PARTICLE PROPERTIES : January, 1969

Table S: STABLE PARTICLES. January, 1969.
(Closing date for data: November 1, 1968)

From Review of Particle Properties, UCRL-8030.
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Quantities in italics have changed by more than one (old) standard deviation since January, 1968.

Particle	$I^G(J^P)C$	Mass (MeV)	Mass difference (MeV)	Mean life (sec) $c\tau$(cm)	Mass² (GeV)²	Partial mode	Fraction	Q(MeV)	p or p_{max} (MeV/c)†
γ	0, 1(1⁻)⁻	0 (<2.×10⁻²¹ MeV)		stable	0	stable			
ν	ν_e, ν_μ J = ½	0(<0.2 keV) 0(<1.6MeV)		stable	0	stable			
e	J = ½	0.511006 ±.000002		stable (>2×10²¹y)	0.000	stable	μ_e = 1.001159557 ±.000000030 $e\hbar/2m_ec$		
μ	J = ½; μ_μ = 1.00116614 ±.00000031 $e\hbar/2m_\mu c$	105.659 ±.002	-33.920 ±.013	2.1983×10⁻⁶ ±.0008 cτ = 6.592×10⁴	0.011	$e\nu\bar\nu$	100 %	105	53
						eγγ	(<1.6)10⁻⁵	105	53
						3e	(<1.3)10⁻⁷	104	53
						eγ	(<6)10⁻⁹	105	53
π±	1 (0⁻)	139.578 ±.013	4.6041 ±.0037	2.604×10⁻⁸ ±.007, S=2.3* cτ = 781	0.019	μν	100 %	34	30
				$(\tau^+-\tau^-)/\bar\tau$ = (0.09±0.07) %		eν	(1.24±0.03)10⁻⁴	139	70
				(test of CPT)		μνγ	c(1.24±0.25)10⁻⁴	34	30
						π⁰eν	(1.02±0.07)10⁻⁸	4	5
						eνγ	c(3.0 ±0.5)10⁻⁸	139	70
π⁰	1⁻(0⁻)⁺	134.975		c 0.89×10⁻¹⁶ ±.18, S=1.6* cτ = 2.67×10⁻⁶	0.018	γγ	(98.83±0.04)%	135	67
						γe⁺e⁻	(1.17±0.04)%	134	67
						γγγ	(<5)10⁻⁶	135	67
						e⁺e⁻e⁺e⁻	b(3.47)10⁻⁵	133	67
K±	½(0⁻)	493.82 ±0.11	-3.94 ±0.13	1.235×10⁻⁸ ±.004 S = 1.8* cτ = 370	0.244	μν	(63.65±0.29)% S = 1.1*	388	236
				$(\tau^+-\tau^-)/\bar\tau$=(.09±.12)%		ππ⁰	(21.03±0.30)% S = 1.2*	219	205
				(test of CPT) S=1.3*		πππ⁺	(5.57±0.04)% S = 1.2*	75	126
						ππ⁰π⁰	(1.70±0.05)%	84	133
						μπ⁰ν	(*3.18 ± 0.11*)% S = 2.4*	253	215
						eπ⁰ν	(4.86±0.07)% S = 1.2*	358	228
						ππ±e∓ν	(3.4 ±0.3)10⁻⁵	214	203
						ππ±e±ν	(<7)10⁻⁷	214	203
						ππ∓μ±ν	(1.0 ±0.5)10⁻⁵	109	151
						ππ±μ±ν	(<3)10⁻⁶	109	151
						eν	(1.24±0.40)10⁻⁵	493	247
						ππ⁰γ	c(2.2 ±0.7)10⁻⁴	219	205
						ππ⁺π⁻γ	c(10 ±4)10⁻⁵	75	126
						πeνγ	c(6 ±4)10⁻⁴	354	227
						πe⁺e⁻	(<1.1)10⁻⁶	353	227
						πμ⁺μ⁻	(<2.4)10⁻⁶	143	172
						πγγ	(<1.1)10⁻⁴	354	227
K⁰	½(0⁻)	497.76 ±0.16		50% K_{short}, 50% K_{long}					
K^0_S	½(0⁻)	S = 1.5*	-0.469×$\frac{1}{\tau_S}$ ±.015	0.862×10⁻¹⁰ ±.006 S=1.2* cτ = 2.59	0.248	π⁺π⁻	(68.4 ±1.1)%	219	206
						π⁰π⁰	(31.6 ±1.1)%	228	209
K^0_L	½(0⁻)			5.38×10⁻⁸ ±.20 cτ = 1614 S = 1.7*	0.248	π⁰π⁰π⁰	(*21.5 ± 0.7*)% S=1.2*	93	139
						π⁺π⁻π⁰	(*12.7 ± 0.3*)% S=1.1*	84	133
						πμν	(28.1 ±0.8)%	253	216
						πeν	(*37.7 ± 0.9*)%	358	229
						π⁺π⁻	(*0.157 ± 0.004*)%	219	206
						c) π⁰π⁰	still uncertain	228	209
						π⁺π⁻γ	c(< 0.4)10⁻³	219	206
						γγ	(5.2 ±0.5)10⁻⁴	498	249
						eμ	(< 0.6)10⁻⁵	392	238
						μ⁺μ⁻	(< 1.5)10⁻⁶	286	225
						e⁺e⁻	(< 1.7)10⁻⁵	497	249
η	0⁺(0⁻)⁺	548.8 ±0.6		Γ=(2.63±.64)keV	Neutral decays 71.1%	γγ	(*38.1 ± 2.3*)%	549	274
						π⁰γγ	(*2.5 ± 2.8*)% S=1.3*	414	258
						3π⁰	(*29.4 ±2.8*)% (c)	144	179
					Charged decays 28.9%	π⁺π⁻π⁰	(23.3 ±1.1)% S=1.2*	135	174
						π⁺π⁻γ	(5.5 ±0.5)%	269	236
						π⁰e⁺e⁻	(< 0.01)%	413	258
						π⁺π⁻e⁺e⁻	(0.1 ±0.1)%	268	236
p	½(½⁺)	938.256 ±0.005	-1.2933 ±.0001	stable (>2×10²⁸ y)	0.880				
n	½(½⁺)	939.550 ±0.005		*(0.932 ± 0.014)10³* cτ = 2.80×10¹³	0.882	pe⁻ν	100 %	1	1
Λ	0(½⁺)	*1115.60 ±0.08* S=1.3*		2.51×10⁻¹⁰ ±.03 S=1.3* cτ = 7.54	1.245	pπ⁻	(65.3 ±1.2)%	38	100
						nπ⁰	(34.7 ±1.2)%	41	104
						peν	(0.85±0.08)10⁻³ S=1.3*	177	163
						pμν	(1.35±0.60)10⁻⁴	72	131
Σ⁺	1(½⁺)	1189.40 ±0.19 S=1.7*	-7.92 ±.13	0.810×10⁻¹⁰ ±.013 cτ = 2.43	1.412	pπ⁰	(52.8 ±1.5)%	116	189
						nπ⁺	(47.2 ±1.5)%	110	185
						pγ	(1.9 ±0.5)10⁻³	251	225
						nπ⁺γ	c(0.9 ±0.3)10⁻⁴	110	185
						Λe⁺ν	(2.11±0.45)10⁻⁵	73	72
				$\frac{\Gamma(\Sigma^+\to\ell^+n\nu)}{\Gamma(\Sigma^-\to\ell^-n\nu)}$ = < .04		nμ⁺ν	(< 0.36)10⁻⁴	144	202
						ne⁺ν	(< 0.19)10⁻⁴	249	224
Σ⁰	1(½⁺)	1192.46 ±0.12 S=1.2*	-4.86 ±.07	<1.0×10⁻¹⁴ cτ<3×10⁻⁴	1.422	Λγ	100 %	77	75
						Λe⁺e⁻	b(5.45)10⁻³		
Σ⁻	1(½⁺)	1197.32 ±0.11 S=1.3*		1.64×10⁻¹⁰ ±.06 S=2.4* cτ = 4.92	1.434	nπ⁻	100 %	118	193
						ne⁻ν	(1.08±0.05)10⁻³	257	230
						nμ⁻ν	(*0.48 ± 0.06*)10⁻³	152	210
						Λe⁻ν	(0.60±0.06)10⁻⁴	81	79
						nπ⁻γ	c(<1)10⁻³	118	193
Ξ⁰	½(½⁺)§	1314.7 ±0.7	-6.6 ±.7	3.03×10⁻¹⁰ ±.18 cτ = 9.10	1.728	Λπ⁰	100 %	64	135
						pπ⁻	(< 0.9)10⁻³	237	299
						pe⁻ν	(< 1.3)10⁻³	376	323
						Σ⁺e⁻ν	(< 1.5)10⁻³	125	119
						Σ⁻e⁺ν	(< 1.5)10⁻³	117	112
						Σ⁺μ⁻ν	(< 1.5)10⁻³	20	64
						Σ⁻μ⁺ν	(< 1.5)10⁻³	12	49
						pμ⁻ν	(< 1.3)10⁻³	271	309
Ξ⁻	½(½⁺)§	1321.25 ±0.18		*1.66×10⁻¹⁰ ±.03* S=1.1* cτ = 4.98	1.746	Λπ⁻	100 %	66	139
						Λe⁻ν	d(0.67±0.23)10⁻³	205	190
						Σ⁰e⁻ν	(<0.5)10⁻³	128	122
						Λμ⁻ν	(<1.3)10⁻³	100	163
						Σ⁰μ⁻ν	(<0.5)%	23	70
						nπ⁻	(<1.1)10⁻³	242	303
						ne⁻ν	(<1.0)%	381	327
Ω⁻	0(3/2⁺)§	1672.4±.6 S=1.1*		$1.3^{+0.4}_{-0.3}$×10⁻¹⁰ cτ = 3.9	2.797	Ξ⁰π⁻	8 events seen	217	293
						Ξ⁻π⁰	3 events seen	216	289
						ΛK⁻	13 events seen	63	210

General Atomic and Nuclear Constants[a]

N	= 6.02252×10²³mole⁻¹(based on A_C12=12)
c	= 2.997925×10¹⁰cm sec⁻¹
e	= 4.80298×10⁻¹⁰ esu = 1.60210×10⁻¹⁹ coulomb
1 MeV	= 1.60210×10⁻⁶ erg
ħ	= 6.5819 ×10⁻²² MeV sec = 1.05449×10⁻²⁷ erg sec
ħc	= 1.9732 ×10⁻¹¹ MeV cm = 197.32 MeV fermi
$k_{Boltzmn}$	= 8.6171×10⁻¹¹ MeV/° = 1 eV/11605°K
α	= e²/ħc = 1/137.0388
m_e	= 0.511006 MeV/c² = 1/1836.10 m_p
m_p	= 938.256 MeV/c² = 1836.10 m_e = 6.721 $m_{\pi\pm}$ = 1.00727663 m_1(where m_1 = 1 amu = $\frac{1}{12}m_C$12 = 931.478 MeV/c²)
r_e	= e²/m_ec^2 = 2.81777 fermi (1 fermi = 10⁻¹³cm)
$ƛ_e$	= ħ/m_ec = $r_e\alpha^{-1}$ = 3.86144×10⁻¹¹ cm
a_∞ Bohr	= ħ²/m_ee^2 = $r_e\alpha^{-2}$ = 0.529167 A(1 A=10⁻⁸ cm)
$\sigma_{Thomson}$	= $\frac{8}{3}\pi r_e^2$ = 0.66516×10⁻²⁴ cm² = 0.66516 barn
μ_{Bohr}	= eħ/2m_ec = 0.578817×10⁻¹⁴ Mev gauss⁻¹
μ_{nucl}	= eħ/2m_pc = 3.1524×10⁻¹⁸ Mev gauss⁻¹

½$\omega_{cyclotron}$ (rad sec⁻¹gauss⁻¹) = e/2m_ec = 8.79404×10⁶
= e/2m_pc = 4.7895 ×10³

(continued on other sheet)

[a] Based mainly on E. R. Cohen and J. W. M. DuMond, Rev. Mod. Phys. 37, 537 (1965). Note that recent fine structure and ac Josephson effect measurements currently indicate that α is ~20 ppm larger than the value given above. The corresponding readjustment of the fundamental constants will increase e by 60 ppm, ħ by 100 ppm, m_e by 60 ppm, and decrease N by 60 ppm. For a preliminary analysis see W. H. Parker, B. N. Taylor, and D. N. Langenberg, Phys. Rev. Lett. 18, 287 (1967).

