a value for $\alpha(E)$ which is about 40 times larger than the values shown in Fig. 1. It can also be shown that in the region $E > 100$ MeV [where (6) is certainly valid], the empirical equation (2) should be replaced by

$$\alpha(E)E^{+4.5} = \text{const.} \qquad (2a)$$

This can be seen as follows: The "natural" volume varies as $E^{2.5}$, δq as E^2, Δp^2 as E, and the factor L can be approximated in this region by const $\times E$.

This qualitative disagreement can be completely removed if one assumes that the actual volume of the bunches is about 40 times bigger than the natural one and that this enlargement of the volume is due to a coupling between radial and vertical betatron oscillations. The last assumption makes sure that (2) holds and not (2a), since in the case of coupling V varies as $E^{3.5}$ and not as $E^{2.5}$.

The function $L(x)$ also gives a qualitative description of the deviation from (2) for energies below 100 MeV. This deviation is due to the fact that with decreasing energy the width of the momentum distribution also decreases (x goes as E^{-5}). For $x = 1$ (at about 38 MeV in AdA) the longitudinal momentum transferred in the majority of collisions is too small to lead to the loss of particles. A detailed comparison with experiment is rendered difficult by the following circumstances: The momentum distribution of the natural beam is not Gaussian; if $x \approx 1$, it is further modified by the fact that it is no longer true that most Coulomb collisions lead to the loss of particles. The behavior of $\alpha(E)$ at high energies is not subject to these uncertainties, since all the processes which lead to the loss of electrons come from the center—and not as in the case of low energies from the tail—of the momentum distribution.

There is further supporting evidence for assuming that the actual volume is considerably larger than the natural one. At about 200 MeV the natural height of the beam should be about 2 μ. Multiple electron-positron scattering should bring this to about 10 if the ring is charged with two beams containing 10^7 particles each. Since we have shown that $\alpha(E)$ is insensitive to the presence of the other beam, it follows that the effective height of a single beam must be considerably larger than 10 μ. This is also borne out by the—as yet not conclusive—evidence on the frequency of 2γ annihilations, which strongly suggests an effective beam height of more than 25 μ.

Our thanks are due to Dr. F. Lacoste for his help in installing AdA at Orsay and his collaboration during the first runs of the machine. We want also to acknowledge the help of the linac crew and particularly of Dr. B. Millman and Dr. L. Burnod. Finally we thank Professor A. Blanc Lapierre for his hospitality and interest.

[1]C. Bernardini, G. F. Corazza, G. Ghigo, and B. Touscheck, Nuovo Cimento 18, 1293 (1960).

[2]C. Bernardini, U. Bizzarri, G. F. Corazza, G. Ghigo, R. Querzoli, and B. Touscheck, Nuovo Cimento 23, 202 (1962).

DETERMINATION OF THE RELATIVE Σ - Λ PARITY*

H. Courant,† H. Filthuth, P. Franzini,‡ R. G. Glasser,|| A. Minguzzi-Ranzi, A. Segar,⊥ and W. Willis**
CERN, Geneva, Switzerland

and

R. A. Burnstein, T. B. Day, B. Kehoe, A. J. Herz, M. Sakitt,†† B. Sechi-Zorn, N. Seeman, and G. A. Snow‡‡
University of Maryland, College Park, Maryland and U. S. Naval Research Laboratory, Washington, D. C.
(Received 11 April 1963)

To determine the relative Σ - Λ parity, we have measured the invariant mass spectrum of Dalitz pairs from the decay of unpolarized Σ^0 hyperons, $\Sigma^0 \rightarrow \Lambda^0 + e^- + e^+$. This method has been suggested by Feinberg[1] and Feldman and Fulton.[2] The problem is to establish the decay as an electric (odd Σ - Λ parity) or a magnetic (even parity) dipole transition. Under the even-parity hypothesis, the radiative matrix element is proportional to the momentum of the Dalitz pair, whereas with odd parity the matrix element is independent of the pair momentum. This has the consequence that for odd parity more Dalitz pairs exhibiting large invariant mass would be expected to occur than for even parity. Our data favor even Σ - Λ parity.

