

Signals of a superlight gravitino at e^+e^- colliders when the other superparticles are heavy¹

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Abstract

If the gravitino \tilde{G} is very light and all the other supersymmetric particles are above threshold, supersymmetry may still be found at colliders, by looking at processes with only gravitinos and ordinary particles in the final state. We compute here the cross-section for the process $e^+e^- \rightarrow \tilde{G}\tilde{G}\gamma$, whose final state can give rise to a distinctive photon plus missing energy signal at present and future e^+e^- colliders. We describe how the present LEP data can be used to establish a lower bound on the gravitino mass of order 10^{-5} eV. We conclude with a critical discussion of our results, comparing them with related ones and outlining possible generalizations.

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1 Introduction

Low-energy supersymmetry (for reviews and references, see e.g. [1]) is the most popular and probably the best motivated extension of the Standard Model, at energy scales accessible to present or future colliders. At a fundamental level, however, the present dynamical understanding of supersymmetry breaking is still quite unsatisfactory. Denoting by Δm the scale of the supersymmetry-breaking mass splittings between the Standard Model particles and their superpartners, and by Λ_S the microscopic supersymmetry-breaking scale (in one-to-one correspondence with the gravitino mass $m_{3/2}$), from a phenomenological viewpoint both these scales should be taken as arbitrary parameters. Different possibilities then arise, as reviewed in a recent paper of ours [2], to which we refer for more details on the general theoretical framework: we denoted these possibilities by ‘heavy’, ‘light’ and ‘superlight’ gravitino. In the heavy gravitino case, reactions involving the gravitino are never important for collider physics. In the light gravitino case, the gravitino can be relevant in the decays of other supersymmetric particles, if there is sufficient energy to produce the latter. In the superlight gravitino case, also the direct production of gravitinos (with or without other supersymmetric particles) can become relevant. In this paper, we concentrate on the superlight gravitino case, where the two scales Δm and Λ_S are both close to the electroweak scale and to each other, the gravitino mass $m_{3/2}$ is several orders of magnitude below the eV scale and, as will be clear soon, the resulting phenomenology can be quite different from the extensively studied cases of heavy and light gravitino.

Many aspects of the superlight gravitino phenomenology at colliders have been discussed long ago in the pioneering works by Fayet [3, 4], and also in more recent papers [5]. All these authors, however, assumed that some other supersymmetric particle, for example a neutralino or one of the spin-0 partners of the gravitino, is light enough to be produced on-shell in some reaction. Here we take an orthogonal point of view: there may be experiments where the available energy is still insufficient for the on-shell production of other supersymmetric particles, but nevertheless sufficient to give rise to final states with only gravitinos and ordinary particles, at measurable rates.

As discussed in [2], particularly powerful processes to search for a superlight gravitino \tilde{G} (when the supersymmetric partners of the Standard Model particles and the spin-0 superpartners of the gravitino are above threshold) are:

$$e^+e^- \longrightarrow \tilde{G}\tilde{G}\gamma, \quad (1)$$

and $q\bar{q} \longrightarrow \tilde{G}\tilde{G}\gamma$, which would give rise to a distinctive (*photon + missing energy*) signal. The first process can be studied at e^+e^- colliders such as LEP or the proposed NLC, the second one at hadron colliders such as the Tevatron or the LHC. At hadron colliders, we can also consider the process $q\bar{q} \longrightarrow \tilde{G}\tilde{G}g$, which, together with other partonic subprocesses such as $qg \rightarrow q\tilde{G}\tilde{G}$ and $gg \rightarrow g\tilde{G}\tilde{G}$, contributes to the (*jet + missing energy*) signal. In this letter, we compute the cross-section and the relevant angular distributions for the process of eq. (1), in the limit in which the supersymmetric particles of the Mini-

mal Supersymmetric extension of the Standard Model (MSSM), such as sleptons, squarks and gauginos, and all other exotic particles, such as the spin-0 partners of the gravitino, are heavy. We also show how the present LEP data, in the absence of a signal over the Standard Model background, should allow to establish the lower bound $\Lambda_S \gtrsim 200$ GeV, or, equivalently, $m_{3/2} \gtrsim 10^{-5}$ eV. In contrast with other collider bounds discussed in the literature [5], the ones discussed in the present paper cannot be evaded by modifying the mass spectrum of the other supersymmetric particles: making some additional supersymmetric particle light leads in general to stronger bounds. The study of the processes involving quarks and/or gluons and the discussion of the phenomenology at hadron colliders will be presented in a companion paper [6].

