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PHASE-SHIFT ANALYSIS OF p-p AND n-p ELASTIC SCATTERING AT 735 MeV

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ABSTRACT

Phase-shift analysis of the elastic N-N scattering at 735 MeV has been performed taking $1_{max} = 4$. It was supposed that the pion production gives a main contribution in the P, D, and F states. The search for the solutions was performed only for the isospin T = 1. Ten solutions with the negative sign of the $^1S_0$ phase shift were found in the interval $\chi^2 \leq 1.5\ \overline{\chi^2}$.

All these solutions were used as initial conditions for the specification of the simultaneous p-p and n-p phase-shift analysis. The values of phase shifts are given in the tables and the most important angular dependences of experimental quantities for the four first solutions are shown in the graphs.

The new statistical criterion has been performed for the discrimination between the phase-shift sets. Using this criterion, the whole number of solutions was reduced to 4, from which only two for $^1S_0 < 0$.

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†) On leave of absence from CEN-Saclay.

1. INTRODUCTION

Previously it was impossible to perform the phase-shift analysis at 735 MeV, since the number of the experimental points was quite insufficient. The polarized proton target made it possible to complete the experimental data, mainly with the C_{nn}^{pp} measurement. These data were obtained in Saclay$^{1)}$ and using them the conditions for the performance of the phase-shift analysis were fulfilled.

At first the phase-shift analysis was performed for the elastic p-p scattering at $l_{max} = 4$ with the imaginary parts of the P, D, and F waves. The considerable part of scattering amplitude corresponding to the interaction in high orbital momentum states $l > l_{max}$ was taken into account with the one pion exchange approximation. The limit of application of this approximation (l_{max}) was determined according to the best agreement with experimental data.

The phase-shift analysis was carried out by the program described in detail in Ref. 2. The obtained solutions for the p-p system were used as initial parameters in the simultaneous n-p and p-p phase-shift analysis.

The aim of this work is to obtain all possible solutions on the basis of the known data and to use it for the planning of other experiments at 735 MeV.

2. EXPERIMENTAL DATA

Until now, only three experimental quantities have been measured: the effective cross-section and polarization for p-p and n-p scattering and the spin correlation coefficient C_{nn} in p-p scattering. The data used in the phase-shift analysis are given$^{1,3-10)}$ in Tables 1 and 2.

The values of P_{pp} were measured by five author groups$^{1,4-7)}$ in the energy region 700-750 MeV. For the good description of all measured data it was necessary to normalize the measurements from the different works.

The approximation function of the angular dependence of the polarization was taken as:

$$N_i f(\theta,a_n) = \sum_n a_n \sin\theta \cos^n\theta , \qquad (1)$$

where a_n are the distribution coefficients, θ is the scattering angle in CM and N_i are the normalizing factors for the various sets of data $(i = 1\text{-}5)$.

The coefficients a_n and N_i were obtained by using the least squares method in which the following function was minimized:

$$\chi^2 = \sum_{i=0}^{5} \sum_{m=1}^{M} \left(\frac{F_{\exp}^{i,m} - N_i f(\theta, a_n)}{\Delta_{i,m}} \right)^2 . \qquad (2)$$

Here $F_{\exp}^{i,m}$ is the value of the experimental point with the experimental error $\Delta_{i,m}$, the index m denoting simply the number of points.

The maximal number of coefficients was determined from the condition that the increasing of this number does not decrease the value $\chi^2/\overline{\chi^2}$.

The results of the normalization procedure are given in Table 3.

3. PHASE-SHIFT ANALYSIS

In Table 4 is shown the maximal number of parameters, which we can obtain from the different experiments. From this table it follows that at $l_{\max} = 4$ the performance of the phase-shift analysis is reasonable, since we must determine only 21 real and 3 imaginary partial waves and one binding constant f^2.

The search for the solutions of the phase-shift analysis with random initial parameters was made in the following way: for the system with isospin T = 1 the waves S, P, D, and F were varied, the value of the binding constant f^2 was fixed at 0.08 and the phase shift $^1D_2$ was fixed to 10°. All other phase shifts were fixed to zero.

From 157 sets of initial parameters, 54 solutions were specified and only 10 of them had a χ^2 in the region $\chi^2 \leq 1.5\,\overline{\chi^2}$ with a negative sign for the $^1S_0$ phase shift*).

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*) Only negative sign of $^1S_0$ phase shift is acceptable at this energy.

After this, the experimental data of differential and total cross-sections and polarization for the n-p scattering were added[5,8-10] (see Tables 1 and 2) and the solutions chosen for p-p scattering were taken as the initial conditions for the simultaneous n-p and p-p phase-shift analysis. The phase shifts of ^3G waves and the imaginary parts of P, D, and F waves were fixed, since they gave no decrease of the χ^2 value. The obtained nine solutions with $\chi^2 \leq 1.5\ \overline{\chi^2}$ are given in Table 5. The stability of the first solution with regard to different numbers of varying parameters is shown in Table 6.

Solutions previously obtained by MacGregor in the energy dependent phase-shift analysis[11] and solutions in Dubna at 630 MeV [12] were also used as initial conditions.

Only one of the latter solutions gave a χ^2 in the region $\chi^2 \leq 1.5\ \overline{\chi^2}$ [solution of Kazarinov et al.[12] with $\chi^2 = 244$ at 630 MeV, which corresponds to the solution No. VII of the present work]. The specified solutions of MacGregor at 720 MeV and 750 MeV give after minimization a χ^2 value of more than 140 but a positive sign for the $^1S_0$ phase shift (see Table 7).

From Table 5 it follows that for only five solutions, specified for the n-p and p-p data, the sign of $^1S_0$ remains negative.

For discriminating between the nine obtained solutions (see Table 5) the so-called τ-criterion[13] was used. This criterion allows us to calculate the probability α of the type I error which we make by rejecting the i^{th} solution, with respect to the error which we make by rejecting the solution I with the lowest χ^2 and which we take equal to 100%. The probabilities α are given in Table 8.

If we accept the limit of the probability α equal to 1% or 0.1% we obtain 3 or 4 solutions respectively, from which only one or two have the negative sign of $^1S_0$ phase shift.

With the first four solutions we have calculated the angular dependence of the most important experimental quantities (graphs 1-18).

4. RESULTS

a) Ten solutions of the p-p phase shift analysis with negative signs of $^1S_0$ were obtained. Only nine of these are specified solutions for the p-p and n-p phase shift analysis, five of which have the negative sign of the $^1S_0$ phase shift.

b) Using the τ criterion the number of solutions decreases to four, of which only two have a negative sign for the $^1S_0$ phase shift.

c) The calculated phase shifts coincide within two errors with the values obtained by extrapolating the energy dependences of the phase shifts (see Refs. 14 and 15) with exception of $^3P_2$, ε_2, and $^3F_4$ parameters for the first and fourth solutions.

Acknowledgements

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Table 1

Experimental data used in the phase-shift analysis at 735 MeV

Measured quantity	Energy MeV	Angular Range (CM)	Number of points	Authors	Refs.
σ_{pp}	725	8 -45	11	McManigal et al.	3
P_{pp}	735	8°-45°	11	McManigal et al.	4
	735	6°-75°	14	Cozzika et al.	1
	700	30°-87°	8	Cheng et al.	5
	750	43°-87°	5	Neal et al.	6
	736	32°-84°	17	Betz et al.	7
C_{nn}^{pp}	735	35°-93°	15	Cozzika et al.	1
σ_{np}	710	158°-180°	9	Larsen	8
	775	90°	1	Martelli et al.	9
P_{pn}	700	29°-143°	8	Cheng et al.	5
σ_{tot}^{np}	730		1	Palevsky et al.	10

Table 2

The experimental data used in phase-shift analysis for p-p and n-p scattering at energy 735 MeV

Measured quantity	Energy MeV	$\theta^\circ_{\text{lab}}$	Measured values	Statistical error ±	Refs.
$\left(\frac{d\sigma}{d\Omega}\right)_{\text{lab}}$	725	4.5	57.9	*)	3
		6.0	55.7		
		7.3	51.3		
[mb/sr]		8.6	46.2		
		10.0	45.3		
		11.5	38.5		
		13.0	37.7		
		15.3	31.7		
		16.4	26.5		
		18.0	25.2		
		20.5	19.7		
P_{pp}	735	4.5	0.248	± 0.013	4
		6.0	0.352	± 0.014	
		7.3	0.387	± 0.011	
		8.6	0.421	± 0.015	
		10.0	0.468	± 0.013	
		11.5	0.530	± 0.017	
		13.0	0.543	± 0.015	
		15.3	0.547	± 0.018	
		16.4	0.602	± 0.023	
		18.0	0.599	± 0.023	
		20.5	0.591	± 0.027	

(contd.)

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*) These numbers have a reproducibility error of ±5% plus an additional error due to uncertainty in the energy dependence of the p-C cross-section at 6° (see Ref. 3).

Table 2 (contd.)

Measured quantity	Energy MeV	$\theta^\circ_{c.m.}$	Measured value	Statistical error ±	Refs.
P_{pp}	700	30.8	0.555	± 0.019	5
		35.7	0.522	± 0.012	
		43.2	0.558	± 0.016	
		52.4	0.529	± 0.022	
		60.1	0.474	± 0.011	
		68.7	0.354	± 0.039	
		77.0	0.255	± 0.015	
		86.1	0.109	± 0.038	
P_{pp}	735	6.7	0.118	± 0.160	1
		18.3	0.367	± 0.032	
		25.4	0.490	± 0.030	
		29.8	0.520	± 0.040	
		34.2	0.530	± 0.040	
		39.0	0.570	± 0.030	
		43.6	0.550	± 0.030	
		48.0	0.500	± 0.030	
		54.2	0.486	± 0.030	
		58.2	0.460	± 0.200	
		62.4	0.404	± 0.030	
		66.2	0.364	± 0.030	
		70.4	0.295	± 0.050	
		74.4	0.268	± 0.015	
P_{pp}	750	43.85	0.541	± 0.075	6
		47.19	0.513	± 0.044	
		53.25	0.530	± 0.029	
		63.98	0.470	± 0.067	
		86.29	0.097	± 0.078	

(contd.)

Table 2 (contd.)

Measured quantity	Energy MeV	$\theta^\circ_{\text{c.m.}}$	Measured value	Statistical error ±	Refs.
P_{pp}	736	32.5	0.579	± 0.049 *)	7
		35.6	0.579	± 0.028	
		38.8	0.553	± 0.017	
		42.0	0.560	± 0.014	
		45.1	0.559	± 0.015	
		48.3	0.528	± 0.011	
		51.4	0.520	± 0.011	
		54.6	0.497	± 0.013	
		57.7	0.498	± 0.013	
		60.9	0.473	± 0.014	
		65.5	0.419	± 0.018	
		68.4	0.365	± 0.017	
		71.4	0.342	± 0.018	
		74.3	0.304	± 0.018	
		77.2	0.231	± 0.018	
		80.2	0.180	± 0.018	
		83.2	0.144	± 0.023	
C^{pp}_{nn}	735	92.1	0.88	± 0.21	1
		88.8	0.55	± 0.34	
		85.5	0.88	± 0.29	
		82.0	0.65	± 0.25	
		78.5	0.83	± 0.23	
		74.8	0.47	± 0.16	
		71.0	0.66	± 0.19	
		66.7	0.51	± 0.10	
		62.3	0.45	± 0.09	
		58.4	0.50	± 0.09	
		53.9	0.43	± 0.09	

(contd.)

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*) A systematic error of $\begin{pmatrix} +6.5 \\ -5.8 \end{pmatrix} \times P_{pp}$ must be added in quadrature to the above errors ΔP_{pp} (see Ref. 7).

Table 2 (contd.)

