calculation by using other models: phenomenological Lagrangian model for the GT case and the Chew-Low model for the MM case. For the HME, we consider the ρ- and ω-exchange graphs.

1. ONE-PION EXCHANGE CONTRIBUTION

For the benefit of those readers who have not quite appreciated the power and relevance of low-energy theorems, we summarize here some important points to bear in mind. For both the e.m. and axial currents, we have divided the pion production amplitudes (and therefore the exchange current) into a well-defined Born term, and the rest into the non-Born term. Then the low-energy theorems tell us essentially the following: (i) for the isovector e.m. current, the NB term is negligible; (ii) for the isoscalar e.m. current, the Born term is small, while the NB term is undetermined; and (iii) for the axial current, the NB term is given in terms of off-shell πN scattering amplitudes, not necessarily small.

These results rest on the applicability of the soft-pion theorems, and the question is then: to what extent can the exchanged pions in the OPE process be considered as soft? In practice, there are limitations in Eq. (III.12) on the range of integration over the pion momentum. The long-range parts of the interactions are only sensitive to small values of q. The longer the range, the smaller the q involved. From energy-momentum conservation at the πNN vertex, we also have: $q = p_2' - p_2$. Thus for uncorrelated nucleons near the Fermi surface, $q_0 = p_{20}' - p_{20} \simeq 0$ and q^2 roughly in the range between m_π^2 and $4m_\pi^2$ with the peak at m_π^2. This is not far from the soft-pion limit, and thus renders the PCAC applicable. Once this is established, the power of the theorems is that they give us non-trivial relations exact to all orders in strong interaction coupling constants connecting the required amplitudes to either calculable quantities or measurable quantities.

1.1 Non-Born contribution

We reverse the order and treat the non-Born part first. The reason is that the NB part permits the full pion propagator and has no danger of double counting which the Born term does. Thus the discussion of Chapter III is directly applicable.

In following this part of the Chapter, Tables 2 and 3 are of practical importance to us, since once the amplitudes are calculated explicitly, the associated GT and MM operators can be read off from the tables. The assumption that the amplitudes are independent of the invariants applies to most of the situations discussed below, as the relevant point corresponds to $\nu = \nu_B = k^2 = q^2 = 0$.

1.1.1 Axial current

Let us consider the NB parts (denoted by a bar) of the pion production matrix element given in Eq. (II.16) in the limit $k \to 0$. They are regular, so we have

$$\bar{M}^A_\lambda \equiv \left\langle \pi^n(q)N(p_1')\,|J^{Aj}_\lambda|\,N(p_1)\right\rangle_{NB} = \bar{u}(p_1')\left\{\left[\bar{A}_1^{(-)} + \bar{A}_2^{(-)}\right] a^{(-)}_{nj} O_\lambda(A_2)\right.$$

$$\left. + \bar{A}_3^{(+)} a^{(+)}_{nj} O_\lambda(A_3) + \left[\bar{A}_4^{(-)} - 2\bar{A}_1^{(-)}\right] a^{(-)}_{nj} O_\lambda(A_4)\right\} u(p_1) + O(k)$$

$$\equiv \bar{u}(p_1')\left\{-\beta(q^2)a^{(-)}_{nj} O_\lambda(A_2) + \gamma(q^2)a^{(+)}_{nj} O_\lambda(A_3) -\right.$$

$$\left. - 2\alpha(q^2)a^{(-)}_{nj} O_\lambda(A_4)\right\} u(p_1) + O(k)\ . \qquad \text{(IV.1)}$$

This equation defines α, β, and γ for arbitrary q^2. Note that α and β are related by the Adler-Weisberger sum rule at $q^2 = 0$, as can be seen from Eq. (II.19). The soft-pion limit is a good approximation for the case we are considering; hence we need only $\alpha(0)$, $\beta(0)$, and $\gamma(0)$. The corrections for the case $q^2 \simeq m_\pi^2$ can be safely neglected. From Eq. (III.5) we have

$$H_j^{(2)} = \frac{G}{2} \Bigg\{ \left[\vec{\tau}(1) \times \vec{\tau}(2)\right]_j \left[(\vec{\sigma}_1 \times \vec{\sigma}_2) g_I + \vec{T}_{12}^{(x)} \, g_{II}\right] +$$

$$+ \left[\tau(1) - \tau(2)\right]_j \left[(\vec{\sigma}_1 - \vec{\sigma}_2)(h_I + h_I^{\tau} \, P_{12}^{\tau}) +\right.$$

$$\left. + \vec{T}_{12}^{(-)} \, (h_{II} + h_{II}^{\tau} \, P_{12}^{\tau})\right] +$$

$$+ \left[\tau(1) + \tau(2)\right]_j \left[(\vec{\sigma}_1 + \vec{\sigma}_2) \, j_I +\right.$$

$$\left. + T_{12}^{(+)} \, j_{II}\right] + H_j^{N.L.} \Bigg\} , \qquad \text{(IV.2)}$$

where

$$g_I = \frac{2}{3} \, \xi\alpha(0) Y_0(x_\pi) , \qquad g_{II} = -\xi\alpha(0) Y_2(x_\pi)$$

$$h_I = j_I = -\frac{1}{6} \, \xi\gamma(0) Y_0(x_\pi) , \qquad h_{II} = j_{II} = -\frac{1}{2} \, \xi\gamma(0) Y_2(x_\pi)$$

$$h_I^{\tau} = -\frac{1}{3} \, \xi\left[\alpha(0) + \beta(0)\right] Y_0(x_\pi) , \qquad h_{II}^{\tau} = -\, \xi\left[\alpha(0) + \beta(0)\right] Y_2(x_\pi)$$

(IV.3)

where

$$\xi = \frac{1}{8\pi} \left(\frac{g_r}{g_A}\right)\left(\frac{m_\pi}{M}\right) m_\pi^2 , \qquad Y_0(x) = \frac{e^{-x}}{x} ,$$

$$Y_2(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right) Y_0(x) .$$

There is no translationally non-invariant term, but there is a non-local term:

$$(H_j)_{NL} = - \left[\vec{\tau}(1) \times \vec{\tau}(2)\right]_j \; 2\left[\alpha(0) + \beta(0)\right] \frac{\xi}{m_\pi} \left[(\vec{\sigma}_1 \cdot \hat{r})\vec{p}_2 + \right.$$

$$\left. + (\vec{\sigma}_2 \cdot \hat{r})\vec{p}_1\right] \left(1 + \frac{1}{x_\pi}\right) Y_0(x_\pi) \; . \qquad \text{(IV.4)}$$

The PCAC implies that

$$\alpha(0) = \frac{g_A}{g_r(0)} \left[\bar{B}^{\pi N(-)}\right]_0 ,$$

$$\beta(0) = \frac{g_A}{g_r(0)} \left[\frac{\partial \bar{A}^{\pi N(-)}}{\partial \nu}\right]_0 , \qquad \text{(IV.5)}$$

$$\gamma(0) = \frac{g_A}{g_r(0)} \left[\frac{\partial \bar{A}^{\pi N(+)}}{\partial \nu_B}\right]_0 ,$$

where the subscript 0 means that $\nu = \nu_B = k^2 = q^2 = 0$. These values have been evaluated by Adler[8)] by extrapolating from on-shell πN scattering amplitudes. They are given in Table 4, row 1, denoted for short as "Adler". The errors involved in these numbers are hard to assess, but we would guess that they are of the same magnitudes as the errors usually quoted in the test of the Adler consistency condition and the Alder-Weisberger sum rule.

We shall now turn to an alternative calculation of the amplitudes which may be considered as some sort of book-keeping of the global predictions of the low-energy theorems. The low-energy theorems can be viewed as sum rules derived from current algebra by insertion of a complete set of intermediate states in the equal-time commutators[15)]. Cheng and Kim[16)] exploited this idea in relation to the Adler-Weisberger sum rule. We shall apply here a similar idea and approximate the pion-production process by pole diagrams with intermediate states corresponding to nucleon isobars (N^*) and vector mesons (V). These are illustrated by the diagrams given in Figs. 6a to 6c.

Nucleon isobars

We describe the intermediate resonances in particle representation. Effective couplings are used for the vertices πNN^* and ANN^*, where A is a shorthand notation for the axial current. Coupling constants for the former vertices are determined from available experimental widths and for the latter by using one-pion-pole dominance in the axial form factors.

The isobars with angular momentum and parity $J^P = (n + \frac{1}{2})^{\pm}$, $n > 0$, are described by Rarita-Schwinger spinors $u_{\mu_1 \ldots \mu_n}(P)$. For $n = 0$, the usual Dirac spinors are used. We give the πNN^* vertices in terms of the matrix elements of the Lagrangian density[16]:

$$\left\langle \pi(q)N(p')\,|\mathcal{L}|\,N^*(p)\right\rangle = g_{\pi NN^*}\, i^n\, q_{\mu_1} \ldots q_{\mu_n}\, \bar{u}(p') \begin{pmatrix} 1 \\ i\gamma_5 \end{pmatrix} u_{\mu_1 \ldots \mu_n}(p) \;,$$

$$\left[(J^P)_{N^*} = (n + \tfrac{1}{2})^{\pm},\; n = 1, 2, \ldots \right], \qquad \text{(IV.6)}$$

$$\left\langle \pi(q)N(p')\,|\mathcal{L}|\,N^*(p)\right\rangle = g_{\pi NN^*}\, \bar{u}(p') \begin{pmatrix} i\gamma_5 \\ 1 \end{pmatrix} u(p) \;,$$

$$\left[(J^P)_{N^*} = \tfrac{1}{2}^{\pm} \right]. \qquad \text{(IV.7)}$$

Introduction of isospin is straightforward. Let T be the isospin of the isobar, s′ its projection along isospin Z-axis and let α (spherical component) and s specify the isospin projections of π and N, respectively. Inserting the Clebsch-Gordan coefficient $\langle 1\ \tfrac{1}{2}\ \alpha s|Ts'\rangle$ in the r.h.s. of Eqs. (IV.6) and (IV.7) takes care of the isospin dependence by making them isoscalars, as they should be.

The ANN^* couplings involve, in general, a number of invariant operators. In the limit $k \to 0$, relevant to our problem, only one invariant operator survives. We write the vertices in terms of the matrix elements of the axial current:

$$\langle N(p')|J_\lambda^A|N^*(p)\rangle = \frac{1}{2} g_A^* \, i^{n-1} \, \delta_{\lambda\mu_1} q_{\mu_2} \cdots q_{\mu_n}$$

$$\times \bar{u}(p') \begin{pmatrix} 1 \\ i\gamma_5 \end{pmatrix} u_{\mu_1\mu_2\ldots\mu_n}(p) + O(k) \; ,$$

$$\left[J^P = (n + \tfrac{1}{2})^{\pm}, \; n = 1, 2, \ldots \right] \qquad \text{(IV.8)}$$

$$\langle N(p')|J_\lambda^A|N^*(p)\rangle = \frac{i}{2} g_A^* \, \bar{u}(p')\gamma_\lambda \begin{pmatrix} \gamma_5 \\ 1 \end{pmatrix} u(p) + O(k) \; ,$$

$$\left[J^P = \tfrac{1}{2}^{\pm} \right] . \qquad \text{(IV.9)}$$

The requirement of time-reversal invariance implies that all the coupling constants introduced so far must be real. The inclusion of isospin dependence is done in a similar way to that for the πNN^* vertices.

The PCAC hypothesis places constraints on the coupling constants g_A^* that we shall use in the following. They are connected to the coupling constants $g_{\pi NN^*}$ through the relation[16]:

$$\left\langle N(p')|\partial_\lambda J_\lambda^A|N^*(p)\right\rangle = \frac{M\, g_A}{g_r(0)} \; \frac{m_\pi^2}{q^2 + m_\pi^2} \left\langle \pi(q)N(p')|\mathcal{L}|N^*(p)\right\rangle \; . \qquad \text{(IV.10)}$$

These Goldberger-Treiman-type relations may be written in general form:

$$\frac{g_A^*}{f_{\pi NN^*}} = \frac{2M\, g_A}{g_r(0)} \qquad \text{(IV.11)}$$

$$f_{\pi NN^*} = \begin{cases} g_{\pi NN^*} \left[J^P = (n + \tfrac{1}{2})^{\pm}, \quad n = 1,2, \ldots\right] \\[2ex] \dfrac{g_{\pi NN^*}}{M^* \pm M} \left[J^P = \tfrac{1}{2}^{\pm}\right] . \end{cases} \qquad \text{(IV.12)}$$

In particular, they fix relative signs of the coupling constants which would otherwise be difficult to know.

We have now the necessary ingredients for calculating the diagrams given in Figs. 6a and 6b. We restrict to the lowest-excited nucleon resonances $N^*_{3+3}(1236,120)$, $N^*_{1+1}(1400,120)$, and $N^*_{3-1}(1525,40)$, where the numbers in paranthesis specify masses and partial widths (in MeV) for $N^* \to N + \pi$ decay modes, and the subscripts stand for $(2J^P, 2T)$. Corresponding to the Feynman propagator for spin-½ particles $(i\,\gamma \cdot Q + M)^{-1}$, we use the following expression for the spin-3/2 propagator:

$$\left(\delta_{\lambda\mu} - \frac{1}{3}\gamma_\lambda\gamma_\mu\right)\Big/(i\,\gamma \cdot Q + M) \;,$$

where terms of order $\vec{Q}/M$ which vanish in the static limit and unknown off-mass-shell terms have been dropped.

The results will be expressed in terms of the contribution to the amplitudes $A_s^{(\pm)}$. Terms of order q^2/M^2 appear in the calculations but will not be written down.

$$N^*_{3+3}: \quad \bar{A}_1^{(-)} = \frac{\bar{A}_3^{(+)}}{4} = \frac{2}{9}\,\frac{g_{\pi NN^*}\,g_A^*}{M^* - M} \;,$$

$$\alpha(0) = -\beta(0) = \frac{1}{4}\gamma(0) = \frac{2}{9}\,\frac{g_{\pi NN^*}\,g_A^*}{M^* - M} \qquad \text{(IV.13)}$$

$$N^*_{1+1}: \quad \bar{A}_2^{(-)} = \bar{A}_3^{(+)} = -\frac{M}{M^* + M}\,\bar{A}_4^{(-)} = \frac{2g_{\pi NN^*}\,g_A^*}{M^{*2} - M^2}$$

$$\alpha(0) = \frac{g_{\pi NN^*}\,g_A^*}{M(M^* - M)}$$

$$\beta(0) = -\gamma(0) = \frac{-2g_{\pi NN^*}\,g_A^*}{M^{*2} - M^2} \;. \qquad \text{(IV.14)}$$

$$N^*_{3\,1}: \quad \bar{A}_1^{(-)} = -\frac{1}{2}\,\bar{A}_3^{(+)} = -\frac{2}{3}\,\frac{g_{\pi NN^*}\,g_A^*}{M^* + M}\,,$$

$$\alpha(0) = -\beta(0) = -\frac{1}{2}\,\gamma(0) = -\frac{2}{3}\,\frac{g_{\pi NN^*}\,g_A^*}{M + M}\,. \qquad \text{(IV.15)}$$

The numerical values are given in Table 4 where the low-energy theorem values are also given for comparison. In the last column of the table we consider a particular combination of the amplitudes given by $[2\bar{A}_1^{(-)} - \bar{A}_4^{(-)} - \frac{1}{2}\,\bar{A}_3^{(+)}]$ within a factor. This is precisely the combination which determines the GT matrix element of triton beta-decay if one restricts to the S-state. Note that the contribution from the N^*_{3+3} is exactly cancelled

Vector mesons

A strict polology treatment of the vertex correction given by Fig. 6c presents delicate points because of the vector-meson propagators $(q^2 + m_V^2)^{-1}$ and their non-trivial dependence on the pion mass $(-q^2)$. Adler[8] has given a general treatment of these vector-meson exchange amplitudes by a method implementing the vector-meson dominance (VMD) hypothesis with the current algebra. It includes the effects of all vector mesons with quantum numbers $J^{PG} = 1^{-+}$. We shall not enter into the details of his method, but shall simply enumerate some definitions and quote his results. They are strictly valid in the soft-pion limit $q \to 0$, but we shall also retain the q^2-dependence originating from the vector-meson propagator.