For the theoretically oriented readers, we summarize the framework of our calculation. The $\pm 1/2$ helicity components of the superlight but massive gravitino, corresponding to the would-be goldstino, have effective couplings to the MSSM fields comparable in strength with the MSSM gauge couplings. Exploiting the supersymmetric version of the equivalence theorem [7], we start from an effective theory where global supersymmetry is linearly realized but spontaneously broken. Its degrees of freedom are just the MSSM superfields and the superfields containing the goldstino \tilde{G} (denoted here, in the spirit of the equivalence theorem, with the same symbol as the gravitino). Assuming pure F -breaking, as we will do in the following, it is sufficient to add to the MSSM states a single neutral chiral superfield. Such an effective theory is non-renormalizable, and its Lagrangian can be parametrized (up to higher-derivative and Fayet-Iliopoulos terms, assumed here to be negligible) by a Kähler potential K , a superpotential w and a gauge kinetic function f . However, the terms that contribute to the on-shell scattering amplitudes for the process of eq. (1) do not depend on the details of these defining functions, but only on suitable combinations of Δm and Λ_S . We then move to a ‘more effective’ theory by explicit integration of the heavy superpartners in the low-energy limit. The residual degrees of freedom are only the goldstino and the Standard Model particles, and at this level supersymmetry is non-linearly realized [8] in a non-standard way [9]. We finally compute the relevant scattering amplitudes and the cross-section. As we shall see, the final result is independent of Δm , and is function only of the centre-of-mass energy \sqrt{s} and of the supersymmetry-breaking scale Λ_S (or, equivalently, of the gravitino mass $m_{3/2}$).

The paper is organized as follows. In sect. 2 we define in full detail the theoretical framework of our calculation, identifying the explicit form of the relevant lagrangian terms, at the level where supersymmetry is linearly realized but spontaneously broken. In sect. 3 we show how to take the low-energy limit in the case of heavy superpartners, ending up with an effective lagrangian where supersymmetry is non-linearly realized. We also list the independent amplitudes contributing, in this limit, to the process under consideration. In sect. 4 we compute the differential and the integrated cross-section, and discuss the implications of the present LEP data. Finally, in sect. 5 we give a critical discussion of our results and outline further generalizations.

Before concluding this introduction, we should mention some related literature. The

process of eq. (1) was already considered by Fayet [4], who gave the scaling laws of the cross-section with respect to the gravitino mass and the centre-of-mass energy in the low-energy limit. However, at the time the possibility of heavy gauginos was not considered, and, in the limit of light photino, other processes with photinos in the final state become more important. The process of eq. (1) was also considered in [10], in the same kinematical limit as in the present work, but relying on the non-linear realization of supersymmetry proposed in [11]. As will be discussed in the concluding section, the two treatments are not equivalent and lead to different results, both consistent with supersymmetry. The effective interactions of a superlight gravitino with the Standard Model fermions and gauge bosons have been recently discussed in [12], which identified possible new effects associated with Fayet-Iliopoulos terms: these effects do not arise in our calculational framework.

2 Theoretical framework

To discuss the process of eq. (1), given the present collider energies and the typical cuts on the photon spectrum, we can safely neglect the electron mass. Our starting point is an $N = 1$ globally supersymmetric theory with gauge group $U(1)_{em}$. Besides the vector supermultiplet, describing the photon A_μ and the photino λ , the model contains three chiral superfields, with spinor and scalar components ψ_i and φ_i , respectively ($i = 0, 1, 2$). Two of them ($i = 1, 2$) describe the Dirac electron field e together with the selectrons (complex scalar fields) $\tilde{e} \equiv \tilde{e}_L$ and $\tilde{e}^c \equiv (\tilde{e}_R)^*$. The remaining chiral multiplet, ($i = 0$), is electrically neutral, and describes a Majorana fermion, \tilde{G} , and a complex scalar $z \equiv (S + iP)/\sqrt{2}$. The theory is specified, up to higher-derivative and Fayet-Iliopoulos terms, to be neglected here, in terms of three defining functions: the superpotential w , the Kähler potential K and the gauge kinetic function f . The explicit form of the lagrangian in terms of w , K and f is standard: we refer to the Appendix of ref. [2] for its expression in component fields. We assume that the defining functions of the theory are such that, at the minimum of the scalar potential,

$$\langle F^0 \rangle \neq 0, \quad \langle F^{1,2} \rangle = 0, \quad (2)$$

where F^i denote the auxiliary fields of the chiral supermultiplets. Therefore, supersymmetry is spontaneously broken and the goldstino coincides with \tilde{G} . We will also assume $\langle \varphi^i \rangle = 0$ ($i = 1, 2$), so that $U(1)_{em}$ remains an exact local symmetry.