In our experiment Σ^0 hyperons were produced by K^- mesons at rest in hydrogen. The 81-cm Saclay hydrogen bubble chamber was exposed to a K^--meson beam at the CERN proton synchrotron.[3] About 10^6 K^- mesons were stopped in the chamber producing about 3×10^5 Σ^0 hyperons via the reaction $K^-+P\to\Sigma^0+\pi^0$. The Σ^0 Dalitz decays are recognized in scanning by the presence of an electron pair originating at the stopping point of an incident K^- meson together with a Λ^0 hyperon decay in the proximity of the K^- stopping point.

There are three reactions of stopped K^- mesons capable of producing such events:

$$K^-+P\to\Sigma^0+\pi^0 \quad \hookrightarrow\Lambda^0+e^-+e^+, \tag{1a}$$

$$K^-+P\to\Sigma^0+\pi^0 \quad \hookrightarrow\Lambda^0+\gamma \quad \hookrightarrow\gamma+e^-+e^+, \tag{1b}$$

$$K^-+P\to\Lambda^0+\pi^0 \quad \hookrightarrow\gamma+e^-+e^+. \tag{1c}$$

The theoretical branching ratios for π^0 and Σ^0 Dalitz decays are approximately 1/80 and 1/180, respectively. The experimental production rates for (Σ^0, π^0) and (Λ^0, π^0) from (K^-, P) at rest[4] are 0.28 and 0.064, respectively. This implies that, of 3.6 events with a Λ^0 and Dalitz pair, one corresponds to Reaction (1a).

To date we have obtained 353 events which after measurement and analysis have proved to be examples of type (1a). These events came from a sample of 1430 measured and analyzed events. The remaining events in the present sample have been identified as being either types (1b) and (1c) or events in flight. Events of type (1a) were accepted in the sample if their χ^2 probability exceeded 5% for a five-constraint fit. Events of type (1b) and (1c) were also identified by kinematic fitting and events in flight rejected on the basis of the Λ^0 or Σ^0 momentum in the decay fit.

There is a small region of overlap between the allowed configurations of Reaction (1b) and Reaction (1a). Figure 1 shows a sample of events from Reactions (1a) and (1b). The ordinate and abscissa are the sum of the momenta and the invariant mass, respectively, of the constrained Λ^0 and the measured e^+ and e^-. Good events cluster about a value of momentum (182.2 MeV/c) and invariant mass (1191.5 MeV) characteristic of Σ^0's produced by K^- interacting at rest. It is

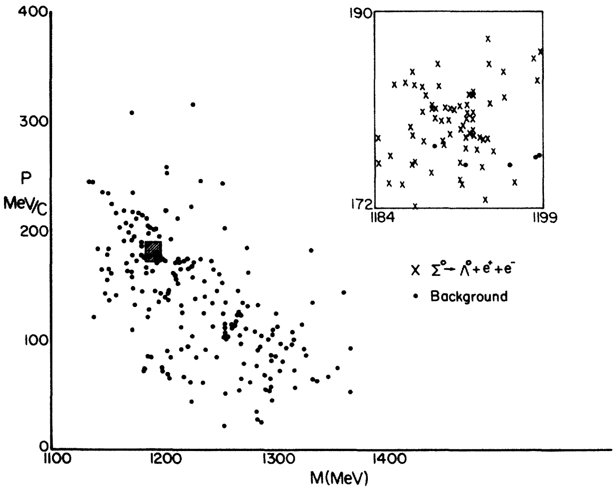

FIG. 1. The momentum and invariant mass of the constrained Λ^0 and the measured e^+ and e^- for a sample of events from Reactions (1a) and (1b). Crosses have satisfied and dots have failed to satisfy the selection criterion for good events. The inset is an enlargement of the shaded region.

evident from Fig. 1 and the inset that the density of points in a region about the expected Σ^0 momentum and mass is much larger than the general density. Therefore, the fraction of events from Reaction (1b) contained in the sample of "good" $\Sigma^0\to\Lambda^0+e^-+e^+$ events is only a few percent. [Moreover, the invariant mass spectrum from Reaction (1b) is very close to that expected for Reaction (1a) with either $\Sigma\Lambda$ parity.] Thus, we conclude that background effects are negligible.