Measured quantity	Energy MeV	$\theta^{\circ}_{c.m.}$	Measured value	Statistical error ±	Refs.
C_{nn}^{pp}	735	49.6	0.38	± 0.06	1
		45.3	0.53	± 0.06	
		40.9	0.35	± 0.09	
		35.5	0.46	± 0.11	
σ_{np} [mb/sr]	775	90	0.89	± 0.05	9
σ_{np} [mb/sr]	710	180.00	6.15	± 0.54	8
		175.89	5.04	± 0.51	
		172.94	4.39	± 0.39	
		170.60	3.97	± 0.38	
		168.25	3.84	± 0.31	
		165.90	3.12	± 0.25	
		163.25	3.35	± 0.28	
		161.37	2.71	± 0.24	
		158.90	2.65	± 0.23	
P_{np}	700	29.5	0.334	± 0.027	5
		44.3	0.305	± 0.017	
		60.4	0.157	± 0.033	
		77.1	-0.068	± 0.030	
		93.8	-0.352	± 0.026	
		110.9	-0.411	± 0.032	
		127.1	-0.247	± 0.019	
		143.2	-0.146	± 0.019	
σ_{np}^{tot} [mb]	730		35.8	± 1.6	10

Table 3

Distribution coefficients and normalization factors for the P_{pp} data normalization (55 points)

Distribution coefficients	Normalization factors	Authors	Refs.
$a_0 = 0$ fix	$N_1 = 1.022 \pm 0.011$	McManigal et al.	4
$a_1 = +1.354 \pm 0.089$	$N_2 = 0.954 \pm 0.018$	Cozzika et al.	1
$a_2 = -1.253 \pm 0.283$	$N_3 = 0.989 \pm 0.012$	Cheng et al.	5
$a_3 = +1.297 \pm 0.215$	$N_4 = 1.008 \pm 0.033$	Betz et al.	7
$\chi^2 = 31.27$	$N_5 = 1.006 \pm 0.042$	Neal et al.	6
$\chi^2/\bar{\chi}^2 = 0.6$			

Table 4

Experimental quantity	Number of parameters	
	p-p	n-p
σ	$l_{max} + 1$	$2\,l_{max} + 1$
P	$l_{max} - 1$	$2\,l_{max} - 1$
C_{nn}	$l_{max} + 1$	
Σ	$3\,l_{max} + 1$	$4\,l_{max}$

Table 5

The obtained N-N phase-shift sets at
735 MeV in the region $\chi^2 \leq 1.5\ \overline{\chi^2}$

	I	II	III	IV
	$\delta^0 \pm \Delta\delta^0$	$\delta^0 \pm \Delta\delta^0$	$\delta^0 \pm \Delta\delta^0$	$\delta^0 \pm \Delta\delta^0$
$^1S_0$	-17.97 ± 3.13	28.76 ± 12.71	10.83 ± 12.13	-41.65 ± 5.93
$^3S_1$	-38.13 ± 4.27	-32.52 ± 3.85	-51.31 ± 9.56	-40.07 ± 13.71
$^3P_0$	-12.95 ± 2.27	-10.82 ± 16.56	- 5.33 ± 13.53	- 9.13 ± 3.56
$^1P_1$	-94.74 ± 9.69	71.72 ± 16.69	-57.27 ± 12.72	74.41 ± 16.92
$^3P_1$	- 0.89 ± 2.68	-11.04 ± 4.92	-14.86 ± 3.48	2.13 ± 3.10
$^3P_2$	-33.93 ± 1.92	-33.19 ± 3.20	-33.39 ± 1.43	-35.10 ± 1.72
ε_1	-21.84 ± 4.40	-25.23 ± 3.63	16.66 ± 3.53	-19.77 ± 17.52
$^3D_1$	-30.04 ± 5.92	-19.44 ± 7.93	- 7.81 ± 15.62	-31.08 ± 10.10
$^1D_2$	-16.77 ± 2.46	16.28 ± 1.25	17.82 ± 2.84	11.06 ± 5.18
$^3D_2$	- 9.34 ± 5.05	- 6.04 ± 6.31	-26.89 ± 12.16	-14.16 ± 21.13
$^3D_3$	8.44 ± 1.38	10.29 ± 2.68	8.93 ± 2.81	7.62 ± 2.68
ε_2	-11.52 ± 0.71	2.82 ± 7.00	4.31 ± 2.05	-13.36 ± 0.78
$^3F_2$	- 3.05 ± 0.97	- 3.68 ± 1.28	- 7.72 ± 1.33	- 6.78 ± 1.44
$^1F_3$	-13.86 ± 4.93	-10.98 ± 6.35	8.80 ± 3.94	-10.39 ± 10.14
$^3F_3$	5.65 ± 0.64	- 0.23 ± 1.47	- 1.31 ± 1.33	0.51 ± 2.68
$^3F_4$	20.29 ± 1.50	21.22 ± 1.71	21.67 ± 2.01	19.15 ± 1.36
ε_3	0 fix	0 fix	0 fix	0 fix
$^3G_3$	0 fix	0 fix	0 fix	0 fix
$^1G_4$	5.0 fix	5.00 fix	5.00 fix	5.00 fix
$^3G_4$	0 fix	0 fix	0 fix	0 fix
$^3G_5$	0 fix	0 fix	0 fix	0 fix
Im $^1D_2$	10.0 ± fix	10.00 ± fix	10.00 ± fix	10.00 ± fix
f^2	0.067 ± 0.009	0.051 ± 0.016	0.049 0.017	0.074 0.009
χ^2	75.89	77.45	82.08	84.90
$\chi^2/\overline{\chi^2}$	0.91	0.93	0.99	1.02

Table 5 (contd.)

	V	VI	VII
$^1S_0$	-18.74 ± 3.56	-37.87 ± 3.30	-29.15 ± 5.36
$^3S_1$	-28.38 ± 4.80	-30.28 ± 4.71	38.95 ± 4.88
$^3P_0$	-88.02 ± 4.63	96.07 ± 5.61	45.61 ± 10.78
$^1P_1$	-70.62 ± 8.97	93.22 ± 7.09	100.05 ± 15.08
$^3P_1$	28.75 ± 2.77	33.69 ± 3.00	25.81 ± 6.60
$^3P_2$	3.60 ± 2.39	5.58 ± 2.43	16.59 ± 1.09
ε_1	-25.18 ± 4.30	-20.69 ± 4.37	- 6.61 ± 6.27
$^3D_1$	-36.85 ± 4.51	-35.31 ± 3.97	2.37 ± 5.62
$^1D_2$	-15.96 ± 2.54	4.78 ± 4.89	10.54 ± 3.29
$^3D_2$	0.83 ± 6.61	2.85 ± 5.60	-13.38 ± 3.95
$^3D_3$	8.67 ± 1.95	9.38 ± 1.49	9.80 ± 1.87
ε_2	4.22 ± 1.27	7.70 ± 1.19	- 5.92 ± 1.80
$^3F_2$	-23.95 ± 1.41	-22.60 ± 1.42	-29.15 ± 2.00
$^1F_3$	-11.24 ± 4.09	- 7.89 ± 3.38	10.85 ± 7.19
$^3F_3$	13.25 ± 0.79	8.24 ± 1.97	3.74 ± 2.13
$^3F_4$	9.99 ± 0.54	10.22 ± 0.49	7.69 ± 0.63
ε_3	0 fix	0 fix	0 fix
$^3G_3$	0 fix	0 fix	0 fix
$^1G_4$	5.00 fix	5.00 fix	5.00 fix
$^3G_4$	0 fix	0 fix	0 fix
$^3G_5$	0 fix	0 fix	0 fix
Im $^1D_2$	10.00 fix	10.00 fix	10.00 fix
f^2	0.051 ± 0.013	0.052 ± 0.014	0.057 ± 0.011
χ^2	84.90	92.25	92.61
$\chi^2/\overline{\chi^2}$	1.02	1.11	1.11

(contd.)

Table 5 (contd.)

	VIII	IX
$^1S_0$	24.78 ± 11.11	6.81 ± 15.75
$^3S_1$	40.36 ± 5.36	-58.73 ± 5.25
$^3P_0$	43.42 ± 13.27	15.05 ± 3.35
$^1P_1$	64.98 ± 10.48	31.41 ± 13.02
$^3P_1$	28.51 ± 8.29	61.09 ± 5.54
$^3P_2$	14.73 ± 1.31	5.72 ± 2.05
ε_3	- 9.80 ± 6.35	24.15 ± 2.17
$^3D_1$	- 5.24 ± 6.02	- 8.69 ± 8.11
$^1D_2$	17.44 ± 4.13	4.07 ± 5.56
$^3D_2$	- 0.80 ± 7.28	- 32.85 ± 6.46
$^3D_3$	11.90 ± 2.10	7.19 ± 2.13
ε_2	6.24 ± 1.84	8.36 ± 5.19
$^3F_2$	-25.91 ± 2.29	- 23.55 ± 1.35
$^1F_3$	-18.25 ± 4.43	- 9.33 ± 5.03
$^3F_3$	-13.53 ± 1.63	8.73 ± 2.02
$^3F_4$	7.46 ± 0.56	9.53 ± 0.92
ε_3	0 fix	0 fix
$^3G_3$	0 fix	0 fix
$^1G_4$	5.00 fix	5.00 fix
$^3G_4$	0 fix	0 fix
$^3G_5$	0 fix	0 fix
Im $^1D_2$	10.00 fix	10.00 fix
f^2	0.013 ± 0.012	0.030 ± 0.028
χ^2	97.15	120.72
$\chi^2/\overline{\chi^2}$	1.17	1.46

Table 6

The stability of the 1st solution with respect to the number of variable parameters

χ^2	75.885	75.301	75.791
f^2	0.067 ± 0.009	0.067 ± 0.009	6.851 ± 0.012
$^1S_0$	-17.97 ± 3.18	-18.19 ± 3.22	-18.51 ± 4.30
$^3S_1$	-38.08 ± 4.22	-38.16 ± 4.38	-38.41 ± 5.57
$^3P_0$	-12.94 ± 2.22	-12.31 ± 9.43	-12.67 ± 2.69
$^1P_1$	-94.56 ± 9.61	-93.72 ± 9.61	-95.65 ±11.70
$^3P_1$	- 0.87 ± 2.68	- 1.37 ± 7.63	- 1.12 ± 3.14
$^3P_2$	-33.92 ± 1.92	-33.83 ± 1.93	-33.63 ± 2.93
ε_1	-21.87 ± 4.38	-21.80 ± 4.34	-20.87 ± 8.46
$^3D_1$	-29.98 ± 5.91	-29.93 ± 5.77	-30.35 ± 6.88
$^1D_2$	-16.78 ± 2.46	-16.81 ± 2.58	-16.71 ± 2.82
$^3D_2$	- 9.37 ± 5.86	- 9.11 ± 6.49	- 9.51 ± 5.98
$^3D_3$	8.44 ± 1.38	8.47 ± 1.49	8.41 ± 1.54
ε_2	-11.51 ± 0.71	-11.74 ± 2.58	-11.43 ± 0.80
$^3F_2$	- 8.05 ± 0.57	- 7.94 ± 1.19	-79.71 ± 1.02
$^1F_3$	-13.80 ± 4.98	-13.69 ± 4.99	-14.00 ± 5.01
$^3F_3$	5.63 ± 0.60	5.51 ± 1.22	5.37 ± 1.59
$^3F_4$	20.30 ± 1.50	20.28 ± 1.67	20.38 ± 1.53
ε_3	0 fix	0 fix	0 fix
$^3G_3$	0 fix	0 fix	0 fix
$^1G_4$	5.00 fix	5.00 fix	5.00 fix
$^3G_4$	0 fix	0 fix	0 fix
$^3G_5$	0 fix	0 fix	0 fix
Im P	0 fix	0.28 ± 1.16	0 fix
Im $^1D_2$	10.00 fix	10.00 fix	11.51 ± 7.33
Im F	0 fix	0 fix	0 fix

(contd.)

Table 6 (contd.)

