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NEUTRON ANGULAR DISTRIBUTION FROM HALO BREAKUP

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Abstract

In this paper we study the neutron elastic breakup in reactions initiated by ^{11}Be . The neutron angular distribution is obtained from a general formalism which can be applied to any heavy-ion transfer to the continuum reaction at intermediate energies. With the same method momentum distributions can be obtained. Our equations can be reduced to the incident free particle limit and the validity of this approximation is discussed. Theoretical results are compared to recent experimental data.

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A number of heavy-ion reactions at intermediate and high energies have shown that a large part of the total cross section for peripheral collisions is due to the breakup of light particles from the projectile. According to the circumstances in which these particles are studied they have been called pre-equilibrium particles (cf.fragmentation reactions), 3-body physical background (transfer to the continuum reactions), halo-neutron breakup (reactions with exotic beams). This component of the cross section can be determined experimentally from exclusive measurements of the light particle in coincidence with the ejectile.

In previous works [1–5] we have shown that the transfer to the continuum cross section is due to the sum of absorption on resonant and non-resonant states of the target, and elastic breakup. This paper is concerned with the elastic breakup of the halo neutron in ^{11}Be. This nucleus has a very loosely bound neutron whose single-particle $2s_{1/2}$ wave function has a long tail. Peripheral reactions induced by a halo nucleus will have characteristics similar to those of other heavy-ions, but in a more extreme form because of the very weak binding of the halo nucleons. The authors of Ref. [6–8], used a version of the Serber and Glauber model [9–11] to describe the breakup. Other authors have developed similar models based on the stationary scattering theory [12–17]. More information on the present experimental and theoretical situation can be found in a recent report [18].

In the present paper the break up amplitude is calculated in time dependent perturbation theory from a formula

$$A_{12} = \frac{1}{i\hbar}\int_{-\infty}^{\infty} dt < \Phi_2(t)|V_2(\mathbf{r})|\Phi_1(t) > \qquad (1)$$

which has been used before to discuss neutron transfer and breakup reactions [1,2,19–21]. The relative motion of the projectile core and the target nucleus is treated classically in Eq.(1). The target nucleus is assumed to be at rest and the projectile core follows a classical path $\mathbf{s}(t)$. The interaction of the breakup neutron with the target is represented by a potential $V_2(r)$ and with the projectile by a moving potential $V_1(\mathbf{r} - \mathbf{s}(t))$. For the reaction studied in this paper, with a light projectile and target and a high incident energy, $\mathbf{s}(t)$ can be approximated by a constant velocity path with impact parameter $\mathbf{d}$.

The initial neutron state is represented by the wave function $\Phi_1(t)$ which is a bound state in the moving potential V_1. The final breakup state of the neutron $\Phi_2(t)$ is a continuum state in the potential V_2. Elastic breakup arises from peripheral collisions and we assume that the only orbits which contribute are those with impact parameter greater than some strong absorption radius R_s. The strong absorption model is a well established and widely used method to deal with peripheral heavy-ion reactions [19,22–24]. We assume here that the same is true for halo projectiles. In the present case R_s is the strong absorption radius for the projectile core and the target nucleus. In Ref. [22] it is given as $R_s = r_0[(A_P - 1)^{\frac{1}{3}} + A_T^{\frac{1}{3}}]$ with $r_0 = 1.4 fm$.

In Eq.(1) the breakup neutron is treated quantum mechanically. Effects related to the momentum distribution of the neutron in the projectile and its final state interaction with the target are included in the wave functions Φ_1 and Φ_2. The final state interaction of the neutron with the projectile is neglected.

There are different ways of treating the breakup wave function Φ_2 depending on the physics involved. Resonance scattering and absorption of the breakup neutron by the target has been studied by taking V_2 to be an appropriate neutron-nucleus optical potential and the state Φ_2 as a neutron optical model wave function satisfying the appropriate boundary conditions [1,3,25]. When the outgoing neutron has a high energy it is often sufficient to approximate Φ_2 by a plane wave [2]. In the present paper we approximate Φ_2 by an eikonal wave function because we expect this to be a useful approach when the projectile is a halo nucleus. In fact one gets the correct free neutron scattering limit when the halo nucleus is very weakly bound. There is also a clear connection with our previous work and with simple diffraction models of breakup.