We will first analyze our data neglecting terms proportional to the electric form factor, the effect of which will be considered later. Under this assumption, the differential invariant-mass distribution of the Dalitz pairs has the approximate form

$$w(x)=C(x)(1-x) \text{ for even parity,} \tag{2}$$

$$w(x)=C(x)(1+\tfrac{1}{2}x) \text{ for odd parity,} \tag{3}$$

where

$$x=[(E_++E_-)^2-(\vec{P}_++\vec{P}_-)^2]/\Delta^2$$

is the square of the invariant mass in units of Δ^2, and

$$\Delta=M_\Sigma-M_\Lambda=76.1 \text{ MeV}.$$

The quantities $E_\pm$ and $\vec{P}_\pm$ are the electron energy and momentum, and

$$C(x)=(1/x)(1-x)^{1/2}(1-x_0/x)^{1/2}(1+x_0/2x), \tag{4}$$

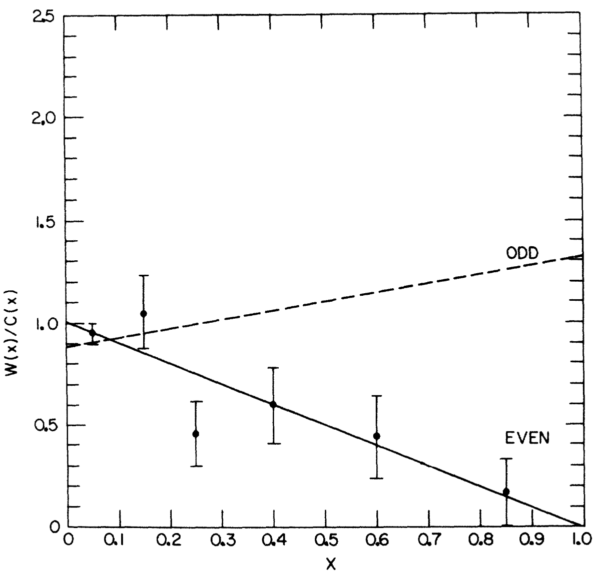

FIG. 2. The ratio of the number of events to the function $C(x)$ plotted against x, the square of the invariant mass. The theoretical predictions for odd and even parity are shown, assuming $|f_1| \lesssim |f_2|$ and $f_2(x) = f_2(0)$. Spectra are normalized to the same number of events.

with

$$x_0 = (4m_e^2/\Delta^2) = 1.80 \times 10^{-4}.$$

Figure 2 shows the measured distribution of the invariant mass, divided by $C(x)$. It also shows the mass distributions calculated using Eqs. (2) and (3) normalized to the number of measured events. The measurements agree well with the even-parity spectrum.[5]

The full expression for the invariant-mass spectrum, from Evans,[6] shows the following dependence on the electric and magnetic form factors, $x(\Delta/M)^2 \times f_1(x)$ and $f_2(x)$, of the Σ - Λ electromagnetic transition:

$$\frac{dw^\pm}{dx\,dy} = \frac{\alpha^2}{\pi}\frac{q}{2}\frac{1}{M_\Sigma^2 M^2} Q^\pm(x,y);$$

$$\frac{Q^\pm}{M_\Sigma} = |f_1|^2\frac{\Delta^2}{M^2}\left\{M_\Sigma q^2(1-y^2) + \Delta^2 x\left(1+\frac{x_0}{2x}\right)(q_0 \mp M_\Lambda)\right\}$$

$$-2\,\mathrm{Re} f_1 f_2^* \frac{\Delta^2}{M}\left(1+\frac{x_0}{2x}\right)\{(M_\Sigma - q_0)(M_\Sigma \mp M_\Lambda) - \Delta^2 x\}$$

