[9]W. Czyz, G. C. Sheppey, and J. D. Walecka, to be published.

[10]The expression for the Fourier transform of the Fermi distribution given by R. Blankenbecler, Am. J. Phys. 25, 279 (1957), with parameters given by R. Hofstadter, Ann. Rev. Nucl. Sci. 7, 231 (1957), was used.

[11]L. N. Hand, D. G. Miller, and R. Wilson, Rev. Mod. Phys. 35, 335 (1963).

[12]R. Hofstadter, F. A. Bumiller, and M. R. Yearian, Rev. Mod. Phys. 30, 482 (1958).

[13]See, in this connection, T. D. Lee, P. Markstein, and C. N. Yang, Phys. Rev. Letters 7, 429 (1961).

[14]For the numerical significance of these effects in the neutrino production of intermediate bosons, see A. C. T. Wu, C.-P. Yang, K. Fuchel, and S. Heller, Phys. Rev. Letters 12, 57 (1964).

SIGMA LEPTONIC DECAYS AND CABIBBO'S THEORY OF LEPTONIC DECAY*

W. Willis†
Brookhaven National Laboratory, Upton, New York

and

H. Courant,‡ H. Filthuth,§ P. Franzini,‖ A. Minguzzi-Ranzi,** and A. Segar††
CERN, Geneva, Switzerland

and

R. Engelmann, V. Hepp, and E. Kluge
University of Heidelberg, Heidelberg, Germany

and

R. A. Burnstein, T. B. Day, R. G. Glasser,‡‡ A. J. Herz, B. Kehoe, B. Sechi-Zorn, N. Seeman, and G. A. Snow§§
University of Maryland, College Park, Maryland and U. S. Naval Research Laboratory, Washington, D. C.
(Received 17 July 1964)

The purpose of this Letter is to present the final results on Σ^+ and Σ^- leptonic decays from the CERN stopping-K^- experiment and to compare these results with Cabibbo's theory of weak interactions.

A sample of about 400 000 Σ^+ and Σ^- hyperons were studied for leptonic decays of the following six types:

$$\left.\begin{array}{l}\Sigma^- \to n + e^- + \nu \\ \Sigma^- \to n + \mu^- + \nu\end{array}\right\} \frac{\Delta S}{\Delta Q} = +1, \quad \begin{array}{r}(1a)\\(1b)\end{array}$$

$$\left.\begin{array}{l}\Sigma^- \to \Lambda + e^- + \nu \\ \Sigma^+ \to \Lambda + e^- + \nu\end{array}\right\} \Delta S = 0, \quad \begin{array}{r}(1c)\\(1d)\end{array}$$

$$\left.\begin{array}{l}\Sigma^+ \to n + e^+ + \nu \\ \Sigma^+ \to n + \mu^+ + \nu\end{array}\right\} \frac{\Delta S}{\Delta Q} = -1. \quad \begin{array}{r}(1e)\\(1f)\end{array}$$

This report is based on the observation of 130 $\Sigma^\pm$ leptonic decays. The Σ hyperons were produced by K^- mesons, from the CERN proton synchrotron, coming to rest in the Saclay 81-cm hydrogen bubble chamber. The details of the experimental method will be published in an extended paper elsewhere.[1]

(A) Relative strength of $\Delta S = +\Delta Q$ and $\Delta S = -\Delta Q$ transitions.—No definite event of the type $\Delta S = -\Delta Q$ has been seen.

We have found 52 unambiguous $\Sigma^- \to n + e^- + \nu$ events and 22 unambiguous $\Sigma^- \to n + \mu^- + \nu$ events versus zero $\Sigma^+ \to n + (e^+, \mu^+) + \nu$ events. Given the differences in production ratios of Σ^- and Σ^+ hyperons from (K^-, p) reactions at rest[2] and the criteria[1] imposed on our events to eliminate background, the ratio of $\Sigma^+ \to n + \pi^+$ decays to $\Sigma^- \to n + \pi^-$ decays is calculated to be 1/3.8. If we define

$$\rho = \frac{\text{rate of } \Delta Q = -\Delta S \text{ transitions}}{\text{rate of } \Delta Q = +\Delta S \text{ transitions}},$$

we find that the upper limit with 90% confidence for ρ is 12%. The Columbia-Rutgers-Princeton collaboration independently obtains a similar upper limit of 15%.[3] It is quite certain that these decays are at least considerably suppressed in rate, and perhaps absent altogether.