The eikonal approximation for the neutron final state wave function is [26]

$$\Phi_2{}^*(\mathbf{r}, \mathbf{k}_f) = e^{-i\mathbf{k}_\perp \cdot \mathbf{b}} e^{-ik_z z} e^{\frac{i}{\hbar v}\int_{+\infty}^{z} V_2(x,y,z')dz'} e^{\frac{i}{\hbar}\varepsilon_f t} \tag{2}$$

where b and $\mathbf{k}_\perp$ are the projections of $\mathbf{r}$ and $\mathbf{k}_f$ on the xy-plane. The breakup amplitude can be evaluated by substituting (2) into (1) and using the bound state wave function of the neutron in the projectile transformed by a Galilean transformation for Φ_1. The t- and z-integrals can be evaluated exactly by defining $k_1 = -(\varepsilon_i - \varepsilon_f + \frac{1}{2}mv^2)/(\hbar v)$ and $k_2 = -(\varepsilon_i - \varepsilon_f - \frac{1}{2}mv^2)/(\hbar v)$, these definitions come naturally in the phases in Eq.(1) and take into account neutron recoil effects. In the final step of the calculation, which involves an integration by parts, $k_z - k_2$ is replaced by zero. This is a good approximation for a halo nucleus because the distribution of $k_z - k_2$ is sharply peaked around zero. The breakup amplitude is finally

$$A_{12}(\mathbf{k_f}, \mathbf{d}) = \int d\mathbf{b} e^{-i\mathbf{k}_\perp \cdot \mathbf{b}}(1 - e^{-i\chi(\mathbf{b})})\tilde{\psi}_1(\mathbf{d} - \mathbf{b}, k_1) \tag{3}$$

where $\mathbf{k}_\perp$ and k_z are the perpendicular and parallel directions of $\mathbf{k_f}$ with respect to the beam direction and the Glauber phase χ is given by $\chi(\mathbf{b}) = \tilde{V}_2(\mathbf{b}, 0)/(v\hbar)$ where $\tilde{V}_2(\mathbf{b}, 0)$ is the Fourier transform of the target potential in the z-direction taken at $k_z - k_2 = 0$.

The Glauber formula (3) includes final state interactions of the neutron with the target potential V_2 in an approximate way. It is valid for high energy neutrons when the angular distribution is concentrated in the forward direction. We can understand its relation with approaches which use a plane wave final state by noting that in that case the amplitude obtained is the same as (3) but instead of the Glauber operator $(1 - e^{-i\chi(\mathbf{b})})$ we get $\tilde{V}_2(\mathbf{b}, k_z - k_2)/(v\hbar)$ (cf. Eq.(3.1) of [2], where we studied breakup from normal nuclei). It is clear that the two approaches are equivalent when $\chi(\mathbf{b})$ is small in the region where the integrand is large so that the exponential in Eq.(3) can be approximated by its expansion to first order in χ and when $k_z - k_2$ can be replaced by zero. The approximation that $\chi(\mathbf{b})$ is small fails for an extreme halo nucleus because the tail of the neutron bound state wave function extends into the region where the neutron-target optical potential is large.

There is an interesting limiting case of the amplitude and the cross-section which holds for a halo nucleus with a very weak binding. Then $\tilde{\psi}(\mathbf{d} - \mathbf{b}, k_1)$ is a very slowly varying function of $\mathbf{b}$ and the integral (3) for A_{12} can be factorized

$$A_{12}(k_f, d) = \left[\int d\mathbf{b} e^{-i\mathbf{k}_\perp \cdot \mathbf{b}}(1 - e^{-i\chi(\mathbf{b})})\right] \tilde{\psi}_1(d, k_1). \tag{4}$$

The first factor is proportional to a Glauber amplitude for neutron elastic scattering by the target. The physical interpretation of the factorized form (4) is that the target sees the neutron bound weakly to the projectile as an incident plane wave which is scattered by the target nucleus in the usual way.

In the above approximation the neutron final energy dependence is contained mainly in $\tilde{\psi}_1$, which contains also the information on the momentum distribution of the neutron in the initial state. On the other hand the neutron angular distribution will be just given by the Glauber-like amplitude which is a factor of Eq.(4), thus showing explicitly the diffractive nature of the elastic breakup. An expression similar to Eq.(4) has been recently obtained in Ref. [8], where the neutron-target rescattering was estimated by using an experimental cross section while we calculate it using an optical potential.