The VNN vertex is taken to be

$$i\,\bar{u}(p_1')\,[\gamma_\lambda\, f_1 - \sigma_{\lambda\mu}\,(k - q)_\mu\, f_2]\,\frac{\tau_e}{2}\,u(p_1)\,, \qquad \text{(IV.16)}$$

where f_1 and f_2 are form factors, functions of $(k - q)^2$. The ρ-propagator has the form $[\delta_{\lambda\mu} + (k - q)_\lambda (k - q)_\mu / m_V^2]/[(k - q)^2 + m_V^2]$. The matrix element of the axial current between vector meson and pion states is found to be (up to order k)

$$\langle V_\mu^k | J_\lambda^{Aj} | \pi_n \rangle = \frac{g_r(0)}{M\, g_A}\, F_V\, \delta_{\lambda\mu}\, \varepsilon^{jkn}\,, \qquad \text{(IV.17)}$$

where unknown off-shell terms are neglected*). The factor F_V is the quantity used in the field-current identity[17] to relate the current $J_\lambda^{e.m.}$ to the vector-meson fields V_λ, namely:

$$J_\lambda^{e.m.} = F_V V_\lambda \equiv \frac{m_V^2}{2\gamma_V} V_\lambda \ . \tag{IV.18}$$

The amplitudes, as $k \to 0$ are

$$\begin{aligned} \bar{A}_2^{(-)}(q^2) &= \frac{g_r(0)\kappa_V}{2M^2 g_A} \frac{m_\rho^2}{q^2 + m_\rho^2} \\ A_4^{(-)}(q^2) &= -\frac{g_r(0)}{M^2 g_A}(1+\kappa_V)\frac{m_\rho^2}{q^2+m_\rho^2} \end{aligned} \tag{IV.19}$$

where we have used the hypothesis of ρ dominance of the isovector e.m. form factor $F_{1,2}^V(k^2) = f_{1,2}F_\rho/(k^2 + m_\rho^2)$. If one neglects q^2 as compared to m_ρ^2, one obtains

$$\bar{A}_2^{(-)} = \frac{g_r(0)\kappa_V}{2M^2 g_A} \ , \qquad \bar{A}_4^{(-)} = -\frac{g_r(0)}{M^2 g_A}(1+\kappa_V) \ .$$

Or

$$\begin{aligned} \alpha(0) &= \frac{g_r(0)}{2M^2 g_A}(1+\kappa_V) \ , \\ \beta(0) &= -\frac{g_r(0)\kappa_V}{2M^2 g_A} \end{aligned} \tag{IV.20}$$

which are almost identical to the results of the low-energy theorems except for an additional term in $\bar{A}_4^{(-)}$ **).

*) The result (17) is related through the KSFR relation to the result obtained by the application of the PCAC hypothesis to the $V\pi A$ vertex. The PCAC constraint specifies the coupling constant $g_{V\pi A}$ in terms of $f_\pi \equiv \sqrt{2}\,[M\, g_A/g_r(0)]$ (pion decay coupling constant) and $g_{V\pi\pi}$ (the coupling constant for $V \to \pi\pi$ decay) by the relation $g_{V\pi A} = \sqrt{2}\, g_{V\pi\pi} f_\pi$. The KSFR relation gives on the other hand $g_{V\pi\pi}^2 = (m_V/f_\pi)^2$, which leads to $g_{V\pi A} = [g_r(0)/g_A\, M] \times (m_V^2/2\gamma_V)$.

**) Note that the PCAC value for $\bar{A}_4^{(-)} = -\dfrac{g_r(0)}{M^2 g_A}(1+\kappa_V) + \dfrac{g_r\, g_A}{M^2}$.

The numerical values corresponding to this assumption are given in Table 4, fifth row.

If the model used to get Eq. (IV.19) were correct, then the q^2 dependence would just reflect the off-shell extrapolation of the amplitudes $\bar{A}_2^{(-)}$ and $\bar{A}_4^{(-)}$. However, even though the $q^2 \to 0$ limit is in agreement with the PCAC prediction, it is not clear whether the q^2 dependence in Eq. (IV.19) is correct. For example, the hard-pion theories give quite different behaviour, with a disagreement with the PCAC results in the $q^2 = 0$ limit. As already noted, when transformation into coordinate space is made, keeping the factor $m_\rho^2(q^2 + m_\rho^2)^{-1}$ in Fig. 7 amounts to replacing the Yukawa function $e^{-m_\pi r}/m_\pi r$ by

$$\frac{m_\rho^2}{m_\rho^2 - m_\pi^2}\left[\frac{e^{-m_\pi r}}{m_\pi r} - \left(\frac{m_\rho}{m_\pi}\right)^3 \frac{e^{-m_\rho r}}{m_\rho r}\right] . \qquad \text{(IV.21)}$$

Note that this is dominated by the ρ-meson term at small r (< 1 fm), and for r >>1 fm by the pion term: it is negative for $r \lesssim 1$ fm, and changes sign to positive for $r \gtrsim 1$ fm. This is quite different from what one would expect from taking Eq. (IV.20). For a sensible nuclear wave function, Eq. (IV.21) gives expectation values with signs opposite to the PCAC results, and is very sensitive to its inner parts. Thus if the extrapolation from $q^2 = 0$ to $q^2 \neq 0$ is indeed smooth, then we must conclude that a calculation based on Eq. (IV.21) is a suspect. It might be appropriate at this point to note that this sort of ambiguities also appears in treating $K(q^2)$ extrapolation in coordinate space.

1.1.2 Vector current

Unlike the axial case, the limit $k \to 0$ cannot be taken directly in the case of magnetic moment, since the latter may depend on terms linear in k in the matrix elements of the vector current [see Eq. (III.16)]. Regardless of what k takes, we can choose to work with the soft-pion limit ($q \to 0$), in which case, as we have seen in Chapter II, only the generalized Born terms survive. This does not mean that the non-Born terms cannot be given by the soft-pion technique. According to Adler and Gilman[18), the terms determined in this way are

$$\bar{V}_1^{(+)} = \frac{g_r(0)\kappa_V}{2M^2}$$
$$\bar{V}_1^{(0)} = \frac{g_r(0)\kappa_S}{2M^2} , \qquad \text{(IV.22)}$$

while the other amplitudes are not determined, since they are associated with the invariants $O_\lambda(V_S)$ linear in q. But these contributions are already of order M^{-2} and, in accordance with the general results of Table 1, the exchange MM (expressed in units of nuclear magneton) corresponding to these terms is of order M^{-3}. Thus we conclude that the NB terms are small.

The question remains, however, as to what happens when q^2 moves away from zero. In Chapter II we have given an argument that for the axial current, an extrapolation to some finite value $q^2 << M^2$ introduces only a small modification; but what about the e.m. current? There is a systematic way of resolving this question as discussed by Furlan et al.[11], and extensively used by M. Ericson[19] in photoproduction of pion on nuclei. We shall not do such analysis for the reason that even extrapolated to $q^2 \sim m_\pi^2$, the NB terms remain small for the isovector e.m. current, and hence an elaborate theory is not warranted. Therefore we shall use models to evaluate them. This is perhaps dangerous for the isoscalar moment for the reason that the NB terms are not small corrections any more. Our analysis for this, therefore, may not be as meaningful as for the isovector case.

In the same way as for the axial vector current, we shall assume that the NB terms can be described by the N^* graphs, and the vector meson (ρ,ω) graphs.

Nucleon isobars

Unlike the β-decay case, coupling constants are not readily available for the phenomenological Lagrangian approach. We shall instead use the celebrated dispersion theoretical method of Chew, Goldberger, Low and Nambu[20] (CGLN) for treating the pion photoproduction amplitudes. We take the static limit for the nucleons and the M1 approximation. The latter amounts to neglecting all partial waves other than the magnetic dipole coming from the nucleon magnetic moment and is expected to take into account the contributions from low-lying even-parity πN resonances with $(2J^P, 2T) \leq (3^+, 3)$.

The procedure goes as follows. We use the expressions connecting the invariant amplitudes V_s to the C.M. amplitudes $\mathcal{F}_s^V$. These are given in Appendix 1 of Adler's paper[8]. We then take the static limit for the nucleon $M \to \infty$, and find

$$V_1 = \frac{1}{\omega}\left[\mathcal{F}_1^V + (\vec{q}\cdot\vec{k})\left(\frac{\mathcal{F}_3^V}{|\vec{q}|\,|\vec{k}|} + \frac{\mathcal{F}_4^V}{|\vec{q}|^2}\right)\right]$$

$$V_2 = -\,\mathcal{F}_4^V/\omega|\vec{q}|$$

$$V_3 = \frac{1}{|\vec{q}|\,|\vec{k}|}\left(\mathcal{F}_2^V + \mathcal{F}_3^V\right) + \frac{\mathcal{F}_4^V}{|\vec{q}|^2} \qquad \text{(IV.23)}$$

$$V_4 = \frac{1}{|\vec{q}|\,|\vec{k}|}\,\mathcal{F}_2^V ,$$

where $|\vec{k}|$ and $|\vec{q}|$ are the C.M. momenta of the current and the pion, respectively, and $\omega = W - M$, where W is the total energy. From partial wave decomposition of the amplitudes $\mathcal{F}_j^V$, keeping only the magnetic dipole M_{1j}, $j = 1/2, 3/2$, we get

$$\mathcal{F}_1^V \simeq 3(\hat{k}\cdot\hat{q})\,M_{1\,3/2} , \qquad \mathcal{F}_2^V \simeq 2M_{1\,3/2} + M_{1\,1/2} ,$$
$$\mathcal{F}_3^V \simeq -3M_{1\,3/2} , \qquad \mathcal{F}_4^V \simeq 0 . \qquad \text{(IV.24)}$$

The amplitudes M_{1j} of course depend upon ω. It is a well-known result of the CGLN theory that the solution for M_{1j} is

$$M_{1j}^{(\pm)} = \frac{\sqrt{4\pi}}{M}\,\frac{\gamma_p - \gamma_n}{2f_{\pi NN}}\,|\vec{k}|\,|\vec{q}|\,h_{1j}^{(\pm)} ,$$

$$f_{\pi NN} \equiv \frac{g_r}{\sqrt{4\pi}}\left(\frac{m_\pi}{2M}\right) , \qquad \text{(IV.25)}$$

where h_{1j} is related to the partial wave amplitude for elastic π-N scattering f_{1j} by

$$h_{1j} = \frac{f_{1j}}{|q|^2} = \frac{e^{i\delta_j}\sin\delta_j}{|q|^3} . \qquad \text{(IV.26)}$$

Here δ_j is the p-wave phase shift for $j = \frac{1}{2}$ or $\frac{3}{2}$. To avoid double-counting, the Born contributions to M_{1j} must be subtracted out. However, the same equation (IV.25) which connects the full M_{1j} and h_j, also connects their Born parts. Thus the NB part of M_{1j} is connected to the NB part of h_j by the same equation.

To state the results for the functions g and h of Eq. (III.5), it is convenient to introduce the following combinations of p-wave scattering amplitudes

$$\begin{aligned} h_1(0) &\equiv (h_{11} - h_{13} - h_{31} + h_{33})\big|_{\omega=0} \\ h_2(0) &\equiv \tfrac{1}{4}(h_{11} + 2h_{13} + 2h_{31} + 4h_{33})\big|_{\omega=0} , \end{aligned} \qquad \text{(IV.27)}$$

where the subscripts on h stand for (2J,2T). We evaluate these quantities at $\omega = 0$ since as $\nu, \nu_B \to 0$, $\omega = \left[2M(\nu - \nu_B) + M^2\right]^{\frac{1}{2}} - M \to 0$. Explicitly we get

$$\begin{aligned} h_1(0) &= \frac{2}{\pi}\int_0^\infty \frac{dp}{\omega_p^2 p^2}\left[\sin^2\delta_{33}(p) + \sin^2\delta_{11}(p)\right] \simeq 0.074/m_\pi^3 \\ h_2(0) &= \frac{2}{\pi}\int_0^\infty \frac{dp}{\omega_p^2 p^2}\left[\sin^2\delta_{33}(p) + \frac{1}{4}\sin^2\delta_{11}(p)\right] \simeq 0.0658/m_\pi^3 , \end{aligned} \qquad \text{(IV.28)}$$

where the numerical values are obtained by putting in the known values for δ_{33} and the Roper[21] parametrization for δ_{11}.

In terms of $h_{2J,2T}$, the desired amplitudes are*)

*) Note that $\bar{V}_1$ is proportional to $1/\omega$ but that since the associated invariant $O_\lambda(V_1)$ is itself proportional to ω (see Table 1), the contribution $O_\lambda(V_1)\bar{V}_1$ to the matrix element of the current is regular in the limit $\omega \to 0$ as it should be. In view of this fact, there may be a contribution to the MM operator associated with $\bar{V}_1$. However, the M1 approximation gives exactly $\bar{V}_1 = 0$.

$$\bar{V}_1^{(\pm)} = 0 \ , \quad \bar{V}_3^{(+)} = \bar{V}_4^{(-)} = \frac{\xi}{3}\left[h_{11} + h_{13} - 2h_{33}\right]\Big|_{\omega=0}$$

$$\bar{V}_3^{(-)} = \frac{\xi}{3}\left[h_{11} - h_{13} - h_{31} + h_{33}\right]\Big|_{\omega=0} \qquad \text{(IV.29)}$$

$$\bar{V}_4^{(+)} = \frac{\xi}{3}\left[h_{11} + 2h_{13} + 2h_{31} + 4h_{33}\right]\Big|_{\omega=0}$$

$$\xi \equiv \frac{m_\pi}{M}\sqrt{4\pi}\,\frac{\gamma_p - \gamma_n}{2f_{\pi NN}} \ .$$

Since $(h_{11} + h_{13} - 2h_{33})\big|_{\omega=0} = 0$, $\bar{V}_3^{(+)} = \bar{V}_4^{(-)} = 0$; so we need the combinations h_1 and h_2 only.

Now from Table 3, the following contributions to the MM operator are obtained

$$g_I = \frac{4}{9}\,\frac{\gamma_p - \gamma_n}{2}\,h_1(0)Y_0(x_\pi)m_\pi^3 \ , \quad g_{II} = -\frac{6}{9}\,\frac{\gamma_p - \gamma_n}{2}\,h_1(0)Y_2(x_\pi)m_\pi^3$$

$$h_I = j_I = -\frac{1}{9}\,\frac{\gamma_p - \gamma_n}{2}\,4\,h_2(0)Y_0(x_\pi)m_\pi^3 \ ,$$

$$h_{II} = j_{II} = -\frac{3}{9}\,\frac{\gamma_p - \gamma_n}{2}\,4\,h_2(0)Y_2(x_\pi)m_\pi^3 \ .$$

Note that we have not considered N^* contributions to the isoscalar amplitudes $V_s^{(0)}$, since the dispersion integral cannot contribute much because of the factor $(\gamma_p + \gamma_n)/2$ and of the fact that only $(\frac{1}{2},\frac{1}{2})$ resonance (Roper) is allowed.

Vector mesons (ρ and ω)

The CGLN theory based on fixed momentum-transfer dispersion relations overlooks a piece of the pion photoproduction amplitude associated with the t-channel singularities. It is known that the neglected part is well approximated by a resonating $J^P = 1^-$ wave in the t-channel, provided the M1-approximation is made and the dispersion integrals are restricted to the 3,3 resonance region.

We shall study these effects in an approach similar to the one developed in the axial case. We assume them to be described by the vector meson exchange processes given by Fig. 6(c). Both ρ and ω are allowed intermediate states and contribute to the matrix elements of the isoscalar and isovector currents, respectively. The $V\pi\gamma$ vertices, where V is ρ or ω, are taken to be

$$\left\langle \pi^n(q)\,|\,J_\lambda^s\,|\,\rho_\mu^m(k')\right\rangle = -i\,\frac{g_{\rho\pi\gamma}}{m_\rho}\,\varepsilon_{\lambda\nu\sigma\mu}\,k'_\nu\,q_\sigma\,\delta^{nm}\,,$$
$$\left\langle \pi^n(q)\,|\,J_\lambda^{Vj}\,|\,\omega_\mu(k')\right\rangle = -i\,\frac{g_{\omega\pi\gamma}}{m_\omega}\,\varepsilon_{\lambda\nu\sigma\mu}\,k'_\nu\,q_\sigma\,\delta^{nj}\,, \tag{IV.31}$$

where $\varepsilon_{\lambda\nu\sigma\mu}$ is the fully antisymmetric tensor ($\varepsilon_{1234} = +1$). These are the only possible gauge-invariant couplings if gradient terms are ignored. In a recent dispersion theory analysis of the nucleon magnetic moments, made by Abarbanel, Callan and Sharp[22], the ρ and ω pole-contributions were included and vertices of the form (IV.31) used. This study, as well as the analysis of the inelastic form factors of the nucleon by Walecka and Zucker[23], give us some clue to the signs of the coupling constants.

The VNN vertices are written as in Eq. (IV.16) for the axial vector current

$$\left\langle N(p_1')\,|\,J_\mu^{\rho j}\,|\,N(p_1)\right\rangle = ig_{\rho NN}\,\bar{u}(p_1')\,\tau_j\left[\gamma_\mu - \frac{\kappa_V}{2M}\,\sigma_{\mu\nu}\,(p_1' - p_1)_\nu\right] u(p_1)$$
$$\left\langle N(p_1')\,|\,J_\mu^{\omega}\,|\,N(p_1)\right\rangle = ig_{\omega NN}\,\bar{u}(p_1')\left[\gamma_\mu - \frac{\kappa_s}{2M}\,\sigma_{\mu\nu}\,(p_1' - p_1)_\nu\right] u(p_1)\,. \tag{IV.32}$$

Note that

$$g_{\rho NN} = 2f_1(0)\,, \qquad \kappa_V = 2M\,\frac{f_2(0)}{f_1(0)}\,,$$

where f_1 and f_2 are the form factors used before [see Eq. (IV.16)].