(B) $\Delta S = 0$ and $\Delta S = +\Delta Q$ hyperon decay rates; and tests of $\Delta I = 1$ rule and (μ, e) universality.—The rates, or branching ratios, of the decay modes that we have observed are summarized in Table I, together with the predictions of the original universal Fermi interaction (UFI),[4] conserved vector current (CVC) theory.[5] Our experimental branching ratio $R_{e^-} = (\Sigma^- \to n + e^- + \nu)/(\Sigma^- \to n + \pi^-) = (1.4 \pm 0.3) \times 10^{-3}$ is in excellent agreement with two other recent measurements that yielded[3]

Table I. Summary of branching ratios of $\Sigma^{\pm}$ leptonic decays.

Decay	UFI prediction	$R = \frac{\Sigma^{\pm} \to \text{leptonic}}{\Sigma^{\pm} \to \pi^{\pm} + n}$	No. of events used
$\Sigma^- \to e^- + n + \bar{\nu}$	580×10^{-4}	$(14 \pm 3) \times 10^{-4}$	$31(p_e \leq 60$ MeV/$c)$
$\Sigma^- \to \mu^- + n + \bar{\nu}$	260×10^{-4}	$(6.6 \pm 1.4) \times 10^{-4}$	$22(\mu^-$ stop)
$\Sigma^- \to \Lambda + e^- + \bar{\nu}$	1.8×10^{-4}	$(0.75 \pm 0.28) \times 10^{-4}$	$11(\Lambda \to p + \pi^-)$
$\Sigma^+ \to \Lambda + e^+ + \nu$	1.1×10^{-4}	$(0.66 \pm 0.35) \times 10^{-4}$	$1(\Lambda \to p + \pi^-)$, $3(\Lambda$ not observed)
$\Sigma^+ \to e^+ + n + \nu$	0	$<2.3 \times 10^{-4}$	0
$\Sigma^+ \to \mu^+ + n + \nu$	0	$<2.6 \times 10^{-4}$	0

$R_{e^-} = (1.37 \pm 0.34) \times 10^{-3}$ and[6] $(1.0 \pm 0.5) \times 10^{-3}$, respectively. As has long been known,[7] the $\Sigma^- \to n + e^- + \nu$ rate is lower than the UFI rate by one or two orders of magnitude. This reduction is also well established[8] for $\Lambda \to p + e^- + \nu$ decays. On the other hand, the $\Sigma^{\pm} \to \Lambda + e^{\pm} + \nu$ rates are apparently of the same order of magnitude but about half as large as the UFI prediction, assuming CVC. The hypothesis[9] that the interaction in $\Sigma^{\pm} \to \Lambda + e^{\pm} + \nu$ decay transforms as $\Delta I = 1$ predicts[10] that the ratio $(\Sigma^- \to \Lambda + e^- + \nu)/(\Sigma^+ \to \Lambda + e^+ + \nu)$ equals 1.6. The data in Table I yield $1.1_{-0.4}^{+1}$ for this ratio, in agreement with this prediction, within the large statistical errors.

A test of the hypothesis of equal (μ, ν) and (e, ν) couplings in hyperon decay can be made by comparing the decay rates of $\Sigma^- \to n + \mu^- + \nu$ and $\Sigma^- \to n + e^- + \nu$. Ignoring all interactions other than allowed vector and axial vector, (μ, e) universality predicts that $w(\Sigma^- \to n + \mu^- + \nu)/w(\Sigma^- \to n - e^- + \nu) = 0.45$, essentially the ratio of phase space for the two decay modes. From Table I we can see that our experimental result for this ratio is 0.47 ± 0.14, in excellent agreement with (μ, e) universality.