Eq. (3) can be simplified by using the asymptotic form for $\tilde{\psi}_1$. From the appendix to Ref. [20] we have

$$\begin{aligned}\tilde{\psi}_{l_1 m_1}(\mathbf{d} - \mathbf{b}, k_1) &= 2C_1 Y_{l_1,m_1}(\hat{k}_1) K_{m_1}(\eta\rho) \\ &\approx C_1 Y_{l_1,m_1}(\hat{k}_1) e^{-\eta\rho} \left(\frac{2\pi}{\eta\rho}\right)^{\frac{1}{2}}\end{aligned} \tag{5}$$

where $\tilde{\psi}_1$ is the Fourier transform of the asymptotic part of the initial state wave function

$$\psi_{l_1 m_1}(\mathbf{r}) = C_1 \ \gamma_i \ h_{l_1}^{(+)}(i\gamma_i r) Y_{l_1 m_1}(\Omega) \tag{6}$$

with respect to the z-coordinate, C_1 is the asymptotic normalization constant [21,27] and K_{m_1} is a modified Bessel function of the second kind [28]. In Eq.(5) $\rho = |(\mathbf{d} - \mathbf{b})|$ and $\eta^2 = k_2^2 - k_f^2 = k_1^2 + \gamma_i^2$ where $\gamma_i = \sqrt{-2m\varepsilon_i}/\hbar$.

Combining Eq.(3) with Eq.(5), we get the breakup probability distribution $d^3P(d)/d\mathbf{k}_\perp dk_z = |A_{12}(\mathbf{k}_f, d)|^2/8\pi^3$ in terms of the components of the neutron momentum and then the cross section is obtained by integration over the impact parameter **d** of the center of mass of the projectile relative to the target [1].

$$\frac{d^3\sigma}{d\mathbf{k}_\perp dk_z} = C^2S\int_0^\infty d^2\mathbf{d}\frac{d^3P(d)}{d\mathbf{k}_\perp dk_z}P_{el}(d). \tag{7}$$

There are no interference terms from different values of **d** in Eq.(7) because the relative motion of the two nuclei is treated classically. C^2S is a spectroscopic factor for the initial neutron single particle state state. P_{el} is the probability that the projectile core-target system remains in the ground state during the reaction. We suppose that the strong absorption hypothesis is a good approximation so that that $P_{el} = 1$ for $d > R_s$ and $P_{el} = 0$ for $d < R_s$, where R_s is the strong absorption radius already discussed. One could use a smooth cut-off approximation for P_{el} as proposed in [29] but in the case of a weakly bound nucleus it gives a negligible modification to the absolute value of the cross section.

To our knowledge there have been only two attempts [7,14] to calculating the neutron angular distribution from halo breakup, which is in principle a difficult problem unless one makes several simplifying assumptions, as it is discussed in [17]. Our work can be considered as a development on these approaches [7,14] in that we use a more general form of the initial state wave function, which they both took of the Yukawa form, and an improvement on the final state wave function since ours contains final state interaction with the target via an optical potential.

Fig.(1) shows the neutron angular distribution from the reaction $^9Be(^{11}Be,^{10}Be)^9Be + n$ at $E_{inc} = 41A.MeV$ calculated with Eq.(3) averaging over the azimuthal angle ϕ. The parameter values for the $2s_1/2$ state of ^{11}Be are ε_i = -0.5 MeV, γ_i= 0.155 fm^{-1}, C_i = 0.94 $fm^{-1/2}$ and C^2S=0.77. Experimental points from Ref. [7] are also shown. We have used the neutron-^9Be optical potential of [30]. In order to compare to the experimental data we have integrated the energy spectra in the range $\varepsilon_f = 26 - 80MeV$. All calculations have been done at a strong absorption radius of $R_s = 6.2fm$. The longdashed line is the calculation done starting from the amplitude Eq.(4) while the solid line has been obtained from the more exact form of the amplitude, Eq.(3). The results of both calculations are very similar at small θ, where the peak in the data is due to the sum of nuclear plus Coulomb breakup [7,31]. The large angle data seem to be slightly better reproduced by the more exact calculation from Eq.(3) but due to the error bars definite conclusions cannot be drawn. The lower dotted line, dashed and upper dotted line are the neutron angular distributions $d\sigma/\sin\theta d\theta$ for fixed values of ϕ = 0, $\pi/2$ and π respectively. They show that at forward angles the distribution in ϕ is uniform, while there are noticeable differences for large θ. The total elastic breakup cross section, integrated over energies and angles, is $\sigma^{el} = 0.23b$, in good agreement with the results of Ref. [7] where it was found $\sigma(^{10}Be + n) = 0.24 \pm 0.05b$.