Partial Rates (sec⁻¹)

Particle	Mode	Partial rate
K±	μν	(51.54±0.30)10⁶ S = 1.2*
	ππ⁰	(17.03±0.26)10⁶ S = 1.2*
	πππ⁺	(4.52±0.03)10⁶ S = 1.1*
	ππ⁰π⁰	(1.38±0.04)10⁶
	μπ⁰ν	(*2.58 ± 0.11*)10⁶ S = 2.4*
	eπ⁰ν	(3.93±0.06)10⁶ S = 1.2*
K^0_S	π⁺π⁻	(0.794±0.013)10¹⁰
	π⁰π⁰	(0.367±0.013)10¹⁰
K^0_L	π⁰π⁰π⁰	(*4.00 ± 0.20*)10⁶ S=1.3*
	π⁺π⁻π⁰	(2.36 ±0.10)10⁶ S=1.4*
	πμν	(5.22 ±0.25)10⁶ S=1.3*
	πeν	(*7.01 ± 0.30*)10⁶ S=1.4*
	π⁺π⁻	(0.029±0.001) 10⁶

CP violation parameters

$\eta_{+-\,(00)} = \frac{A(K_L\to\pi^+\pi^-\,(\pi^0\pi^0))}{A(K_S\to\pi^+\pi^-\,(\pi^0\pi^0))} = |\eta_{+-\,(00)}|\,e^{i\Phi_{+-\,(00)}}$

$|\eta_{+-}|$ = (1.92±0.04)×10⁻³

Φ_{+-} = (50±8)°

$|\eta_{00}|$ still uncertain, see data listings for published values

Decay Parameters †

Particle	Magnetic Moment (eħ/2m_pc)	Measured α	Measured Φ(degree)	Derived γ	Derived Δ(degree)
p	2.792763 ±.000030				
n	-1.913148 ±.000066	g_A/g_V = *-1.231 ± .010* †			δ = (176.1±6.4)° †
Λ	-0.73 ±.16 S=1.2*	0.646±0.016 0.71 ±0.18	(-6.3±3.5)°	0.76	(7.5±3.9)°
	For Λ → peν, g_A/g_V = -0.97 $^{+0.14}_{-0.22}$ †				
Σ⁺	2.5 ±.5	-.955±0.070 S=1.1* +.018±0.039	(161±21)°	-0.95	(-88$^{+175}_{-2}$)°
Σ⁻		-.06 ±0.05 S=1.6*	(4±17)°	1.0	(230$^{+31}_{-125}$)°
	For Σ⁻ → ne⁻ν, g_A/g_V = 0.28±0.16†				
	For Σ⁻ → Λeν, g_V/g_A = 0.3±0.3†				
Ξ⁰		-.35±0.08	(25±21)° S=1.3*	0.85	(229$^{+14}_{-38}$)°
Ξ⁻		-.41 ±.04	(-3±9)° S=1.3*	0.90	(172±18)°

† The definition of these quantities is as follows: [for more details on sign convention, see text]

$\alpha = \frac{2|s||p|\cos\Delta}{|s|^2+|p|^2}$; $\beta = \sqrt{1-\alpha^2}\sin\Phi$; $\beta = \frac{-2|s||p|\sin\Delta}{|s|^2+|p|^2}$; $\gamma = \sqrt{1-\alpha^2}\cos\Phi$

g_A/g_V defined by: $\langle B_f|\gamma_\lambda(g_V - g_A\gamma_5)|B_i\rangle$

δ defined by: $g_A/g_V = |g_A/g_V|e^{i\delta}$

* S = Scale factor = $\sqrt{\chi^2/(N-1)}$ where N ≈ number of experiments. S should be ≈ 1. If S > 1, we have enlarged the error of the mean, δx, i.e., δx→S δx. This new convention is still inadequate, since if S > 1, the real uncertainty is probably even greater than Sδx. See text of January 1967 edition.

† In decays with more than two bodies, P_{max} is the maximum momentum that any particle can have. § Predicted from SU(3).

b. Theoretical value, see also data card listings. d. Assumes rate for Ξ⁻ → Σ⁰e⁻ν small compared with Ξ⁻ → Λe⁻ν.

c. See note in data card listings.

e. See data card listings for energy limits used in measuring this branching ratio.

(f) Reported values range between 1% and 10%, and depend on assumptions on ρ-ω interference.

(g) This $\omega \to e^+e^-$ value is the average from a $\pi^- p \to e^+e^- n$ experiment (giving 0.0040±.0015% [±.0014% from possible $\rho\omega$ interference]) and an $e^+e^- \to \pi^+\pi^-\pi^0$ experiment (giving 0.0085±.0016%).

(h) This 0^- meson was named η' on discovery, when it looked as if it completed the 0^- nonet. With the recent evidence that the E(1420) is probably also 0^-, it is no longer clear whether η' or E or both are mixed in with the π, η, K octet; so the name η' may be misleading.

(i) Empirical limits on fractions for other decay modes of η'(958): $\pi^+\pi^-$ < 7%, 3π < 7%, 4π < 1%, 6π < 1%, $\pi^+\pi^-e^+e^-$ < 0.6%, $\pi^0e^+e^-$ < 1.3%, ηe^+e^- < 1.1%, $\pi^0\rho^0$ < 4%, $\pi^0\omega$ < 8%.

(j) Empirical limits on fractions for other decay modes of ϕ(1019): $\pi^+\pi^-$ < 20%, $\eta\gamma$ < 8%, η + neutrals < 13%, $\pi^+\pi^-\gamma$ < 4%, $\omega\gamma$ < 5%, $\rho\gamma$ < 2%.

(k) Width of $\eta_{0+}(1070) \to K_S K_S$: Average value from two bubble chamber experiments is Γ = (72 ± 13) MeV, whereas two spark chamber experiments give Γ>100 MeV. The latter also allow a scattering length fit. It is not clear whether the reported narrow ($\Gamma \lesssim$ 25 MeV) $\pi^+\pi^-$ enhancements near 1070 MeV have anything to do with the η_{0+}(1070).

(ℓ) $\rho\pi$ fraction of 3π mode difficult to distinguish because ρ bands cover most of the Dalitz plot.

(m) Empirical limits on fractions for decay modes of B(1220): $\pi\pi$ < 30%, $K\bar{K}$ < 2%, 4π < 50%, $\phi\pi$ < 1.5%, $\eta\pi$ < 25%, $(KK)^\pm \pi^0$ < 8%, $K_S K_S \pi^\pm$ < 2%, $K_S K_L \pi^\pm$ < 6%.

(n) Although the splitting of the A2 needs further confirmation, we give the results from the two published experiments that have observed a split A2. Since most experiments have only seen one, rather wide, A2 enhancement, we here list its ("combined") properties: $I^G(J^P)C_n = 1^-(2^+)+$; M = 1297±10 MeV (S=1.8*) (§), Γ = 91±10 MeV (S=1.1*) (§); partial decay modes: $\rho\pi$ 86±2%, $K\bar{K}$ 2.4 ± 0.5%, $\eta\pi$ 11±2%, $\eta'\pi$ 0.5±0.4% (S=2.1*); $\pi^+\pi^-\pi^0$ (excl. $\rho\pi$) < 17%.

(o) There is only a weak indication for a $K^*\bar{K} + \bar{K}^*K$ mode of the f'(1514). If this mode does not exist, the $K\bar{K}$ branching fraction will have to be reported as (80 ± 13)% (rather than (72 ± 12)% as given in the table), and $\eta\pi\pi$ as (20 ± 13)%.

(p) See the listings for many statistically weak Y = 0 bumps with M ≥1700 MeV, seen in bubble chambers. We tabulate here 9 statistically strong bumps seen with a missing mass spectrometer ($\pi^- p \to p(MM)^-$) or in HBC or counter experiments on $N\bar{N}$ elastic scattering or total cross sections.

	Name	I	M (MeV)		Γ (MeV)	Decay Modes Observed	
	R1(1630)	≥1	1630 ± 15		≤ 21	1/3/>3 charg. part. ≈ .37/.59/.04	(MMS)$^-$
	R2(1700)	≥1	1700 ± 15		≤ 30	1/3/>3 charg. part. ≈ .43/.56/.01	(MMS)$^-$
	R3(1750)	≥1	1748 ± 15		< 38	1/3/>3 charg. part. > .14/<.80/.15	(MMS)$^-$
?	$N\bar{N}$(1925)	0, 1	≈ 1925		≈ 10	structure in $p\bar{p}$ backw. el. scatt.	(MMS)$^-$
	S (1930)	≥1	1929 ± 14		< 35	1/3/>3 charg. part. ≈ 0/.92/ 0	(MMS)$^-$
?	$N\bar{N}$(1945)	0, 1	≈ 1945		≈ 22	structure in $p\bar{p}$ backw. el. scatt.	
?	$N\bar{N}$(2190)	1	2190 ± 10	! ⇄	≈ 85	structure in $N\bar{N}$ total cross section	
	T(2200)	≥1	2195 ± 15		≤ 13	(MM)$^-$ → 3 charged particl. ≈ 94%	
?	$N\bar{N}$(2345)	1	2345 ± 10	! ⇄	≈ 140	structure in $N\bar{N}$ total cross section	
	U(2380)	≥1	2382 ± 24		≤ 30	(MM)$^-$ → 1/3/>3 chrgd part. ≈30/45/25	
	$N\bar{N}$(2380)	0	2380 ± 10		≈ 140	structure in NN total cross section	

There is no evidence on the G, J, or P quantum numbers of these bumps (apart from the suggestion of ℓ = odd for $N\bar{N}$(1925), ℓ = even for $N\bar{N}$(1945)), nor is there satisfactory agreement between them and the other bubble chamber claims. Further, the σ_{tot} ($N\bar{N}$) bumps are broader than the (MM)$^-$ bumps, and there is no evidence for or against their interpretation as resonances.

(q) Taken from compilation by T. Ferbel, Proc. 1968 Philadelphia Conf. See the data listings for averages of the values given in the literature. Also see B. French's review of Mesons (Proc. 14th International Conf. High Energy Physics, Vienna (1968), p. 91) for possible differences between M and Γ of charged and neutral ρ_N(1650).

(r) See note in listings. Some investigators see a broad enhancement in mass ($K\pi\pi$) from 1200 - 1350 MeV, and others see structure. A further bump at 1280 MeV, Γ = 80 MeV, has been suggested. In light of this confusion, the masses, widths, quantum numbers, and branching ratios are at best tentative. For the mass region 1200 - 1350 MeV, the decay rate into $K^*(890)\,\pi$ is large, and a $K\rho$ decay is seen. The $K\eta$, $K\omega$ and $K\pi$ rates are less than a few percent.

(s) This $\eta' \to \gamma\gamma$ value is from a constrained fit under the assumption that $\eta\pi\pi$, $\pi^+\pi^-\gamma$ (inclusive $\rho^0\gamma$), and $\gamma\gamma$ are the only existing decay modes. Note that direct measurement of the $\eta' \to \gamma\gamma$ branching fraction gave the slightly different result of $(5.5^{+3.6}_{-3.0})$%.

<u>Mixing angles from Quadratic SU(3) Mass Formula:</u> 0^- nonet (π, K, η, η') θ = 10.4° ±0.2°; alternative 0^- nonet (π, K, η, E) θ = 6.2° ±0.1°; 1^- nonet (ρ(m = 765±15 MeV), K^*, ϕ, ω) θ = 39.9±1.1°; 2^+ nonet ($A2_H$, K_N(1420), f', f) θ = 29.9°±2.2°.

Footnotes for the BARYON Table

* Quoted error includes an S(scale) factor. See footnote to Table S.

‡ For decay modes into ≥ 3 particles p_{max} is the maximum momentum that any of the particles in the final state can have. The momenta have been calculated using the averaged central mass values, without taking into account the widths of the resonances.

a. Square brackets indicate a sub-reaction of the previous unbracketed decay mode.

b. J is not known; x is Γ_{el}/Γ.

c. This is only an educated guess; the error given is larger than the error of the average of the published values (see listings for the latter).

† For the baryon states, the name [such as N(1470)] contains the mass, which shifts by 5 or 10 MeV with each new analysis. We can't keep up with changing labels in the card-listing section, so we don't try. The name (col. 1) is the same as can be found in large print in the listings. The best current value of the mass (col. 4) is what we use to determine the beam parameters, $M^2 \pm \Gamma M$, c.m. decay momenta, etc., that are found in other columns.

→ An arrow at the left of the Table indicates a candidate that has been omitted because the evidence for the existence of the effect and (or) for its interpretation as a resonance is open to considerable question. See listings for information on the following: $\Delta(1690P_{33}$, $N(1730)D_{13}$, $N(1860)P_{13}$, $N(1980)D_{13}$, N(2080), $N_?(3245)$, N(3690), N(3755), Z_0(1865), Z_1(1900), Λ(1327), $\Lambda(1745)P_{01}$, $\Lambda(1750)S_{01}$, $\Lambda(1860)F_{07}$, Σ(1440), $\Sigma(1650)S_{11}$, Σ(1780), Σ(1880), and Ξ(1705).