(C) Comparison of Cabibbo's theory of weak interactions with experiment.—In brief, Cabibbo[11] postulates that the currents of strongly interacting charged particles coupled to leptons transform under SU(3) transformations like members of an octet. It follows that there are only two types of currents, $\Delta S = 0$, $\Delta I = 1$, and $\Delta S = +\Delta Q$, $\Delta I = \frac{1}{2}$. The currents are further assumed to be linear combinations of vectors and axial vectors with respect to space-time. In the limit of exact SU(3) symmetry, all the vector currents are conserved. The original idea of a universal four-fermion interaction is modified by assuming that the sum of the squares of unrenormalized vector coupling constants for the $\Delta S = 0$ and $\Delta S = +\Delta Q$ currents equals the square of the $\mu \to e + \nu + \bar{\nu}$ coupling constant. Hence one can define an angle θ such that $j_{\mu}(\Delta S = 0) \sim \cos\theta$ and $j_{\mu}(\Delta S = +\Delta Q) \sim \sin\theta$. The axial vector currents, which are not conserved, are assumed to be proportional to the same angle factors, and to have the same renormalization factors for all hyperon decays as for the $n \to p$ beta decay. Cabibbo[11] has shown with preliminary hyperon leptonic decay data that with the assumptions outlined above, there exists an angle, $\theta \cong 0.26$, that fits roughly not only the baryon leptonic decays but also the $K^+ \to \mu^+ + \nu$ and $K^+ \to \pi^0 + e^+ + \nu$ decays.

In what follows we carry out a least-squares fit to all the pertinent data using Cabibbo's theory. As experimental input, we take the eight pieces of data listed in Table II. We find two distinct acceptable fits to the data. (Cabibbo obtained only one solution because he singled out the $n \to p$ and $\Sigma^- \to \Lambda$ beta decays as input and

Table II. Data used to test Cabibbo' theory of leptonic decays.

Quantity	Value	Source
$(\Lambda \to p + e^- + \nu)$/all Λ	$(1.0 \pm 0.1) \times 10^{-3}$	Reference 8, weighted average
$(\Sigma^- \to n + e^- + \nu)/(\Sigma^- \to n + \pi^-)$	$(1.39 \pm 0.2) \times 10^{-3}$	Table I and reference 3
$(\Sigma^- \to \Lambda + e^- + \nu)/(\Sigma^- \to n + \pi^-)$	$(0.75 \pm 0.28) \times 10^{-4}$	Table I
$(\Xi^- \to \Lambda^0 + e^- + \nu)$/all Ξ^-	$(2.4 \pm 1.4) \times 10^{-3}$	Reference 12
$(G_V^{n \to p}/G_V^{\mu \to e})$	0.974 ± 0.010	Reference 13
$(G_A/G_V)_{n \to p}$	1.15 ± 0.05	Reference 14
$(K^+ \to \mu^+ + \nu)$/all K^+	0.60 ± 0.04	Reference 15
$(K^+ \to \pi^0 + e^+ + \nu)$/all K^+	0.050 ± 0.005	Reference 15

then predicted the $\Delta S=+\Delta Q$ hyperon decay rates. This method does not allow the full variation of experimental values within their errors to play a role in the comparison with theory.)

The parameters that enter into the theory are the angle θ, defined above, and the strengths of the F and D reduced matrix elements of the axial vector current. Two independent matrix elements arise since in SU(3) an $\underline{8}\otimes\underline{8}$ can combine to form two types of octets, which we call F and D in a notation suggested by that of Gell-Mann.[16] The CVC hypothesis implies that only F_V terms exist for the vector part. The equations that must be satisfied are the following:

$$\left(\frac{\Lambda^0\to p+e^-+\nu}{\text{all }\Lambda}\right)=0.37\times10^{-2}\times\sin^2\theta(\tfrac{3}{2})[1+2.98(F+\tfrac{1}{3}D)^2], \quad (2)$$

$$\left(\frac{\Sigma^-\to n+e^-+\nu}{\text{all }\Sigma^-}\right)=1.52\times10^{-2}\times\sin^2\theta(1)[1+2.95(D-F)^2], \quad (3)$$

$$\left(\frac{\Sigma^-\to\Lambda^0+e^-+\nu}{\text{all }\Sigma^-}\right)=0.60\times10^{-4}\times\cos^2\theta(\tfrac{2}{3})[3.00D^2], \quad (4)$$

$$\left(\frac{\Xi^-\to\Lambda^0+e^-+\nu}{\text{all }\Xi^-}\right)=5.7\times10^{-3}\times\sin^2\theta(\tfrac{3}{2})[1+2.98(F+\tfrac{1}{3}D)^2], \quad (5)$$

$$\left(\frac{G_V^{\;n\to p}}{G_V^{\;\mu\to e}}\right)^2=\cos^2\theta, \quad (6)$$