From radiative widths of the vector mesons $\Gamma(\rho \to \pi\gamma) = 0.5$ MeV and $\Gamma(\omega \to \pi\gamma) = 1.3$ MeV,

$$g_{\rho\pi\gamma} = 0.448\,, \qquad g_{\omega\pi\gamma} = 0.758\,. \tag{IV.33}$$

The one-pole approximation to the e.m. form factors of nucleons gives

$$g_{VNN} = \gamma_V ,$$

where γ_V is the coupling constant in the direct V-γ coupling [see Eq. (IV.18)]. The coupling constants γ_V can be obtained from leptonic decays of the vector mesons or storage ring experiments or photoreactions on hadrons. There is some disagreement among various experiments. We shall use the values deduced from leptonic decays[24],

$$\frac{\gamma_\rho^2}{4\pi} = 0.52 \pm 0.03 , \quad \frac{\gamma_\omega^2}{4\pi} = 3.70 \pm 0.7$$

which give

$$g_{\rho NN} = 2.56 , \quad g_{\omega NN} = 6.82 . \qquad \text{(IV.34)}$$

With the vertices Eqs. (IV.31) and (IV.32), the V amplitudes $V^{(\pm,0)}$ are easily computed:

$$\begin{aligned}
\bar{V}_1^{(0)} &= 2\,\frac{t(g_{\rho\pi\gamma}\ f_{2\rho NN}/m_\rho)}{t + m_\rho^2} \\
\bar{V}_2^{(0)} &= \frac{2(g_{\rho\pi\gamma}\ f_{2\rho NN}/m_\rho)}{t + m_\rho^2} \\
\bar{V}_4^{(0)} &= \frac{2(g_{\rho\pi\gamma}\ g_{\rho NN}/m_\rho)}{t + m_\rho^2} \\
\bar{V}_1^{(+)} &= 2\,\frac{t(g_{\omega\pi\gamma}\ f_{2\omega NN}/m_\omega)}{t + m_\omega^2} \\
\bar{V}_2^{(+)} &= 2\,\frac{(g_{\omega\pi\gamma}\ f_{2\omega NN}/m_\omega)}{t + m_\omega^2} \\
\bar{V}_4^{(+)} &= \frac{2(g_{\omega\pi\gamma}\ g_{\omega NN}/m_\omega)}{t + m_\omega^2} ,
\end{aligned} \qquad \text{(IV.35)}$$

where we use the abbreviations $t = (q - k)^2$, $f_{2\rho NN} = g_{\rho NN}\,(\kappa_V/2M)$, $f_{2\omega NN} = g_{\omega NN}\,(\kappa_s/2M)$. At the symmetry point and to the lowest order in M^{-1}, we have

$$\bar{V}_4^{(0)} = \frac{2g_{\rho NN}\, g_{\rho\pi\gamma}}{m_\rho^3}\,, \qquad \bar{V}_4^{(+)} = \frac{2g_{\omega NN}\, g_{\omega\pi\gamma}}{m_\omega^3}\,. \tag{IV.36}$$

The sign $g_{VNN}\, g_{V\pi\gamma} > 0$ is favoured in various analyses[22,23]. Referring to Table 3, the exchange MM operators coming from the vector mesons are

$$m_I = 2\zeta_\rho \left(\frac{m_\pi}{m_\rho}\right)^3 Y_0(x_\pi)\,, \qquad m_{II} = 6\zeta_\rho \left(\frac{m_\pi}{m_\rho}\right)^3 Y_2(x_\pi)$$

$$h_I = j_I = \zeta_\omega \left(\frac{m_\pi}{m_\omega}\right)^3 Y_0(x_\pi)\,, \qquad h_{II} = j_{II} = 3\zeta_\omega \left(\frac{m_\pi}{m_\omega}\right)^3 Y_2(x_\pi)\,, \tag{IV.37}$$

where

$$\zeta_V = - g_{VNN}\, g_{V\pi\gamma}\, g_r/12\pi\,.$$

Let us emphasize that the results (37) are simple since q^2 is taken to be zero. However, if we were to keep the q^2 dependence, we would have a situation analogous to that discussed near Eq. (IV.21). If one keeps the q^2 dependence then the following modification should be made to Eq. (IV.37):

$$Y_{0,2}(x_\pi) \rightarrow \frac{m_V^2}{m_V^2 - m_\pi^2} \left[Y_{0,2}(x_\pi) - \left(\frac{m_V}{m_\pi}\right)^3 Y_{0,2}(x_V) \right]. \tag{IV.38}$$

1.2 Born contributions

In principle, the Born contributions to the amplitudes are uniquely defined, as it is through the definition of the Born terms as in Figs. 4 and 5 that the non-Born terms are given their precise meaning. However, in the spirit of deriving an equivalent operator which is to be sandwiched between nuclear wave functions, there is a problem of double-counting, which is a unique feature of a many-body system. In as far as the nuclear wave function already contains some parts of meson exchanges through the N-N potential, not all the graphs of Figs. 4 and 5 can be ascribed to the additional meson exchange effects we are talking about. This can be best illustrated in terms of the time-dependent picture. The pieces already included in the wave function are given in Fig. 8, in which emission and reabsorption of pions is completed before or after the interaction of a nucleon with the external field.

Subtracting these graphs from the Born terms, one obtains the legitimate nucleon Born terms of Fig. 9. One notes that in Figs. 9a and 9b a neutron β-decays(for example) to a proton before the pion gets reabsorbed by the spectator nucleon. Figures 9c and 9d correspond to $N\bar{N}$ pair intermediate states. Here of course, the pion can propagate with both positive and negative frequencies and has the Feynman propagator. Figure 9e contributes only to the isovector e.m. current.

Because the pion is present in the intermediate states in Figs. 9a and 9b, an introduction of such terms as corrections would necessitate a normalization correction also. We do not know at this moment how to treat this satisfactorily, the trouble being that our formulation is not a perturbation theory. (It is important to realize that despite the terminology "Born term", our Born terms are not perturbative objects.) In perturbation theory, one may be able to formulate a systematic theory to take into account terms such as those given by Figs. 9a and 9b and corresponding normalization corrections. These questions are discussed in Appendix A from a general point of view. The practical aspects pertinent to our application will be left to the next section.

We divide the Born terms into three groups. Referring to Fig. 9, we shall call the current associated with Fig. 9e a "pionic current", Fig . 9c and 9d a "pair excitation current", and Figs. 9a and 9b a "nucleon recoil current". Both the axial and e.m. currents will be discussed simultaneously. Most of the arguments baving been presented already[25], the discussion will be kept as brief as possible.

1.2.1 Pionic current

All the time-ordered diagrams are allowed here so that the Feynman rules are to be used in getting the matrix element of the exchange current in momentum space. There cannot be any contributions to the Gamow-Teller operator because of the Lorentz invariance, nor to the isoscalar magnetic moment operator because of the G-parity invariance. The isovector matrix element in momentum space is

$$J_{\lambda}^{\pi j} = \frac{-g_r^2}{(2\pi)^3} \delta(\vec{p}_1 + \vec{p}_2 + \vec{k} - \vec{p}_1{}' - \vec{p}_2{}') \frac{1}{(p_1 - p_1')^2 + m_\pi^2} \times$$

$$\times (p_1 - p_1' + p_2' - p_2)_\lambda \frac{1}{(p_2 - p_2')^2 + m_\pi^2}$$

$$\left[\bar{u}(\vec{p}_1{}')\gamma_5 \tau_m u(p_1)\right] i\varepsilon_{jmn} \left[\bar{u}(p_2')\gamma_5 \tau_n u(p_2)\right] . \qquad \text{(IV.39)}$$

After some standard calculations, the two-body MM operator associated with pionic current $\vec{\mathcal{M}}_{12}^{\pi}$ is found to be [in units of nuclear magneton (e/2M)]

$$\vec{\mathcal{M}}_{12}^{\pi} = \frac{-M}{m_\pi} f_{\pi NN}^2 \left[\vec{\tau}(1) \times \vec{\tau}(2)\right]_3 (\vec{\sigma}_1 \cdot \vec{\nabla}_1)(\vec{\sigma}_2 \cdot \vec{\nabla}_2) \times$$

$$\left[(\vec{r} \times \vec{R})\, Y_0(x_\pi)\right] , \qquad \text{(IV.40)}$$

where

$$f_{\pi NN}^2 = \frac{1}{4\pi} \left(g_r \frac{m_\pi}{2M}\right)^2 .$$

Carrying out the differentiation indicated by $\vec{\nabla}_1$ and $\vec{\nabla}_2$ in Eq. (IV.40), we find among other terms the well-known Sachs' space-exchange moment:

$$\vec{\mathcal{M}}_{\text{Sachs}} = \frac{M}{3m_\pi} f_{\pi NN}^2 \left[\tau(1) \times \tau(2)\right]_3 (\vec{r} \times \vec{R}) \times$$

$$\times \left[(\vec{\sigma}_1 \cdot \vec{\sigma}_2) Y_0(x_\pi) + S_{12} Y_2(x_\pi)\right] ,$$

$$S_{12} \equiv 3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2) , \qquad \text{(IV.41)}$$

which can be identified with the 4th term in Eq. (III.8); i.e.,

$$F_{IV} = \frac{1}{i}\frac{M}{3m_\pi} f^2_{\pi NN} |\vec{r}||\vec{R}| \left[\tau(1) \times \tau(3)\right]_3 \times$$

$$\times \left[(\vec{\sigma}_1 \cdot \vec{\sigma}_2)Y_0(x_\pi) + S_{12}Y_2(x_\pi)\right] . \qquad (IV.42)$$

In addition to $\vec{m}_{Sachs}$, we have other operators which can be written in terms of the classifications (III.8) and (III.5),

$$g_I = -\frac{2}{3}\frac{M}{m_\pi} f^2_{\pi NN} (2 - x_\pi)Y_0(x_\pi) , \quad g_{II} = -\frac{M}{m_\pi} f^2_{\pi NN} (1 + x_\pi)Y_0(x_\pi) ,$$

$$F_{III} = - f^2_{\pi NN} \left[\vec{\tau}(1) \times \vec{\tau}(2)\right] MR\, Y_1(x_\pi) , \qquad (IV.43)$$

$$Y_1(x) \equiv (1 + x)\, Y_0(x) = (1 + x)\, \frac{e^{-x}}{x} .$$

1.2.2 Pair excitation current

The relevant diagrams include all possible time-orderings for the πNN vertex, but the current is allowed to create or annihilate a nucleon-antinucleon pair only. Thus in writing down the contributions to the exchange current, we can still make use of the Feynman rules except that in the nucleon propagator $(i\gamma \cdot Q + M)^{-1}$ only the part which propagates with negative frequency must be kept. This part is the second term in

$$(i\gamma \cdot Q + M)^{-1} = -\frac{1}{2E_{\vec{Q}}} \left\{ \frac{\gamma_4 E_{\vec{Q}} - i\vec{\gamma} \cdot \vec{Q} + M}{Q_0 - E_{\vec{Q}} + i\varepsilon} - \frac{-\gamma_4 E_Q - i\vec{\gamma} \cdot \vec{Q} + M}{Q_0 + E_{\vec{Q}} - i\varepsilon} \right\} .$$

This term contributes to both the axial and vector current. The contribution to the isovector e.m. vertex leads to the large seagull term which is just Fig. 5a*),

$$\left\langle \pi^n(q)N(p_1')\,|J_\ell^{Vj}|\,N(p_1)\right\rangle_{\text{pair}} \rightarrow -\frac{g_r}{2M}\,\varepsilon_{njk}\,\tau_k\,\sigma_\ell\,, \qquad \ell = 1,2,3 \qquad \text{(IV.44)}$$

whereas there is no contribution to the isoscalar vertex to the order 1/M. On the other hand, due to the fact that the axial vector current of the nucleons has an odd Dirac operator, we have to go to one more power of M^{-1}, with the result

$$\left\langle \pi^n(q)N(p_1')\,|J_\ell^{Aj}|\,N(p_1)\right\rangle_{\text{pair}} \rightarrow -g_A\frac{g_r}{2M^2}\times$$

$$\left\{a_{nj}^{(-)}(\vec{\sigma}\times\vec{q})_\ell + a_{nj}^{(+)}\left[\vec{\sigma}\times(\vec{p}_1+\vec{p}_1{}')\right]_\ell\right\}. \qquad \text{(IV.45)}$$

We shall keep this, since terms of that order also occur in the NB parts. It should be clear to everyone at this point that the exchange currents must be basically different between the β-decay and the MM. The notion which is still invoked, that one can determine one matrix element from another independently of exchange corrections, is clearly erroneous.

The pair contributions expressed in terms of our classification are:

Axial current

$$g_I = 2h_I^\sigma = \frac{2}{3}\frac{m_\pi}{M}f_{\pi NN}^2\,Y_0(x_\pi)$$

$$g_{II} = 2h_{II}^\sigma = -\frac{2}{3}\,j_{III} = -\frac{m_\pi}{M}f_{\pi NN}^2\,Y_2(x_\pi)\,. \qquad \text{(IV.46)}$$

Vector current

$$g_I = -\frac{2}{3}\,g_{II} = \frac{2}{3}\frac{M}{m_\pi}f_{\pi NN}^2\,Y_1(x_\pi)$$

$$F_{III} = f_{\pi NN}^2\left[\tau(1)\times\tau(2)\right]_3 MR\,Y_1(x_\pi)\,. \qquad \text{(IV.47)}$$

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*) The equivalence between the seagull term (sometimes called Kroll-Ruderman term) and the pair term in the NR limit can be checked explicitly by noting that the negative frequency propagator becomes $-\left[(\beta-1)+\beta\vec{\alpha}\cdot\vec{Q}/M\right]/4M \rightarrow -(\beta-1)/4M$ to the lowest order in M^{-1}.

Comparing with Eq. (IV.43), we see that the sum of R-dependent terms from the pionic and pair currents vanishes, i.e. $F^{\pi}_{III} + F^{pair}_{III} = 0$.

1.2.3 Nucleon recoil current

The graphs to be considered are the time-ordered ones given by Fig. 8. These contribute in the adiabatic limit, since the nucleon recoils due to the emission or absorption of the exchanged virtual pion. The matrix element of the exchange current can be calculated in the non-covariant perturbation theory, which we do not reproduce here. We simply quote the results.

Axial current

$$h_I = j_I = -\frac{1}{4} g_I = -\frac{1}{3\pi} f^2_{\pi NN} \left[K_0(x_\pi) - \frac{K_1(x_\pi)}{x_\pi} \right]$$

$$h_{II} = j_{II} = \frac{1}{2} g_{II} = -\frac{1}{\pi} f^2_{\pi NN} K_2(x_\pi) \qquad \text{(IV.48)}$$

Vector current

$$m_I = 2j_I = 2h_I = -\frac{1}{2} g_I = -\frac{2}{3\pi} f^2_{\pi NN} \left[K_0(x_\pi) - \frac{K_1(x_\pi)}{x_\pi} \right]$$

$$m_{II} = 2j_{II} = 2h_{II} = -2g_{II} = -\frac{2}{\pi} f^2_{\pi NN} K_2(x_\pi) , \qquad \text{(IV.49)}$$

where $K_\ell(x_\pi)$ are the Bessel functions of the second kind.

2. HEAVY-MESON EXCHANGE CONTRIBUTIONS

It is a formidable task to calculate systematically the exchanges of more than one pion. Such calculations have been performed for the meson theoretic N-N potential with reasonable success. They were, however, limited to two-pion exchanges. The two-pion exchange graphs are usually supplemented by exchanges of resonances such as ρ,ω. Apart from the complexity, clearly there would be a need for care because of the double counting.

There have been calculations in fourth-order perturbation theory for the GT [26)] and MM [27)] exchange operators, but the reliability of such calculations is not clear. In our approach, we shall neglect the uncorrelated multipion exchanges, but assume as in nuclear force calculations that the correlated part of multipion exchanges is important. Thus we shall consider only the ω and ρ exchanges*) and use the vector meson dominance model for their evaluations. The results are given in Table 5.

A strict adherence to the procedures used in the calculation of nuclear potentials would require that VNN coupling constants and m_V be determined phenomenologically by fitting the scattering data etc. We have already touched upon this question in connection with the pionic form factor of nucleons. In view of rather large discrepancies between parameters used by different people, we shall not do the (in principle) correct procedure. We shall just use the coupling constants already determined in the previous subsection.

Even restricted to the vector meson exchanges, a complete calculation is difficult because of the vector nature of the particles. For the nucleon-recoil graphs (equivalent to Figs. 9a and 9b with the pion replaced by the vector mesons), the time-time components of the vector meson vertices give the leading terms, while those coming from the space components are of order M^{-2} relative to the former. However, our calculation of other contributions in the MM case is not complete for the following reasons. We have disregarded the mesonic-current processes in which the currents interact with the intermediate mesons (i.e. graphs equivalent to Fig. 9c with π replaced by the vector mesons), partly because of the complicated structure of the radiative vertices and partly because the relevant form factors (associated with monopole, dipole, and quadrupole interactions) are not at

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*) We have also looked into the η-meson exchange graphs but found them to have unimportant effects. They have essentially the same radial dependence as the OPE diagrams but do not compete significantly with them because of the smaller ηNN coupling constant [exact SU(3) with a ratio f/d = 0.6 gives $g_{\eta NN}/g_{\pi NN} \simeq 0.3$] and the shorter range. They also turn out to be small compared to the vector-meson exchange diagrams. The role that the η plays in meson-exchange currents appears therefore to be very similar to the one it plays in the NN potentials.

present available. It is worth while to note here that a part of the contributions from these processes must be related to the NN potentials due to the vector meson exchanges in the same way as the Sachs moment [Eq. (IV.41)] is related to the OPEP. For the pair excitation graphs in the MM case, both the time-time and space-space vertices of the vector mesons give contributions of the same order. The results given in Table 5 refer only to the latter, where only the spin currents were taken (neglecting the convection current part).

V. APPLICATIONS TO ^{3}H AND ^{3}He

The isospin doublet ^{3}H and ^{3}He is the simplest nuclear system with all the many-body characteristics, such as off-shell effects, etc. Next to the deuteron, its wave function is the best known, and it has a complete set of experiments we are interested in -- i.e., magnetic moments and β-decay. Deuteron possesses only an isoscalar moment, which is too tiny and cannot make a bound-to-bound β-decay transition, making a clear-cut calculation unfeasible. Thus the trinucleon system is better suited for our purpose. Historically, the anomaly in its magnetic moment was one of the first pieces of evidence of meson-exchange effects[28].