Change in Notation. The subscript N stands for "normal spin-parity series" ($J^P=0^+, 1^-, 2^+, \ldots$), A for "abnormal" ($J^P=0^-, 1^+, 2^-, \ldots$).

Quantities in italics have changed by more than one (old) standard deviation since January, 1968.

Symbol	(J^P)	$I^G(J^P)C_n$ (├─┤ estab., ? = guess)	Mass M (MeV)	Width Γ (MeV)	$M^2 \pm \Gamma M$ (GeV)^2 (a)	Partial decay modes: Mode	Fraction %	Q (MeV)	p or p_{max} (b) (MeV/c)
$\pi^\pm$(140)	$\pi(0^-)$	$1^-(0^-)+$	139.58	7.2 eV	0.019483	See Table S			
π^0(135)			134.97	±1.2 eV	0.018217				
η(549)	$\eta(0^-)$	$0^+(0^-)+$	548.8	2.63 keV	0.301	all neutral	71 } See Table S		
			±0.6	±.64 keV	±.000	$\pi^+\pi^-\pi^0+\pi^+\pi^-\gamma$	29		
ρ (765)	$\rho(1^-)$	$1^+(1^-)-$	765 (c)	125 (c)	0.585	ππ	≈ 100	491	359
			±10 (c)	±20 (c)	±.095	$\pi^\pm\pi^+\pi^-\pi^0$	< 0.2	212	247
						$\pi^+\pi^-\pi^+\pi^-$	< 0.15	212	247
						$\pi^\pm\gamma$	< 0.4	630	372
						$\eta\pi^\pm$	< 0.8	82	146
						e^+e^-	.0061±.0005 (d)	769	385
						$\mu^+\mu^-$	.0066±.0015 (e)	559	370
ω(783)	$\phi'(1^-)$	$0^-(1^-)-$	783.4	12.6	0.614	$\pi^+\pi^-\pi^0$	≈ 90	369	328
			±0.7	±1.1	±.010	$\pi^+\pi^-$	seen (f)	504	366
			S=2.0*			$\pi^0\gamma$	9.3±0.8	648	380
						η+ neutral	< 1.5	234	199
						$\pi^+\pi^-\gamma$	< 5	504	366
						$\pi^0\pi^0\gamma$	< 1	513	368
						e^+e^-	0.0068±.0022(g)	782	392
						$\mu^+\mu^-$	< 0.10	572	377
						$\pi^0\mu^+\mu^-$	< 0.2	437	350
η'(958) or X^0	$\eta'(0^-)$	$0^+(0^-)+$	958.3	< 4	0.918	ηππ	71 ± 4	131	232
		$J^P = 2^-$ not yet excluded	±0.8		<.004	$\pi^+\pi^-\gamma$(incl. $\rho^0\gamma$)	22 ± 3	958	479
See note (h), on name η'.						γγ	6.2 ± 3.1 (s)		
						for upper limits see footnote (i)			
δ(962)	?( )	≥1 ( )	962	< 5	0.927	ηπ possibly seen		278	306
	These two could be related, see listings		±5		<.005				
π_N(1016) → $K\bar{K}$	$\pi(0^+)$	$1^-(0^+)+$	1016	~25 } if res.	1.032	$K^\pm K^0$	only mode seen	24	110
			±10		±.025	ηπ	< 80	328	342
Resonance, virtual bound state, or antibound state, still not distinguished.									
φ(1019)	$\phi(1^-)$	$0^-(1^-)-$	1019.5	3.7	1.039	K^+K^-	47.6 ± 1.8	32	126
			±0.6	±0.6	±.004	K_LK_S	*32.8 ± 1.9 S = 1.5**	24	109
			S = 1.5*			$\pi^+\pi^-\pi^0$ (incl. ρπ)	*19.6 ± 2.3 S = 1.5**	605	462
						e^+e^-	0.036±.006	1018	510
						$\mu^+\mu^-$	$0.035^{+.035}_{-.018}$	808	499
						for upper limits see footnote (j)			
η_{0+}(1070) → K_SK_S	$\eta(0^+)$	$0^+(0^+)+$ if res.	1070§ ±20§ S=2.3*	~ 80 (?) see note (k)	1.14 ±.09	ππ	< 65	793	516
Resonance or scattering length both possible.						$K\bar{K}$	> 35	76	198
A1(1070)	$\pi(1^+)$	$1^-(1^+)+$	1070§	80§	1.14	3π see note (ℓ)	≈ 100	651	488
Existence still in doubt		($J^P = 2^-$ not yet excluded)	±20§	±35§	±.09	$K\bar{K}$	< 0.25[$G=(-1)^{\ell+1}$ forbids this]		
B(1220)	ρ(A)	$1^+(1^+)-$	1221	123	1.46	ωπ	≈ 100	297	339
	?		±16	±16	±.14	ππ	< 30 } suggests	950	600
	$J^P = 2^+, 3^-, \ldots$ still poss.		S=1.2*	S=1.3*		$K\bar{K}$	< 2 } $J^P \neq 3^-, 5^-, \ldots$	240	360
						for other upper limits see footnote (m)			
f(1260)	$\eta'(2^+)$	$0^+(2^+)+$	1264	145§	1.57	ππ	large	984	616
			±10§	±25§	±.15	$2\pi^+2\pi^-$	< 4	705	552
				S=2.7*		$K\bar{K}$ indic. seen,	< 2.5	267	388
D(1285)	η'(A)	$0^+(A)+$	1285	31	1.65	$K\bar{K}\pi$(mainly $\pi_N(1016)\pi$)	seen	154	304
			±4	±4	±.04	$K^*\bar{K}+\bar{K}^*K$		-100	
	$J^P = 0^-, 1^+, 2^-$ with 1^+ favored					ππρ	not seen	256	356
$A2_L$(1270)	π(N)	$1^-(N)+$	1269	26	1.61	ρπ (and π+neutrals)	dominant	364	387
see footnote(n)			±5	±7	±.03	ηπ indication suggests $J^P = N$		580	504
						$K\bar{K}$ indication seen			
$A2_H$(1315)	$\pi(2^+)$	$1^-(2^+)+$	1315	24	1.73	ρπ (and π+neutrals)	dominant	410	420
see footnote(n)			±4	±12	±.04	$K\bar{K}$	seen	327	434
				S=1.6*		ηπ	indication seen	626	532
E(1420)	η(A)	$0^+(0^-)+$	1424	71	2.03	$K^*\bar{K}+\bar{K}^*K$	50 ± 10	38	157
			±6	±10	±.10	$\pi_N(1016)\pi$	50 ± 10	268	328
				S=1.2*		ππη	< 88	596	569
	$J^P = 1^+$ not yet excluded					ππρ	not seen	384	455
f'(1515)	$\eta(2^+)$	$0^+(2^+)+$	1514	73	2.29	$K\bar{K}$	72 ± 12 (o)	518	570
			±5	±23	±.11	$K^*\bar{K}+\bar{K}^*K$	10 ± 10 (o)	128	294
				S=1.8*		ππ	< 14	1235	744
						ηππ	18 ± 10	686	624
						ηη	< 40	417	522

Symbol	(J^P)	$I^G(J^P)C_n$	Mass M (MeV)	Width Γ (MeV)	$M^2 \pm \Gamma M$ (GeV)^2	Mode	Fraction %	Q (MeV)	p or p_{max} (MeV/c)
π_A(1640) →3π	π(A) ?	$1^-(A)+$?	*1633* *±9* S=1.2*	93 ±24	2.73 ±.18	3π	dominant	1245	797
						†[$\pi^\pm\rho^0$/all $\pi^\pm\pi^+\pi^-$	< 40	745	637
						†[$\pi^\pm f(\to\pi^+\pi^-)$/all $\pi^\pm\pi^+\pi^-$	35±20	252	320
Low signal/background; may be partly						πη	< 9	966	727
Deck effect and/or several resonances.						$K\bar{K}$	not seen	654	658
						$\pi^\pm 2\pi^+ 2\pi^-$	< 10	935	714
ρ_N(1650) ("g" → 2π)	ρ(N)	$1^+(N)-$	1650 (q) ±20	120 (q) ±30	2.72 ±.20	2π	dominant	1385	820
($J^P = 1^-, 3^-, \ldots$ with 3^- favored)						$K\bar{K}$ indication seen,	< *16*	668	667
						(other possible modes see under ρ(1700) below)			
ρ(1700)? → 4π	ρ( )	$1^+(\ )-$	1700 ±20 (q)	110 ±25 (q)	2.89 ±.19	4π	dominant	1141	792
						†[$\pi^\pm A_2^0(\to\pi^+\pi^-\pi^0)$/all $\pi^\pm\pi^+\pi^-\pi^0$	40±20]	263	333
						†[$\pi^\pm\omega(\to\pi^+\pi^-\pi^0)$/all $\pi^\pm\pi^+\pi^-\pi^0$	25±10]	777	661
						†[$\rho^\pm\rho^0$	seen]	170	371
						$\pi^\pm\phi/\pi^\pm\pi^+\pi^-\pi^0$	< 11	540	532
						$\pi^\pm 2\pi^+ 2\pi^-\pi^0$	< 15	862	699
						2π (if ≠ ρ_N(1650))	< 10	1420	838

Not yet clear whether this is just an alternative mode of the ρ_N(1650), or a different resonance. The branching ratios are therefore only tentative.

→ There is no well-established Y = 0 meson resonance with M > 1700 MeV. For a list of some possible states → see footnote (p).

Symbol	(J^P)	$I^G(J^P)C_n$	Mass M (MeV)	Width Γ (MeV)	$M^2 \pm \Gamma M$ (GeV)^2	Mode	Fraction %	Q (MeV)	p or p_{max} (MeV/c)
K^+(494)	$K(0^-)$	$1/2(0^-)$	493.82		0.244	See Table S			
K^0(498)			497.76		0.248				
K^*(890)	$K(1^-)$	$1/2(1^-)$	891.4	49.7	0.797	Kπ	≈ 100	259	289
			±0.6	±1.1	±.044	Kππ	< 0.2	120	216
	$m_0 - m_\pm = 6.3 \pm 4.1$		↳ for charged K^*						
K_A(1240) or C	K(A)	$1/2(1^+)$	≈ 1240	≈ 60	1.54 ±.07	Q-Region } Kππ dominant. See footnote (r).			
Not well established (footnote (r)). $J^P=2^-$ not yet ruled out completely									
K_A(1320)	K(A)	$1/2(1^+)$	≈ 1330	≈ 70	1.77 ±.09				
	$J^P = 2^-$ not yet completely ruled out.								
K_N(1420)	$K(2^+)$	$1/2(2^+)$	*1422*	90	2.022	Kπ	51 ± 5	787	615
			±4	±6	±.128	$K^*\pi$	33 ± 3	388	413
			S=1.3*			Kρ	11 ± 4 } S = 1.5*	156	318
	$J^P = 3^-$ still possible.					Kω	3.4 ± 1.2	143	304
						Kη	2.0 ± 1.1	378	482
K_A(1780) or L	K(A) ?	$1/2(A)$?	1775	72	3.17	Kπ	< *6*	1167	825
			±12	±26	±.13	$K^*\pi$	34 ± 12	772	670
			S=1.3*	S=1.4*		$K_N(1420)\pi$	19 ± 15	245	317
						Kρ	11 ± 9	532	632
	$J^P = 1^+, 2^-, 3^+$ favored					Remaining Kππ	*28 ± 13*	1032	807
						Kω	*8 ± 5*	519	624
						Upper limits; Kη < 1%, $K^*\eta$ < 5; Kφ < 5.			

→ The following bumps, excluded above, are listed among the data cards: σ(410); η_{0+}(720); H(990); A1.5(1170); ρρ(1410); K_SK_S(1440); π/ρ(1550), φ(1650), R(1750); η or ρ(1830)→4π; φ or π(1830)→ωππ; S(1930); T(2200); $N\bar{N}_{I=0}$(2380), U(2380); κ(725); K_N(1100-1200); $K_{A(I=3/2)}$(1175); $K_{A(I=3/2)}$(1265); $K_{A(I=1/2)}$(1280); $K_{N(I=1/2)}$(1660); K^*(2240)→$\bar{Y}$N. (See note (p).)

* Quoted error includes scale factor $S = \sqrt{\chi^2/(N-1)}$. See footnote to Table S.

† Square brackets indicate a subreaction of the previous (unbracketed) decay mode.

§ This is only an educated guess; the error given is larger than the error of the average of the published values (see listings for the latter).

(a) ΓM is the half-width of the resonance when plotted against M^2.

(b) For decay modes into ≥ 3 particles p_{max} is the maximum momentum that any of the particles in the final state can have. The momenta have been calculated using the averaged central mass values, without taking into account the widths of the resonances.