In Fig.(2) we show the calculations for a fixed neutron final energy equal to the incident beam energy per nucleon. The top and bottom solid lines correspond to $\phi = \pi$, and 0 respectively. The dotted line is the calculation averaged over ϕ while the dashed line is the result of the calculation from Eq. (4). The calculation from Eq.(4) has a first minimum and a secondary diffractional maximum at $\theta \simeq 64^o$ and $\theta \simeq 86.5^o$ in accord with a cross section proportional to $(J_1(k_\perp R_s)/k_\perp R_s)^2$. The difference between the minimum and secondary maximum is very small and cannot be noticed in the figure. This is because in our calculation the surface of the target nucleus which is diffracting the neutrons is smooth.

Diffraction oscillations appear also in the calculation for fixed $\phi = 0$ (bottom line), from Eq.(3), and they disappear progressively increasing ϕ and increasing the neutron final energy to values larger than the beam energy per nucleon. The physical interpretation of these results is that the neutrons with high final energy do not suffer much interaction with the target potential and fly away at very forward angles. The slow neutrons interact instead with the target potential and are diffracted by it having the final momentum component perpendicular to the beam pointing away from the target.

Our results can be discussed also in terms of momentum distributions. In that case we get that the widths of the in-plane and out-of-plane momentum distributions, $d\sigma/dk_x$, $d\sigma/dk_y$ for the same reaction, shown in Fig.(3) (solid and dashed line respectively) are very close to each other. They were obtained from Eq.(7) after integration over k_z. We get in fact $\hbar\Delta k_y \simeq \hbar\Delta k_x = 187MeV/c$. These values agree with those estimated in [32]. The in-plane momentum distributions k_x are not symmetrical around $k_x = 0$ because of a slight non-uniform ϕ-dependence.

By the dot line we show the k_x (or k_y) distribution from the approximate amplitude Eq.(4). Eq.(4) assumes that the halo neutron behaves as a plane wave in the incident channel and therefore gives identical forms of the in-plane and out-of-plane distributions since the diffractive source represented by the potential of the target is spherically symmetric. In this case $\hbar\Delta k_y \simeq \hbar\Delta k_x = 177MeV/c$. We get distributions narrower than those obtained from (3) because the whole target nucleus plays the role of diffraction source while Eq.(3) shows that the source of diffraction [33] is the overlap region between the asymptotic initial wave function of the neutron and the potential representing the target.

In this paper we have developed a diffractional model to study neutron breakup from the projectile in a heavy-ion reaction. It takes into account final state interactions of the neutron with the target nucleus using an eikonal wave function. The method is especially suited to cases where the projectile is a halo nucleus but goes over continuously to a breakup model which we have used before for normal heavy ions where the neutron binding energy is several MeV. It can be used to calculate angular distributions of the breakup neutron as well as the distributions of longitudinal and transverse momenta. Furthermore the method allows for the description of the scattering out of the reaction plane, which is not uniform. A simple limiting case of our transition amplitude valid in the case of very weak binding and almost constant tail of the halo wave function has been discussed in connection to the application of the model to the breakup of ^{11}Be on a light ^{9}Be target . It consists in considering the neutron re-scattering on the target as that of an initial plane-wave. The two formulas describe equally well the small angle scattering but they give slightly different results at large angles. As a consequence the transverse momentum distributions of the halo neutrons from Eq.(1) are similar to, but somewhat broader than the corresponding transverse momentum distributions for neutron free scattering by the target nucleus (cf. [34]). The method presented in this paper can be used also to calculate the neutron energy spectra and related parallel momentum distribution [5].