Before going into details of calculation, let us review both theoretical and experimental situations.

1. MAGNETIC MOMENT AND GAMOW-TELLER MATRIX ELEMENTS

1.1 One-body operators

$$\text{MM:} \quad \mu_i = \frac{\gamma_p + \gamma_n}{2}\vec{\sigma}_i + \frac{\gamma_p - \gamma_n}{2}\vec{\sigma}_i \tau_3(i) + \text{orbital}$$

$$\text{GT:} \quad H_+^{(1)} = -g_A\, \vec{\sigma}_i \tau^{(+)}(i) \qquad \text{(V.1)}$$

where

$$(\gamma_p \pm \gamma_n)/2 = \begin{pmatrix} 0.440 \\ 2.353 \end{pmatrix} .$$

The classification of allowed states: ($J^P = \frac{1}{2}^+$, $T = \frac{1}{2}$) of three nucleons gives 10 distinct states[29] corresponding to the spectroscopic terms: three $^2S_{\frac{1}{2}}$, $^2P_{\frac{1}{2}}$ and $^4D_{\frac{1}{2}}$ and one $^4P_{\frac{1}{2}}$. If we assume that of all these only the S (full space-symmetry), S′ (mixed space-symmetry), D and T = 3/2 (mixed space-symmetry) states are significant which seems to be borne out by most recent analyses[30], then the GT matrix elements and the MM are given in terms of the state probabilities $P_S\cdots$ and the amplitudes $A_S\cdots$ by

$$M_A^{(1)} = \frac{|\langle {}^3\mathrm{He}||H_+^{(1)}||{}^3\mathrm{H}\rangle|}{\sqrt{2}} = \sqrt{3}\left[P_S - \frac{1}{3}P_{S'} + \frac{1}{3}P_D + \frac{2}{3}P_{3/2} + \frac{2}{3}A_{S'}A_{3/2}\right] ,$$

$$\mu_S^{(1)} = \frac{\gamma_p + \gamma_n}{2}\left(P_S + P_{S'} - P_D + \frac{1}{2}P_{3/2}\right) + \frac{1}{2}P_D , \qquad \text{(V.2)}$$

$$\mu_V^{(1)} = \frac{\gamma_p - \gamma_n}{2}\left(P_S - \frac{1}{3}P_S' + \frac{1}{3}P_D + \frac{1}{6}P_{3/2}\right) - \frac{1}{6}P_D ,$$

where all contributions are independent of the radial forms chosen for the states*). The last terms in $\mu_{S,V}^{(1)}$ are the small contributions from the orbital magnetic moment. The isoscalar and isovector moments (μ_S and μ_V) are related to ^{3}H and ^{3}He magnetic moments by

$$\mu_S = \frac{\mu(^3\mathrm{H}) + \mu(^3\mathrm{He})}{2}$$
$$\mu_V = \frac{\mu(^3\mathrm{H}) - \mu(^3\mathrm{He})}{2} \qquad \text{(V.3)}$$

so that

$$\mu = \mu_S - (2t)\mu_V; \qquad 2t = +1(-1) \quad \text{for} \quad {}^3\mathrm{He}\ ({}^3\mathrm{H}) .$$

1.2 Two-body operators

In evaluating the exchange current correction, we shall not use the full wave function, but retain only the symmetric S-state. This brings with it a considerable simplification in the calculation.

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*) Similar radial forms were assumed for $T = 3/2$ and S' states. Note that breakdown of mirror symmetry which may be due to the $T = 3/2$ state or to different state probabilities in ^{3}H and ^{3}He has only a slight effect. We find, for example, that a 10% difference in P_S between ^{3}H and ^{3}He results in a change in μ_V and μ_S by only 1% and 20%, respectively.

The spin-isospin part of the S-state wave function is given by

$$\Psi^{mt} = \frac{1}{\sqrt{2}} (\phi^m \bar{\eta}^t - \bar{\phi}^m \eta^t) ,$$

$$\phi^m = \chi_0(1,2)\chi_m(3) , \qquad (V.4)$$

$$\bar{\phi}^m = \frac{1}{\sqrt{12}}(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \vec{\sigma}_3 \, \phi^m ,$$

where m,t are the z-components of the total spin and isospin, respectively. A more explicit form for $\phi^{m=\frac{1}{2}}$ and $\bar{\phi}^{m=\frac{1}{2}}$ is convenient for calculation;

$$\phi^{m=\frac{1}{2}} = \frac{1}{\sqrt{2}} \left[(+-+) - (-++)\right]$$

$$\bar{\phi}^{m=\frac{1}{2}} = \sqrt{\frac{2}{3}} \left[\frac{(-++) + (+-+)}{2} - (++-)\right] \qquad (V.5)$$

and similarly for $\eta^{t=\frac{1}{2}}$ and $\bar{\eta}^{t=\frac{1}{2}}$, where (+) and (-) represent spin up and down for ϕ and proton and neutron for η. The simplifications that Eq. (V.4) brings are that because the S-state is fully space-symmetric any operator odd under interchange of space coordinates has vanishing expectation values; thus the C.M. dependent operators [Eq. (III.8)] do not contribute. Moreover, tensor operators have vanishing matrix elements. Note that the vertex corrections with an intermediate state N^*_{3+3} cannot contribute in fully space-symmetric states by virtue of spin-isospin selection rules.

Unlike the one-body operators of Eq. (V.1), the matrix elements of the exchange current operators depend on radial wave functions. There are various forms involving the coordinates $\vec{r}_1,\vec{r}_2,\vec{r}_3$ which satisfy the symmetry. We shall work exclusively with the analytically simplest Gaussian form exp (-const $\{r_{12}^2 + r_{13}^2 + r_{23}^2\}$) where $r_{ij} \equiv |\vec{r}_i - \vec{r}_j|$. The fact that some of the Born terms and also all heavy meson exchanges are very sensitive to the part of wave functions where r_{ij} is small requires that the short-range correlation be handled with care. Again for simplicity, we shall use the Jastrow type correlation function $\Pi_{i<j} (1 - e^{-\gamma^2 r_{ij}^2})^{\frac{1}{2}}$ to simulate the short-distance behaviour. For

completeness, in Appendix B we examine what happens when a correlation function of different nature used frequently in nuclear matter calculation is used. Although this cannot be a quantitative assessment of what we do below, it gives an idea as to how different correlation functions behave at short distances. The complete form of our radial function is then

$$f(r_{12},r_{13},r_{23}) = N\, e^{-\alpha^2(r_{12}^2+r_{23}^2+r_{13}^2)/2} \prod_{i<j} (1 - e^{-\gamma^2 r_{ij}^2})^{\frac{1}{2}} , \quad (V.6)$$

where the range parameter α takes the value 0.384 fm^{-1} when fitted to Coulomb energy and is taken in our calculation to be 0.337, 0.384 (C.E.), and 0.440 fm^{-1}. The parameter γ is interpreted as indicative of repulsive core radius r_c, the rough relation being $r_c \approx (\gamma\sqrt{2})^{-1}$. The values used in the calculation are

r_c = 0.0	0.2	0.4	0.6	0.8	fm	(V.7)
γ = ∞	3.536	1.768	1.179	0.884	fm^{-1}	

We can now write down the non-vanishing matrix elements with the wave function Eq. (V.4), and these are given in Table 6. They comprise all matrix elements needed, since operators like $[\vec{\tau}(1) \times \vec{\tau}(2)]_j\ (\vec{\sigma}_1 \times \vec{\sigma}_2)$ reduce to $[\vec{\tau}(1) - \vec{\tau}(2)]_j\ (\vec{\sigma}_1 - \vec{\sigma}_2)$ in S-state, and all others vanish. Using Table 6, and Eqs. (III.5) and (III.6), we have

$$\mu_V^{(2)} = 2\left[\langle g_I^M\rangle + \langle h_I^M\rangle\right]$$

$$\mu_S^{(2)} = \langle -k_I + \ell_I - 3m_I\rangle \quad (V.8)$$

$$\delta_A^{(2)} \equiv \frac{\langle {}^3He||H_+^{(2)}||{}^3H\rangle}{\langle {}^3He||H_+^{(1)}||{}^3H\rangle} = 4\langle g_I^\beta + h_I^\beta\rangle$$

where by definition the one-body matrix element in $\delta_A^{(2)}$ is evaluated with the S state only. We use

$$\langle g_I\rangle = N^2 \int |f(r_{12},r_{23},r_{13})|^2\, g_I(r_{12})\, d\vec{r}_1\, d\vec{r}_2\, d\vec{r}_3$$

and superscripts M and β for magnetic moment and β-decay. The functions g, h, ..., are given in terms of amplitudes V and A in Tables 2 and 3 for the case of NB term, and in Chapter IV for other contributions. In order to compute $\mu_{V,S}$ and δ^A, it suffices to collect all contributions to g,h, etc. For example,

$$g_I = (g_I)_{OPE} + (g_I)_{HME} = (g_I)^{Born}_{OPE} + (g_I)^{NB}_{OPE} + (g_I)_{HME} \quad (V.9)$$

and likewise for the others.

1.3 Wave function normalization correction

As we have discussed in Chapter IV, there is a correction quite distinctive from the two-body operators we have been considering and any other one-body correction such as relativistic corrections etc. -- it is the normalization in the wave function due to an explicit dependence on meson degrees of freedom. For the trinucleon system, it adds the following corrections,

$$\begin{aligned} \mu^{norm}_{S,V} &= \mu^{(1)}_{S,V} \langle Z \rangle \\ \delta^{norm} &= \langle Z \rangle \end{aligned} \quad (V.10)$$

where

$$Z = \frac{2}{3} f^2_{\pi NN} \sum_{i<k} (\vec{\tau}_i \cdot \vec{\tau}_k) \left\{ (\vec{\sigma}_i \cdot \vec{\sigma}_k) \left[K_0(x_\pi) - \frac{K_1(x_\pi)}{x_\pi} \right] + S_{ik} K_2(x_\pi) \right\}$$

and $\langle Z \rangle$ means an expectation value which is independent of the spin and isospin projections. Since

$$\left\langle \psi^{mt} \right| \sum_{i<k} (\vec{\tau}_i \cdot \vec{\tau}_k)(\vec{\sigma}_i \cdot \vec{\sigma}_k) \left| \psi^{mt} \right\rangle = -9$$

we find

$$\mu_{S,V}^{norm} = \mu_{S,V}^{(1)} \left[- \frac{6}{\pi} f_{\pi NN}^{2} \left\langle K_0(x_\pi) - \frac{K_1(x_\pi)}{x_\pi} \right\rangle \right]$$

$$\delta^{norm} = - \frac{6}{\pi} f_{\pi NN}^{2} \left\langle K_0(x_\pi) - \frac{K_1(x_\pi)}{x_\pi} \right\rangle . \qquad (V.11)$$

There are two points we must emphasize here. Firstly we are only considering the normalization correction coming from the pion exchanges whereas vector meson exchanges are also included in our calculation. In principle, we could do a similar calculation for the exchanges of other mesons, but it is numerically small and perhaps the meaning of such a calculation is not clear because of the difficulty of extending renormalization procedure to two-particle systems. We find it more appropriate in numerical calculations to lump the normalization correction into the OPE Born term. The second point is that practical calculation of $\langle Z \rangle$ gives a positive result instead of negative as the interpretation of a normalization would suggest. In other words, it enhances rather than reduces the matrix elements.

We also note here that this normalization correction cancels only partially the nucleon recoil term. The exact cancellation occurs only in the extreme case of the neutral scalar meson theory.

2. EXPERIMENTAL SITUATIONS

The magnetic moments are known very accurately and are given by

$$\mu(^3H) = 2.97893 , \quad \mu(^3He) = -2.12815 . \qquad (V.12)$$

Or in terms of the isoscalar and isovector moments,

$$\mu_S^{exp} = 0.426 , \quad \mu_V^{exp} = +2.553 . \qquad (V.13)$$

The discrepancies between the experimental values [Eq. (V.13)] and the single-particle values [Eq. (V.2)] as a function of various P's are given in Table 7, where $\delta\mu$ denotes

$$\delta\mu_{S,V} = \mu^{exp}_{S,V} - \mu^{(1)}_{S,V} \tag{V.14}$$

The experimental situation for M_A is quite controversial, because of the various experimental quantities involved in its extraction. This is seen in the formula

$$|M_A^{exp}|_{^3H\to{}^3He} = 3\left[\frac{2(ft)_{^{14}O\to{}^{14}N} - (ft)_{^3H\to{}^3He}}{2(ft)_{^{14}O\to{}^{14}N} - (ft)_{n\to p}}\right]\frac{(ft)_{n\to p}}{(ft)_{^3H\to{}^3He}} \quad , \tag{V.15}$$

which shows that, besides information on $(ft)_{^3H\to{}^3He}$, the super-allowed decay $^{14}O \to {}^{14}N$ needs to be used to determine G_V (Fermi constant) and neutron beta-decay to determine $g_A \equiv G_A/G_V$. For all these decays, uncertainties are also introduced by radiative and Coulomb corrections. The former correction, though small, is not unambiguously known. The major uncertainties lie however in ft-values for n and ^3H decays. Previous determinations based on the neutron lifetime measurement by Sosnovsky et al.[31], $(t_{\frac{1}{2}})_n$ = (11.7 ± 0.3) min, gave $|M_A|^2$ ranging between 3.30 and 3.17. The more recent value reported by Christensen et al.[32], $(t_{\frac{1}{2}})_n$ = (10.80 ± 0.16) min, decreases $|M_A|^2$ down to 2.88. All these estimates are done with the same $(ft)_{^3H\to{}^3He}$, namely the value quoted by Goldhaber[33], 1132 ± 90, which is close to Porter's[34] 1137 ± 20.

In recent re-examinations of the triton beta-spectrum by Salgo and Staub[35] and by Bergkvist[35], the end-point of the spectrum was found to extend farther to the right than in the older data. This has the effect of increasing the ft values and therefore of reducing still further $|M_A|^2$. Using almost similar inputs they find

$$(ft)_{^3H} = (1159 \pm 11)\ \text{sec}\ , \quad |M_A|^2 = 2.84 \pm 0.06 \quad \text{Salgo-Staub}$$
$$(ft)_{^3H} = (1143 \pm 3)\ \text{sec}\ , \quad |M_A|^2 = 2.94 \pm 0.05 \quad \text{Bergkvist}\ . \tag{V.16}$$

A summary of the early and recent status of this important question is given in Table 8. The discrepancy between the value of Salgo and Staub and the single-particle operator value is tabulated as a function of P's in Table 7.

We also give in Table 9 the variations in δ(Gibson) $\equiv$ $(|M_A|_{exp} - |M_A^{(1)}|_{Gibson})/ M_{A\,Gibson}^{(1)}$ and similarly in δ(Blatt-Delves) corresponding to four values of $|M_A|_{exp}$ given in Table 8 and probability amplitudes listed in Table 7.