(c) The values given for M(ρ) and Γ(ρ) and their errors are not average values from various experiments, but rather are intended to give the range where we believe the actual values are most likely to fall. Contrast the results tabulated in this note:

	M (MeV)	Γ (MeV)	
ρ^0	760±6	112±12	from $e^+e^-\to\pi^+\pi^-$, fits with energy-independent width
ρ^0	754±9	105±20	
ρ^0		105±15	from πN→ππN, ππ phase shift analysis with Chew-Low extrapolation, and energy-independent width
ρ^-	755±5	110± 9	
ρ^-	767±2	135±10	similar, but energy-dependent width and off-shell corrections
ρ^-	764±2	147±4	from πN→ππN, fits in phys. region, energy-dependent width.

Energy-independent width is a narrow-resonance approximation which tends to give lower mass and width. Thus an analysis of all $e^+e^-\to\pi^+\pi^-$ data with Omnès' formula and energy-dependent width gives M = 770±4 MeV, Γ = 122±6 MeV (M. Roos, priv. comm.).

(d) The quoted value of the rate $\rho^0\to e^+e^-$ is the average from two $e^+e^-\to\pi^+\pi^-$ experiments (which alone give an average of (0.0060±0.0006)%) and one photoproduction experiment of high mass resolution. Interference effects with ω decay are hoped to be small.

(e) Warning: The values given in the literature, and in our table, for the rate $\rho^0\to\mu^+\mu^-$ may be somewhat too high, due to possible interference with ω decay.

(Footnotes continued on the back of Table S)

Particle or resonance †	I(J^P) —— = estab.	Beam π, K (BeV) (BeV/c)	Mass (MeV)	Γ (MeV)	M^2±ΓM (BeV^2)	Partial decay modes: Mode	Fraction (%)	p or p$^†_{max}$ (MeV/c)	4πƛ^2 (mb)
p n	1/2(1/2$^+$)		938.3 939.6		0.880 0.883	See Table S			
N′(1470)	1/2(1/2$^+$) P_{11}	T=0.52πp p=0.64	1460	260	2.13 ±0.38	Nπ Nππ [Nσ][a] [Δπ][a]	55 45 [domin] [seen]	412 360 271 162	28.8
N(1518)	1/2(3/2$^-$) D_{13}	T=0.60 p=0.73	1515	115	2.30 ±0.17	Nπ Nππ [Δ(1236)π][a] Nη	50 50 [domin] ~ 0.5	452 406 219 137	23.9
N(1550)	1/2(1/2$^-$) S_{11}	T=0.62 p=0.75	1525	80	2.33 ±0.12	Nπ Nη Nππ	35 65 small	459 161 414	23.1
N(1680)	1/2(5/2$^-$) D_{15}	T=0.88 p=1.01	1675	145	2.81 ±0.24	Nπ Nππ [Δ(1236)π][a] ΛK Nη	45 55 [?] <1.6 <2.5	564 530 361 209 374	15.4
N(1688)	1/2(5/2$^+$) F_{15}	T=0.90 p=1.03	1690	125	2.86 ±0.21	Nπ Nππ [Δ(1236)π][a] ΛK Nη	60 40 [?] <.13 <1.5	574 540 374 234 390	14.9
→ N′(1710)	1/2(1/2$^-$) S_{11}	T=0.95 p=1.08	1715	280	2.94 ±0.48	Nπ	65	590	14.0
⇉ N″(1750)	1/2(1/2$^+$) P_{11}	T=1.08 p=1.21	1785	405	3.19 ±0.72	Nπ ΛK	34 seen	636	12.1
N(2190)	1/2(7/2$^-$) G_{17}	T=1.94 p=2.07	2190	300	4.80 ±0.66	Nπ	35	894	6.21
N(2650)	1/2(?$^-$)	T=3.12 p=3.26	2650	360	7.02 ±0.95	Nπ	(J+1/2)x=0.45[b]	1154	3.67
⇉ N(3030)	1/2(?)	T=4.26 p=4.40	3030	400	9.18 ±1.21	Nπ	(J+1/2)x=0.05[b]	1377	2.62
Δ(1236)	3/2(3/2$^+$) P_{33} m_0-m_{++} =	T=0.195 p=0.304 0.45±0.85	(++) 1236.0 ±0.6 m_--m_{++} = 7.9±6.8	120 ±2	1.53 ±0.15	Nπ Nπ$^+$π$^-$ Nγ	100 0 ~ 0.6	231 89 262	91.9
Δ(1640)	3/2(1/2$^-$) S_{31}	T=0.80 p=0.93	1630	160	2.69 ±0.26	Nπ Nππ	25 75	533	17.2
→ Δ(1690)	3/2(3/2$^-$) D_{33}	T=0.87 p=1.00	1670	225	2.79 ±0.38	Nπ	15	560	15.6
Δ(1910)	3/2(5/2$^+$) F_{35}	T=1.27 p=1.40	1880	250	3.53 ±0.47	Nπ	20	697	10.1
Δ(1930)	3/2(1/2$^+$) P_{31}	T=1.37 p=1.50	1905	300	3.63 ±0.57	Nπ	25	713	9.62
Δ(1950)	3/2(7/2$^+$) F_{37}	T=1.39 p=1.52	1940	210	3.76 ±0.41	Nπ ΣK Σ(1385)K Δ(1236)π Δ(1236)ρ	40 2.4 1.4 ≈ 50 seen	735 450 215 564	8.90
Δ(2420)	3/2(11/2$^+$)	T=2.50 p=2.64	2420	310	5.86 ±0.75	Nπ Nππ	11 > 20	1024 1007	4.67
Δ(2850)	3/2(?$^+$)	T=3.71 p=3.85	2850	400	8.12 ±1.14	Nπ	(J+1/2)x=0.25[b]	1266	3.05
Δ(3230)	3/2(?)	T=4.94 p=5.08	3230	440	10.4 ±1.4	Nπ	(J+1/2)x=0.05[b]	1475	2.24
Λ	0(1/2$^+$)		1115.6		1.24	See Table S			
→ Λ(1405)	0(1/2$^-$) S_{01}	p<0 K$^-$p	1405 ±5[c]	40 ±10[c]	1.97 ±0.06	Σπ	100	140	
Λ(1520)	0(3/2$^-$) D_{03}	p=0.392	1518.8 ±1.5	16 ±2	2.31 ±0.02	NK̄ Σπ Λππ Λγ	45±4 45±4 }S=1.8* 10±1 0.9±0.2	235 258 251 350	83.6
Λ′(1670)	0(1/2$^-$) S_{01}	p=0.74	1670	25	2.79 ±0.04	NK̄ Λη Σπ	14 33 45	410 66 387	28.5
⇉ Λ′(1700)	0(3/2$^-$) D_{03}	p=0.78	1690	40	2.86 ±0.07	NK̄ Σπ Λππ Σππ	25 35 ≈ 20 ≈ 20	429 403 419 350	26.1
Λ(1815)	0(5/2$^+$) F_{05}	p=1.05	1815 ±5[c]	75 ±10[c]	3.30 ±0.14	NK̄ Σπ Σ(1385)π Λη	65 11 9 ~ 1	538 500 359 346	16.7
→ Λ(1830)	0(5/2$^-$) D_{05}	p=1.08	1830	80	3.35 ±0.15	NK̄ Σπ	10 35	550 510	16.0
Λ(2100)	0(7/2$^-$) G_{07}	p=1.68	2100	140	4.41 ±0.29	NK̄ Σπ Λη ΞK Λω	30 4 <3 ~ 1 <10	748 699 617 483 443	8.68
Λ(2350)	0(?) — Seen in total c. s.	p=2.29	2350	210	5.52 ±0.49	NK̄	(J+1/2)x=0.6[b]	913	5.85
Σ	1(1/2$^+$)		(+)1189.4 (0)1192.5 (−)1197.3		1.41 1.42 1.43	See Table S			
→ Σ(1385)	1(3/2$^+$) P_{13}	p<0 K$^-$p S=4.8*! ←→(−) 1388±3	(+)1382±1	(+)36±3 S=2.1* (−)38±8, S=3.7*!	1.92 ±0.05	Λπ Σπ	90±3 10±3 S=1.4*	208 117	
Σ(1610)	1(?)	p=0.62	1615	65	2.61 ±0.10	NK̄ Λπ Σ(1385)π	small domin. mode seen	355 406 171	37.9
Σ(1660)	1(3/2$^-$) D_{13}	p=0.72	1660	50	2.76 ±0.08	Λ(1405)π NK̄	large small for both	197 400	29.9
	Decay modes of these two states not separated yet.								
Σ(1700)	1(? ?) —	p=0.80	1700	110	2.89 ±0.19	Λπ Σπ	large not disentangled	470 411	25.1
Σ(1765)	1(5/2$^-$) D_{15}	p=0.94	1765 ±5[c]	100 ±15[c]	3.13 ±0.18	NK̄ Λπ Λ(1520)π Σ(1385)π Ση Σπ	46 16 15 15 ~ 1 ~ 1	497 519 190 317 140 463	19.4
Σ(1915)	1(5/2$^+$) F_{15}	p=1.24	1905	60	3.63 ±0.11	NK̄ Λπ	10 5	608 615	12.9
Σ(2030)	1(7/2$^+$) F_{17}	p=1.52	2030	120	4.12 ±0.24	NK̄ Λπ Σπ ΞK	10 35 10 <2	700 700 652 412	9.92
Σ(2250)	1(?) — Seen in total c. s.	p=2.04	2250	200	5.06 ±0.45	NK̄	(J+1/2)x=0.4[b]	849	6.76
Σ(2455)	1(?) — Seen in total c. s.	p=2.57	2455	120	6.03 ±0.29	NK̄	(J+1/2)x=0.3[b]	979	5.08
Σ(2595)	1(?) Seen in total c. s.	p=2.95	2595	~ 140	6.73 ±0.36	NK̄	(J+1/2)x=0.25[b]	1064	4.30
Ξ	1/2(1/2$^+$)		(0)1314.7 (−)1321.3		1.73 1.75	See Table S			
Ξ(1530)	1/2(3/2$^+$) p-wave		(0)1528.9±1.1 (−)1533.8±1.9	7.3 ±1.7	2.34 ±0.01	Ξπ	100	145	
→ Ξ(1820)	1/2(?)		1820	≈20	3.31 ±0.04	ΛK̄ Ξπ Ξ(1530)π ΣK̄	≈60 small ≈10 ≈30(?)	396 413 234 306	
Ξ(1930)	1/2(?)		1930	110	3.72 ±0.21	Ξπ ΛK̄	large small	499 502	
Ξ(2030)	1/2(?)		2030	50	4.12 ±0.11	Ξπ ΛK̄ ΣK̄ Ξ(1530)π	small ≈50 ≈50 small	573 587 524 421	
Ω$^-$	0(3/2$^+$)		1672.4		2.80	See Table S			

(Footnotes are on the back of Table S)

CLEBSCH-GORDAN COEFFICIENTS AND SPHERICAL HARMONICS

Note: A $\sqrt{\ }$ is to be understood over every coefficient; e.g., for -8/15 read $-\sqrt{8/15}$.

Notation:

	J	J	...
	M	M	...
m_1 m_2	Coefficients		
m_1 m_2			
. .			