χ^2	75.840	75.163	73.6
f^2	0.067 ± 0.009	0.0677 ± 0.012	0.0567 ± 0.018
$^1S_0$	-17.82 ± 3.14	-18.21 ±	-20.77 ± 4.49
$^3S_1$	-37.99 ± 4.23	-38.09 ± 5.68	-38.74 ± 12.73
$^3P_0$	-13.60 ± 6.25	-12.86 ± 37.43	-17.99 ± 4.15
$^1P_1$	-94.07 ± 9.86	-93.10 ± 12.95	-84.06 ± 29.91
$^3P_1$	- 0.39 ± 5.37	- 0.94 ± 29.25	- 2.54 ± 6.96
$^3P_2$	-34.13 ± 3.22	-33.98 ± 6.83	-33.15 ± 2.29
ε_1	-21.58 ± 5.49	-21.33 ± 12.11	-10.90 ± 21.67
$^3D_1$	-29.78 ± 5.85	-29.76 ± 6.98	-28.19 ± 17.88
$^1D_2$	-16.92 ± 2.44	-16.92 ± 2.99	-18.69 ± 3.06
$^3D_2$	- 9.43 ± 5.75	- 9.11 ± 10.23	- 5.56 ± 7.41
$^3D_3$	8.42 ± 1.44	8.45 ± 1.52	7.93 ± 2.27
ε_2	-11.12 ± 3.54	-11.36 ± 14.61	-10.00 ± 1.54
$^3F_2$	- 7.99 ± 1.08	- 7.86 ± 1.28	- 8.44 ± 1.97
$^1F_3$	-13.62 ± 5.05	-13.49 ± 5.82	-15.39 ± 19.48
$^3F_3$	5.59 ± 0.83	5.12 ± 1.96	5.39 ± 1.39
$^3F_4$	20.27 ± 1.60	20.27 ± 1.75	20.69 ± 2.14
ε_3	0 fix	0 fix	4.90 ± 9.47
$^3G_3$	0 fix	0 fix	2.80 ± 24.29
$^1G_4$	5.00 fix	5.00 fix	3.02 ± 2.38
$^3G_4$	0 fix	0 fix	3.01 ± 18.92
$^3G_5$	0 fix	0 fix	4.69 ± 12.12
Im P	0 fix	0.39 ± 3.00	0.40 fix
Im $^1D_2$	10.00 fix	10.24 ± 7.97	10.24 fix
Im F	0.14 ± 1.21	0.14 ± 3.06	0.14 fix

Table 7

	Our solutions obtained by using the MacGregor et al. phase shifts as initial conditions		Solution obtained by MacGregor et al. for the energy dependent p-p phase-shift analysis	
	$\delta^0 \pm \Delta\delta^0$	$\delta^0 \pm \Delta\delta^0$	δ^0	δ^0
$^1S_0$	38.45 ± 11.81	30.07 ± 9.77	-42.65	-44.78
$^3S_1$	-42.72 ± 7.89	-32.79 ± 6.07		
$^3P_0$	-101.62 ± 20.49	-71.65 ± 7.56	-41.87	-43.27
$^1P_1$	78.81 ± 13.33	74.90 ± 9.78		
$^3P_1$	-23.34 ± 10.11	-23.98 ± 4.47	-49.89	-51.05
$^3P_2$	18.12 ± 1.99	18.46 ± 1.86	15.56	15.30
ε_1	22.81 ± 2.35	18.23 ± 2.21		
$^3D_1$	1.59 ± 14.00	- 0.13 ± 14.16		
$^1D_2$	13.41 ± 1.96	9.94 ± 2.34	9.72	9.52
$^3D_2$	- 1.77 ± 5.56q	0.91 ± 5.24		
$^3D_3$	19.38 ± 2.87	22.29 ± 2.91		
ε_2	3.20 ± 2.35	12.94 ± 2.56	0.99	1.28
$^3F_2$	-16.18 ± 2.39	-13.00 ± 1.79	- 5.52	- 6.00
$^1F_3$	-12.76 ± 6.35	-16.68 ± 5.40		
$^3F_3$	7.41 ± 1.63	4.52 ± 1.41	- 2.23	- 2.11
$^3F_4$	7.87 ± 0.96	7.00 ± 1.21	4.45	4.50
ε_3	0 fix	0 fix		
$^3G_3$	0 fix	0 fix		
$^1G_4$	5.00	5.00	3.72	3.84
$^3G_4$	0 fix	0 fix		
$^3G_5$	0 fix	0 fix		

(contd.)

Table 7 (contd.)

	Our solutions obtained by using the MacGregor et al. phase shifts as initial conditions		Solution obtained by MacGregor et al. for the energy dependent p-p phase-shift analysis	
	$\delta^0 \pm \Delta\delta^0$	$\delta^0 \pm \Delta\delta^0$	δ^0	δ^0
Im $^1D_2$	10.0 fix	10.0 fix		
f^2	0.013 ± 0.011	0.012 ± 0.010		
χ^2	140.98	140.81		
Energy	720 MeV	750 MeV	720 MeV	750 MeV

Table 8

The probability α of Type I error for the first seven solutions

Solution	I	II	III	IV	V	VI	VII
χ^2	75.89	77.45	82.08	84.90	84.90	92.25	92.61
α%	100	4.7	2.2	0.27	0.10	0.02	0.0005

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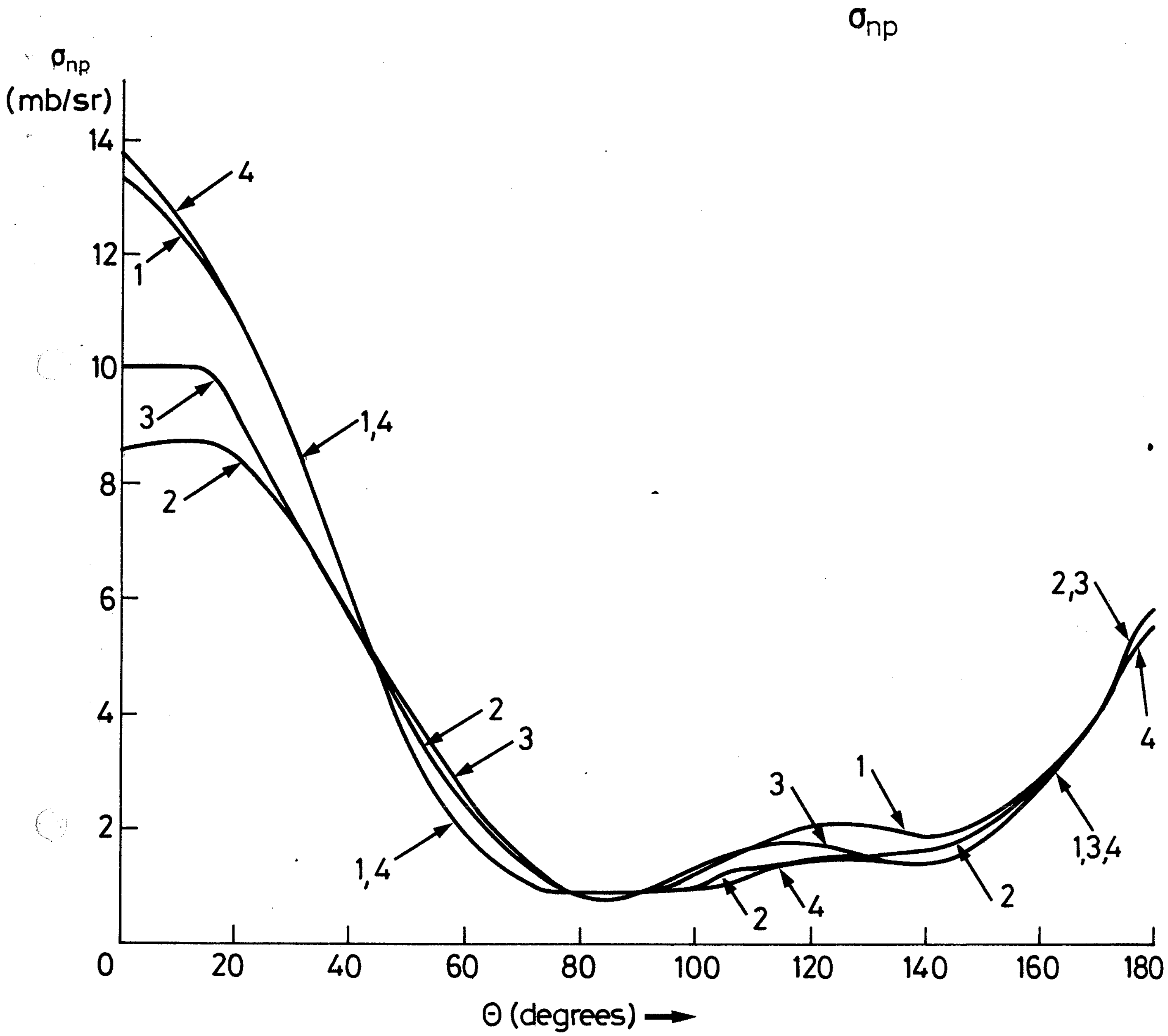

σ_{np}
σ_{np}
(mb/sr)
14
12
10
8
6
4
2
0
20
40
60
80
100
120
140
160
180
Θ (degrees)
4
1
3
2
1,4
2
3
1,4
3
1
2
4
2
2,3
4
1,3,4

Diagram 1

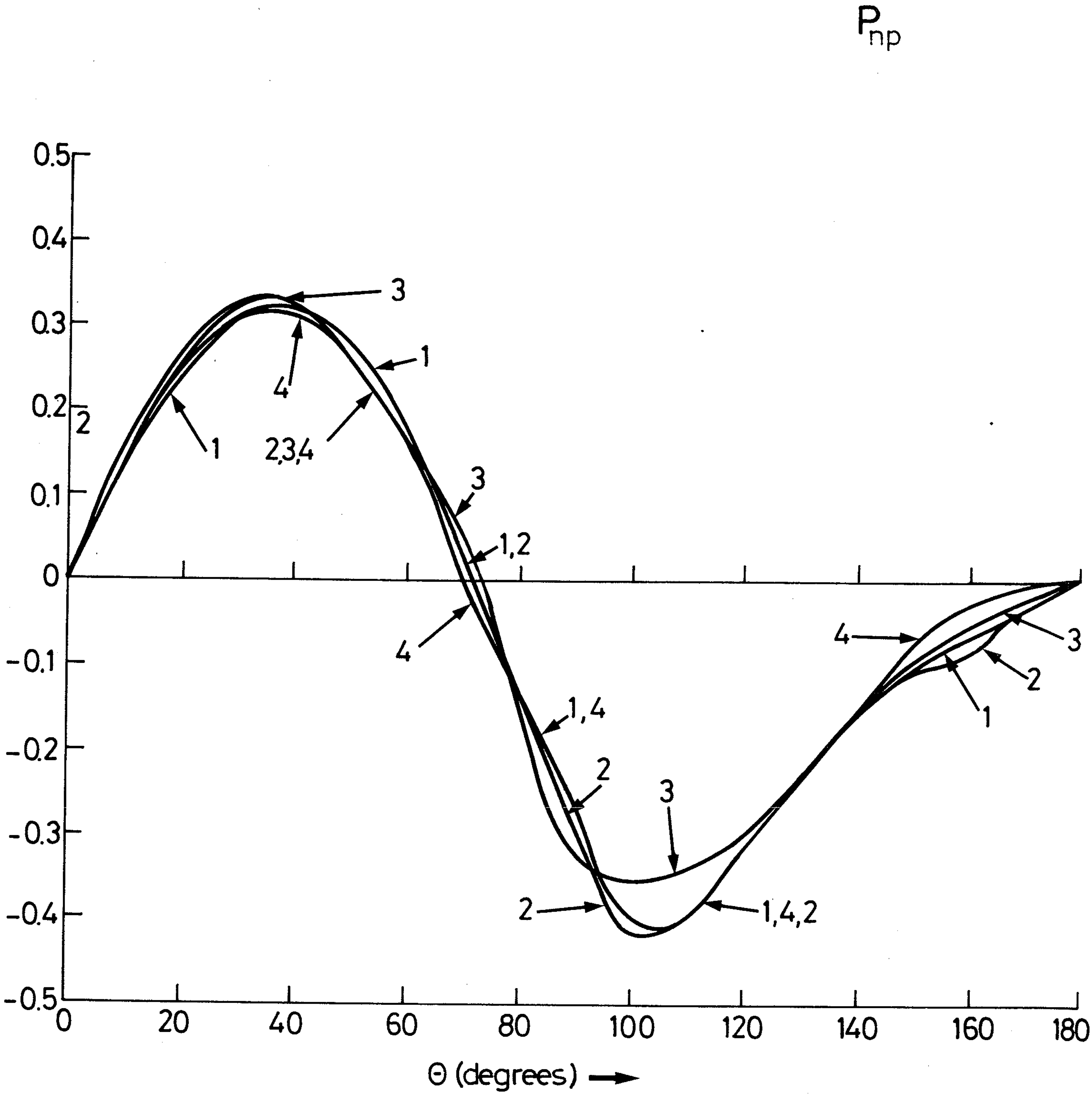

P_np
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
20
40
60
80
100
120
140
160
180
Θ (degrees)
3
1
4
2
1
2,3,4
3
1,2
4
1,4
2
3
2
1,4,2
4
3
2
1