3. NUMERICAL RESULTS

We now write down the necessary formulae explicitly and evaluate them:

$$\mu_V^{(2)} = \mu_{V,\pi}^{pionic} + \mu_{V,\pi}^{pair} + \mu_{V,\pi}^{recoil} + \mu_{V,\pi}^{norm} + \mu_{V,\pi}^{NB}$$

$$+ \mu_{V,\rho}^{pair} + \mu_{V,\rho}^{recoil} + \mu_{V,\omega}^{pair} + \mu_{V,\omega}^{recoil} \tag{V.17}$$

$$\mu_S^{(2)} = \mu_{S,\pi}^{recoil} + \mu_{S,\pi}^{norm} + \mu_{S,\pi}^{NB} + \mu_{S,\rho}^{pair} + \mu_{S,\rho}^{recoil} + \mu_{S,\omega}^{pair} + \mu_{S,\omega}^{recoil}$$

$$\delta_A^{(2)} = \delta_{A,\pi}^{pair} + \delta_{A,\pi}^{recoil} + \delta_{A,\pi}^{norm} + \delta_{A,\pi}^{NB} + \delta_{A,\rho}^{recoil} + \delta_{A,\omega}^{recoil} ,$$

where subscripts π,ρ,ω represent the mesons exchanged, the superscripts the kinds of terms: pionic current, pair excitation current, normalization correction, Non-Born (NB) term, nucleon recoil term etc.,

$$\mu_{V,\pi}^{pionic} + \mu_{V,\pi}^{pair} = \frac{4}{3}\frac{M}{m_\pi} f_{\pi NN}^2 \left\langle (2x_\pi - 1)Y_0(x_\pi)\right\rangle = 0.7257 \left\langle (2x_\pi - 1)Y_0(x_\pi)\right\rangle$$

$$\mu_{V,\pi}^{recoil} = \frac{2}{\pi} f_{\pi NN}^2 \left\langle K_0(x_\pi) - \frac{K_1(x_\pi)}{x_\pi}\right\rangle = 0.05093 \left\langle K_0(x_\pi) - \frac{K_1(x_\pi)}{x_\pi}\right\rangle$$

$$\mu_{V,\pi}^{norm} = -\frac{6}{\pi} f_{\pi NN}^2 \left\langle \mu_V^{(1)}\right\rangle_S \left\langle K_0(x_\pi) - \frac{K_1(x_\pi)}{x_\pi}\right\rangle =$$

$$= -0.3595 \left\langle K_0(x_\pi) - \frac{K_1(x_\pi)}{x_\pi}\right\rangle$$

$$\mu^{NB}_{V,\pi} = \frac{8}{9}\left(\frac{1+\kappa_V}{2}\right)\left[\frac{3}{4}(h_{11} - 4h_{13})\right]\left\langle Y_0(x_\pi)\right\rangle -$$

$$- \frac{g_{\omega NN}\, g_{\omega\pi\gamma}\, g_{\pi NN}}{6\pi}\left(\frac{m_\pi}{m_\omega}\right)^3 \left\langle Y_0(x_\pi)\right\rangle = -0.00336 \left\langle Y_0(x_\pi)\right\rangle$$

$$\mu^{pair}_{V,\rho} = -\frac{2M}{m_\rho} f^2_{\rho NN}(1+\kappa_V)\left\langle Y_1(x_\rho)\right\rangle = -0.9755 \left\langle Y_1(x_\rho)\right\rangle$$

$$\mu^{recoil}_{V,\rho} = 4\left(\frac{g_{\rho NN}}{4\pi}\right)^2 \left\langle (1+2\kappa_V)K_0(x_\rho) - \frac{1}{6}x_\rho K_1(x_\rho)\right\rangle = 1.394 \left\langle K_0(x_\rho)\right\rangle -$$

$$-0.02767 \left\langle x_\rho K_1(x_\rho)\right\rangle$$

$$\mu^{pair}_{V,\omega} = \frac{2}{3}\frac{M}{m_\omega} f^2_{\omega NN}(1+\kappa_S)\left\langle Y_1(x_\omega)\right\rangle = 0.4523 \left\langle Y_1(x_\omega)\right\rangle$$

$$\mu^{recoil}_{V,\omega} = -4\left(\frac{g_{\omega NN}}{4\pi}\right)^2 \left\langle (1+2\kappa_V)K_0(x_\omega) - \frac{1}{6}x_\omega K_1(x_\omega)\right\rangle =$$

$$- \left[9.897 \left\langle K_0(x_\omega)\right\rangle - 0.1964 \left\langle x_\omega K_1(x_\omega)\right\rangle\right]$$

$$\mu^{recoil}_{S,\pi} = \frac{2}{\pi} f^2_{\pi NN} \left\langle K_0(x_\pi) - K_1(x_\pi)/x_\pi\right\rangle = 0.05093 \left\langle K_0(x_\pi) - K_1(x_\pi)/x_\pi\right\rangle$$

$$\mu^{norm}_{S,\pi} = -\frac{6}{\pi} f^2_{\pi NN} \left\langle \mu^{(1)}_S\right\rangle_S \left\langle K_0(x_\pi) - K_1(x_\pi)/x_\pi\right\rangle =$$

$$- 0.06723 \left\langle K_0(x_\pi) - K_1(x_\pi)/x_\pi\right\rangle$$

$$\mu^{NB}_{S,\pi} = \frac{g_{\rho NN}\, g_{\rho\pi\gamma}\, g_{\pi NN}}{2\pi}\left(\frac{m_\pi}{m_\rho}\right)^3 \left\langle Y_0(x_\pi)\right\rangle = 0.01551 \left\langle Y_0(x_\pi)\right\rangle$$

$$\mu^{\text{pair}}_{S,\rho} = 2\,\frac{M}{m_\rho}\,f^2_{\rho NN}\,(1+\kappa_V)\,\langle Y_1(x_\rho)\rangle = 0.9755\,\langle Y_1(x_\rho)\rangle$$

$$\mu^{\text{recoil}}_{S,\rho} = 12\left(\frac{g_{\rho NN}}{4\pi}\right)^2\left\langle\left[(1+2\kappa_S)K_0(x_\rho) - \frac{1}{6}\,x_\rho\,K_1(x_\rho)\right]\right\rangle =$$

$$= 0.3785\,\langle K_0(x_\rho)\rangle - 0.08300\,\langle x_\rho\,K_1(x_\rho)\rangle$$

$$\mu^{\text{pair}}_{S,\omega} = -\frac{2}{3}\,\frac{M}{m_\omega}\,f^2_{\omega NN}\,(1+\kappa_S)\,\langle Y_1(x_\omega)\rangle = -0.4523\,\langle Y_1(x_\omega)\rangle$$

$$\mu^{\text{recoil}}_{S,\omega} = -4\left(\frac{g_{\omega NN}}{4\pi}\right)^2\left\langle\left[(1+2\kappa_S)K_0(x_\omega) - \frac{1}{6}\,x_\omega\,K_1(x_\omega)\right]\right\rangle =$$

$$= -0.8954\,\langle K_0(x_\omega)\rangle + 0.1964\,\langle x_\omega\,K_1(x_\omega)\rangle$$

$$\delta^{\text{pair}}_{A,\pi} = \frac{8}{3}\,\frac{m_\pi}{M}\,f^2_{\pi NN}\,\langle Y_0(x_\pi)\rangle = 0.03136\,\langle Y_0(x_\pi)\rangle$$

$$\delta^{\text{recoil}}_{A,\pi} = \frac{4}{\pi}\,f^2_{\pi NN}\,\langle K_0(x_\pi) - K_1(x_\pi)/x_\pi\rangle = 0.1019\,\langle K_0(x_\pi) - K_1(x_\pi)/x_\pi\rangle$$

$$\delta^{\text{norm}}_{A,\pi} = -\frac{6}{\pi}\,f^2_{\pi NN}\,\langle K_0(x_\pi) - K_1(x_\pi)/x_\pi\rangle = -0.1528\,\langle K_0(x_\pi) - K_1(x_\pi)/x_\pi\rangle$$

$$\delta^{\text{NB}}_{A,\pi} = \frac{1}{6\pi}\,\frac{g_{\pi NN}}{g_A}\,\frac{m_\pi}{M}\,m^2_\pi\left[2\bar{A}^{(-)}_1 - \bar{A}^{(-)}_4 - \tfrac{1}{2}\bar{A}^{(+)}_3\right]\langle Y_0(x_\pi)\rangle =$$

$$= -0.0144\,\langle Y_0(x_\pi)\rangle \;\; ^{*)}$$

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*) This formula was quoted erroneously in Eq. (5) of our previous paper[1)] and elsewhere[37)] with a misprint of a factor of two and a sign mistake inside the brackets. Due to the sign mistake the conclusions drawn there about the NB contributions are not correct.

$$\delta^{\text{recoil}}_{A,\rho} = 4\left(\frac{g_{\rho NN}}{2\pi}\right)^2 \left\langle K_0(x_\rho)\right\rangle = 0.6640 \left\langle K_0(x_\rho)\right\rangle$$

$$\delta^{\text{recoil}}_{A,\omega} = -4\left(\frac{g_{\omega NN}}{2\pi}\right)^2 \left\langle K_0(x_\omega)\right\rangle = -4.713 \left\langle K_0(x_\omega)\right\rangle \quad , \qquad \text{(V.18)}$$

where the numerical values are obtained from coupling constants listed in Table 10. Other quantities needed are the nucleon mass M = 938.9 MeV, κ_V = 3.70, κ_S = −0.12, the matrix elements $\left\langle \mu_V^{(1)}\right\rangle_S$ = 2.353, $\left\langle \mu_S^{(1)}\right\rangle_S$ = 0.440. We have taken positive sign for $g_{\rho\pi\gamma}\, g_{\rho NN}$ and $g_{\omega\pi\gamma}\, g_{\omega NN}$ following Adler and Drell[38], and Abarbanel et al.[22]. We have also used $\frac{3}{4}[h_{11} - 4h_{13}] = 0.8197 \times 10^{-2}/m_\pi^3$ obtained by using the Roper resonance parametrization[21].

The results for $\mu^{(2)}_{V,S}$ and $\delta^{(2)}_A$ are given in Table 11 in terms of the range parameter α [Eq. (V.6)], the hard core radius r_c [Eq. (V.7)], and individually for the OPE and HME contributions. For reasonable values of α and r_c, α = 0.384 and $r_c \approx 0.4$, our results are

$$\mu_V^{(2)} = 0.193 \left({}^{+0.041}_{-0.041}\right)$$

$$\mu_S^{(2)} = 0.0093 \left({}^{+0.0077}_{-0.0053}\right) \qquad \text{(V.19)}$$

$$\delta_A^{(2)} = 0.053 \left({}^{+0.18}_{-0.043}\right) \% ,$$

where we have indicated in parenthesis the range of values they can take when α and r_c are varied. We may summarize the results in the following way.

$\mu_V^{(2)}$: The low-energy theorem implied that the pion-exchange contribution is dominated by the Born term. This is confirmed explicitly. Even taking some sizeable errors in the method of calculation, the NB term is definitely negligible, even less than ½% of the Born term. The heavy meson exchange is small (about 10% of the Born term), but needs to be treated carefully since it subtracts. As one can see from Eq. (V.18) there is a large cancellation between ρ and ω terms; thus the result could be quite sensitive to the relative size of the coupling constants $g_{\rho NN}$ and $g_{\omega NN}$. In our calculation $g^2_{\omega NN}/g^2_{\rho NN} \sim 7$. However, the more recent value quoted by Ting[39] has the ratio 8.6 ± 1.0, while Partovi and Lomon[40] prefer in their nuclear force calculation a ratio of about 12. Furthermore, the normalization correction which we made only for the pion may be necessary for other mesons if more accuracy is needed. Thus there is room for improvement in the HME contributions. It should be emphasized that the normalization correction has roughly the same magnitude as the OPE contribution, but is more strongly suppressed when r_c is increased.

$\mu_S^{(2)}$: Clearly the low-energy theorem is useless here as all the model-dependent corrections are as large as the Born term. The non-Born term coming from the vertex correction due to the ρ meson is of about the same size as the Born term. Here not only the magnitude but also the sign of the coupling constants $g_{\rho NN}\, g_{\rho\pi\gamma}\, g_{\pi NN}$ are uncertain. Although the positive sign is favoured by other evidences, there is no fundamental reason why it cannot be negative. If it is negative there will be almost complete cancellation within the pion-exchange term, leaving everything to the HME contribution. The same discussion as for the $\mu_V^{(2)}$ leaves $\mu_S^{(2)}$ in considerable doubt. Clearly more refined and if possible model-independent calculations are called for.

$\delta_A^{(2)}$: The Born terms are small as expected, but in principle the NB term could be substantial. In triton β-decay, the latter calculated in S-state is very small because of the fact that $N^*_{3,3}$ cannot contribute. Table 4 shows that the Adler values are mostly accounted for by $N^*_{3,3}$, the remaining part presumably coming from $N^*_{1,1}$, $N^*_{1,3}$ and ρ-terms. The small NB contribution is most probably a fortuitous situation due to the cancellation of $N^*_{3,3}$ terms, typical of the S-state of triton, but perhaps the situation could be different in other systems. Because of this cancellation, the OPE term is as small as the HME term, with the resulting value consistent with zero exchange contribution.

3. COMPARISON WITH EXPERIMENTS

In Table 7, we have given the expected corrections for different choices of state probabilities (S, S′ and D). Let us compare these values with our results for $\mu_S^{(2)}$, $\mu_V^{(2)}$ and $M_A^{(2)}$ given in Table 11. For the isovector moment and the GT matrix element, our results would be compatible with the wave functions with anomalously small P_D. The Blatt-Delves[41] wave functions (P_D = 9%, $P_{S'}$ = 2%), for example, would require much larger corrections than our calculated meson exchange corrections would provide. With the Gibson wave function[30] (P_D = 6%, $P_{S'}$ = 2%), the corrections needed are smaller.

With the GT matrix element, and isovector and isoscalar magnetic moments, we have three linear relations for the state probabilities:

$$\mu_S = \frac{\gamma_p + \gamma_n}{2}\left(P_S + P_{S'} - P_D + \frac{1}{2} P_{3/2}\right) + \frac{P_D}{2} + P_S\, \mu_S^{(2)}$$

$$\mu_V = \frac{\gamma_p - \gamma_n}{2}\left(P_S - \frac{1}{3} P_{S'} + \frac{1}{3} P_D + \frac{1}{6} P_{3/2}\right) - \frac{P_D}{6} + P_S\, \mu_V^{(2)} \qquad \text{(V.20)}$$

$$M_A = \sqrt{3}\left(P_S - \frac{1}{3} P_{S'} + \frac{1}{3} P_D + \frac{2}{3} P_{3/2} + \frac{2}{3} A_{S'} A_{3/2} + \delta_A^{(2)}\right) .$$

One may ask what conclusions about the state probabilities can be drawn from our values of $\mu_{V,S}^{(2)}$ and $\delta_A^{(2)}$. μ_V and M_A appear in a similar combination of P's; therefore we may consider μ_S and μ_V only. Neglecting $P_{3/2} \approx 0.25\%$ which has negligible effect, we find with $P_D + P_S + P_{S'} = 1$,

$$P_D = \frac{1 - a}{2}$$

$$P_{S'} = \frac{3}{4}\left[1 - b - \frac{2}{3} P_D\right] , \qquad \text{(V.21)}$$

where

$$a \equiv \left[\mu_S - \mu_S^{(2)}\right] \Big/ \left(\frac{\gamma_p + \gamma_n}{2}\right) = P_S + P_{S'} - P_D$$

$$b = \left[\mu_V - \mu_V^{(2)}\right] \Big/ \left(\frac{\gamma_p - \gamma_n}{2}\right) = P_S - \frac{1}{3} P_{S'} + \frac{1}{3} P_D . \qquad \text{(V.22)}$$

Note that whereas $P_{S'}$ depends on the isovector and isoscalar moments, P_D depends only on the isoscalar moment. Thus a precise calculation of $\mu_S^{(2)}$ is needed since it would unambiguously fix P_D. (However, the isoscalar moment is a difficult object to calculate.) Taking our calculated values for $\mu_{S,V}^{(2)}$, the solutions of Eq. (V.21), for the range of α and r_c considered, are $P_{S'} \approx 0$, $P_D \lesssim 6\%$.

4. REMARKS

To summarize, we have found that our calculated values for $\mu_{V,S}^{(2)}$ are consistent with the wave functions different from what the presently accepted nuclear force dictates. $\mu_S^{(2)}$ directly gives P_D, but its calculation is model-dependent and uncertain because of ambiguities in the coupling constants and normalization correction. $\mu_V^{(2)}$ is fairly model-independent, although the exact magnitude can also be affected by the model-dependent correction terms. As for the $\delta_A^{(2)}$, the corrections considered are small, consistent with zero. As one can see in Table 12 there are, however, some differences between the PCAC and the phenomenological Lagrangian methods. If in the latter one treats the ρ-propagator as suggested by the PCAC [i.e. replacement of $1/(q^2 + m_\rho^2)$ by $1/m_\rho^2$], the difference in the two methods is substantial. However, if one keeps $(q^2 + m_\rho^2)^{-1}$ in Fourier-transforming, the ρ-contribution becomes negligible for $r_c \simeq 0.4$ and the net result is small, close to the PCAC value.

Since a 3-5% reduction is expected from the relativistic corrections to the GT one-body operator[42)], a still larger exchange correction than given in Table 11 would be needed to eliminate the discrepancy. There are a number of effects that can resolve the discrepancies. Firstly, although the D-states may enter with small probabilities, they can compete substantially through the S-D matrix elements. The reason is that the S-D matrix elements can receive contributions from the vertex correction with an intermediate $N_{3,3}^*$ and in general from tensor-like operators [see Eq. (III.6a)] . Recent calculations by Blomqvist[43)], and Brown and Riska[43)] which include the D-states by approximate methods indeed suggest the importance of the S-D terms. Secondly, a possibility of having a substantial contribution from three-body currents cannot be excluded as suggested by the calculation of the isovector MM by Padgett et al.[44)].

Finally, there is the dependence of the two-body operators on the shapes of the radial wave function and on the types of core correlations (soft-core or hard-core). The Gaussian function is perhaps not very adequate in view of its too rapid fall-off. The dependence is insignificant for two-body operators of long-range character, but may not be for operators of the forms $[Y_0(x_\pi) - (m_\rho/m_\pi)^3\, Y_0(x_\rho)]$ and $\{K_0(x_\pi) - K_1(x_\pi)/x_\pi\}$. Their matrix elements depend also on whether soft- or hard-core correlations are used. These matters are further discussed in Appendix B.