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

1/2 × 1/2

m_1 m_2	J=1, M=+1
+1/2 +1/2	1

m_1 m_2	J=1, M=0	J=0, M=0
+1/2 -1/2	1/2	1/2
-1/2 +1/2	1/2	-1/2

m_1 m_2	J=1, M=-1
-1/2 -1/2	1

1 × 1/2

m_1 m_2	J=3/2, M=+3/2
+1 +1/2	1

m_1 m_2	J=3/2, M=+1/2	J=1/2, M=+1/2
+1 -1/2	1/3	2/3
0 +1/2	2/3	-1/3

m_1 m_2	J=3/2, M=-1/2	J=1/2, M=-1/2
0 -1/2	2/3	1/3
-1 +1/2	1/3	-2/3

m_1 m_2	J=3/2, M=-3/2
-1 -1/2	1

2 × 1

m_1 m_2	J=3, M=+3
+2 +1	1

m_1 m_2	J=3, M=+2	J=2, M=+2
+2 0	1/3	2/3
+1 +1	2/3	-1/3

m_1 m_2	J=3, M=+1	J=2, M=+1	J=1, M=+1
+2 -1	1/15	1/3	3/5
+1 0	8/15	1/6	-3/10
0 +1	6/15	-1/2	1/10

m_1 m_2	J=3, M=0	J=2, M=0	J=1, M=0
+1 -1	1/5	1/2	3/10
0 0	3/5	0	-2/5
-1 +1	1/5	-1/2	3/10

m_1 m_2	J=3, M=-1	J=2, M=-1	J=1, M=-1
0 -1	6/15	1/2	1/10
-1 0	8/15	-1/6	-3/10
-2 +1	1/15	-1/3	3/5

m_1 m_2	J=3, M=-2	J=2, M=-2
-1 -1	2/3	1/3
-2 0	1/3	-2/3

m_1 m_2	J=3, M=-3
-2 -1	1

1 × 1

m_1 m_2	J=2, M=+2
+1 +1	1

m_1 m_2	J=2, M=+1	J=1, M=+1
+1 0	1/2	1/2
0 +1	1/2	-1/2

m_1 m_2	J=2, M=0	J=1, M=0	J=0, M=0
+1 -1	1/6	1/2	1/3
0 0	2/3	0	-1/3
-1 +1	1/6	-1/2	1/3

m_1 m_2	J=2, M=-1	J=1, M=-1
0 -1	1/2	1/2
-1 0	1/2	-1/2

m_1 m_2	J=2, M=-2
-1 -1	1

2 × 1/2

m_1 m_2	J=5/2, M=+5/2
+2 1/2	1

m_1 m_2	J=5/2, M=+3/2	J=3/2, M=+3/2
+2 -1/2	1/5	4/5
+1 +1/2	4/5	-1/5

m_1 m_2	J=5/2, M=+1/2	J=3/2, M=+1/2
+1 -1/2	2/5	3/5
0 +1/2	3/5	-2/5

m_1 m_2	J=5/2, M=-1/2	J=3/2, M=-1/2
0 -1/2	3/5	2/5
-1 +1/2	2/5	-3/5

m_1 m_2	J=5/2, M=-3/2	J=3/2, M=-3/2
-1 -1/2	4/5	1/5
-2 +1/2	1/5	-4/5

m_1 m_2	J=5/2, M=-5/2
-2 -1/2	1

3/2 × 1/2

m_1 m_2	J=2, M=+2
+3/2 +1/2	1

m_1 m_2	J=2, M=+1	J=1, M=+1
+3/2 -1/2	1/4	3/4
+1/2 +1/2	3/4	-1/4

m_1 m_2	J=2, M=0	J=1, M=0
+1/2 -1/2	1/2	1/2
-1/2 +1/2	1/2	-1/2

m_1 m_2	J=2, M=-1	J=1, M=-1
-1/2 -1/2	3/4	1/4
-3/2 +1/2	1/4	-3/4

m_1 m_2	J=2, M=-2
-3/2 -1/2	1

3/2 × 1

m_1 m_2	J=5/2, M=+5/2
+3/2 +1	1

m_1 m_2	J=5/2, M=+3/2	J=3/2, M=+3/2
+3/2 0	2/5	3/5
+1/2 +1	3/5	-2/5

m_1 m_2	J=5/2, M=+1/2	J=3/2, M=+1/2	J=1/2, M=+1/2
+3/2 -1	1/10	2/5	1/2
+1/2 0	3/5	1/15	-1/3
-1/2 +1	3/10	-8/15	1/6

m_1 m_2	J=5/2, M=-1/2	J=3/2, M=-1/2	J=1/2, M=-1/2
+1/2 -1	3/10	8/15	1/6
-1/2 0	3/5	-1/15	-1/3
-3/2 +1	1/10	-2/5	1/2

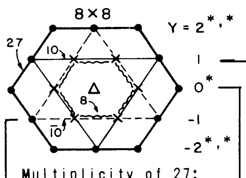
m_1 m_2	J=5/2, M=-3/2	J=3/2, M=-3/2
-1/2 -1	3/5	2/5
-3/2 0	2/5	-3/5

m_1 m_2	J=5/2, M=-5/2
-3/2 -1	1

SU(3) ISOSCÀLAR FACTORS

adapted from J.J. de Swart, Rev. Mod. Phys. 35, 916 (1963)

{8} ⊗ {8} = {27} ⊕ {10} ⊕ {10*} ⊕ {8}₁ ⊕ {8}₂ ⊕ {1}.



Multiplicity of 27;

● = 1, × = 2, Δ = 3

* Five single-coefficient tables are omitted. The one involving a {10*} has a negative coefficient, i.e. (NK|10*) = -1. The others, involving {27} and {10}, are all +1.

Y = 1 I = 1/2 N

$\xi_1 \rightarrow$	27 (+)	8_s (+)	8_a (−)	10* (−)
KΣ	$\sqrt{5}/10$	$3\sqrt{5}/10$	1/2	−1/2
πN	$-\sqrt{5}/10$	$-3\sqrt{5}/10$	1/2	−1/2
KΛ	$3\sqrt{5}/10$	$-\sqrt{5}/10$	1/2	1/2
ηN	$3\sqrt{5}/10$	$-\sqrt{5}/10$	−1/2	−1/2

Y = 1 I = 3/2 Δ

$\xi_1 \rightarrow$	27 (+)	10 (−)
KΣ	$\sqrt{2}/2$	$-\sqrt{2}/2$
πN	$\sqrt{2}/2$	$\sqrt{2}/2$

Y = 0 I = 0 Λ

$\xi_1 \rightarrow$	27 (+)	8_s (+)	1 (+)	8_a (−)
KΞ	$\sqrt{15}/10$	$\sqrt{10}/10$	1/2	$\sqrt{2}/2$
$\bar{K}$N	$-\sqrt{15}/10$	$-\sqrt{10}/10$	−1/2	$\sqrt{2}/2$
πΣ	$-\sqrt{10}/20$	$-\sqrt{15}/5$	$\sqrt{6}/4$	0
ηΛ	$3\sqrt{30}/20$	$-\sqrt{5}/5$	$-\sqrt{2}/4$	0

Y = 0 I = 1 Σ

$\xi_1 \rightarrow$	27 (+)	8_s (+)	8_a (−)	10 (−)	10* (−)
KΞ	$\sqrt{5}/5$	$-\sqrt{30}/10$	$\sqrt{6}/6$	$-\sqrt{6}/6$	$\sqrt{6}/6$
$\bar{K}$N	$\sqrt{5}/5$	$-\sqrt{30}/10$	$-\sqrt{6}/6$	$\sqrt{6}/6$	$-\sqrt{6}/6$
πΣ	0	0	$\sqrt{6}/3$	$\sqrt{6}/6$	$-\sqrt{6}/6$
πΛ	$\sqrt{30}/10$	$\sqrt{5}/5$	0	1/2	1/2
ηΣ	$\sqrt{30}/10$	$\sqrt{5}/5$	0	−1/2	−1/2

Y = −1 I = 1/2 Ξ

$\xi_1 \rightarrow$	27 (+)	8_s (+)	8_a (−)	10 (−)
$\bar{K}$Σ	$-\sqrt{5}/10$	$-3\sqrt{5}/10$	1/2	1/2
πΞ	$\sqrt{5}/10$	$3\sqrt{5}/10$	1/2	1/2
$\bar{K}$Λ	$3\sqrt{5}/10$	$-\sqrt{5}/10$	−1/2	1/2
ηΞ	$3\sqrt{5}/10$	$-\sqrt{5}/10$	1/2	−1/2

Y = −1 I = 3/2

$\xi_1 \rightarrow$	27 (+)	10* (−)
$\bar{K}$Σ	$\sqrt{2}/2$	$-\sqrt{2}/2$
πΞ	$\sqrt{2}/2$	$\sqrt{2}/2$

The phase factor $\xi_1 = \pm 1$, from de Swart's Table I, enters into his symmetry formula (14.3):

$$\langle \mu_1 \mu_2 | \mu \rangle = \xi_1 (-1)^{I_1 + I_2 - I} \langle \mu_2 \mu_1 | \mu \rangle \quad .$$

This factor is irrelevant if you are doing your own self-consistent calculations; it enters when you try to check someone else who chose $\mu_2 \otimes \mu_1$ instead of $\mu_1 \otimes \mu_2$.

{8} ⊗ {10} = {35} ⊕ {27} ⊕ {10} ⊕ {8}.

* Four single coefficient tables are omitted; only the {27} is -1; the three with {35} are +1.

Y = 1 I = 1/2 N

$\xi_1 \rightarrow$	27 (−)	8 (+)
πΔ	$\sqrt{5}/5$	$-2\sqrt{5}/5$
KΣ	$-2\sqrt{5}/5$	$-\sqrt{5}/5$

Y = 1 I = 3/2 Δ

$\xi_1 \rightarrow$	35 (+)	27 (−)	10 (−)
πΔ	−1/4	$-\sqrt{5}/4$	$\sqrt{10}/4$
ηΔ	$\sqrt{5}/4$	−3/4	$-\sqrt{2}/4$
KΣ	$\sqrt{10}/4$	$\sqrt{2}/4$	1/2

Y = 0 I = 0 Λ

$\xi_1 \rightarrow$	27 (−)	8 (+)
πΣ	$\sqrt{10}/5$	$-\sqrt{15}/5$
KΞ	$-\sqrt{15}/5$	$-\sqrt{10}/5$

Y = 0 I = 1 Σ

$\xi_1 \rightarrow$	35 (+)	27 (−)	10 (−)	8 (+)
πΣ	$-\sqrt{3}/6$	$-3\sqrt{5}/10$	$\sqrt{3}/3$	$\sqrt{30}/15$
ηΣ	$\sqrt{2}/2$	$-\sqrt{30}/10$	0	$-\sqrt{5}/5$
KΞ	$\sqrt{3}/3$	$\sqrt{5}/5$	$\sqrt{3}/3$	$\sqrt{30}/15$
$\bar{K}$Δ	$-\sqrt{3}/6$	$\sqrt{5}/10$	$\sqrt{3}/3$	$-2\sqrt{30}/15$

Y = 0 I = 2

$\xi_1 \rightarrow$	35 (+)	27 (−)
πΣ	$\sqrt{3}/2$	1/2
$\bar{K}$Δ	1/2	$-\sqrt{3}/2$

Y = −1 I = 1/2 Ξ

$\xi_1 \rightarrow$	35 (+)	27 (−)	10 (−)	8 (+)
πΞ	−1/4	$-7\sqrt{5}/20$	$\sqrt{2}/4$	$\sqrt{5}/5$
ηΞ	3/4	$-3\sqrt{5}/20$	$\sqrt{2}/4$	$-\sqrt{5}/5$
KΩ	$\sqrt{2}/4$	$3\sqrt{10}/20$	1/2	$\sqrt{10}/5$
$\bar{K}$Σ	−1/2	$\sqrt{5}/10$	$\sqrt{2}/2$	$-\sqrt{5}/5$

Y = −1 I = 3/2

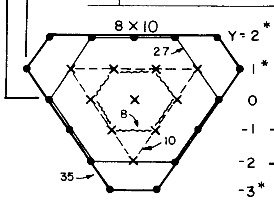
$\xi_1 \rightarrow$	35 (+)	27 (−)
πΞ	$\sqrt{2}/2$	$\sqrt{2}/2$
$\bar{K}$Σ	$\sqrt{2}/2$	$-\sqrt{2}/2$

Y = −2 I = 0 Ω^-

$\xi_1 \rightarrow$	35 (+)	10 (−)
ηΩ	$\sqrt{2}/2$	$\sqrt{2}/2$
$\bar{K}$Ξ	$-\sqrt{2}/2$	$\sqrt{2}/2$

Y = −2 I = 1

$\xi_1 \rightarrow$	35 (+)	27 (−)
πΩ	1/2	$\sqrt{3}/2$
$\bar{K}$Ξ	$\sqrt{3}/2$	−1/2



Multiplicity of 35;

● = 1, × = 2

Special Relativity

Notation. 4-vector in c.m. $p = (w, \vec{p})$; in lab $P = (W, \vec{P})$, $T = W - m$.
Solid-angle element $d\omega = 2\pi d\cos\theta$; $d\Omega = 2\pi d\cos\Theta$.
$p^2 = w^2 - \vec{p}^2 = m^2$ is an invariant. Cross section σ is invariant.

Lorentz Transformation

$$\begin{pmatrix} w \\ p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} \bar\gamma & -\bar\eta & 0 & 0 \\ -\bar\eta & \bar\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} W \\ P_x \\ P_y \\ P_z \end{pmatrix}$$

If θ and Θ are measured with respect to the transformation axis x,

$$\frac{p_\perp}{p_x} = \tan\theta = \frac{P\sin\Theta}{-\bar\eta W + \bar\gamma P\cos\Theta} \quad (1)$$

If particle 1 is beam, 2 is target, $(W_2, \vec{P}_2) = (m_2, \vec{0})$ and

$$\bar\gamma = (W_1 + m_2)/\sqrt{s}, \quad \bar\eta = \bar\gamma\bar\beta = |\vec{P}_1|/\sqrt{s}, \quad |\vec{p}_1| = |\vec{p}_2| = \bar\eta m_2 = |\vec{P}_1| m_2/\sqrt{s}. \quad (2)$$

For $m_1 = m_2$, $\bar\gamma^2 = 1 + T_1/2m_1$.