Diagram 2

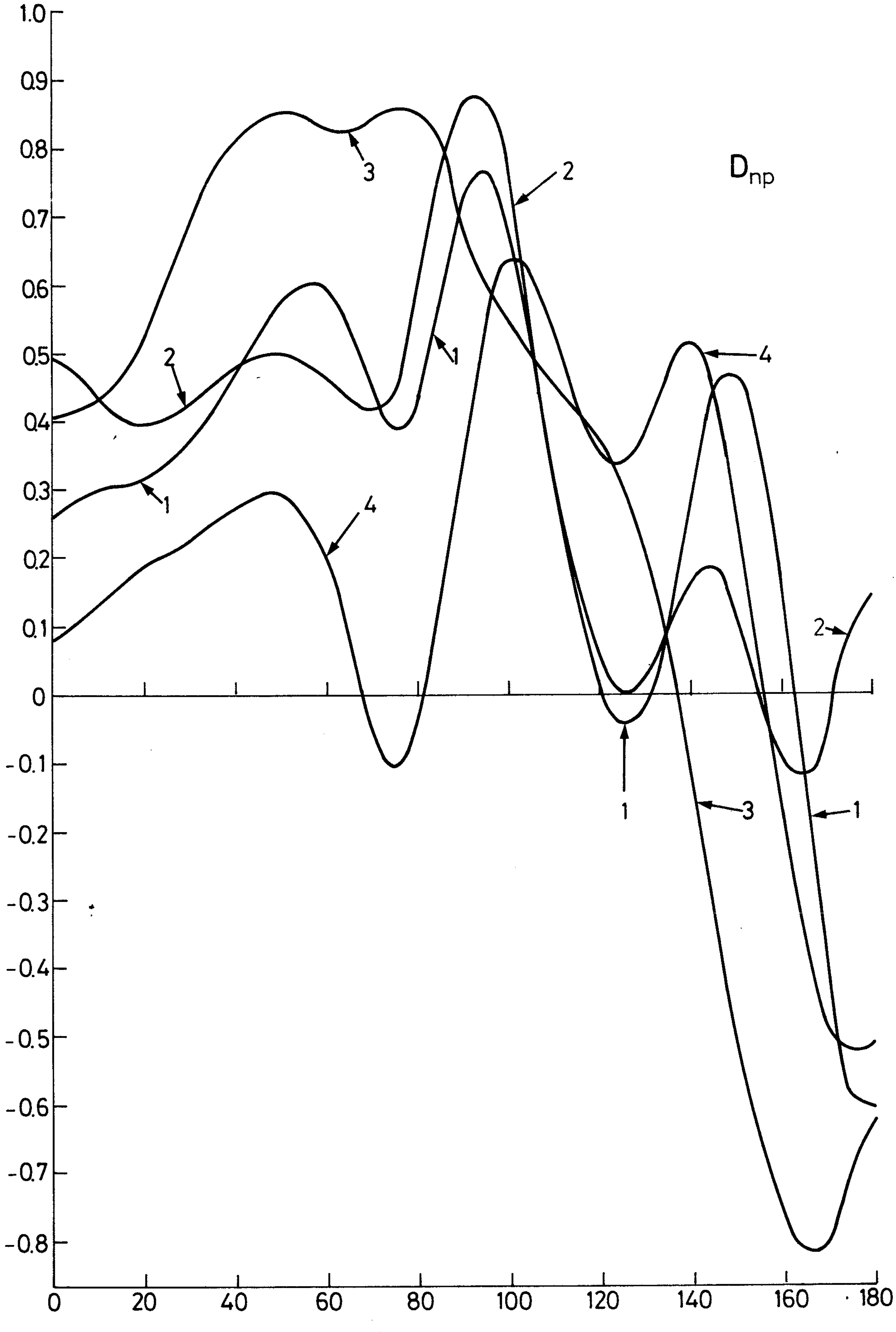

1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
0
20
40
60
80
100
120
140
160
180
Θ (degrees) ⟶
D_np
1
2
3
4

Diagram 3

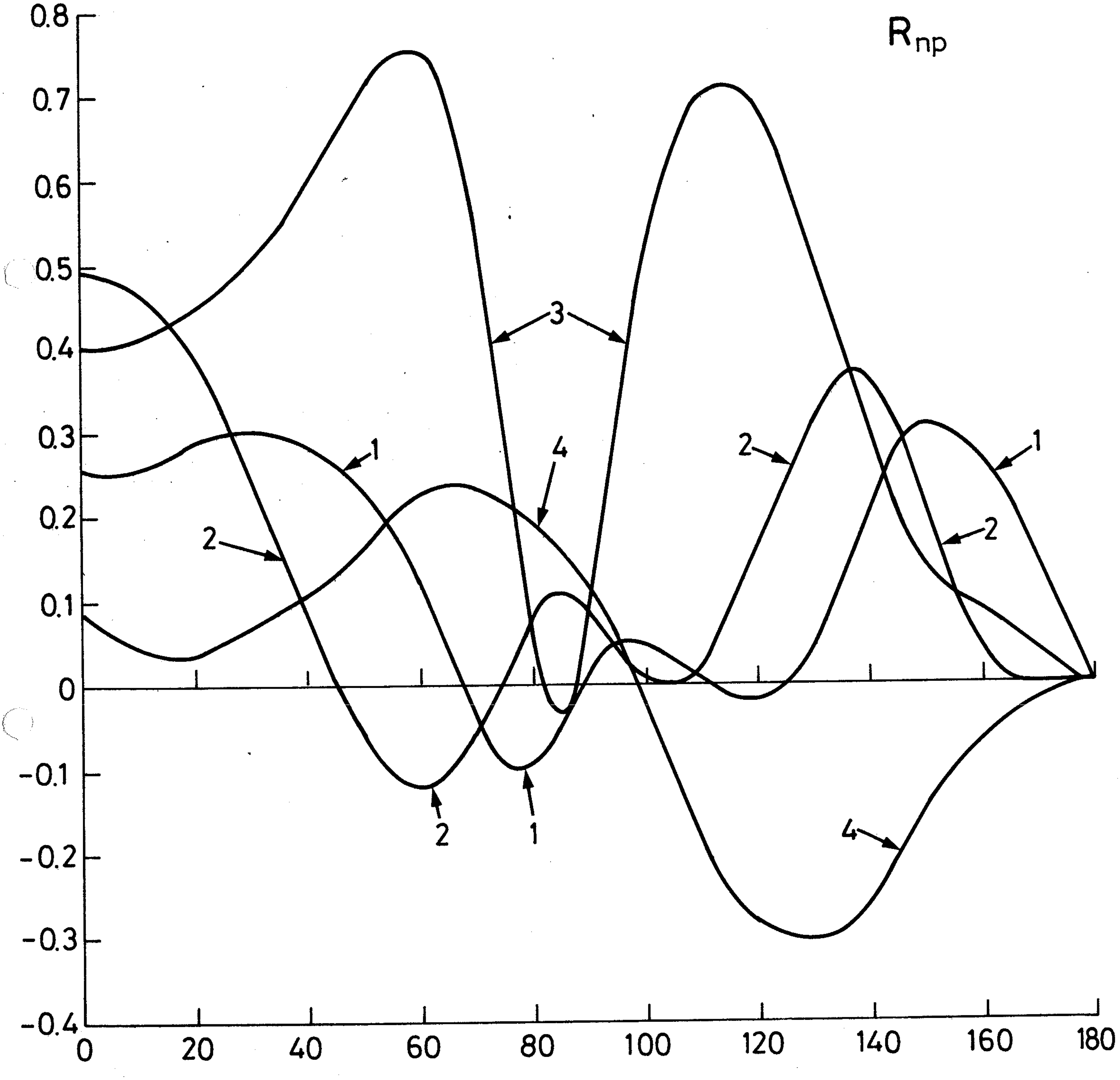

R_np
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
20
40
60
80
100
120
140
160
180
Θ (degrees) →
1
2
3
4
2
1
2
1
2
4

Diagram 4

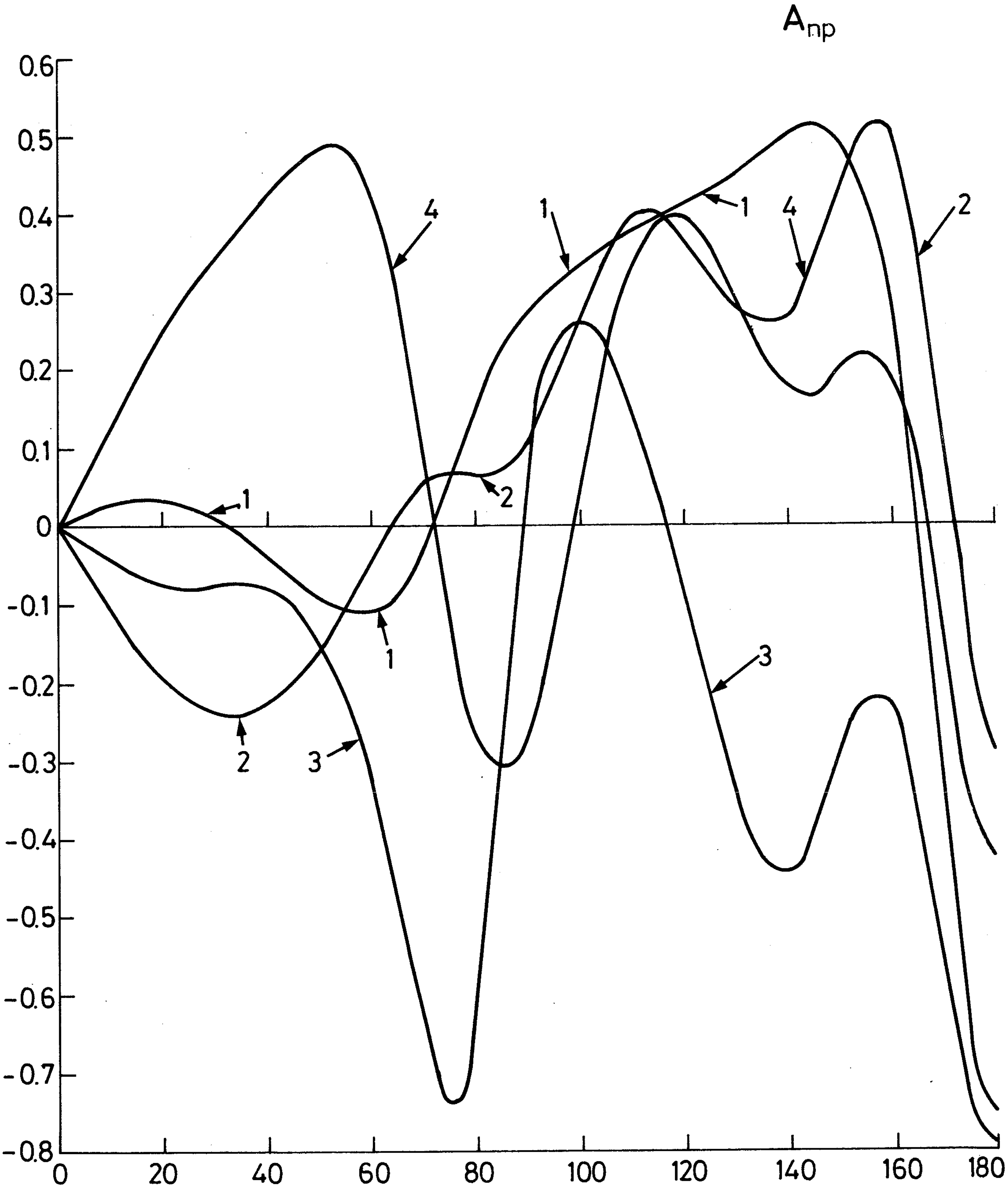

A_{np}
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
0
20
40
60
80
100
120
140
160
180
Θ (degrees)
1
2
3
4

