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CROSSING CONSTRAINTS ON $\pi\pi$ PARTIAL WAVE AMPLITUDES †

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A B S T R A C T

New constraints on the $\pi\pi$ partial wave amplitudes below threshold, derived on the basis of crossing symmetry and isospin invariance alone, are given.

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During the past several years, Martin [1] and his collaborators have derived a great number of restrictions on the $\pi\pi$ partial wave amplitudes below threshold. These restrictions have usually been in the form of inequalities, derived on the basis of analyticity, crossing symmetry and unitarity. Although the work of Atkinson [2] has shown that these general properties can be satisfied by a very large class of functions (labelled by a symmetric function of two variables) the results of Martin have shown that the low partial waves are numerically constrained in the region $0 < s < 4$ (in the units where $m_\pi = 1$).

In this letter, I wish to point out some new constraints that can be derived from crossing symmetry and isospin invariance alone. In contrast to the results mentioned above, these take the form of equalities involving integrals over the partial wave amplitudes in the unphysical region. They have been derived by a systematic application of the ideas of Balachandran and Nuyts [3] to the $\pi\pi$ system.

If $f^{(0)}$, $f^{(1)}$, $f^{(2)}$ denote the $I = 0$ s wave, $I = 1$ p wave and $I = 2$ s wave respectively, one can derive the following relations:

$$\int_0^4 (4-s)(3s-4)\left[f^{(0)}(s)+2f^{(2)}(s)\right] ds = 0 \tag{1}$$

$$\int_0^4 (4-s)\left(2f^{(0)}(s)-5f^{(2)}(s)\right) ds = 0 \tag{2}$$

$$\int_0^4 s(4-s)\left(2f^{(0)}(s)-5f^{(2)}(s)\right) ds = -3\int_0^4 (4-s)^2 f^{(1)}(s)\, ds \tag{3}$$

$$\int_0^4 s(4-s)^2\left(2f^{(0)}(s)-5f^{(2)}(s)\right) ds = -3\int_0^4 s(4-s)^2 f^{(1)}(s)\, ds \tag{4}$$

$$\int_0^4 s(4-s)^3 \left(2f^{(0)}(s)-5f^{(2)}(s)\right)ds = -3\int_0^4 s(4-s)^2(3s-4)f^{(1)}(s)\,ds \tag{5}$$

If one includes higher partial waves, the number of equations increases rapidly. For example, including the $I=0$ and $I=2$ d waves leads to 10 new relations.

The complete set of such relations together with their proof will be described elsewhere [4]. Here I shall give a simple proof (due to S. Nussinov) of the first relation only.

Let $F(s,t,u)$ denote the amplitude for $\pi^o\pi^o$ elastic scattering. By crossing symmetry, $F(s,t,u)$ is a totally symmetric function. If Δ denotes the Mandelstam triangle ($s>0$, $t>0$, $u>0$), the relation

$$\iint_\Delta F(s,t,u)\,(s+t+u-4)\,ds\,dt = 0 \tag{6}$$

is trivially true. By the symmetry of F, one can replace $s+t+u-4$ by $3s-4$ in the integral. Performing the t integral then yields

$$\int_0^4 (4-s)(3s-4)\,f_{oo}(s)\,ds = 0 \tag{7}$$

where f_{oo} is the $\pi^o\pi^o$ s wave. Recalling that

$$f_{oo}(s) = \frac{1}{3}\left(f^{(0)}(s)+2f^{(2)}(s)\right) \tag{8}$$

one obtains (1).

Equations (1)-(5) have been obtained using crossing symmetry and isospin invariance alone. Moreover, it can be shown [4] that these constraints are also sufficient - given any s and p waves satisfying (1)-(5), it is always possible to find amplitudes with the proper crossing properties of which these are the s and p waves.

This suggests that these relations could be useful in models for the low partial waves of $\pi\pi$ scattering, which seem to violate crossing symmetry. These models usually have some freedom, which might be used to satisfy (1)-(5). Then, the violations of crossing symmetry would be pushed up to higher partial waves. For example, Wanders [5)] and his collaborators have taken a particular form for the s wave amplitudes, with certain free parameters, which they choose to be consistent with the constraints of Ref. 1). They find that these constraints strongly restrict the range of their parameters, and as a result, the physical phase shifts are quite well determined, in reasonable agreement with experiment. It is hoped that using their techniques our relations will help to further pin down the low energy $\pi\pi$ phase shifts.

After completing this work, I learned that similar results had been obtained by Piguet and Wanders.

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R E F E R E N C E S

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