$$(G_A/G_V)_{n\to p}=F+D. \quad (7)$$

In obtaining the numbers in Eqs. (2)-(7), we have adopted $G_V^{\mu\to e}=1.025\times10^{-5}/M_p^{\,2}$ as deduced from the μ lifetime, $\tau_\mu=2.200\ \mu\text{sec}$,[17] and masses and lifetimes from Barkas and Rosenfeld[18] and Ticho.[12] For the transition rates for hyperon leptonic decays with arbitrary amounts of V and A coupling we have used the formulas of Yamaguchi[19] and Ryan[20] which agree numerically. These formulas have as a factor the SU(3) Clebsch-Gordan coefficients appropriate to each decay, which follow from the assumption of octet currents.[21] The formulas used by Cabibbo to relate the K^+ decay rates, the seventh and eighth rows of Table II, to θ, are given in reference 11 and are not repeated here.

We have searched for the best values of the parameters θ, F, and D in the sense of the minimum value of the chi-squared function computed with (i) all eight pieces of data in Table II, and (ii) the first six, omitting the K^+ decay data. The second alternative was tried since there have been some theoretical objections, particularly by Sakurai,[13] to the direct comparison of K^+ decays with π^+ decays because of the large mass splittings within the pseudoscalar meson octet. The general minimization program MINFUN of W. H. Humphrey was used.

The results are listed in Table III, along with the probabilities of the chi-squared values for five and three degrees of freedom for the solutions (i) and (ii). The values of some of the experimental quantities deduced from these solutions are also given. The χ^2 probability values indicate two acceptable types of solutions called A and B. The addition of the K^+ data hardly changes the solutions and is in surprisingly good agreement with the hyperon data. Solution A is of the type originally obtained by Cabibbo.[11] It is characterized by large D, small F, small $\Xi^-\to\Lambda$ decay, large $\Sigma^-\to\Lambda$ decay, and an (A/V) $\Sigma^-\to n$

Table III. Least-square solutions to Cabibbo's theory of leptonic decays using the data of Table II as input.

	Solution A		Solution B	
	(i)	(ii)	(i)	(ii)
No. of input data	8	6	8	6
No. of constraints	5	3	5	3
(χ^2) probability	(4.75) 45%	(3.51) 32%	(7.92) 17%	(7.04) 8%
θ	0.264	0.272	0.249	0.246
F	0.437	0.436	0.715	0.749
D	0.742	0.742	0.409	0.377
$(\Lambda\to p+e^-+\bar\nu)$/all Λ	0.91 $\times10^{-3}$	0.96 $\times10^{-3}$	1.08 $\times10^{-3}$	1.10 $\times10^{-3}$
$(\Sigma^-\to n+e^-+\bar\nu)$/all Σ^-	1.32 $\times10^{-3}$	1.38 $\times10^{-3}$	1.19 $\times10^{-3}$	1.28 $\times10^{-3}$
$(\Sigma^-\to\Lambda+e^-+\bar\nu)$/all Σ^-	0.61 $\times10^{-4}$	0.59 $\times10^{-4}$	0.19 $\times10^{-4}$	0.16 $\times10^{-4}$
$(\Xi^-\to\Lambda+e^-+\bar\nu)$/all Ξ^-	0.65 $\times10^{-3}$	0.66 $\times10^{-3}$	1.06 $\times10^{-3}$	1.06 $\times10^{-3}$
A/V for $\Sigma^-\to e^-+n+\bar\nu$	+0.305	+0.292	-0.306	-0.372

ratio that is positive. In contrast, solution B has large F, small D, large $\Xi^- \to \Lambda$ decay, small $\Sigma^- \to \Lambda$ decay, and an (A/V) $\Sigma^- \to n$ ratio that is negative. Solution B is closer to the type of hyperon decay pattern suggested by Zweig[22] based on "aces." Solution A, on the other hand, has the character suggested by the generalized Goldberger-Treiman relations. Figure 1 illustrates the positions of these two solutions in the F, D space and also shows how each baryon leptonic decay experiment serves to impose conditions in this space.

The Cabibbo angle θ is not sensitive to the type of solution, but is always within the limits $0.25 < \theta < 0.27$. The assumption of an octet current is to some extent born out by the relative magnitudes of the $\Lambda \to p$ and $\Sigma^- \to n$ beta decays.