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Figure captions

Fig.1. Angular distribution of the one-neutron breakup in the reactions $^9Be(^{11}Be,^{10}Be)^9Be + n$ at $E_{inc} = 41A.MeV$. The solid and longdashed lines are calculations using Eqs.(3) and (4) respectively, averaged over the azimuthal angle ϕ. The bottom dot line , short dashed and upper dot lines are the neutron angular distribution for fixed values of ϕ, $\phi = 0$, $\pi/2$ and π respectively.

Fig.2. Calculations of the neutron angular distribution for a fixed neutron final energy equal to the incident beam energy per nucleon for the same reaction as Fig.(1). The top and bottom solid lines correspond to $\phi = \pi$, and 0 respectively. The dot line is the calculation averaged over ϕ while the dashed line is the result of the calculation from Eq. (4). All results are normalized to the same value at $\theta = 0$

Fig.3. Differential cross sections $d\sigma/dk_x$ (solid line) and $d\sigma/dk_y$ (dashed line) from Eq.(3), for the same reaction as Fig.(1), after integration over the two other directions. Shortdashed line is calculated from Eq. (4).

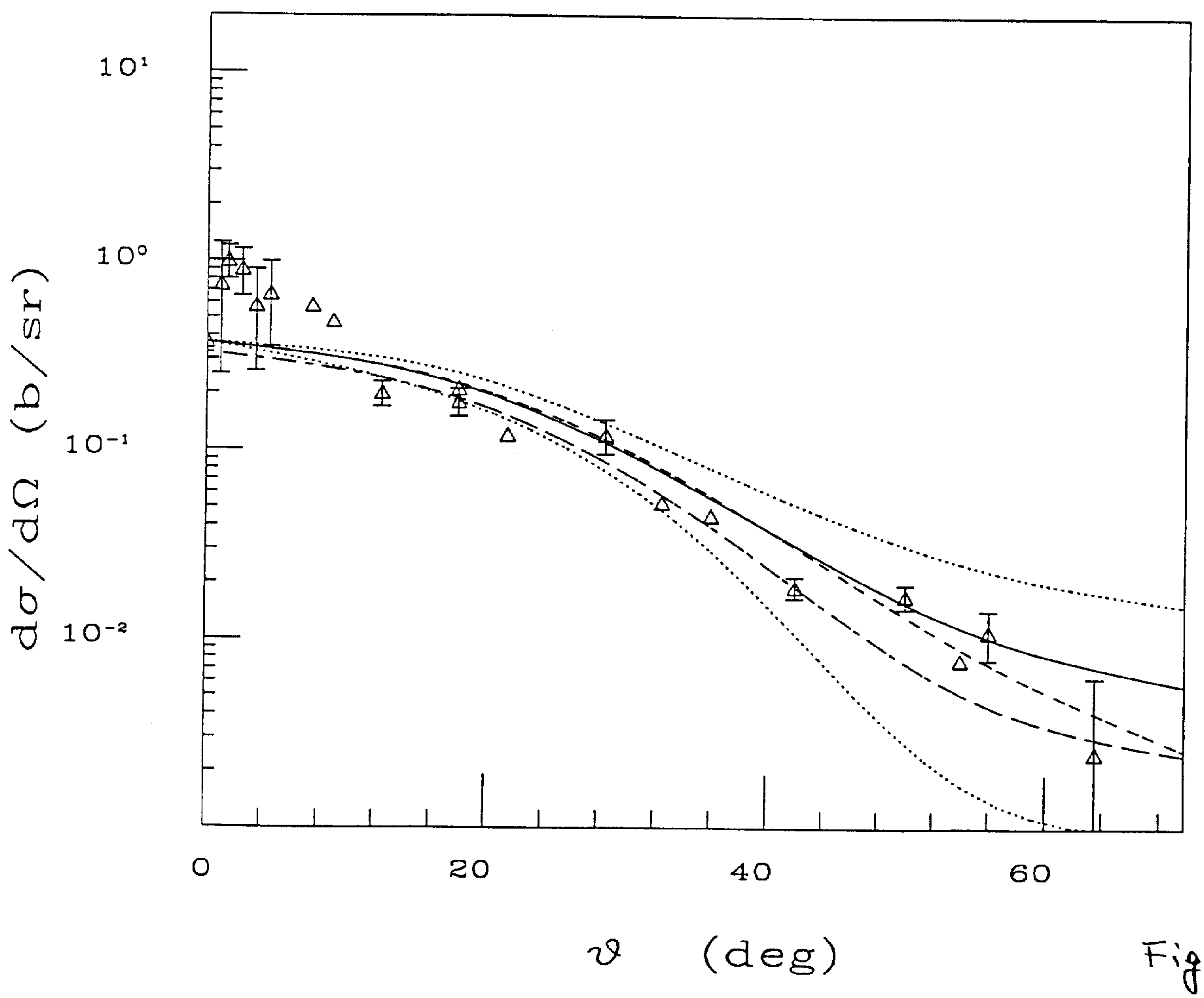
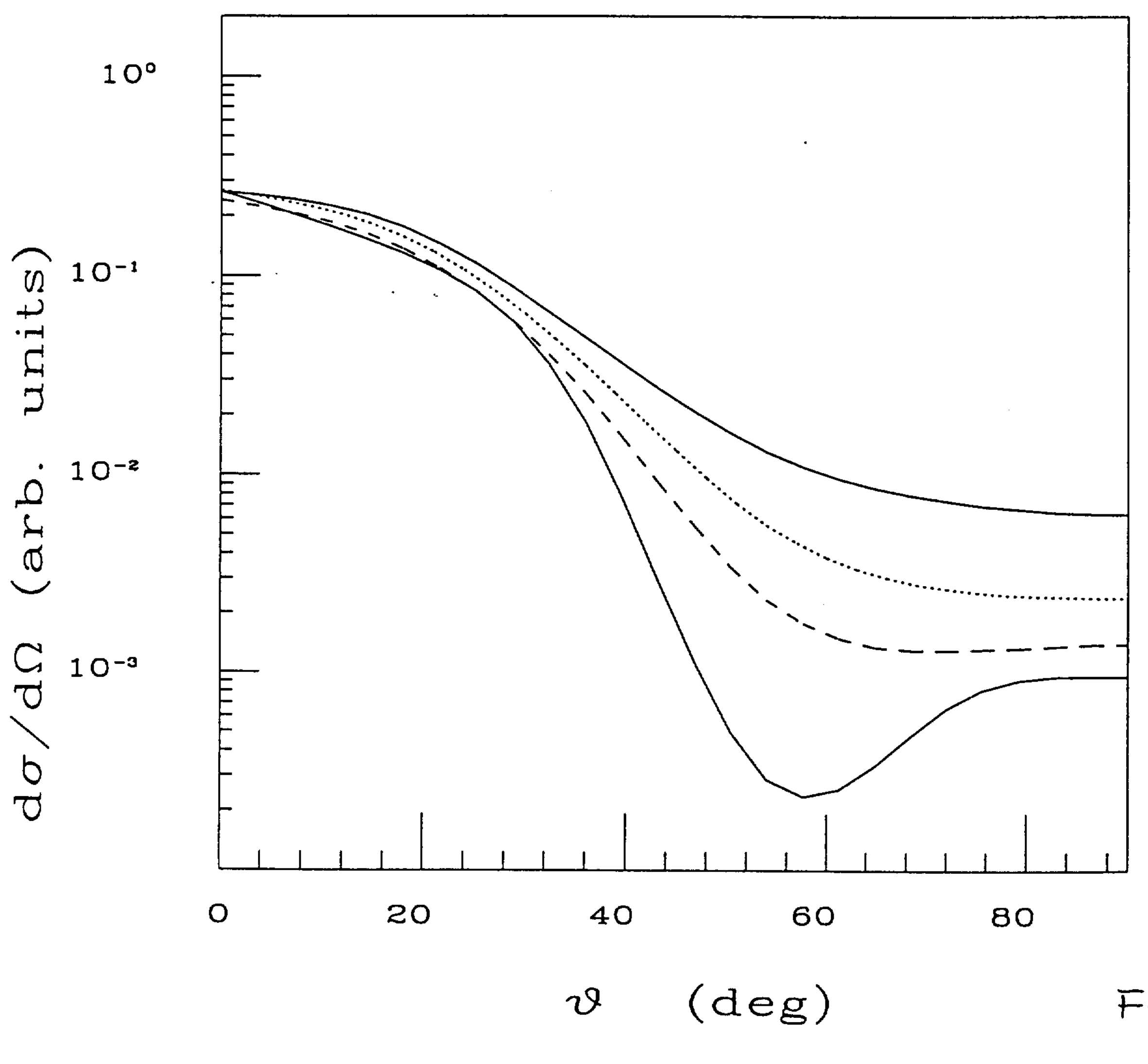