Diagram 5

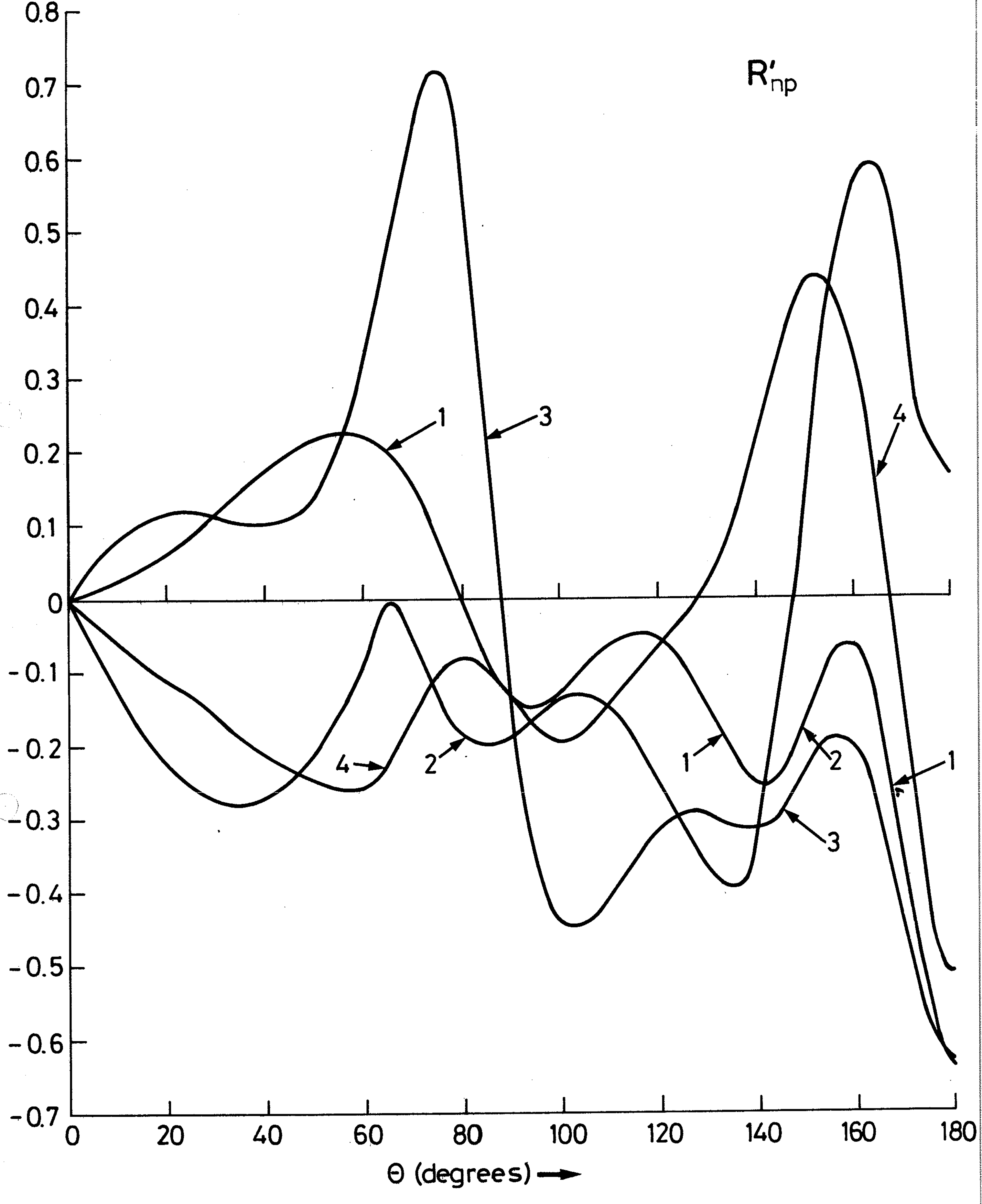

R'np
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
- 0.1
- 0.2
- 0.3
- 0.4
- 0.5
- 0.6
- 0.7
0
20
40
60
80
100
120
140
160
180
Θ (degrees)
1
3
4
2
4
1
2
1
3

Diagram 6

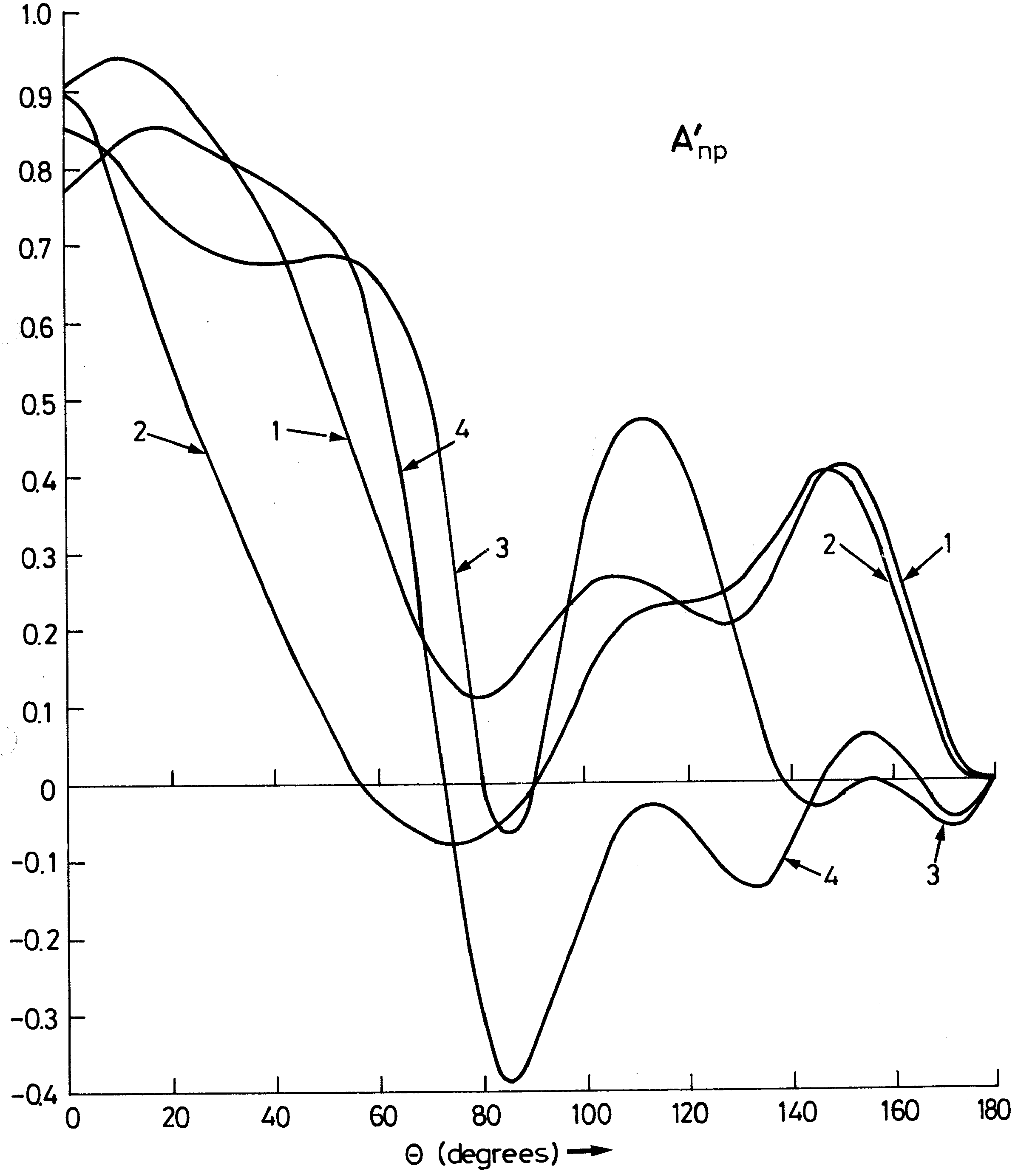

A'np
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
20
40
60
80
100
120
140
160
180
Θ (degrees)
1
2
3
4
2
1
4
3

Diagram 7

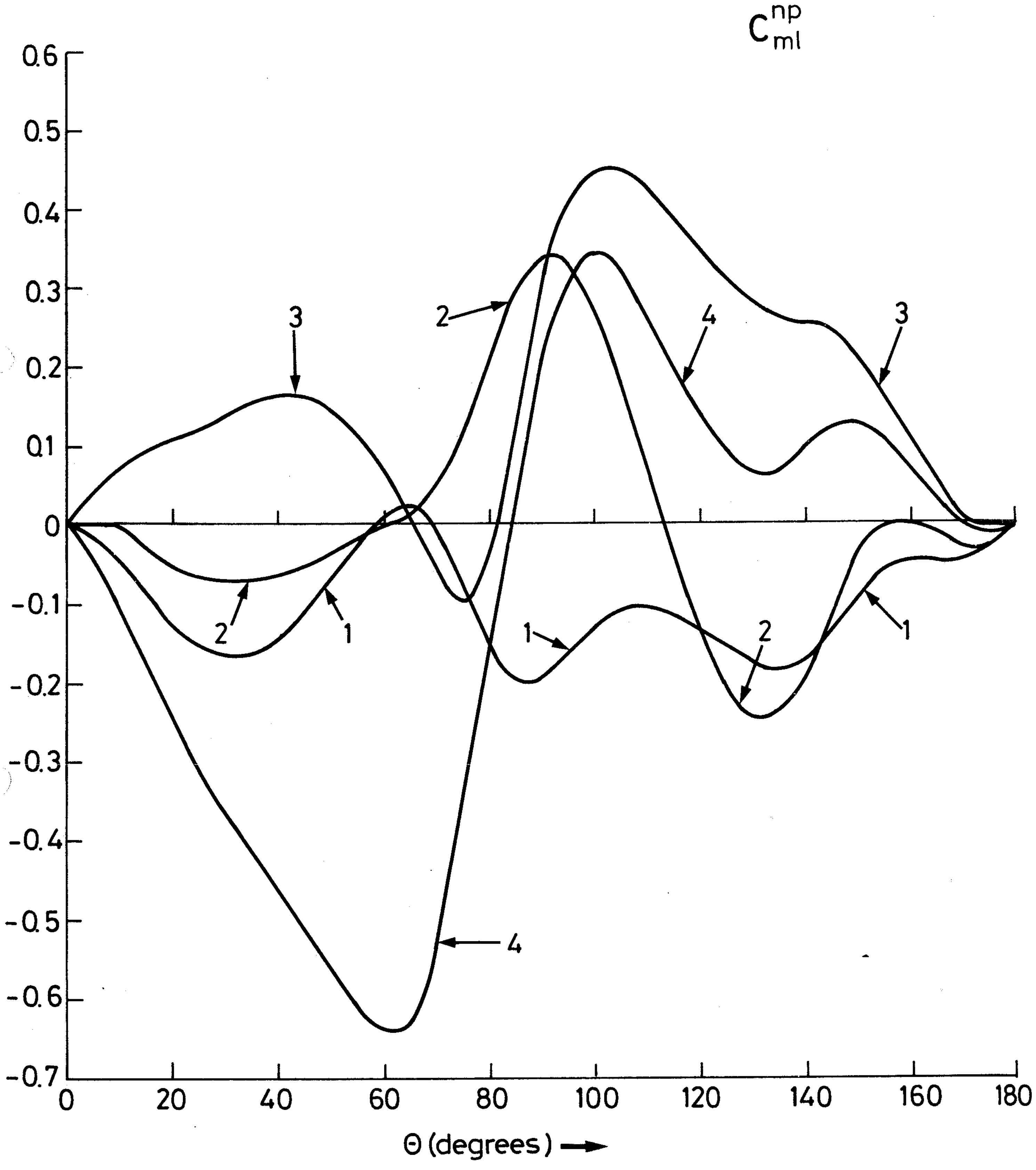
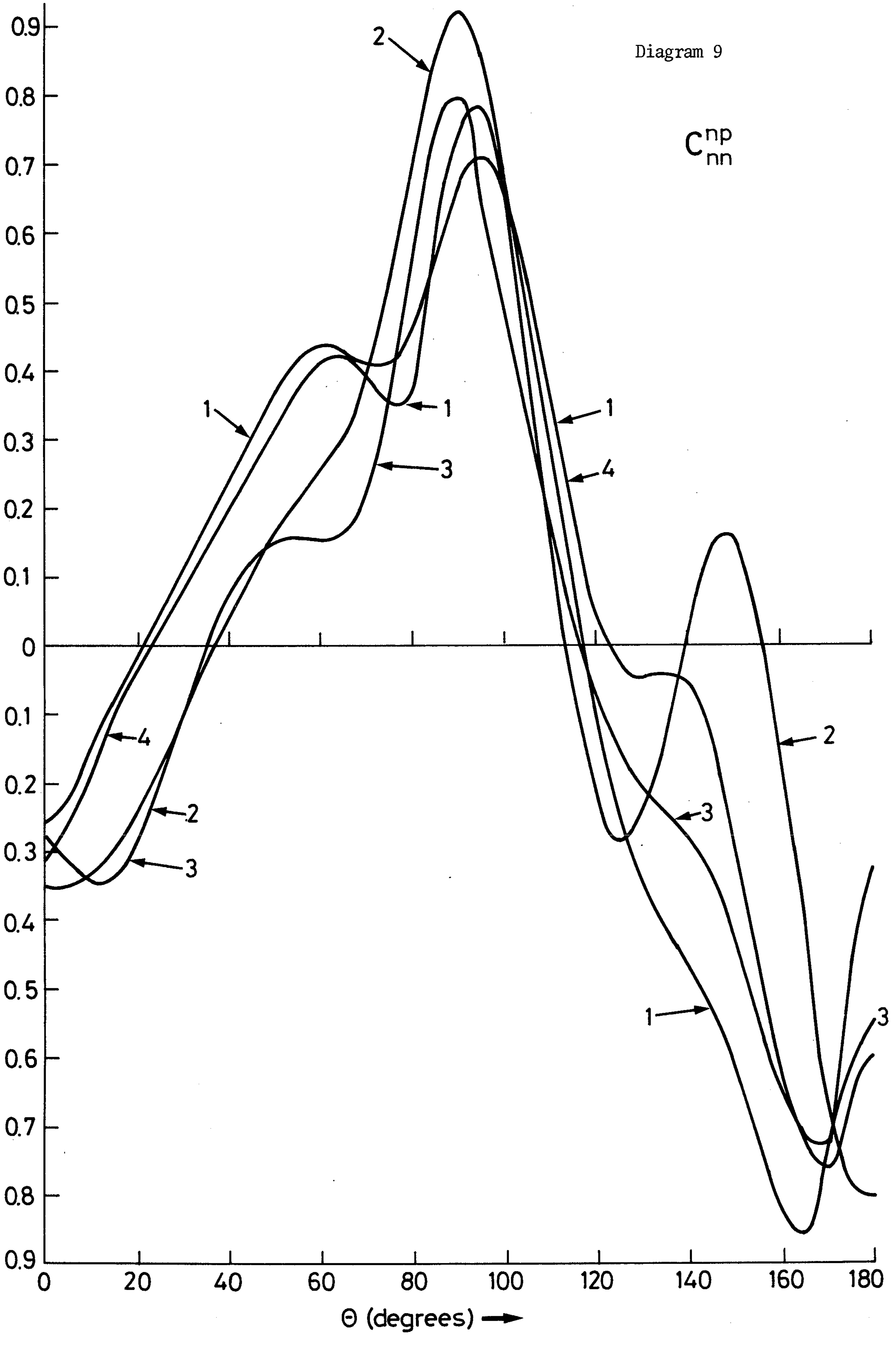