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Table 1

Explicit forms of covariants $O\,(A_s)$ and $O\,(V_s)$ and their non-relativistic expressions, where $\{a,b\}_\lambda \equiv a_\lambda b \cdot k - a \cdot k b_\lambda$, $P \equiv \frac{1}{2}(p_1 + p_1')$, $\lambda = 1,2,3,4$

O_λ	Covariants	NR limits
A_1	$\sigma_{\lambda\mu} q_\mu$	$-(\vec{\sigma} \times \vec{q})$
A_2	$2iP_\lambda$	$2i\vec{P}$
A_3	iq_λ	$i\vec{q}$
A_4	$-M\gamma_\lambda$	$\frac{i}{2}\left[\vec{\sigma}(\vec{\sigma} \cdot \vec{p}_1) + (\vec{\sigma} \cdot \vec{p}_1')\vec{\sigma}\right]$
A_5	$-(\gamma \cdot k)2P_\lambda$	$\frac{i}{M}\vec{P}\left[(\vec{\sigma} \cdot \vec{k})(\vec{\sigma} \cdot \vec{p}_1) + (\vec{\sigma} \cdot \vec{p}_1')(\vec{\sigma} \cdot \vec{k})\right]$
A_6	$-(\gamma \cdot k)q_\lambda$	$\frac{i}{2M}\vec{q}\left[(\vec{\sigma} \cdot \vec{k})(\vec{\sigma} \cdot \vec{p}_1) + (\vec{\sigma} \cdot \vec{p}_1')(\vec{\sigma} \cdot \vec{k})\right]$
A_7	ik_λ	$i\vec{k}$
A_8	$-(\gamma \cdot k)k_\lambda$	$\frac{i}{2M}\vec{k}\left[(\vec{\sigma} \cdot \vec{k})(\vec{\sigma} \cdot \vec{p}_1) + (\vec{\sigma} \cdot \vec{p}_1')(\vec{\sigma} \cdot \vec{k})\right]$
V_1	$\frac{i}{2}\gamma_5\{\gamma,\gamma\}_\lambda$	$ik_0\vec{\sigma} + \frac{1}{2M}\left[(\vec{\sigma} \cdot \vec{p}_1')(\vec{\sigma} \times \vec{k}) - (\vec{\sigma} \times \vec{k})(\vec{\sigma} \cdot \vec{p}_1)\right]$
V_2	$2i\gamma_5\{P,q\}_\lambda$	$i\vec{\sigma} \cdot (\vec{k} - \vec{q})\left[k_0\vec{q} - \frac{1}{2M}\left(\{(\vec{p}_1 + \vec{p}_1') \times \vec{q}\} \times \vec{k}\right)\right]$
V_3	$\gamma_5\{\gamma,q\}_\lambda$	$i\left[(\vec{\sigma} \times \vec{q}) \times \vec{k}\right]$
V_4	$2\gamma_5\left[\{\gamma,P\}_\lambda - \frac{1}{2}iM\{\gamma,\gamma\}_\lambda\right]$	$(\vec{q} \times \vec{k})$
V_5	$i\gamma_5\{k,q\}_\lambda$	$\frac{i}{2M}\vec{\sigma} \cdot (\vec{k} - \vec{q})\left[\vec{k}(\vec{q} \cdot \vec{k}) - \vec{q}\vec{k}^2\right]$
V_6	$\gamma_5\{k,\gamma\}_\lambda$	$-i\left[(\vec{\sigma} \times \vec{k}) \times \vec{k}\right]$

Table 2 a)

Contributions to the pion exchange Gamow-Teller operators in terms of the classification of Eq. (III.5), corresponding to $O_\lambda(A_S)$, S = 1, ..., 4 of Table 1. We use the abbreviated notations $H_I = h_I + h_I^\tau P_{12}^\tau + h_I^\sigma P_{12}^\sigma$, similarly for H_{II}, and $(C_I, C_{II}) = (24\pi)^{-1}(m_\pi/M)m_\pi^2(g_r/g_A)\left[Y_0(x_\pi), 3Y_2(x_\pi)\right]$.

	g_I/C_I	g_{II}/C_{II}	H_I/C_I	H_{II}/C_{II}	j_I/C_I	j_{II}/C_{II}	j_{III}/C_{II}
$O_\lambda(A_1)$	$2A_1^{(-)}$	$-A_1^{(-)}$	$-A_1^{(+)}P_{12}^\sigma$	$\frac{1}{2}A_1^{(+)}P_{12}^\sigma$			$-\frac{3}{2}A_1^{(+)}$
$O_\lambda(A_2)$			$\frac{1}{2}A_2^{(+)} + A_2^{(-)}P_{12}^\tau$	$\frac{1}{2}A_2^{(+)} + A_2^{(-)}P_{12}^\tau$	$\frac{1}{2}A_2^{(+)}$	$\frac{1}{2}A_2^{(+)}$	
$O_\lambda(A_3)$			$-\frac{1}{2}A_3^{(+)} - A_3^{(-)}P_{12}^\tau$	$-\frac{1}{2}A_3^{(+)} - A_3^{(-)}P_{12}^\tau$	$-\frac{1}{2}A_3^{(+)}$	$-\frac{1}{2}A_3^{(+)}$	
$O_\lambda(A_4)$	$-A_4^{(-)}$	$+\frac{1}{2}A_4^{(-)}$	$\frac{1}{2}\left[A_4^{(+)}(P_{12}^\sigma + \frac{1}{2}) + A_4^{(-)}P_{12}^\tau\right]$	$\frac{1}{4}\left[A_4^{(+)}(-P_{12}^\sigma + 1) + 2A_4^{(-)}P_{12}^\tau\right]$	$\frac{1}{4}A_4^{(+)}$	$\frac{1}{4}A_4^{(+)}$	$\frac{3}{4}A_4^{(+)}$

a) $A_1^{(+)} = A_2^{(+)} = A_3^{(-)} = A_4^{(+)} = 0$ at $\nu = \nu_B = k^2 = q^2 = 0$ due to crossing symmetry

Table 3

Contributions to the pion exchange magnetic moment operators in terms of the classification given in Eqs. (III.5) and (III.6), corresponding to $O\,(V_s)$, $S = 3,4$. We use $H_{I,II} = h_{I,II} + h^{\sigma}_{I,II}P^{\sigma}_{12} + h^{\tau}_{I,II}P^{\tau}_{12}$ and $(C'_I, C'_{II}) = -\frac{g_r m_\pi^3}{12\pi}\left[Y\,(x\,),\ 3Y\,(x\,)\right]$

	g_I/C'_I	g_{II}/C'_{II}	H_I/C'_I	H_{II}/C'_{II}	j_I/C'_I	j_{II}/C'_{II}	j_{III}/C'_{II}	m_I/C'_I	m_{II}/C'_{II}	m_{III}/C'_{II}
$O_\lambda(V_3)$	$-2V_3^{(-)}$	$V_3^{(-)}$	$V_3^{(+)}P^{\sigma}_{12}$	$-\frac{1}{2}V_3^{(+)}P^{\sigma}_{12}$			$\frac{3}{2}V_3^{(+)}$			$3V_3^{(0)}$
$O_\lambda(V_4)$			$\frac{1}{2}V_4^{(+)} + V_4^{(-)}P^{\tau}_{12}$	$\frac{1}{2}V_4^{(+)} + V_4^{(-)}P^{\tau}_{12}$	$\frac{1}{2}V_4^{(+)}$	$\frac{1}{2}V_4^{(+)}$		$V_4^{(0)}$	$V_4^{(0)}$	

Table 4

Comparisons between Lagrangian and PCAC methods for calculating the amplitudes $\bar{A}_s^{(\pm)}$ and $\alpha(0)$, $\beta(0)$, $\gamma(0)$ [a)]

		$g_{\pi NN^*}$	$g_A(N^*)$	$\bar{A}_1^{(-)}$	$\bar{A}_2^{(-)}$	$\bar{A}_3^{(+)} \equiv \gamma$	$\bar{A}_4^{(-)}$	$\alpha \equiv \bar{A}_1^{(-)} - \frac{1}{2}\bar{A}_4^{(-)}$	$\beta = -\left[\bar{A}_1^{(-)} + \bar{A}_2^{(-)}\right]$	$\frac{1}{6\pi}\frac{g_r}{g_A}\frac{m_\pi}{M}\left[2\bar{A}_1^{(-)} - A_4^{(-)} - \frac{1}{2}\bar{A}_3^{(+)}\right]$
PCAC	Adler			0.28K(0)	0.47K(0)	3.1K(0)	-0.83K(0)	0.70K(0)	-0.75K(0)	-0.0144
Phenomenological Lagrangians	$N^*_{3,3}$	2.13	2.62	0.578		2.31		0.578	-0.578	0
	$N^*_{1,1}$	5.2	0.38		0.0699	0.0699	-0.174	0.087	-0.0699	0.0120
	$N^*_{3,1}$	1.23	1.51	-0.0693		0.139		-0.0693	0.0693	-0.0180
	ρ				0.442		-1.123	0.562	-0.442	0.0970
	Total			0.509	0.512	2.519	-1.297	1.158	-1.021	0.0910

a) Amplitudes are given in units of m_π^{-2}

Table 5

Heavy meson exchange contributions to the Gamow-Teller (GT) and magnetic moment (MM) exchange operators defined by Eqs. (5), (6), and (8) in Section III. We use $y_V \equiv m_V R$, $x_V = m_V r$ and $f_{VNN} = \frac{1}{\sqrt{4\pi}}\left(\frac{g_{VNN}\, m_V}{2M}\right)$, $V = \rho,\omega$.

		Pair excitation current	Nucleon recoil current
ρ meson	GT	$O(1/M_N^3)$	$j_I = -h_I = -\left(\frac{g_{\rho NN}}{2\pi}\right)^2 K_0(x_\rho)$
	MM	$h_I = -\frac{2}{3} h_{II} = -\frac{1}{3}\frac{M_N}{m_\rho} f^2_{\rho NN} (1+\kappa_V)\, Y_1(x_\rho)$ $h_I = j_I = \frac{1}{2} m_I = \frac{1}{2} g_I\ ,\quad h_{II} = j_{II} = \frac{1}{2} m_{II} = \frac{1}{2} g_{II}$ $\frac{F_I}{\tau_3(1)-\tau_3(2)} = \frac{F_{II}}{\{2(\vec{\tau}_1\cdot\vec{\tau}_2)+[\tau_3(1)+\tau_3(2)]\}} = -\frac{F_{III}}{2[\vec{\tau}(1)\times\vec{\tau}(2)]_3}$ $= i\,\frac{F_{IV}}{4(\vec{\sigma}_1\cdot\vec{\sigma}_2)\,[\vec{\tau}(1)\times\vec{\tau}(2)]_3} = -\frac{1}{2} f^2_{\rho NN}(1+\kappa_V)\, y_\rho\, Y_1(x_\rho)/x_\rho$	$h_I = -j_I = 2\left(\frac{g_{\rho NN}}{4\pi}\right)^2 \left[(1+2\kappa_V) K_0(x_\rho) - \frac{1}{6} x_\rho K_1(x_\rho)\right]$ $m_I = -4\left(\frac{g_{\rho NN}}{4\pi}\right)^2 \left[+(1+2\kappa_S) K_0(x_\rho) - \frac{1}{6} x_\rho K_1(x_\rho)\right]$ $h_I^\tau = -4\left(\frac{g_{\rho NN}}{4\pi}\right)^2 \left[+K_0(x_\rho) - \frac{1}{6} x_\rho K_1(x_\rho)\right]$ $h_{II} = -\frac{1}{2} h_{II}^\tau = -j_{II} = -\frac{1}{2} m_{II} = \frac{1}{2}\left(\frac{g_{\rho NN}}{4\pi}\right)^2 x_\rho K_1(x_\rho)$ $F_I = -\frac{1}{2} F_{IV} = \frac{1}{2}\left(\frac{g_{\rho NN}}{4\pi}\right)^2 [\tau_3(1)-\tau_3(2)]\,(1-2P_{12}^\tau)\, y_\rho K_1(x_\rho)$ $F_{II} = -\frac{1}{2}\left(\frac{g_{\rho NN}}{4\pi}\right)^2 \left\{2\,\vec{\tau}_1\cdot\vec{\tau}_2 + [\tau_3(1)+\tau_3(2)]\right\} y_\rho K_1(x_\rho)$
ω meson	GT	$O(1/M_N^3)$	$j_I = h_I = -\left(\frac{g_{\omega NN}}{2\pi}\right)^2 K_0(x_\omega)$
	MM	$h_I = -j_I = -\frac{\ell_I}{2} = -\frac{2}{3} h_{II} = \frac{2}{3} j_{II} = \frac{1}{3}\ell_{II}$ $= \frac{1}{3}\frac{M_N}{m_\omega} f^2_{\omega NN}(1+\kappa_S)\, Y_1(x_\omega)$ $\frac{F_I}{[\tau_3(1)+\tau_3(2)]} = -\frac{F_{II}}{\{2+[\tau_3(1)+\tau_3(2)]\}} =$ $= \frac{1}{2} f^2_{\omega NN}(1+\kappa_S)\, y_\omega\, Y_1(x_\omega)/x_\omega$	$h_I = j_I = -2\left(\frac{g_{\omega NN}}{4\pi}\right)^2 \left[(1+2\kappa_V) K_0(x_\omega) - \frac{1}{6} x_\omega K_1(x_\omega)\right]$ $\ell_I = -4\left(\frac{g_{\omega NN}}{4\pi}\right)^2 \left[(1+2\kappa_S) K_0(x_\omega) - \frac{1}{6} x_\omega K_1(x_\omega)\right]$ $h_{II} = j_{II} = \frac{1}{2}\ell_{II} = -\frac{1}{2}\left(\frac{g_{\omega NN}}{4\pi}\right)^2 x_\omega K_1(x_\omega)$ $\frac{F_I}{[\tau_3(1)+\tau_3(2)]} = \frac{F_{II}}{\{2+[\tau_3(1)+\tau_3(2)]\}} = -\frac{F_{IV}}{2[\tau_3(1)-\tau_3(2)]} =$ $= -\frac{1}{2}\left(\frac{g_{\omega NN}}{4\pi}\right)^2 y_\omega K_1(x_\omega)$

Table 6

Spin-isospin matrix elements of exchange current operators for trinucleon system in symmetric S-state

	Operator O_{12}	Matrix element
MM	$\left[\tau_3(1) - \tau_3(2)\right]_3 (\vec{\sigma}_1 - \vec{\sigma}_2)_3$	-16mt
	$\left[\vec{\tau}(1) \cdot \vec{\tau}(2)\right] (\vec{\sigma}_1 + \vec{\sigma}_2)_3$	-12m
	$(\vec{\sigma}_1 + \vec{\sigma}_2)_3$	+4m
	$\tau_3(1)\tau_3(2)(\vec{\sigma}_1 + \vec{\sigma}_2)$	-4m
GT	$\left[\tau(1) - \tau(2)\right]_+ (\vec{\sigma}_1 - \vec{\sigma}_2)$	$4\sqrt{3}$

MM : $\left\langle \Psi^{mt} \middle| \sum_{i<j} O_{ij} \middle| \Psi^{mt} \right\rangle$; $m = \pm\frac{1}{2}$, $t = \pm\frac{1}{2}$

GT : $\left| \left\langle {}^3He \middle\| \sum_{i<j} O_{ij} \middle\| {}^3H \right\rangle \right| / \sqrt{2}$

Table 7

Discrepancies between experiments and single-particle operator values in $\mu_{S,V}$ and M_A for ^3H and ^3He. Probabilities P_S, P_D ... are given in %

			Single-particle values			Expected corrections			
P_S	P_D	$P_{S'}$	$\mu_S^{(1)}$	$\mu_V^{(1)}$	$M_A^{(1)}$	$\delta\mu_S$ a)	$\delta\mu_V$ a)	δM_A b)	δ_A
96	4	0	0.423	2.284	1.686	0.003 (0.70%)	0.269 (10.5%)	-0.001	-0.059%
94	6	0	0.417	2.249	1.663	0.009 (2.11)	0.304 (11.9)	0.022	1.306
92.8	6	1.2	0.417	2.211	1.635	0.009 (2.11)	0.342 (13.4)	0.050	2.967
92 c)	6	2	0.417	2.185	1.616	0.009 (2.11)	0.368 (14.4)	0.069	4.094
91 d)	8	1	0.410	2.183	1.617	0.013 (3.05)	0.370 (14.5)	0.068	4.036
89 d)	9	2	0.406	2.134	1.582	0.020 (4.69)	0.419 (16.4)	0.103	6.112

a) Values in parenthesis are % deviations

b) We have used Salgo and Staub's[35] value $|M_A^{exp}| = 1.685 \pm 0.018$

c) Corresponds to Gibson wave function[30] (setting $P_{3/2} = 0$)

d) Correspond to wave functions determined variationally by Blatt and his co-workers[41, 6]

Table 8

ft-values and $|M_A|_{{}^3H \to {}^3He}$ determined therefrom

$(ft)_{n \to p}$	$(ft)_{{}^{14}O \to {}^{14}N}$	$(ft)_{{}^3H \to {}^3He}$	g_A	$\lvert M_A\rvert^2$	$\lvert M_A\rvert$
1190 ± 37 sec a)	3075 ± 15 a)	1132 ± 40 d)	1.18 ± 0.026	3.17 ± 0.19	1.78 ± 0.054
1213 ± 35 b)	3127 ± 31 b)	1132 ± 40 d)	1.18 ± 0.028	3.25 ± 0.19	1.80 ± 0.052
1099 ± 34 c)	3075 ± 15	1132 ± 40	1.24 ± 0.02	2.88 ± 0.11	1.70 ± 0.034
1108 ± 16.45 e)	3117 ± 18 e)	1159 ± 11 e)	1.243 ± 0.011	2.84 ± 0.06	1.685 ± 0.0178

a) and b): Taken from [36] corresponding to values respectively without and with radiative corrections. Neutron data correspond to Sosnovsky et al.[31].

c) : Obtained with the same f-value as in a) but with the new neutron data of Christensen et al.

d) : Value quoted by Goldhaber[33] in 1958, close to Porter's[34] value of $(ft)_{{}^3H}$ = 1137 ± 20 sec.

e) : New analysis by Salgo and Staub[35] based on Christensen neutron data[32] and their own ^3H data.