Both solutions A and B have acceptable chi-squared probabilities, but as one can see from Fig. 1, the present experiments have sufficiently large errors that a fortuitous agreement with Cabibbo's theory cannot be excluded. However, further support for this theory is given by a calculation that we have carried out, minimizing chi-squared with independent angles θ and θ' for the vector and axial vector currents, respectively, using only the baryon leptonic decay data. We find for solutions of both types, A and B, that the best values of θ and θ' are equal within a few percent to 0.26. The same result was found already by Cabibbo[11] to be true for the K^+ decays, but for baryon decays there are fewer theoretical uncertainties due to mass splittings.

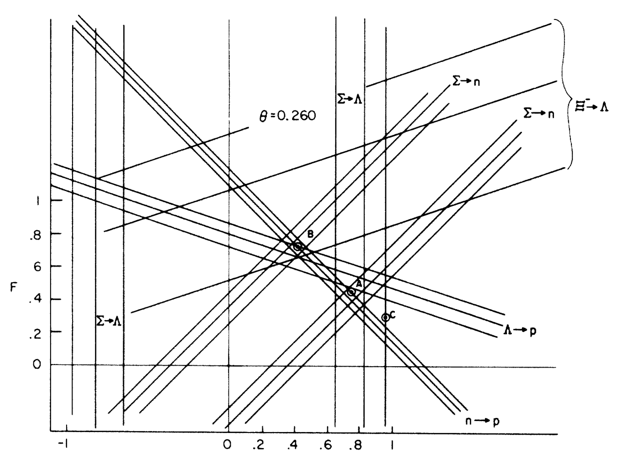

FIG. 1. Comparison of experimental data on baryon leptonic decays with Cabibbo's theory for $\theta = 0.26$. The parameters F and D are the strengths of the two independent reduced matrix elements of the axial vector current. Table II and Eqs. (2)-(7) in the text list the experimental data and formulas used to construct this figure. The points A and B denote the best-fit solutions obtained by minimizing chi-squared. Point C denotes the original solution of Cabibbo based on earlier data.

We are indebted to the personnel of the CERN proton synchrotron and Saclay bubble chamber for their unflagging cooperation throughout the run, and to the persistence and perspicacity of the scanning forces at the University of Maryland, U. S. Naval Research Laboratory, and CERN. We would also like to thank for their invaluable help and support Dr. R. Florent, Professor B. Gregory, and Dr. H. Schneider. We would also like to acknowledge valuable discussions with N. Cabibbo, C. Ryan, P. Kabir, and F. Chilton. One of us (G.A.S.) would also like to thank Brookhaven National Laboratory for its kind hospitality while part of this note was written.

---

*Work supported in part by the U. S. Atomic Energy Commission.

†Ford Fellow at CERN, Geneva, Switzerland, 1961-62.

‡Now at University of Minnesota, Minneapolis, Minnesota.

§At CERN, Geneva, Switzerland, and University of Heidelburg, Heidelburg, Germany.

‖Now at Brookhaven National Laboratory, Upton, New York.

**Now at University of Bologna, Bologna, Italy.

††Now at Rutherford Laboratory, Chilton, Berkshire, England.

‡‡National Science Foundation Senior Postdoctoral Fellow at CERN, Geneva, Switzerland, 1962-63.

§§National Science Foundation Senior Postdoctoral Fellow at CERN, Geneva, Switzerland, 1961-62.

[1]H. Courant et al., to be published.

[2]W. E. Humphrey and R. R. Ross, Phys. Rev. 127, 1305 (1962).

[3]U. Nauenberg, P. Schmidt, J. Steinberger, S. Marateck, R. Plano, H. Blumenfeld, and L. Seidlitz, Phys. Rev. Letters 12, 679 (1964).

[4]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958); and R. Marshak and G. Sudarshan, Phys. Rev. 109, 1860 (1958).

[5]S. S. Gershtein and Ia. B. Zel'dovich, Zh. Eksperim. i Teor. Fiz. 35, 821 (1958) [translation: Soviet Phys.—JETP 8, 570 (1959)]; and Feynman and Gell-Mann, reference 4.

[6]C. T. Murphy, Phys. Rev. 134, B188 (1964).

[7]W. E. Humphrey, J. Kirz, A. H. Rosenfeld, and J. Leitner, Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 442.

[8]V. G. Lind, T. O. Binford, M. L. Good, and D. Stern, to be published; R. P. Ely et al., Phys. Rev. 131, 868 (1963); and C. Baglen et al., to be published.

[9]S. Weinberg, Phys. Rev. 112, 1375 (1958).