C_{ml}^{np}
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
0
20
40
60
80
100
120
140
160
180
Θ (degrees)
1
2
3
4

Diagram 8

Diagram 9


C_{nn}^{np}
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0
20
40
60
80
100
120
140
160
180
Θ (degrees)
1
2
3
4

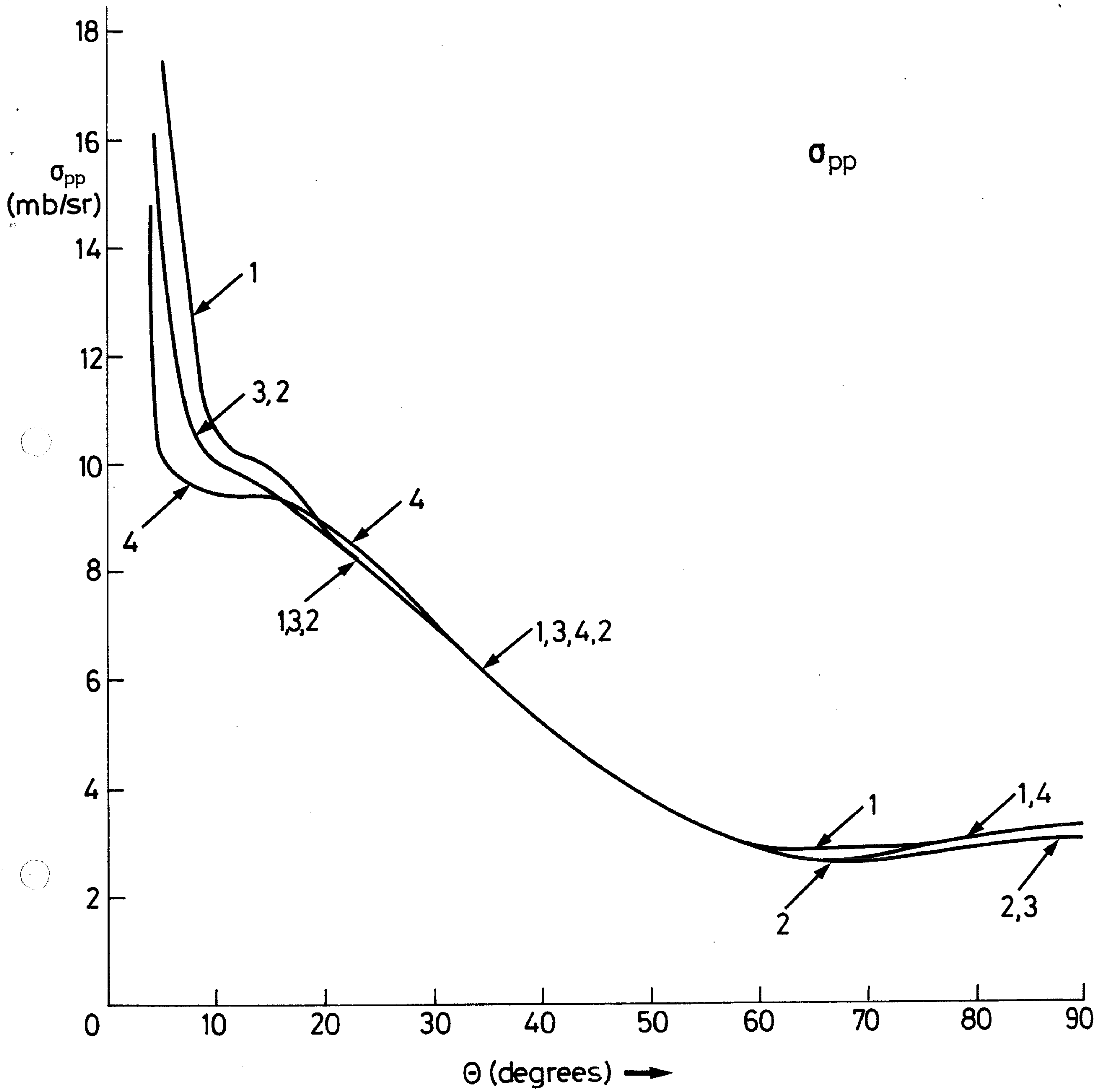

σpp
18
16
14
12
10
8
6
4
2
0
σpp
(mb/sr)
10
20
30
40
50
60
70
80
90
Θ (degrees)
1
3,2
4
4
1,3,2
1,3,4,2
1
1,4
2
2,3

Diagram 10

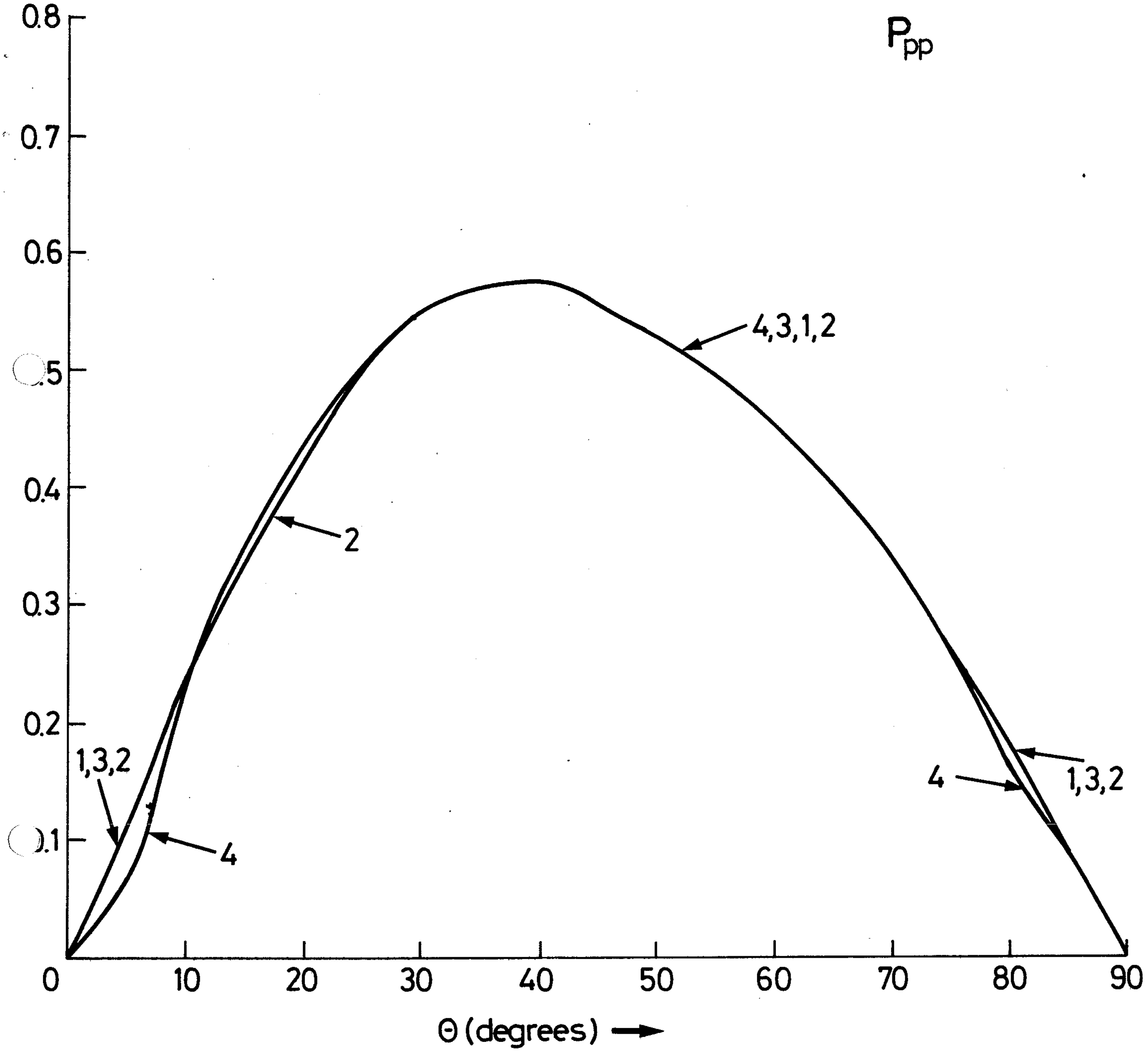
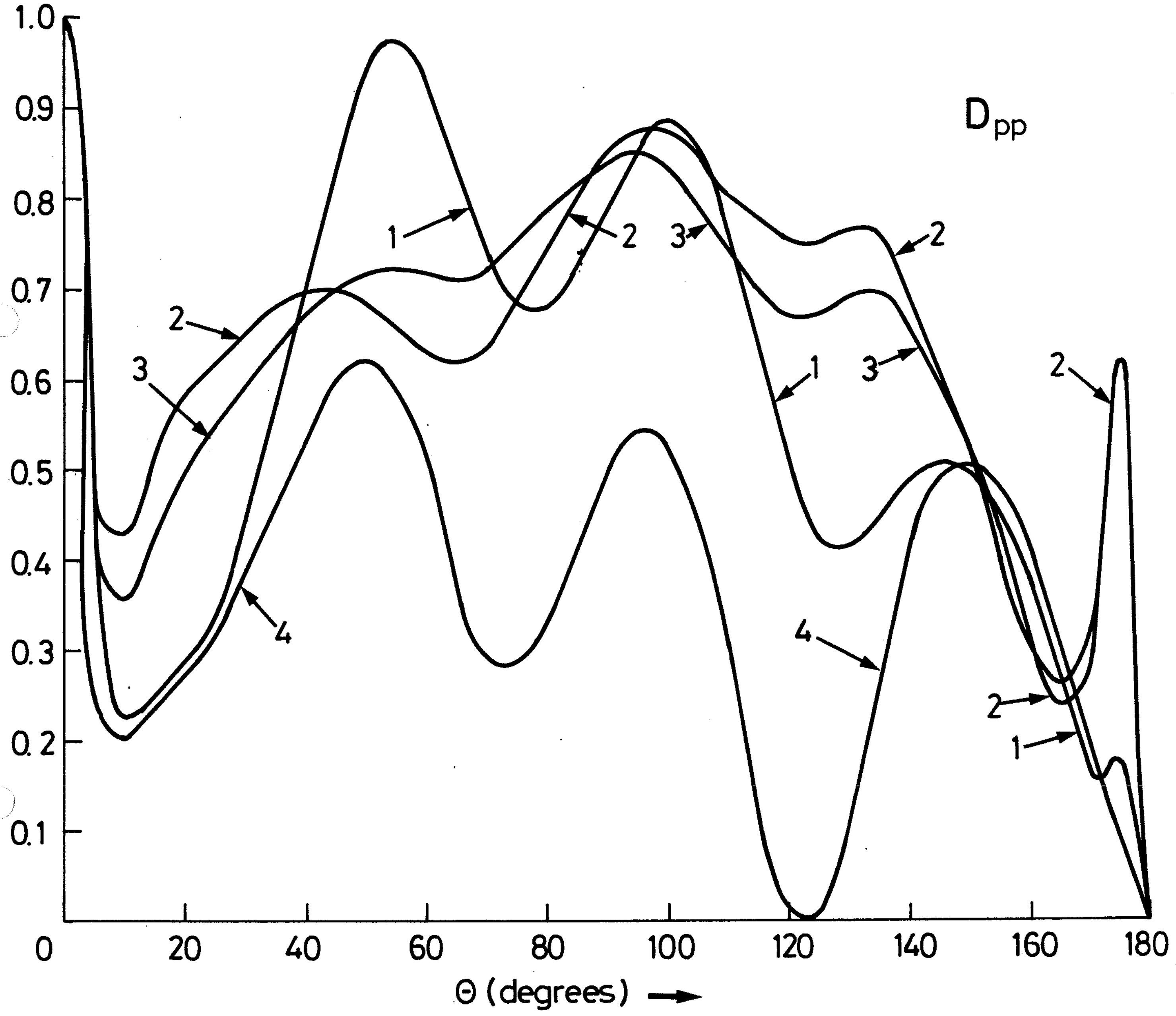