General Lorentz Transformation (characterized by $\vec{\bar\beta}$, with $\bar\gamma = (1-\vec{\bar\beta}^2)^{-1/2}$, $\vec{\bar\eta} = \bar\gamma\vec{\bar\beta}$):
$w = \bar\gamma W - \vec{\bar\eta}\cdot\vec{P}$; $\vec{p} = \vec{P} - \vec{\bar\eta}\,\frac{W+w}{\bar\gamma+1}$

Invariants. Notation: $1 + 2 \to 1' + 2'$.

$$s = (p_1 + p_2)^2 = m_1^2 + m_2^2 + 2(w_1 w_2 - \vec{p}_1\cdot\vec{p}_2) \quad (3)$$

$$t = (p_1' - p_1)^2 = m_1^2 + m_1'^2 - 2(w_1 w_1' - \vec{p}_1\cdot\vec{p}_1') \quad (i = 1, 2) \quad (4)$$

$$u = (p_1' - p_2)^2 = (p_2' - p_1)^2 \quad (5)$$

General relation: $s + t + u = m_1^2 + m_1'^2 + m_2^2 + m_2'^2$. (6)

In lab system $P_2 = (m_2, \vec{0})$, and writing $W = m + T$

$$s = m_1^2 + m_2^2 + 2W_1 m_2 = (m_1 + m_2)^2 + 2T_1 m_2 \quad (3, lab)$$

$$t = m_2^2 + m_2'^2 - 2W_2' m_2 = (m_2 - m_2')^2 - 2T_2' m_2 \quad (4, lab)$$

In c.m. system $dt = +2|\vec{p}_i|\,|\vec{p}_i'|\,d\cos\theta$ (4, cm)

For elastic scattering ($m_1 = m_1'$, $m_2 = m_2'$), (4) in c.m. simplifies to

$$t = -2\vec{p}^2(1 - \cos\theta) = -4\vec{p}^2\sin^2\theta/2 \quad (4, el)$$

$$u = (m_1^2 - m_2^2)^2/s - 2\vec{p}^2(1 + \cos\theta) = (m_1^2 - m_2^2)^2/s - 4\vec{p}^2\cos^2\theta/2$$

For elastic scattering, using (4, lab), (4, el), and (2),

$$T_2' = \frac{2\vec{P}_1^2 m_2}{s}\sin^2\left(\frac{\theta}{2}\right) \quad \text{(useful for calculating } \delta\text{-ray energies).} \quad (7)$$

Two-Body States. Energies and momenta in c.m.

$$w_1 = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}, \quad \vec{p}_1^2 = \vec{p}_2^2 = \frac{1}{4s}\,[s - (m_1 + m_2)^2]\,[s - (m_1 - m_2)^2]. \quad (8)$$

3- and 4-Body States. Let $m_{ij}^2 = (p_i + p_j)^2$, etc., then

$$\sum_{i<j} m_{ij}^2 = \Sigma m_i^2 + m_{123}^2 = \text{const.} \quad (i, j = 1, 2, 3) \text{ (follows from (6)).} \quad (9)$$

$$= 2\Sigma m_i^2 + m_{1234}^2 = \text{const.}$$

$$\sum_{i<j<k} m_{ijk}^2 = \Sigma m_i^2 + 2m_{1234}^2 = \text{const.} \quad (i, j, k = 1, 2, 3, 4.) \quad (10)$$

R_n, Invariant Volume in n-Body Momentum Space

A useful invariant is $\int d^4p\,\delta(p^2 - m^2) = \frac{d^3\vec{p}}{2w} = \frac{\vec{p}^2 d|\vec{p}|d\omega}{2w} = \frac{1}{2}|\vec{p}|\,dw\,d\omega$.

$R_2 = \pi|\vec{p}_1|/\sqrt{s}$, $R_3 = \pi^2\int dw_1 dw_2 = (\pi^2/4s)\int dm_{12}^2\,dm_{23}^2$ (using c.m. quantities)

Recurrence Relation for Factoring R_n (see e.g., Hagedorn, p. 93[a])

Write $N \to 1, 2, \ldots k, k+1, \ldots n$ (R_n)
as $N \to K, k+1, \ldots n$ (R_{n-k+1}), $K \to 1, 2, \ldots k$ (R_k)
then $R_n = \int d(m_K^2) R_k R_{n-k+1}$

Cross Section (or Decay Rate)

For $1 + 2 \to n$ particles (or $1 \to n$ particles)
$\sigma F = |M|^2 R_n$ (or $\Gamma w_1 = |M|^2 R_n$)
where M is an invariant matrix element.
F is (Møller's) invariant flux factor, $F^2 = (p_1 p_2)^2 - m_1^2 m_2^2$.
In every system where $\vec{p}_1$ and $\vec{p}_2$ are collinear, $F = w_1 w_2 |\vec{v}_1 - \vec{v}_2|$ $(\vec{v} = \vec{p}/w)$.
If particle 1 is beam, 2 is target ($\vec{P}_2 = 0$), $F = |\vec{P}_1| m_2 = |\vec{p}_1|\sqrt{s}$ [cf. (2)].
The rate (= number per unit 4-dimensional volume $d^4r = dt\,d^3\vec{r}$) is
$d^4N/d^4r = \sigma\,\rho_1\rho_2\,|\vec{v}_1 - \vec{v}_2|$, ρ_i = volume density of particles (i = 1, 2).

a. R. Hagedorn, Relativistic Kinematics, (W. A. Benjamin, New York, 1964).

C. M. Energy (μ) and Momentum (P_{CMS}) vs. Beam Momentum (P) of e, π, K, or p on p

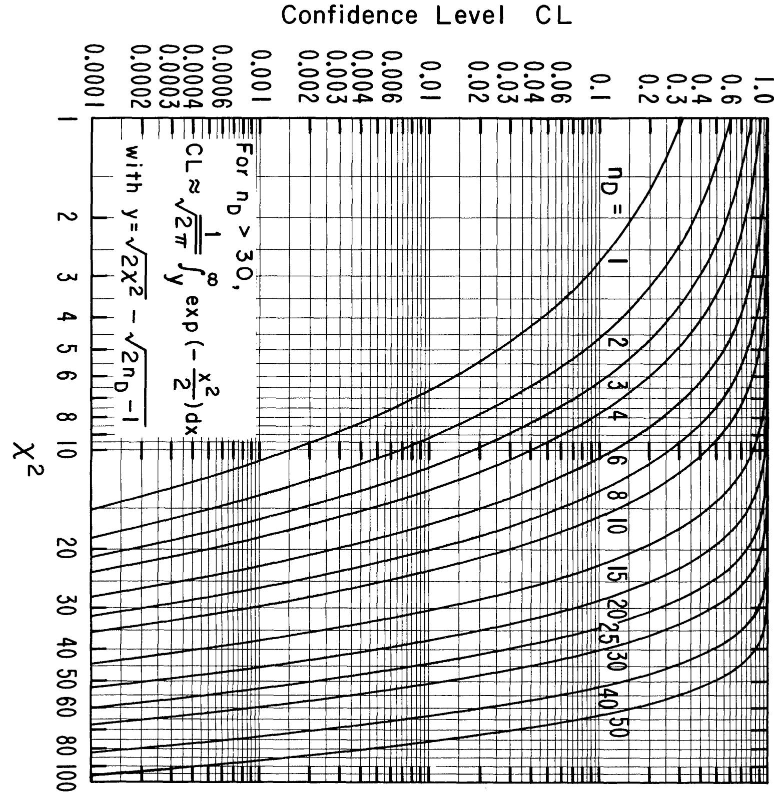
$\mu d\mu = m_p dE = m_p v_{beam,\,lab}\,dP \approx m_p dP$

P(LAB) (MEV/C)	INVARIANT MASS (MEV) ep	πp	Kp	pp	P(CMS) (MEV/C) ep	πp	Kp	pp
0	939	1078	1432	1877	0	0	0	0
20	958	1079	1432	1877	20	17	13	10
40	977	1083	1433	1877	38	35	26	20
60	996	1089	1434	1877	56	52	39	30
80	1015	1096	1436	1878	74	68	52	40
T(PI) = PLAB - 59 MEV								
100	1033	1105	1439	1879	91	85	65	50
120	1051	1116	1441	1880	107	101	78	60
140	1069	1127	1445	1882	123	117	91	70
160	1087	1139	1449	1883	138	132	104	80
180	1104	1152	1453	1885	153	147	116	90
T(PI) = PLAB - 92 MEV								
200	1121	1165	1457	1887	167	161	129	99
220	1137	1178	1462	1889	182	175	141	109
240	1154	1192	1468	1892	195	189	153	119
260	1170	1206	1474	1894	209	202	166	129
280	1186	1219	1480	1897	222	215	178	138
T(PI) = PLAB - 107 MEV								
300	1201	1233	1486	1900	234	228	189	148
320	1217	1247	1493	1903	247	241	201	158
340	1232	1261	1500	1906	259	253	213	167
360	1247	1274	1507	1910	271	265	224	177
380	1262	1288	1514	1913	282	277	235	186
T(PI) = PLAB - 115 MEV								
400	1277	1302	1522	1917	294	288	247	196
420	1292	1315	1530	1921	305	300	258	205
440	1306	1329	1538	1925	316	311	268	214
460	1320	1342	1546	1929	327	322	279	224
480	1335	1356	1554	1933	337	332	290	233
T(PI) = PLAB - 120 MEV								
500	1349	1369	1563	1938	348	343	300	242
520	1362	1382	1572	1943	358	353	310	251
540	1376	1395	1580	1947	368	363	321	260
560	1390	1408	1589	1952	378	373	331	269
580	1403	1421	1598	1957	388	383	341	278
T(PI) = PLAB - 123 MEV								
600	1416	1434	1607	1962	397	393	350	287
620	1430	1446	1616	1967	407	402	360	296
640	1443	1459	1625	1973	416	412	370	304
660	1456	1472	1634	1978	425	421	379	313
680	1468	1484	1644	1984	434	430	388	322
T(PI) = PLAB - 125 MEV								
700	1481	1496	1653	1989	443	439	397	330
720	1494	1509	1662	1995	452	448	406	339
740	1506	1521	1671	2001	461	457	415	347
760	1519	1533	1681	2007	470	465	424	355
780	1531	1545	1690	2013	478	474	433	364
T(PI) = PLAB - 127 MEV								
800	1543	1557	1699	2019	486	482	442	372
820	1555	1569	1709	2025	495	490	450	380
840	1567	1580	1718	2031	503	499	459	388
860	1579	1592	1728	2037	511	507	467	396
880	1591	1604	1737	2043	519	515	475	404
T(PI) = PLAB - 129 MEV								
900	1603	1615	1747	2049	527	523	483	412
920	1615	1627	1756	2056	535	531	492	420
940	1626	1638	1765	2062	542	538	500	428
960	1638	1649	1775	2069	550	546	508	435
980	1649	1661	1784	2075	558	554	515	443
T(PI) = PLAB - 130 MEV								
1000	1660	1672	1794	2082	565	561	523	451
1020	1672	1683	1803	2088	573	569	531	458
1040	1683	1694	1812	2095	580	576	538	466
1060	1694	1705	1822	2102	587	583	546	473
1080	1705	1716	1831	2108	594	591	553	481
T(PI) = PLAB - 131 MEV								
1100	1716	1726	1840	2115	601	598	561	488
1120	1727	1737	1850	2122	609	605	568	495
1140	1738	1748	1859	2129	616	612	575	502
1160	1748	1758	1868	2135	622	619	583	510
1180	1759	1769	1877	2142	629	626	590	517
T(PI) = PLAB - 131 MEV								
1200	1770	1780	1887	2149	636	633	597	524
1220	1780	1790	1896	2156	643	639	604	531
1240	1791	1800	1905	2163	650	646	611	538
1260	1801	1811	1914	2170	656	653	618	545
1280	1812	1821	1923	2177	663	660	624	552
T(PI) = PLAB - 132 MEV								
1300	1822	1831	1932	2184	669	666	631	559
1320	1832	1841	1941	2191	676	673	638	565
1340	1843	1851	1950	2198	682	679	645	572
1360	1853	1862	1959	2205	689	685	651	579
1380	1863	1872	1968	2212	695	692	658	585
T(PI) = PLAB - 133 MEV								