$$+\frac{1}{x}|f_2|^2\{M_\Sigma q^2(1+y^2) + \Delta^2 x(q_0 \mp M_\Lambda)$$

$$+\frac{x_0}{x}M_\Sigma q^2 + \tfrac{1}{2}x_0\Delta^2(q_0 \mp M_\Lambda)\}, \qquad (5)$$

where $M = \frac{1}{2}(M_\Sigma + M_\Lambda)$; q, q_0 = momentum and energy of Λ^0; $y = (T_{e^+} - T_{e^-})/q$; and the upper (lower) sign denotes even (odd) Σ - Λ parity. [In obtaining the expressions (2) and (3) we assumed that $f_2(x) = f_2(0)$ and that $|f_1| \lesssim |f_2|$.]

Figure 3 shows a two-dimensional plot of our data. The energy of the low-energy member of the electron pair is plotted against the invariant mass for each of our events. Contours of constant density are drawn for the two hypotheses using Eq. (5), and neglecting terms proportional to the electric form factor. From the contour lines it is seen that for odd parity an accumulation of points is expected in the upper right-hand region of this plot, whereas for even parity a paucity of events is expected in this region. The preference for even parity is seen. The figure shows a loss of low-energy electron events which is explained by the difficulty of scanning for and measuring such events. It can be shown that the effect of this on the spectrum is small, and correcting for this would make our conclusion in favor of even parity slightly stronger. We have also checked our results for possible biases in the scanning and analysis of the Λ^0's, and have found none.

A single-parameter test of the parity can be made by comparing the average value of the invariant mass with that predicted by the theoretical

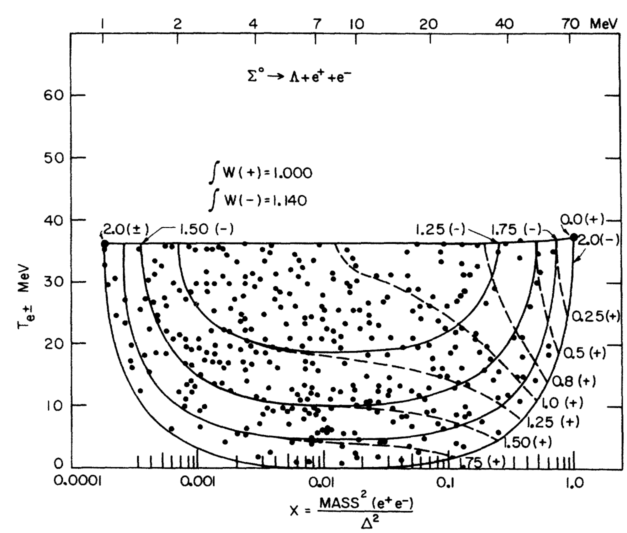

FIG. 3. The energy ($T_e \pm$) of the low-energy member of the electron pair is plotted against x, the square of the invariant mass. Contours of constant density of events are drawn for the predictions of the two hypotheses, normalized so as to be equal for small x. Dashed lines: positive parity; solid lines: negative parity.

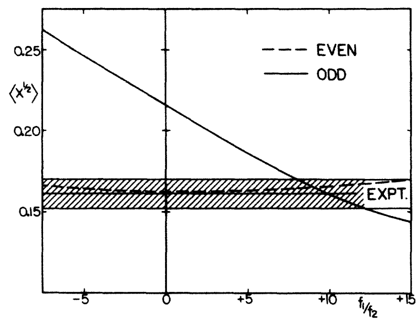

FIG. 4. The theoretical average invariant mass of Σ^0 Dalitz pairs as a function of the ratio of factors f/f_2. The shaded area is the experimental value.

function [Eq. (5)]. We find experimentally

$$\langle x^{1/2}\rangle = 0.161 \pm 0.009.$$

Figure 4 shows the values predicted for even and odd parity as a function of (f_1/f_2). (f_1/f_2) is taken to be a real number,[7] independent of x.[8] It can be seen that this test eliminates odd parity unless $(f_1/f_2) \sim +10$.[9] A perturbation theory estimate[2] of $|(f_1/f_2)|$ gives $\sim\frac{1}{4}$. For $(f_1/f_2)=0$, the predicted values of the invariant mass are

$$\text{for even,}\quad \langle x^{1/2}\rangle = 0.162;$$

$$\text{for odd,}\quad \langle x^{1/2}\rangle = 0.215;$$

and our data strongly favor even parity.