P_pp
0.8
0.7
0.6
.5
0.4
0.3
0.2
.1
0
10
20
30
40
50
60
70
80
90
Θ (degrees)
4,3,1,2
2
1,3,2
4
4
1,3,2

Diagram 11



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20
40
60
80
100
120
140
160
180
Θ (degrees)
D_{pp}
1
2
3
4

Diagram 12

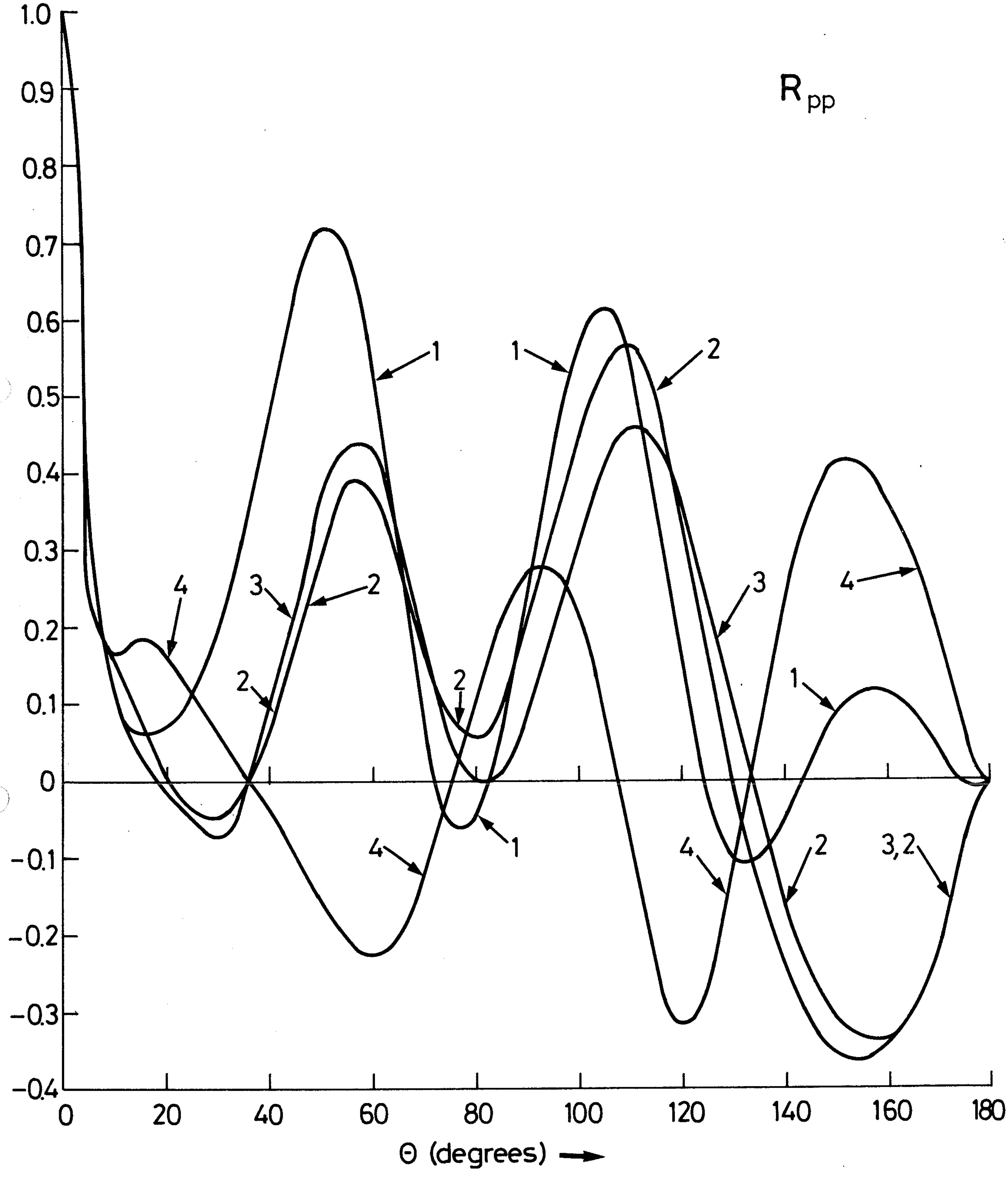

R_pp
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
20
40
60
80
100
120
140
160
180
Θ (degrees)
1
1
2
4
3
2
3
4
2
2
1
1
4
4
2
3,2
1

Diagram 13

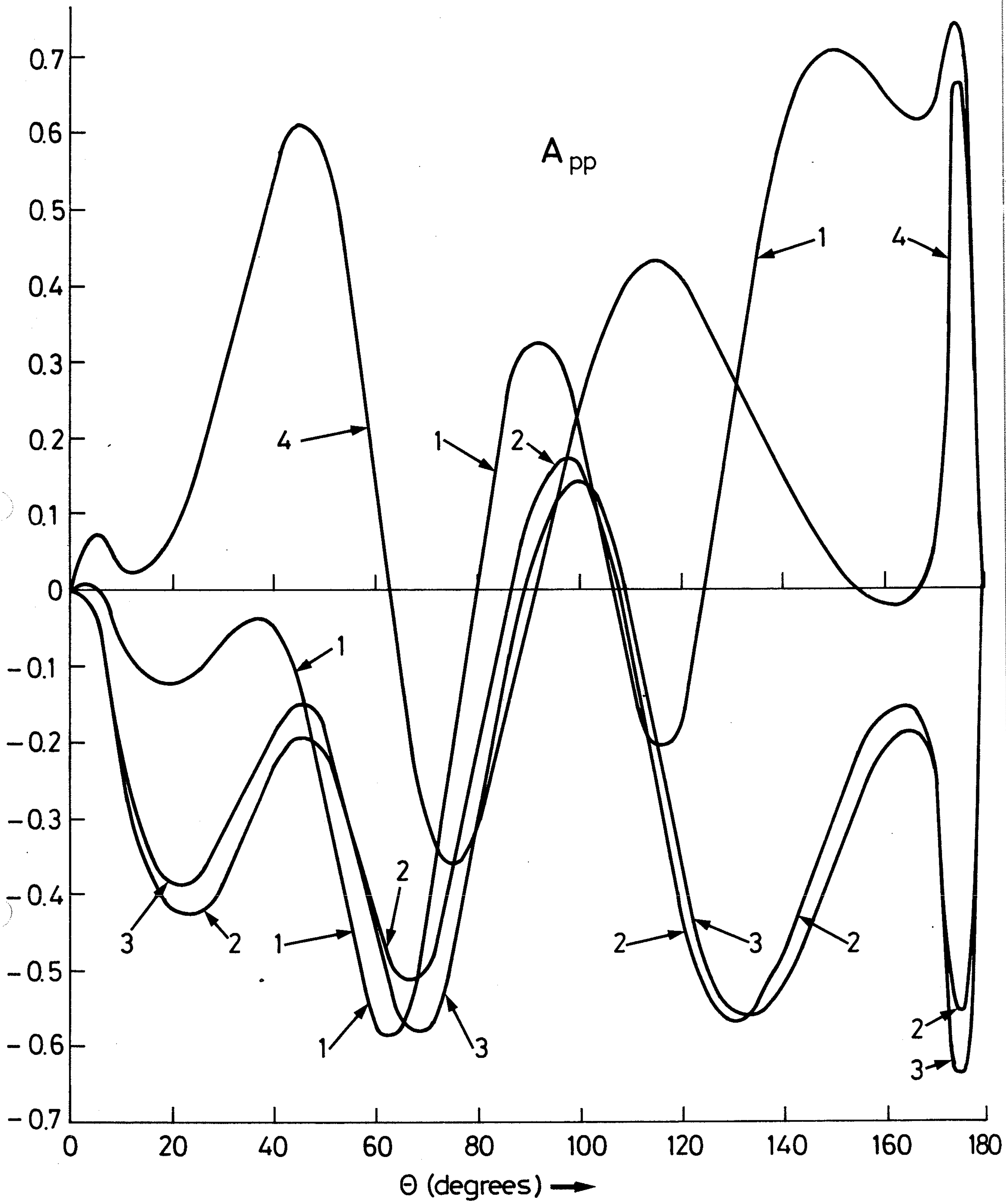

A_pp
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
0
20
40
60
80
100
120
140
160
180
Θ (degrees)
1
2
3
4

Diagram 14

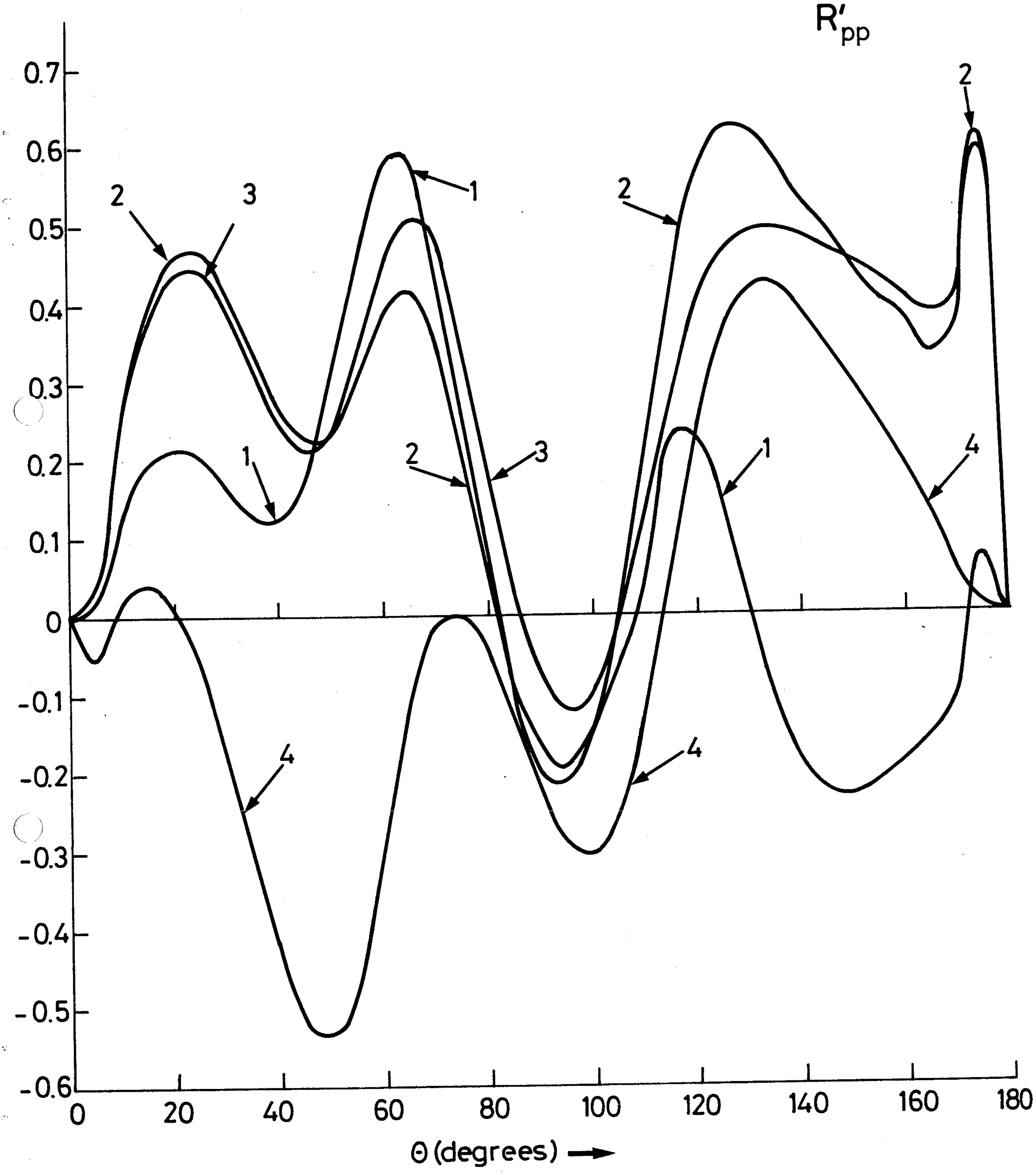

R'pp
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
0
20
40
60
80
100
120
140
160
180
Θ (degrees)
1
2
3
4