P(LAB) (MEV/C)	INVARIANT MASS (MEV) ep	πp	Kp	pp	P(CMS) (MEV/C) ep	πp	Kp	pp
1400	1873	1881	1977	2219	701	698	664	592
1420	1883	1891	1986	2226	708	704	671	599
1440	1893	1901	1995	2233	714	711	677	605
1460	1903	1911	2004	2240	720	717	684	612
1480	1912	1921	2013	2247	726	723	690	618
T(PI) = PLAB - 133 MEV								
1500	1922	1930	2022	2254	732	729	696	624
1520	1932	1940	2031	2261	738	735	702	631
1540	1942	1950	2039	2268	744	741	709	637
1560	1951	1959	2048	2275	750	747	715	643
1580	1961	1969	2057	2282	756	753	721	650
T(PI) = PLAB - 133 MEV								
1600	1970	1978	2065	2289	762	759	727	656
1620	1980	1988	2074	2296	768	765	733	662
1640	1989	1997	2083	2304	773	770	739	668
1660	1999	2006	2091	2311	779	776	745	674
1680	2008	2016	2100	2318	785	782	751	680
T(PI) = PLAB - 134 MEV								
1700	2018	2025	2109	2325	791	788	756	686
1720	2027	2034	2117	2332	796	793	762	692
1740	2036	2043	2126	2339	802	799	768	698
1760	2045	2053	2134	2346	807	805	774	704
1780	2054	2062	2143	2353	813	810	779	710
T(PI) = PLAB - 134 MEV								
1800	2064	2071	2151	2360	818	816	785	716
1820	2073	2080	2159	2367	824	821	791	721
1840	2082	2089	2168	2374	829	827	796	727
1860	2091	2098	2176	2381	835	832	802	733
1880	2100	2107	2184	2388	840	837	808	739
T(PI) = PLAB - 134 MEV								
1900	2108	2115	2193	2395	845	843	813	744
1920	2117	2124	2201	2402	851	848	818	750
1940	2126	2133	2209	2409	856	853	824	756
1960	2135	2142	2217	2416	861	859	829	761
1980	2144	2150	2226	2423	867	864	835	767
T(PI) = PLAB - 135 MEV								
2000	2153	2159	2234	2430	872	869	840	772
2020	2161	2168	2242	2437	877	874	845	778
2040	2170	2176	2250	2444	882	879	851	783
2060	2179	2185	2258	2451	887	885	856	789
2080	2187	2194	2266	2458	892	890	861	794
T(PI) = PLAB - 135 MEV								
2100	2196	2202	2274	2465	897	895	866	799
2120	2204	2211	2282	2472	902	900	872	805
2140	2213	2219	2290	2479	907	905	877	810
2160	2221	2227	2298	2486	912	910	882	815
2180	2230	2236	2306	2493	917	915	887	821
T(PI) = PLAB - 135 MEV								
2200	2238	2244	2314	2500	922	920	892	826
2220	2246	2253	2322	2507	927	925	897	831
2240	2255	2261	2330	2514	932	930	902	836
2260	2263	2269	2338	2520	937	934	907	841
2280	2271	2277	2346	2527	942	939	912	846
T(PI) = PLAB - 135 MEV								
2300	2280	2286	2353	2534	947	944	917	852
2320	2288	2294	2361	2541	951	949	922	857
2340	2296	2302	2369	2548	956	954	927	862
2360	2304	2310	2377	2555	961	959	932	867
2380	2312	2318	2384	2561	966	963	937	872
T(PI) = PLAB - 135 MEV								
2400	2320	2326	2392	2568	970	968	941	877
2420	2328	2334	2400	2575	975	973	946	882
2440	2336	2342	2407	2582	980	977	951	887
2460	2344	2350	2415	2589	984	982	956	892
2480	2352	2358	2423	2595	989	987	960	897
T(PI) = PLAB - 136 MEV								
2500	2360	2366	2430	2602	994	991	965	901
2520	2368	2374	2438	2609	998	996	970	906
2540	2376	2382	2445	2616	1003	1001	975	911
2560	2384	2390	2453	2622	1007	1005	979	916
2580	2392	2398	2460	2629	1012	1010	984	921
T(PI) = PLAB - 136 MEV								
2600	2400	2405	2468	2636	1017	1014	988	926
2620	2408	2413	2475	2643	1021	1019	993	930
2640	2415	2421	2483	2649	1025	1023	998	935
2660	2423	2429	2490	2656	1030	1028	1002	940
2680	2431	2436	2498	2663	1034	1032	1007	944
T(PI) = PLAB - 136 MEV								
2700	2439	2444	2505	2669	1039	1037	1011	949
2720	2446	2452	2512	2676	1043	1041	1016	954
2740	2454	2459	2520	2682	1048	1045	1020	958
2760	2462	2467	2527	2689	1052	1050	1025	963
2780	2469	2474	2534	2696	1056	1054	1029	968
T(PI) = PLAB - 136 MEV								

P(LAB) (MEV/C)	INVARIANT MASS (MEV) πp ep	Kp	pp	P(CMS) (MEV/C) πp ep	Kp	pp
3.0	2.56	2.61	2.77	1.10	1.08	1.02
3.2	2.63	2.68	2.83	1.14	1.12	1.06
3.4	2.70	2.75	2.89	1.18	1.16	1.10
3.6	2.77	2.82	2.96	1.22	1.20	1.14
3.8	2.83	2.88	3.02	1.26	1.24	1.18
T(PI) = PLAB - .137 GEV						
4.0	2.90	2.95	3.08	1.29	1.27	1.22
4.2	2.96	3.01	3.14	1.33	1.31	1.26
4.4	3.03	3.07	3.19	1.36	1.34	1.29
4.6	3.09	3.13	3.25	1.40	1.38	1.33
4.8	3.15	3.19	3.31	1.43	1.41	1.36
T(PI) = PLAB - .138 GEV						
5.0	3.21	3.25	3.36	1.46	1.44	1.40
5.2	3.27	3.31	3.42	1.49	1.48	1.43
5.4	3.32	3.36	3.47	1.53	1.51	1.46
5.6	3.38	3.42	3.52	1.56	1.54	1.49
5.8	3.43	3.47	3.58	1.59	1.57	1.52
T(PI) = PLAB - .138 GEV						
6.0	3.49	3.52	3.63	1.61	1.60	1.55
6.2	3.54	3.58	3.68	1.64	1.63	1.58
6.4	3.59	3.63	3.73	1.67	1.65	1.61
6.6	3.65	3.68	3.78	1.70	1.68	1.64
6.8	3.70	3.73	3.83	1.73	1.71	1.67
T(PI) = PLAB - .138 GEV						
7.0	3.75	3.78	3.87	1.75	1.74	1.70
7.2	3.80	3.83	3.92	1.78	1.76	1.72
7.4	3.85	3.88	3.97	1.81	1.79	1.75
7.6	3.89	3.93	4.02	1.83	1.82	1.78
7.8	3.94	3.97	4.06	1.86	1.84	1.80
T(PI) = PLAB - .138 GEV						
8.0	3.99	4.02	4.11	1.88	1.87	1.83
8.2	4.04	4.07	4.15	1.91	1.89	1.85
8.4	4.08	4.11	4.20	1.93	1.92	1.88
8.6	4.13	4.16	4.24	1.95	1.94	1.90
8.8	4.17	4.20	4.29	1.98	1.96	1.93
T(PI) = PLAB - .138 GEV						
9.0	4.22	4.25	4.33	2.00	1.99	1.95
9.2	4.26	4.29	4.37	2.03	2.01	1.97
9.4	4.31	4.33	4.41	2.05	2.03	2.00
9.6	4.35	4.38	4.46	2.07	2.06	2.02
9.8	4.39	4.42	4.50	2.09	2.08	2.04
T(PI) = PLAB - .139 GEV						
10.0	4.43	4.46	4.54	2.12	2.10	2.07
10.5	4.54	4.57	4.64	2.17	2.16	2.12
11.0	4.64	4.67	4.74	2.22	2.21	2.18
11.5	4.74	4.77	4.84	2.28	2.26	2.23
12.0	4.84	4.86	4.93	2.33	2.31	2.28
T(PI) = PLAB - .139 GEV						
12.5	4.94	4.96	5.03	2.38	2.36	2.33
13.0	5.03	5.05	5.12	2.43	2.41	2.38
13.5	5.12	5.15	5.21	2.47	2.46	2.43
14.0	5.21	5.24	5.30	2.52	2.51	2.48
14.5	5.30	5.32	5.39	2.57	2.56	2.53
T(PI) = PLAB - .139 GEV						
15.0	5.39	5.41	5.47	2.61	2.60	2.57
16.0	5.56	5.58	5.64	2.70	2.69	2.66
17.0	5.73	5.75	5.81	2.78	2.77	2.75
18.0	5.89	5.91	5.97	2.87	2.86	2.83
19.0	6.05	6.07	6.12	2.95	2.94	2.91
T(PI) = PLAB - .139 GEV						
20.0	6.20	6.22	6.27	3.03	3.02	2.99
22.0	6.49	6.51	6.56	3.18	3.17	3.14
24.0	6.78	6.79	6.84	3.32	3.31	3.29
26.0	7.05	7.07	7.11	3.46	3.45	3.43
28.0	7.31	7.33	7.37	3.59	3.59	3.56
T(PI) = PLAB - .139 GEV						
30.0	7.56	7.58	7.62	3.72	3.71	3.69
32.0	7.81	7.82	7.86	3.85	3.84	3.82
34.0	8.04	8.06	8.10	3.97	3.96	3.94
36.0	8.27	8.29	8.33	4.08	4.08	4.06
38.0	8.50	8.51	8.55	4.20	4.19	4.17
T(PI) = PLAB - .139 GEV						
40.0	8.72	8.73	8.77	4.31	4.30	4.28
42.0	8.93	8.94	8.98	4.41	4.41	4.39
44.0	9.14	9.15	9.18	4.52	4.51	4.50
46.0	9.34	9.35	9.39	4.62	4.62	4.60
48.0	9.54	9.55	9.58	4.72	4.72	4.70
T(PI) = PLAB - .139 GEV						
50.0	9.73	9.74	9.78	4.82	4.81	4.80
55.0	10.20	10.21	10.25	5.06	5.05	5.04
60.0	10.65	10.66	10.69	5.28	5.28	5.26
65.0	11.08	11.10	11.12	5.50	5.50	5.48
70.0	11.50	11.51	11.54	5.71	5.71	5.69
T(PI) = PLAB - .139 GEV						

CONFIDENCE LEVEL VS. χ^2 FOR n_D DEGREES OF FREEDOM



Atomic and Nuclear Properties of Materials

Material	Z	A	Cross Section σ[a] barns	Collision Length L_{coll}[b] g cm^{-2}	cm	Minimum $-dE/dx$[c] MeV g^{-1} cm^2	MeV cm^{-1}	Radiation Length L_{rad}[d] g cm^{-2}	cm	Density ρ g cm^{-3}
H_2	1	1.01	0.063	26.5	374	4.13	0.292	58.0	819	0.0708[e]
D_2	1	2.01	0.100	33.4	202	2.07	0.342	116	703	0.165[e]
He	2	4.00	0.16	42.0	336	1.94	0.242	85.4	683	0.125[e]
Li	3	6.94	0.23	50.4	94.3	1.69	0.902	78.7	148	0.534
Be	4	9.01	0.28	55.0	29.9	1.60	2.96	63.7	34.7	1.848
C	6	12.01	0.33	60.4	f	1.78	f	42.4	f	≈1.55[f]
N_2	7	14.01	0.36	63.6	78.9	1.81	1.46	37.8	46.7	0.808[e]
Ne	10	20.18	0.465	72.1	60.1	1.73	2.08	29.1[i]	24.2[i]	1.200[e]
Al	13	26.98	0.57	79.2	29.3	1.62	4.37	24.0	8.9	2.70
Fe	26	55.85	0.92	101.2	12.8	1.48	11.6	13.9	1.8	7.87
Cu	29	63.54	1.00	105.4	11.8	1.44	12.9	12.0	1.34	8.96
Sn	50	118.69	1.55	129.7	17.8	1.28	9.4	8.89	1.22	7.31
W	74	183.85	2.02	150.8	7.80	1.17	22.6	6.89	0.36	19.3
Pb	82	207.19	2.20	156.2	13.8	1.13	12.8	6.52	0.58	11.35
U	92	238.03	2.42	163.6	≈8.63	1.09	≈20.6	6.13	≈0.32	≈18.95
Air				64.6	53620	1.81	0.0022	36.5	30290	0.001205[g]
Freon (CF_3Br)				87.1	≈58.0	1.52	≈2.3	16.6	≈11	≈1.5
H_2 (bubble chamber, 27°K)				26.5	442	4.13	0.248	58.0	970	≈0.060[h]
H–Ne mixture (bubble chamber)[j]				67.3	96.1	1.83	1.28	29.8[i]	42.5[i]	.70
H_2O				57.2	57.2	2.03	2.03	35.7	35.7	1.00
Ilford Emulsion				103.0	27.0		5.49	11.2	2.91	3.815
LiF				63.8	24.2	1.69	4.46	39.0	14.8	2.64
Mylar ($C_5H_4O_2$)				59.1	42.8	1.91	2.64	39.6	28.7	1.38
NaI				119.0	32.4	1.32	4.84	9.58	2.61	3.67
Polyethylene (CH_2)				51.0	≈55.5	2.09	≈1.92	44.1	≈48	≈0.92
Polystyrene (CH) [*]				54.9	≈52.3	2.03	≈2.14	43.4	≈41.3	≈1.05
Propane (C_3H_8, bubble chamber)				48.9	119.3	2.28	0.935	44.6	109	0.41

a. $\sigma = \sigma_{natural} = \pi(\hbar/m_\pi c)^2 \times A^{2/3} = 62.8\ \text{mb} \times A^{2/3}$

b. $L_{coll} = A/(N\sigma_{natural}) = 26.5\ \text{g cm}^{-2} \times A^{1/3}$

c. From W. H. Barkas and M. J. Berger, Tables of Energy Losses and Ranges of Heavy Charged Particles, NASA SP-3013 (1964)

d. Mainly from High Energy and Nuclear Physics Data Handbook, W. Galbraith and W. S. C. Williams, Ed. (N. I. R. N. S., Rutherford Lab., Chilton, Didcot, Berks.) 1964

e. boiling at 1 atmosphere f. density variable g. at 20°C

h. May vary by about ±3%, depending on operation conditions

i. From F. R. Huson, Ionization Loss, Range, Straggling and Multiple Scattering, BNL 11386 (1967)

j. 53.7 atomic percent Ne.