Table 9

The discrepancies between the various experimental M_A's of Table 8 and the values obtained with Gibson and Blatt-Delves wave functions

$\|M_A\|^{exp}$	δ(Gibson) [a)]	δ(Blatt-Delves) [b)]
1.78 ± 0.054	0.107 ± 0.033	0.101 ± 0.033
1.80 ± 0.052	0.120 ± 0.032	0.114 ± 0.032
1.70 ± 0.034	0.054 ± 0.021	0.048 ± 0.021
1.685 ± 0.0178	0.047 ± 0.011	0.042 ± 0.011

a) $|M_A^{(1)}|_{Gibson} = 1.609$

b) $|M_A^{(1)}|_{Blatt\text{-}Delves} = 1.617$

Table 10

Coupling constants used to evaluate Eq. (V.18)

	g_{BNN}	m_B(MeV)	$g_{B\pi\gamma}$	$f^2_{BNN} = \frac{1}{4\pi}\left(\frac{g_{BNN}m_B}{2M}\right)^2$
π	13.64	138		0.080
ρ	2.56	750	0.448	0.0829
ω	6.82	783	0.758	0.643

Table 11

Exchange contributions to magnetic moments and GT matrix element in trinucleon system calculated using Eq. (V.17) and (V.18) as a function of range parameter α and hard core radius r_c. "Born" contains Born graphs + normalization correction

α fm^{-1}	r_c fm	$\mu_S^{(2)}$ OPE BORN	$\mu_S^{(2)}$ OPE NB	$\mu_S^{(2)}$ HME	$\mu_S^{(2)}$ TOTAL	$\mu_V^{(2)}$ OPE BORN	$\mu_V^{(2)}$ OPE NB	$\mu_V^{(2)}$ HME	$\mu_V^{(2)}$ TOTAL	$\delta_A^{(2)}$ OPE BORN	$\delta_A^{(2)}$ OPE NB	$\delta_A^{(2)}$ HME	$\delta_A^{(2)}$ TOTAL
0.337	0.0	0.386E-2	0.292E-2	0.372E-2	0.105E-1	0.217E0	-0.633E-3	-0.321E-1	0.185E0	0.180E-1	-0.271E-2	-0.140E-1	0.124E-2
	0.2	0.278E-2	0.281E-2	0.334E-2	0.892E-2	0.201E0	-0.608E-3	-0.251E-1	0.175E0	0.144E-1	-0.261E-2	-0.109E-1	0.827E-3
	0.4	0.170E-2	0.256E-2	0.245E-2	0.671E-2	0.185E0	-0.554E-3	-0.179E-1	0.166E0	0.105E-1	-0.237E-2	-0.784E-2	0.276E-3
	0.6	0.106E-2	0.227E-2	0.177E-2	0.510E-2	0.173E0	-0.492E-3	-0.121E-1	0.160E0	0.790E-2	-0.211E-2	-0.529E-2	0.496E-3
	0.8	0.632E-3	0.201E-2	0.131E-2	0.395E-2	0.161E0	-0.435E-3	-0.856E-2	0.152E0	0.603E-2	-0.186E-2	-0.374E-2	0.431E-3
0.384	0.0	0.578E-2	0.379E-2	0.532E-2	0.149E-1	0.266E0	-0.821E-3	-0.464E-1	0.219E0	0.257E-1	-0.352E-2	-0.202E-1	0.196E-2
	0.2	0.420E-2	0.362E-2	0.475E-2	0.126E-1	0.243E0	-0.784E-3	-0.361E-1	0.206E0	0.204E-1	-0.336E-2	-0.157E-1	0.135E-2
	0.4	0.262E-2	0.326E-2	0.346E-2	0.934E-2	0.219E0	-0.706E-3	-0.257E-1	0.193E0	0.148E-1	-0.303E-2	-0.112E-1	0.534E-3
	0.6	0.170E-2	0.287E-2	0.250E-2	0.707E-2	0.203E0	-0.622E-3	-0.173E-1	0.185E0	0.111E-1	-0.267E-2	-0.757E-2	0.879E-3
	0.8	0.110E-2	0.253E-2	0.186E-2	0.549E-2	0.189E0	-0.547E-3	-0.123E-1	0.176E0	0.855E-2	-0.235E-2	-0.538E-2	0.826E-3
0.404	0.0	0.674E-2	0.418E-2	0.611E-2	0.170E-1	0.288E0	-0.905E-3	-0.534E-1	0.234E0	0.295E-1	-0.388E-2	-0.233E-1	0.230E-2
	0.2	0.491E-2	0.398E-2	0.544E-2	0.143E-1	0.261E0	-0.862E-3	-0.415E-1	0.218E0	0.234E-1	-0.370E-2	-0.181E-1	0.158E-2
	0.4	0.308E-2	0.357E-2	0.395E-2	0.106E-1	0.234E0	-0.774E-3	-0.294E-1	0.203E0	0.168E-1	-0.332E-2	-0.129E-1	0.631E-3
	0.6	0.202E-2	0.313E-2	0.285E-2	0.800E-2	0.215E0	-0.679E-3	-0.199E-1	0.195E0	0.126E-1	-0.291E-2	-0.869E-2	0.104E-2
	0.8	0.134E-2	0.276E-2	0.212E-2	0.622E-2	0.200E0	-0.597E-3	-0.142E-1	0.185E0	0.975E-2	-0.256E-2	-0.619E-2	0.100E-2

Table 12

Comparison between the PCAC and phenomenological lagrangian methods for $\delta_A^{(2)}$ (in %) calculated with the S-state with α = 0.384 fm.

r_c		0.0 fm	0.4 fm
Adler		-0.35	-0.30
Phenomenological Lagrangian	$N^*_{3,3}$	0	0
	$N^*_{1,1}$	0.29	0.25
	$N^*_{3,1}$	-0.44	-0.38
	ρ [a]	2.4(-3.7)	2.05(-0.64)
	Total	2.2(-4.0)	1.9 (-0.77)

a) The numbers given outside of the parentheses are calculated by setting q^2 = 0 in the ρ propagator whereas those in the parentheses by taking the q^2 dependence into account.

Appendix A

MESON THEORY

In order to have a concise definition of the two-body electromagnetic or weak exchange operators in meson theory, one may follow the same line of reasoning as in the definition of the two body nuclear potential. Some approximation scheme is set up for the evaluation of the field-theoretic S-matrix in the scattering of two nucleons involving interaction with an external field (here, absorption of photon or β-decay). The hypothesis which is then made is that the mutual interaction energy of the nucleons and their interaction energy with the external field have representations in terms of non-relativistic, energy-independent potentials. The identification of transition amplitudes obtained in the meson theory description and the non-relativistic equivalent operator description give then prescriptions of how to define the latter. The non-unique character of the prescriptions arise, in general, from the different choices that can be made for the representation space of the two-nucleon system.

The method used in this work, called the S-matrix method, rests on the representation of the two-nucleon states in the unperturbed Fock basis. It has some specific features which are not shared by others, the unitary transformation method being one example. We shall, therefore, sketch them briefly and refer the reader interested in more details to Ref. 25).

Consider the decomposition of the field theory Hamiltonian of the two-nucleon system $H = H_0 + V_\pi + V_x$ into parts associated to the free fields (H_0), to the pion-nucleon interaction (V_π) and interactions of the pion and nucleon fields with the external field (V_x), all taken to be additive. In the quantum mechanical description the interactions of the nucleons are represented by a potential. We denote its stationary states by $|\phi_\alpha\rangle$. The index α is a label for the state of the two nucleons. If $|\Psi_\alpha\rangle$ denote the corresponding stationary states of the field-theory Hamiltonian H, it can be shown that $|\phi_\alpha\rangle$ are just their projections on the meson vacuum. More precisely, one has

$$|\Psi_\alpha\rangle = |\phi_\alpha\rangle + \frac{\Lambda}{a} R'_\pi \; |\phi_\alpha\rangle \quad , \tag{A.1}$$

where $a = (E - H_0)$, Λ is a projection operator off the meson vacuum and R'_π is the reaction matrix associated to V_π but with all intermediate states containing at least one pion or $N\bar{N}$ pair. The transition amplitude between states labelled by m and n due to one interaction with V_x can be written as

$$\langle m|R_x|n\rangle \equiv \langle \phi_m|H_x|\phi_n\rangle$$

$$= \langle \phi_m|V_x + V_x \frac{\Lambda}{a} R'_\pi + R'_\pi \frac{\Lambda}{a} V_x + R'_\pi \frac{\Lambda}{a} V_x \frac{\Lambda}{a} R'_\pi|\phi_n\rangle \ . \qquad (A.2)$$

The first equation is the definition of H_x, the interaction energy of the system with the external field. In the second equation, the first term stands for the normal one-body operator and the others for the operators describing virtual meson effects. The projection operator Λ shows why processes described by the diagrams of Fig. 8 had to be excluded.

In the perturbation theory expansion of Eq. (A.2) there are a number of terms representing self-action effects which we assumed are already taken into account in using the renormalized forms of the one-body operators and in evaluating the two-body diagrams in renormalized perturbation theory. In principle renormalized quantities are here "system" quantities in the sense that they depend on the state of the system as a whole even though they may refer to the properties of a single nucleon, such as mass, form factors, etc. It was, however, shown by Frantz[45] within the framework of the static model that this system dependence disappears when the adiabatic limit is taken.

There is still another important feature of this approach which deserves some attention -- the wave function normalization correction. The state vectors $|\Psi_\alpha\rangle$ as given by Eq. (A.1) are not normalized. Assuming $|\phi_\alpha\rangle$ to be normalized then the normalization is taken care of by the substitution $|\Psi_\alpha\rangle \to |\Psi_\alpha\rangle / Z'^{\frac{1}{2}}_\alpha$ where $Z'^{\frac{1}{2}}_\alpha = \langle \phi_\alpha|\Psi_\alpha\rangle$. From Eq. (A.1) we get

$$Z'_\alpha = \left[1 + \langle \phi_\alpha|R'_\pi \left(\frac{\Lambda}{a}\right)^2 R'_\pi|\phi_\alpha\rangle\right]^{-1} \qquad (A.3)$$

which, if solved to lowest order in V_π, gives

$$Z'_\alpha \simeq 1 - \left\langle \phi_\alpha \middle| V_\pi \left(\frac{\Lambda}{a}\right)^2 V_\pi \middle| \phi_\alpha \right\rangle . \tag{A.4}$$

The one-body parts of the operator: $V_\pi(\Lambda/a)^2 V_\pi$ are assumed to be incorporated into the renormalization effects mentioned above. The remainder is a two-body part which can be given an operator representation by using essentially the same methods as for the calculation of nuclear potentials. By taking the static limit and transforming to coordinate space, we find

$$Z'(\vec{x}_1,\vec{x}_2) = 1 + \frac{2}{\pi} f^2_{\pi NN} \left[\vec{\tau}(1) \cdot \vec{\tau}(2)\right] (\vec{\sigma}_1 \cdot \vec{\nabla})(\vec{\sigma}_2 \cdot \vec{\nabla}) K_0(x_\pi)$$

$$= 1 + \frac{2}{3\pi} f^2_{\pi NN} \left[\vec{\tau}(1) \cdot \vec{\tau}(2)\right] \left[(\vec{\sigma}_1 \cdot \vec{\sigma}_2) \left(K_0(x_\pi) - \frac{K_1(x_\pi)}{x_\pi} \right) + S_{12} K_2(x_\pi) \right] \tag{A.5}$$

where the derivative is with respect to x_π. The correction to the matrix elements $\langle \phi_m | H_x | \phi_n \rangle$ amounts to multiplying them by the factor $|Z'_m Z'_n|^{\frac{1}{2}}$, where Z'_m (Z'_n) denote the expectation value of Eq. (A.5) in the state m (n). In a second-order perturbation treatment of the normalization correction, it seems adequate to introduce the correction only for the one-body operator, in which case it just adds on the r.h.s. of Eq. (A.2) the term

$$\langle \phi_m | V_x | \phi_n \rangle \left[(Z'_m Z'_n)^{\frac{1}{2}} - 1 \right] . \tag{A.6}$$

Meson-exchange effects reflect the extent to which meson degrees of freedom are eliminated from the nuclear wave functions or, stated equivalently, the extent to which more diagrams appear in a field-theory description than in a non-relativistic description. The normalization correction Eq. (A.6) accounts for these effects as much as the two-body operators.

To conclude, we should mention that the question of normalization has raised much controversy in the past in relation to the definition of nuclear potentials. The difference between the fourth-order parts of the Taketani-Machida-Onuma (TMO) and Brueckner-Watson (BW) potentials lies precisely in its omission in the BW potential [46]. The question is not whether it should be included or excluded but rather how it should be handled. The procedure that we used stays essentially on perturbation theory arguments. It is therefore not very satisfactory, but can perhaps be trusted if Z'_m stays close to one. This turns out to be the case in our applications to the three-nucleon systems.

Appendix B

TRINUCLEON WAVE FUNCTIONS FOR THE S-STATE

We briefly discuss here other forms of wave function which are available and how these different wave functions modify our results. In our calculations, we have used the Gaussian form for the radial wave function of the S-state. The others frequently used are the exponential, Irving, and Irving-Gunn wave functions[6], all of which fall off somewhat less rapidly than the Gaussian form. We have calculated the matrix elements of $Y_0(x_\pi)$, $[K_0(x_\pi) - K_1(x_\pi)/x_\pi]$ and $Y_0(x_\rho)$ with the Irving wave function (multiplied with a Jastrow factor) and found that they were not significantly different from those of Gaussian wave function over the range of r_c we have considered.

Let us now examine the correlation function. We choose a hard-core function frequently used in nuclear matter calculation. Assume that we can write

$$f(r_{12},r_{23},r_{13}) = N\, e^{-\alpha^2(r_{12}^2+r_{13}^2+r_{23}^2)/2} \prod_{i<j} (1 - \eta_{ij}) \,, \qquad (B.1)$$

which seems to be valid when on- and off-shell "defect" wave functions are taken to be the same, and

$$\begin{aligned} \eta_{ij} &= \left(\frac{d - r_{ij}}{d - c}\right)^2 && \text{for} \quad c \leq r_{ij} \leq d \\ &= 1 && \text{for} \quad r_{ij} < c \qquad\qquad (B.2) \\ &= 0 && \text{for} \quad r_{ij} > d \,. \end{aligned}$$

Here d is the separation distance which is about 1 fm for acceptable potentials, and c is the hard-core radius of order of 0.4 - 0.5 fm. For the α given by Coulomb energy $\alpha = 0.384$, Eq. (B.1) and Eq. (V.6) (the soft-core), when properly normalized, are found to give the same matrix element of

$Y_0(x_\pi)$ for each $c = (\gamma\sqrt{2})^{-1}$ if we choose d = 1 fm. For our discussion we may take this as some sort of normalization. Then we have evaluated the matrix elements of $[K_0(x_\pi) - K_1(x_\pi)/x_\pi]$ and $Y_0(x_\rho)$ with both wave functions, and plotted them as a function of $c = r_c = (\gamma\sqrt{2})^{-1}$ in Fig. 10. Note that the correlation function Eq. (B.2) suppresses their matrix elements more than the soft-core factor does. As a consequence, it is possible that our calculations with the soft-core factor obtain somewhat overestimated values for the nucleon recoil, the normalization correction, and HME terms, the bulk contributions of which come from the inner part of the wave function.

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Figure captions

Fig. 1 : Diagram representing the transition amplitude of a two-nucleon system interacting with the axial or vector currents (denoted by the wavy lines).

Fig. 2 : One-pion-exchange (a) and one-boson-exchange (b) contributions to the exchange currents.

Fig. 3 : Pion production amplitude from a single nucleon by the axial or vector currents.

Fig. 4 : Nucleon pole diagrams in the amplitude for pion production from the nucleon by the axial current.

Fig. 5 : Born diagrams representing the amplitude for pion-production by the vector current. The separation into the seagull diagram (a), the nucleon-pole diagrams (b) and (c) and the pion-pole diagram (d) provides, when appropriate expressions for the vertices are used, a book-keeping of the terms describing the full amplitude in the soft-pion limit. These modified vertices also follow from the chiral Lagrangians.

Fig. 6 : Representation of the pion-production amplitude by the axial or vector currents in terms of pole diagrams. Graphs (a) and (b) are for the intermediate nucleon isobars (s-channel singularities) and graph (c) is for the vector-meson intermediate states (t-channel singularities).

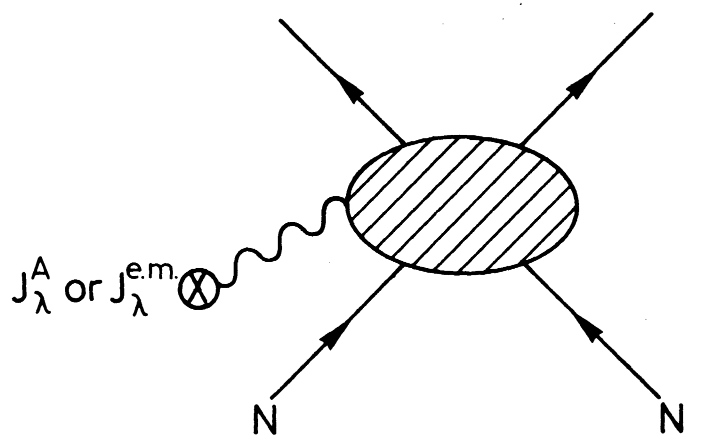
Fig. 7 : One-pion-exchange diagrams representing the vertex corrections due to vector meson intermediate states. Due to its odd G-parity the ω-meson ($J^{PG} = 1^{--}$) contributes only to second-class currents.

Fig. 8 : Time-ordered Feynman diagrams representing mechanisms where the interaction with the current occurs after emission of the pion and its absorption. Together with the diagrams

where the current interacts before the pion is exchanged, they represent effects already included in the wave functions.