[10]T. D. Lee and C. N. Yang, Phys. Rev. 119, 1410 (1960).
[11]N. Cabibbo, Phys. Rev. Letters 10, 531 (1963); see also M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1958).
[12]H. H. Bingham, talk presented to Royal Society Discussion on Leptonic Interactions, Feb. 27, 1964 (unpublished); and H. Ticho, Proceedings of the Sienna International Conference on Elementary Particles (Società Italiana di Fisica, Bologna, Italy, 1963).
[13]J. J. Sakurai, Phys. Rev. Letters 12, 79 (1964).
[14]C. Bhalla, private communication, reinterpretation of neutron and O^{14} lifetime experiments.
[15]Average of data from R. W. Birge, D. H. Perkins, J. E. Peterson, D. H. Stork, and M. H. Whitehead, Nuovo Cimento 4, 834 (1956); G. Alexander, R. H. W. Johnston, and C. O'Ceallaigh, Nuovo Cimento 6, 478 (1957); and B. P. Roe, D. Sinclair, J. L. Brown, D. A. Glaser, J. A. Kadyk, and G. H. Trilling, Phys. Rev. Letters 7, 346 (1961).
[16]M. Gell-Mann, California Institute of Technology Synchrotron Laboratory Report No. CTSL-20, 1961 (unpublished).
[17]G. Feinberg and L. Lederman, Ann. Rev. Nucl. Sci. 13, 431 (1963).
[18]W. Barkas and A. Rosenfeld, University of California Radiation Laboratory Report No. UCRL-8030 (Rev.) (unpublished).
[19]Y. Yamaguchi, CERN Technical Report No. 59-35, 1959 (unpublished), p. 342.
[20]C. Ryan, private communication.
[21]The appropriate SU(3) coefficients are written out explicitly in notes on a lecture by N. Cabibbo at the summer school in Erice, Italy, May 1963 (unpublished).
[22]G. Zweig, to be published.

NATURALLY OCCURRING ZERO-MASS PARTICLES AND BROKEN SYMMETRIES*

G. S. Guralnik†
Imperial College of Science, London, England
(Received 1 June 1964)

In a recent Letter[1] it was pointed out that the Goldstone theorem[2,3] is not valid in superconductivity in the presence of long-range interactions. It was then speculated that the proofs of this theorem in relativistic theories are also misleading and that the consistency of theories with broken symmetry may follow through the existence of spurious states $|0'\rangle$ which are not the limiting states of any branch of the excitation spectrum, and hence cannot be considered as particle states. While we have no quarrel with this possibility, we would like to point out that there is a large class of theories which will support a broken symmetry through relations of the sort

$$\langle 0, \eta | \varphi(x) | \eta, 0\rangle = C(\eta) \tag{1}$$

explicitly because of the natural presence of zero-mass particles in the theory, and where the objections raised in this Letter[1] are not valid. We have introduced a numerical parameter η into the above equation to distinguish explicitly the broken-symmetry vacuum states of this equation from the usual vacuum states $|0\rangle$ for which the expectation value of $\varphi(x)$ vanishes. $C(\eta)$ is a nonvanishing numerical function of η whose exact form depends on the method of construction of $|\eta, 0\rangle$ from $|0\rangle$. Our approach will involve a change in viewpoint from that normally associated with broken symmetries. Usually, one regards the broken symmetry requirements as doing violence to the basic structure of the theory in such a way as to pick an alternative solution to the field equations and in so doing inducing new zero-energy states in order to guarantee the consistency of requirement (1) with the operator symmetries of the problem. It is an explicit mark of usual theories that they will not support condition (1) when the interaction is turned off and that the perturbation solutions exploit the nonlinearity of the interaction. This is not the case in some examples we present.

We shall show that there is a class of theories in which it is possible to construct exactly the state $|\eta, 0\rangle$ satisfying Eq. (1) from the state $|0\rangle$ for which

$$\langle 0 | \varphi | 0\rangle = 0.$$

This is done without modifying the energy spectra or the multiplicity of states and hence without introducing any spurious states of the sort $|0'\rangle$. A remarkable result that we obtain is that ordinary electrodynamics with no bare photon mass belongs to this class. We then conclude as an exact consequence of the field equations that there is a zero-mass particle present in interacting electrodynamics which is identified as the photon.

The mechanism to be examined is quite simple. Suppose that in addition to the normally conserved quantities such as energy-momentum and angular