[*] Typical scintillator, e.g. PILOT B has H/C = 1.1

OTHER ATOMIC AND PHYSICAL CONSTANTS (cont'd)

Hydrogen-like atom (nonrelativistic, μ = reduced mass)

$(\frac{v}{c})_{rms} = \frac{ze^2}{n\hbar c}$; $E_n = \frac{\mu}{2} v^2 = \frac{\mu z^2 e^4}{2(n\hbar)^2}$; $a_n = \frac{n^2\hbar^2}{\mu z e^2}$

$R_\infty = m_e e^4/2\hbar^2 = m_e c^2 \alpha^2/2 = 13.60535$ eV (Rydberg)

$p c = 0.3\ H\rho$ (MeV, kilogauss, cm); 0.3 (which is 10^{-11}c) enters because there are ≈ 300 "volts"/esu volt.

1 year	= 3.1557×10^7 sec ($\approx \pi \times 10^7$ sec)
density of air	= 1.205 mg cm^{-3} (at 20°C, 760 mm)
acceleration by gravity	= 980.62 cm sec^{-2} (sea level, 45°)
gravitational constant	= 6.670×10^{-8} cm^3 g^{-1} sec^{-2}
1 calorie	= 4.184 joules
1 atmosphere	= 1033.2 g cm^{-2}

NUMERICAL CONSTANTS

1 rad = 57.29578 deg	e = 2.71828
ln 2 = 0.69315	1/e = 0.367879
ln 10 = 2.30259	$\log_{10} e$ = 0.43429
$\log_{10} 2$ = 0.30103	

GAUSSIANLIKE DISTRIBUTIONS

For n > -1 but not necessarily integral:

$$\int_0^\infty x^{2n+1} \exp\left[-\frac{x^2}{2\sigma^2}\right] dx = 2^n\, n!\, \sigma^{2n+2};\ (\tfrac{1}{2})! = \sqrt{\pi}/2$$

Relation between standard deviation σ and mean deviation α:

$2\sigma^2 = \pi\alpha^2$; σ = 1.4826 probable error

Odds against exceeding one standard deviation = 2.15:1; two, 21:1; three, 370:1; four, 16,000:1; five, 1,700,000:1.

MULTIPLE COULOMB SCATTERING[b]

The rms projected angle θ due to multiple Coulomb scattering (only) of a particle of charge z, momentum P, velocity V is

$$\theta_{proj} = z\,\frac{15(\text{MeV})}{PV(\text{MeV})}\sqrt{\frac{L}{L_{(rad)}}}\ (1+\epsilon) \text{ radians;}$$

where L = length in scatterer.

For L ⩾ 1/10 L(rad) ϵ is generally < 1/10. The distribution of θ is not truly Gaussian.[c] The rms projected displacement y on traversing an absorber of thickness L is $y_{rms} = L\theta_{proj}/\sqrt{3}$.

RADIOACTIVITY

1 curie = 3.7×10^{10} disintegrations/sec

1 R = 87.8 ergs/g air = 5.49×10^7 MeV/g air

Fluxes (per cm^2) to liberate 1 R in carbon:

3×10^7 minimum ionizing singly charged particles

0.9×10^9 photons of 1 MeV energy.

(These fluxes are actually correct to within a factor of two for all materials.)

1 R of radiation, particularly for neutrons, may produce up to ~ 10 "rem" (R equivalent for man), even 20 "rem" for α and other heavy ions.

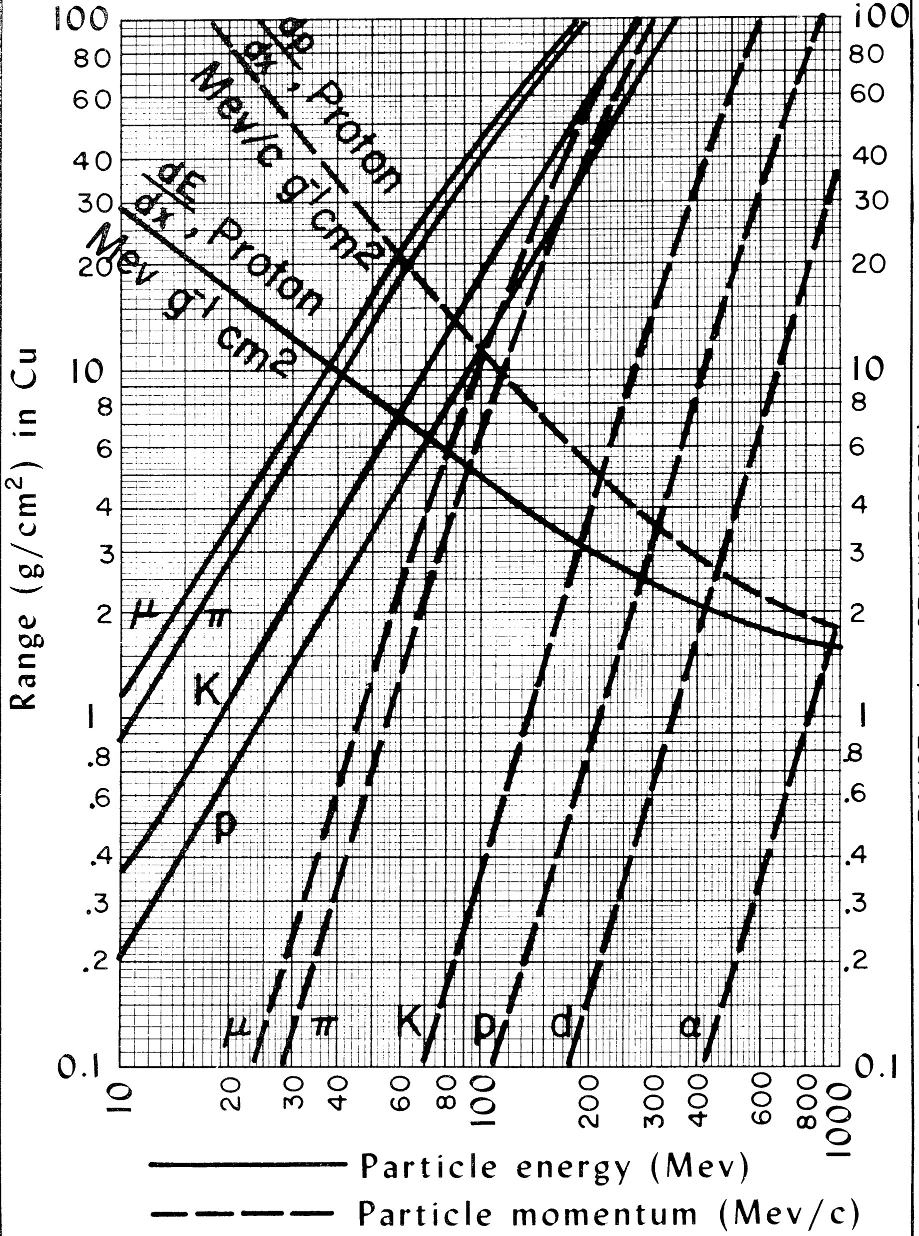
Natural background: 120 → 130 milirem/year divided as follows:

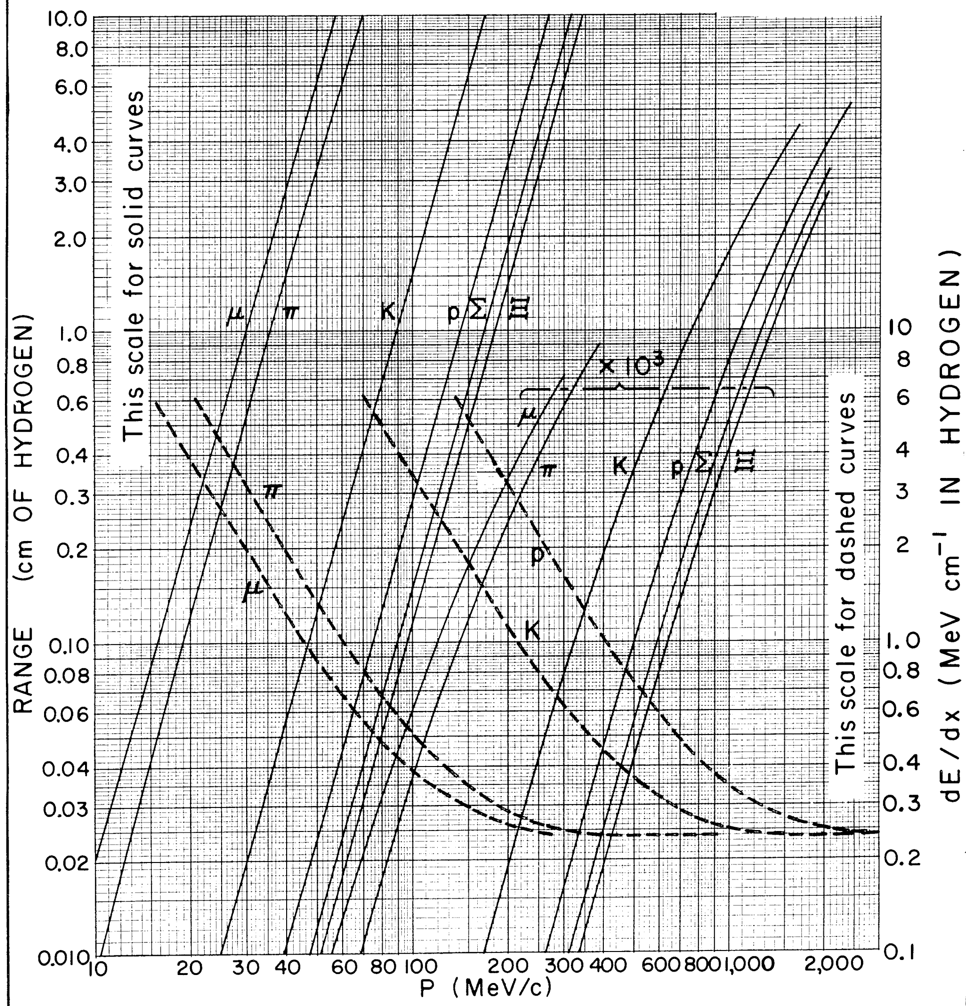
cosmic radiation – charged part. +neutrons		~ 25 milirem/yr
" "	– γ	~ 25 " "
Rock and air	– γ	~ 73 " "

The permissible occupational dose for the whole body: 100 milirem/week, but 1.25 rem per calendar quarter.

b. Mainly from G. Z. Molière, Naturforsch. 3 (a), 78 (1948).

c. See, for example, the experimental work of A. D. Hansen, L. H. Lanzl, E. M. Lyman and M. B. Scott, Phys. Rev. 84, 634 (1951).





Ranges in liquid hydrogen bubble chamber, determined by a μ^+ range of 1.103 ± .003 cm from the $\pi^+ \rightarrow \mu^+\nu$ decay. Liquid hydrogen conditions: T = 27.6 ± .1 °K; P = 48 ± 5 psia; $\rho = (5.86 \pm .06)10^{-2}$ g/cm³. (Data by Clark and Diehl.)