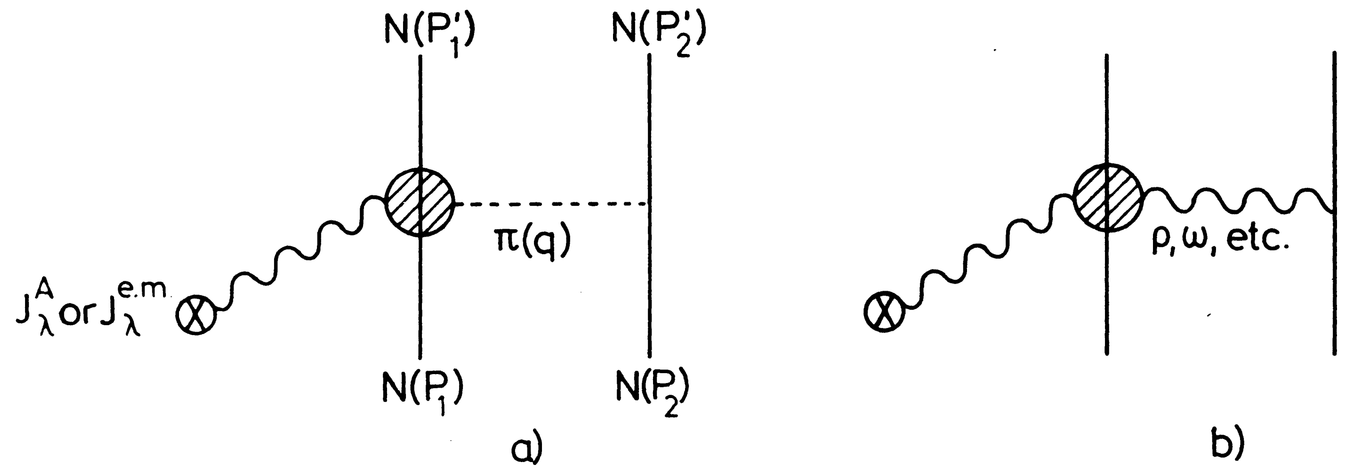
Fig. 9 : The parts of the Born contributions to the OPE process that remain after excluding the unwanted parts given in Fig. 8. The time-ordered diagrams (a) and (b) give the nucleon-recoil exchange current, and (c) and (d) the pair-excitation exchange current. The Feynman diagram (e) gives the pionic exchange current.

Fig. 10 : Plot of the variation of the expectation values of exchange operators having short-range character as a function of core radius. Comparison is given between soft-core and hard-core correlations built with Jastrow factors into the same radial function: Gaussian $\exp\left[-\alpha^2(r_{12}^2 + r_{13}^2 + r_{23}^2)/2\right]$ with $\alpha = 0.384\ \mathrm{fm}^{-1}$.



J_λ^A or $J_\lambda^{e.m.}$
N
N

FIG.1



J_λ^A or $J_\lambda^{e.m.}$
$N(P_1')$
$N(P_2')$
$\pi(q)$
$N(P_1)$
$N(P_2)$
a)
ρ, ω, etc.
b)

FIG.2

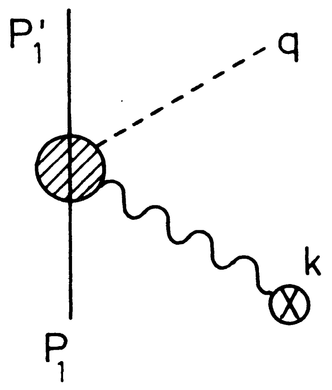


FIG.3

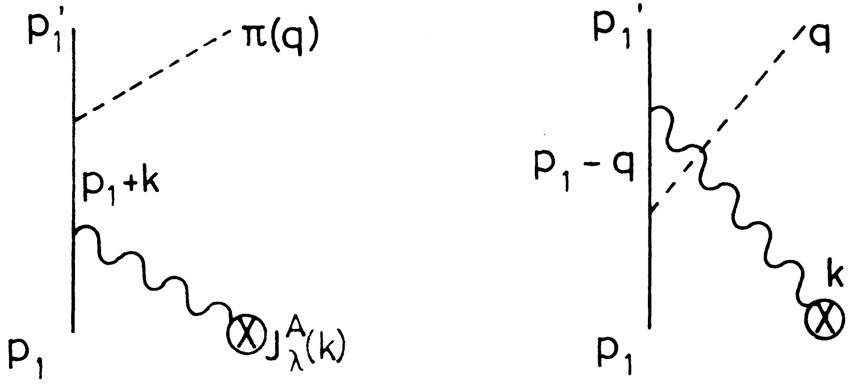


FIG.4

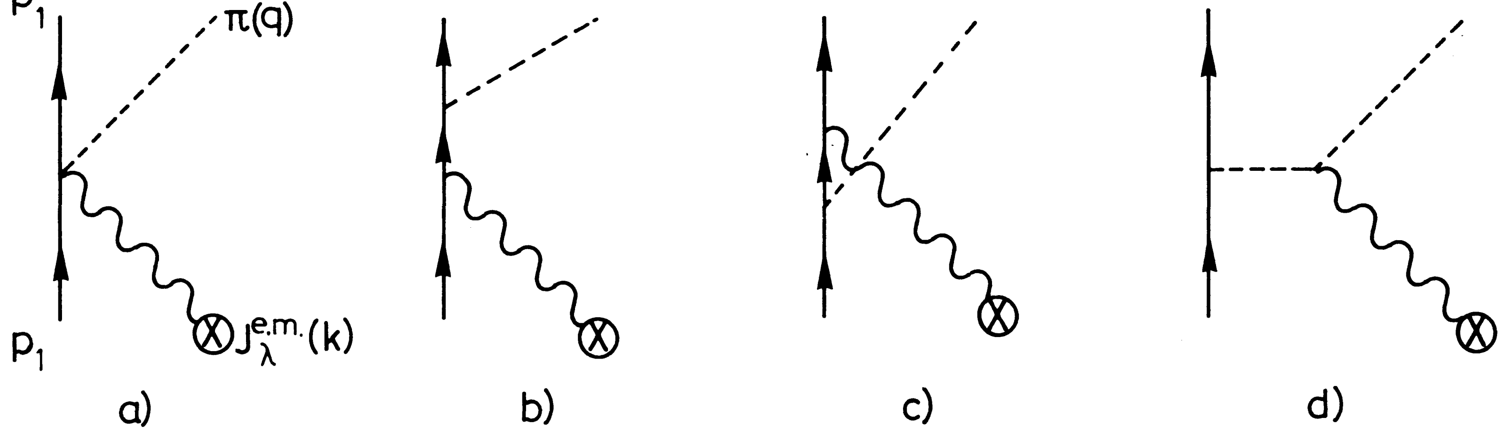


FIG.5

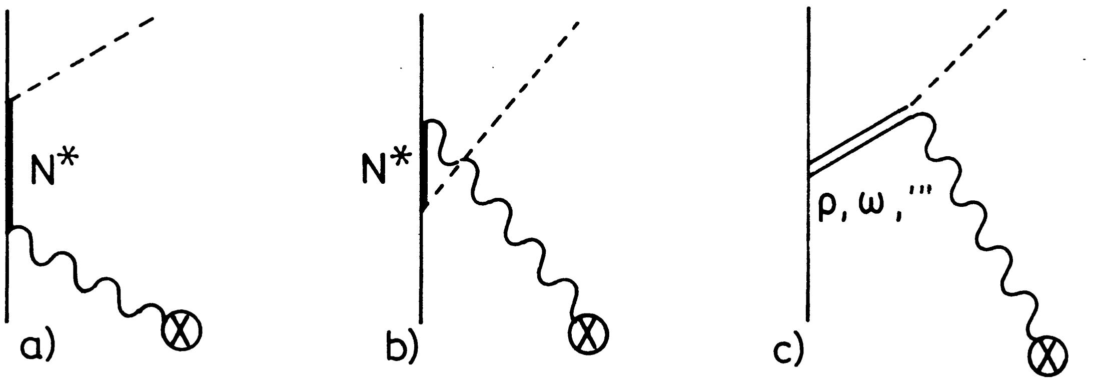


FIG.6

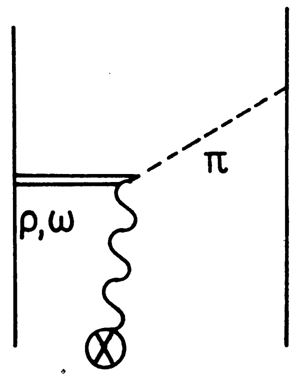


FIG.7

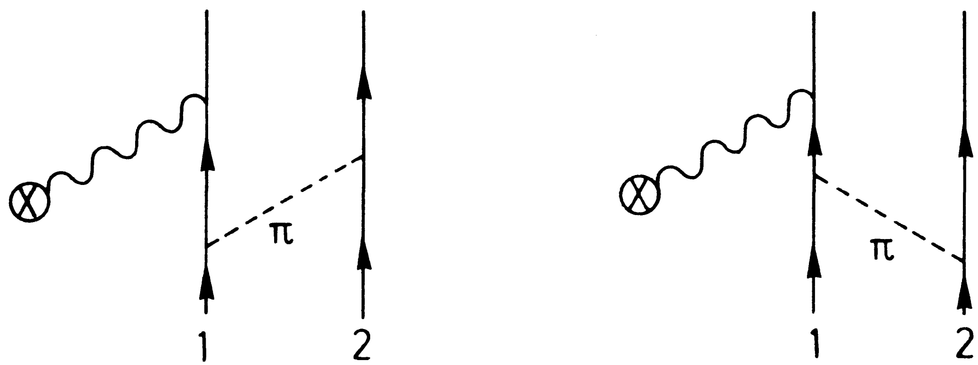


FIG.8

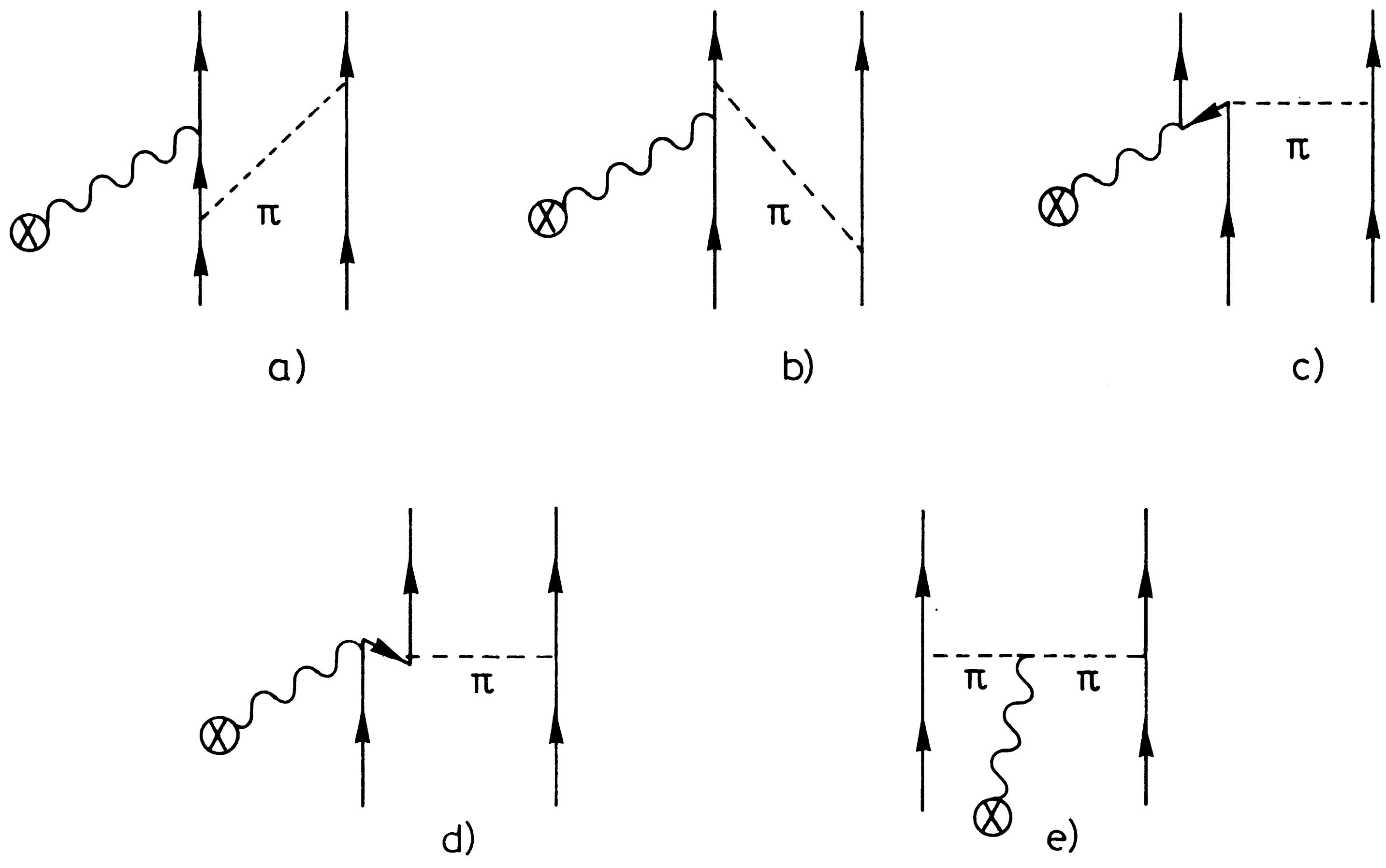
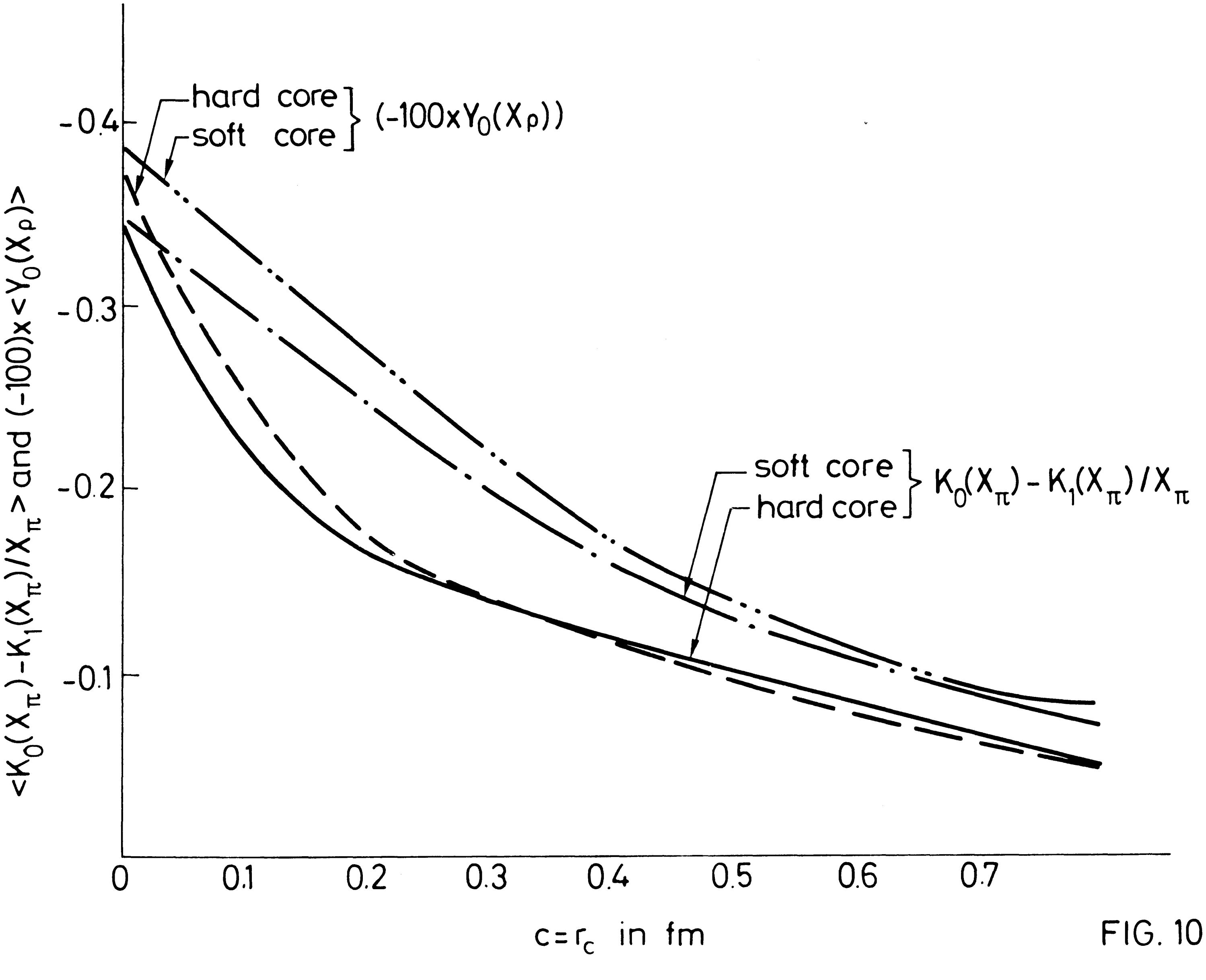


FIG.9



hard core
soft core
(-100xY0(Xρ))
soft core
hard core
K0(Xπ) - K1(Xπ) / Xπ
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
c=rc in fm
<K0(Xπ)-K1(Xπ)/Xπ> and (-100)x<Y0(Xρ)>

FIG. 10