Fig. 1



10^0
10^{-1}
10^{-2}
10^{-3}
$d\sigma/d\Omega$ (arb. units)
0
20
40
60
80
ϑ (deg)

Fig. 2

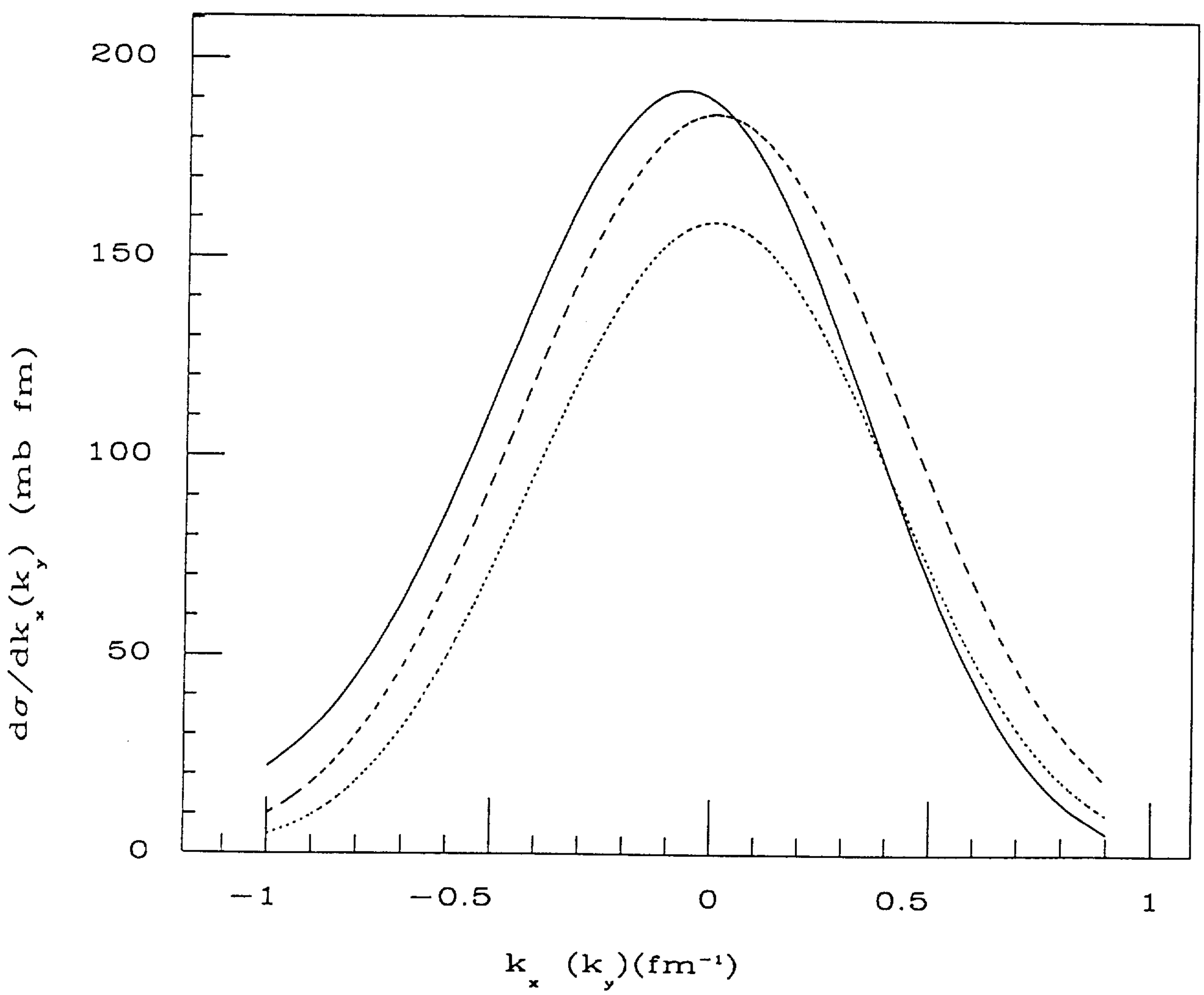


Fig.3