We would like to thank, for their invaluable help and support, Professor R. Adair, Dr. R. Armenteros, Dr. R. Florent, Dr. C. Germain, Professor B. Gregory, Dr. E. Malamud, Professor Ch. Peyrou, H. Schneider, Professor A. Schluter, and R. Tinguely.

We acknowledge very gratefully the efforts of the CERN proton synchrotron machine group, the Saclay hydrogen bubble chamber group, the CERN data analysis group, the CERN track chamber division, and the bubble chamber analysis groups at Maryland and the National Research Laboratory.

---

*Work at University of Maryland supported by the U. S. Atomic Energy Commission.

†Ford Fellow 1961-1962, U. S. National Science Foundation Senior Postdoctoral Fellow 1962-1963.

‡Pisa University and Istituto Nazionale di Fisica Nucleare, Sezione di Pisa, Italy.

‖U. S. National Science Foundation Senior Postdoctoral Fellow on leave from the U. S. Naval Research Laboratory.

⊥National Institute for Research and Nuclear Science, Chilton, England.

**Ford Fellow (CERN) 1961-1962. Present address: Brookhaven National Laboratory, Upton, New York.

††U. S. Steel Fellow.

‡‡U. S. National Science Foundation Senior Postdoctoral Fellow at CERN, 1961-1962.

[1]G. Feinberg, Phys. Rev. 109, 1019 (1958).

[2]G. Feldman and T. Fulton, Nucl. Phys. 8, 106 (1958). Also see R. H. Dalitz, Proceedings of the Aix-en-Provence Conference on Elementary Particles, 1961 (C.E.N. Saclay, France, 1961).

[3]B. Aubert, H. Courant, H. Filthuth, A. Segar, and W. Willis, Proceedings of the International Conference on Instrumentation for High-Energy Physics at CERN (North-Holland Publishing Company, Amsterdam, 1963).

[4]W. E. Humphrey and R. R. Ross, Phys. Rev. 127, 1305 (1962).

[5]Our conclusion in favor of even $\Sigma-\Lambda$ parity supports the conclusion given by R. Tripp, M. Ferro-Luzzi, and M. Watson [Phys. Rev. Letters 8, 175 (1962)] that the $K\Sigma$ parity is odd, since it has been shown that the $K\Lambda$ parity is probably odd [M. M. Block, F. Anderson, A. Pevsner, E. M. Harth, J. Leitner, and H. Cohn, Phys. Rev. Letters 3, 291 (1951); also M. M. Block, L. Lendinara, and L. Monari, Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 371].

[6]L. E. Evans, Nuovo Cimento 25, 580 (1962); University of Wisconsin preprint (unpublished). The complete, correct formula appears only in the preprint.

[7](f_1/f_2) can be taken real since $M_\Sigma - M_\Lambda \ll 2m_\pi$. This follows from dispersion-theory arguments or more generally from the application of time-reversal invariance. Y. S. Kim and J. Sucher (private communication).

[8]The variation of $f_2(x)$ in the range $0 \leq x \leq 1$ is expected to be smaller than two percent. This has been discussed by several authors (see references 1, 2, and 6). Neglecting $f_1(x)$, a large variation in $f_2(x)$, of the order of 100%, would be required to make the odd spectrum fit our results.

[9]With the present statistics, the complete distribution given by Eq. (5) for odd parity and $(f_1/f_2) \cong +10$ cannot be distinguished from the distribution for even parity and $(f_1/f_2)=0$.