Diagram 15

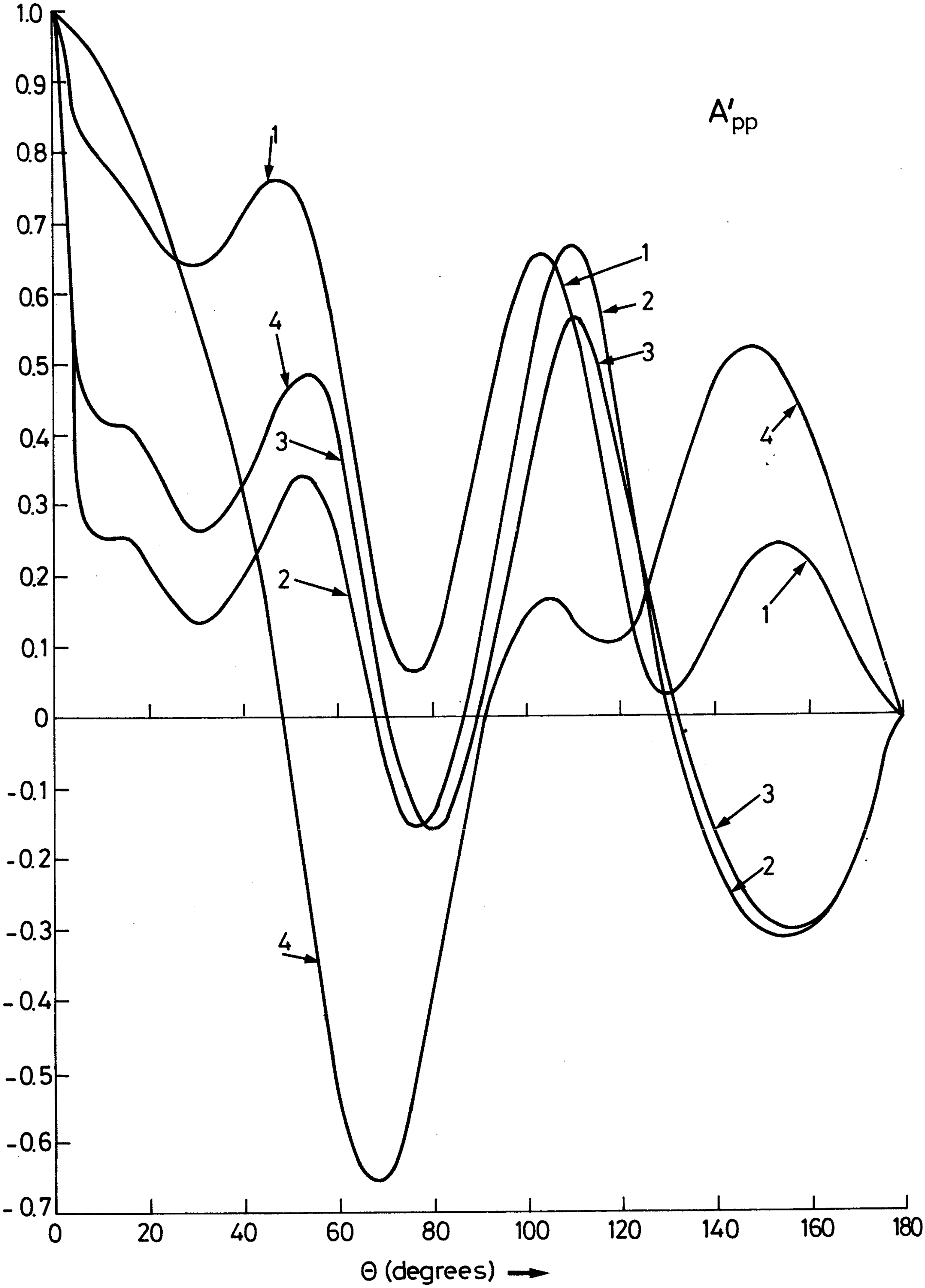

A′pp
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
0
20
40
60
80
100
120
140
160
180
Θ (degrees)
1
2
3
4

Diagram 16

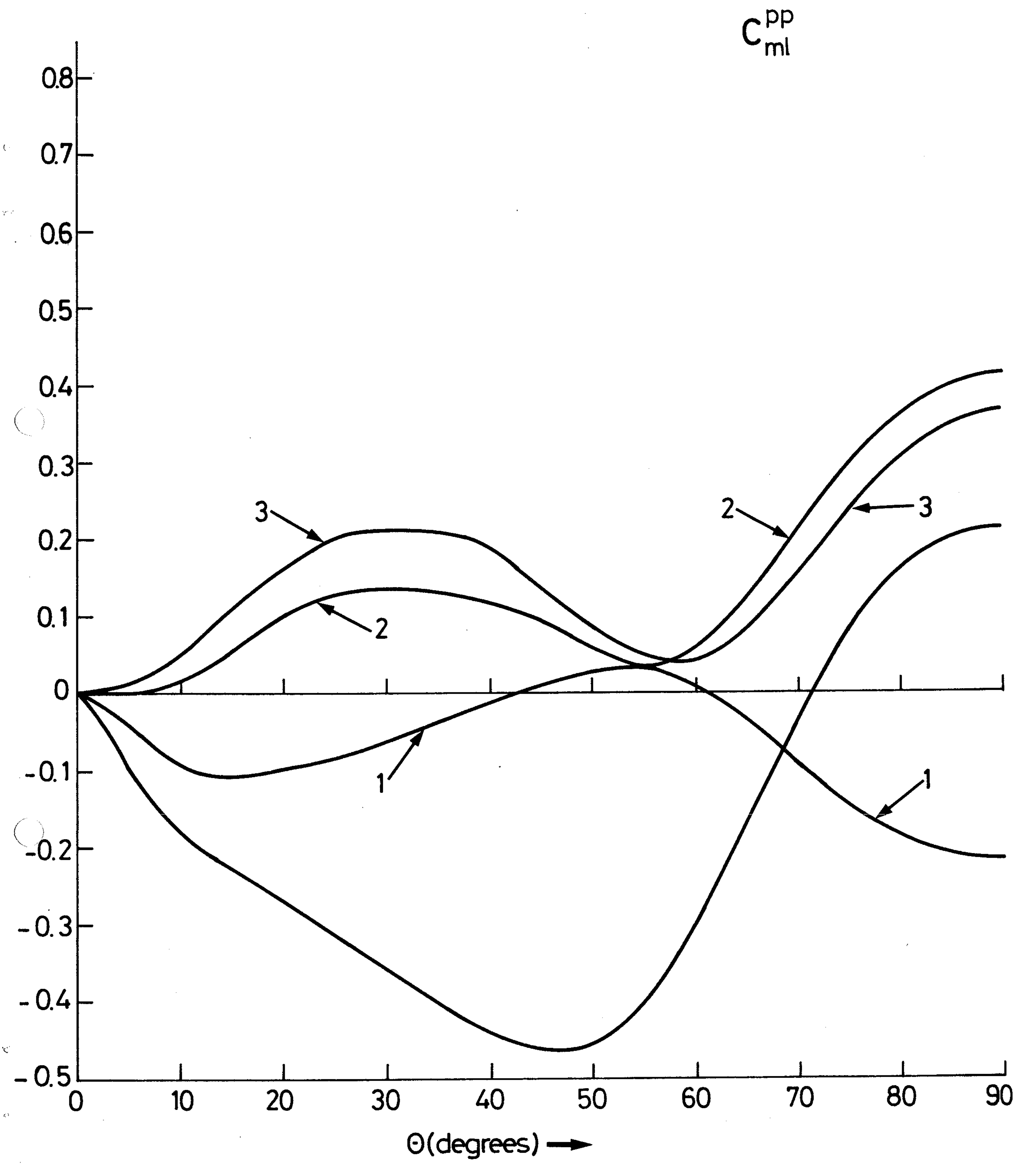
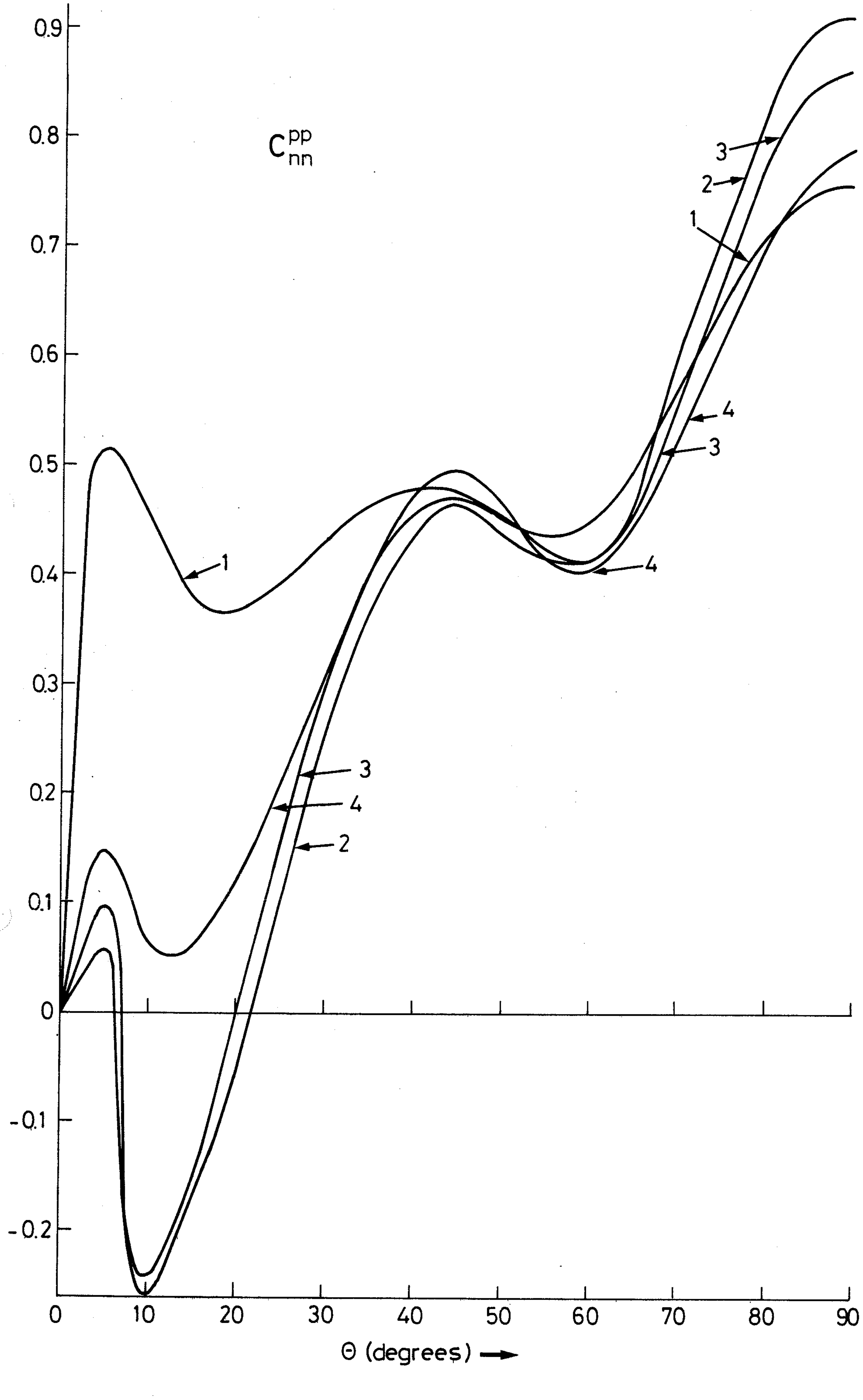

C_{ml}^{pp}
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
10
20
30
40
50
60
70
80
90
Θ(degrees)
1
2
3
1
2
3

Diagram 17



C_{nn}^{pp}
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
10
20
30
40
50
60
70
80
90
Θ (degrees) →
1
2
3
4

Diagram 18