As anticipated, we consider the goldstino interactions in the spirit of the equivalence theorem. The scattering amplitudes involving external goldstinos are identified with corresponding scattering amplitudes with external gravitinos of an $N = 1$ locally supersymmetric theory. This equality holds up to terms that, at fixed scattering angles, are of order $m_{3/2}/\sqrt{s}$, where \sqrt{s} is the centre-of-mass energy of the process. As we shall see, we are always going to consider situations where such correction terms are harmless: typically, \sqrt{s} will be more than $\mathcal{O}(100 \text{ GeV})$, whilst we will be sensitive to values of $m_{3/2}$ less than $\mathcal{O}(10^{-4} \text{ eV})$.

We proceed by expanding the defining functions of the theory around the vacuum, in order to identify the terms potentially relevant for the process under consideration. To carry on such an expansion in a simple way, we characterize the functional dependence of w , K and f as follows:

$$\begin{aligned}
w &= \hat{w}(z) + \dots, \\
K &= \hat{K}(z, \bar{z}) + \hat{K}(z, \bar{z})(|\tilde{e}|^2 + |\tilde{e}^c|^2) + \dots, \\
f &= \hat{f}(z) + \dots,
\end{aligned} \tag{3}$$

where the dots denote terms which will not take part in our discussion. For simplicity, in the Kähler potential of eq. (3) the bilinears in the selectron fields have been chosen to be equal: this will give rise to common supersymmetry-breaking masses for the two selectrons. The reader can easily check that a non-universal choice for the coefficients of $|\tilde{e}|^2$ and $|\tilde{e}^c|^2$ in K , giving rise to different supersymmetry-breaking masses for \tilde{e} and \tilde{e}^c , would not affect the low-energy behaviour of the amplitudes considered here. Also, following the standard practice, we have assumed negligible mixing in the selectron sector. Taking into account eqs. (2) and (3), it is straightforward to evaluate the mass spectrum of the model and the relevant interactions, along the lines of [2]. The photon, the goldstino and the electron remain massless, whereas all the other particles acquire non-vanishing masses proportional to $\langle F^0 \rangle$ and expressed in terms of w , K , f and their derivatives, evaluated on the vacuum. Moreover, even in the presence of non-renormalizable interactions, the expansion of the lagrangian in (canonically normalized) component fields can be rearranged in such a way that all the terms of interest for our calculation are expressed in terms of the mass parameters ($m_{S,P}^2, m^2, M$), associated with the spin-0 partners of the goldstino, the selectrons and the photino respectively, and the scale F of supersymmetry breaking, without explicit reference to w , K and f :

$$\begin{aligned}
\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\bar{\lambda}\not{\partial}\lambda + \frac{1}{2}|M|\bar{\lambda}\lambda + \frac{i}{2}\bar{G}\not{\partial}\tilde{G} \\
&+ \frac{1}{2}(\partial^\mu S)(\partial_\mu S) - \frac{1}{2}m_S^2 S^2 + \frac{1}{2}(\partial^\mu P)(\partial_\mu P) - \frac{1}{2}m_P^2 P^2 \\
&+ i\bar{e}\not{D}e + (D^\mu\tilde{e}_L)^*(D_\mu\tilde{e}_L) - m^2|\tilde{e}_L|^2 + (D^\mu\tilde{e}_R)^*(D_\mu\tilde{e}_R) - m^2|\tilde{e}_R|^2 \\
&- \sqrt{2}i g \left(\tilde{e}_L^* \bar{\lambda}P_L e - \tilde{e}_L \bar{e}P_R \lambda - \tilde{e}_R \bar{e}P_L \lambda + \tilde{e}_R^* \bar{\lambda}P_R e \right) \\
&- \frac{1}{2\sqrt{2}}\frac{m_S^2}{|M|}S \left[\text{Re}\left(\frac{M}{F}\right)\bar{G}\tilde{G} - i\text{Im}\left(\frac{M}{F}\right)\bar{G}\gamma_5\tilde{G} \right] \\
&- \frac{1}{2\sqrt{2}}\frac{m_P^2}{|M|}P \left[\text{Im}\left(\frac{M}{F}\right)\bar{G}\tilde{G} + i\text{Re}\left(\frac{M}{F}\right)\bar{G}\gamma_5\tilde{G} \right] \\
&- \frac{1}{2\sqrt{2}} \left[\text{Re}\left(\frac{M}{F}\right)S - \text{Im}\left(\frac{M}{F}\right)P \right] F_{\mu\nu}F^{\mu\nu} + \frac{1}{2\sqrt{2}} \left[\text{Im}\left(\frac{M}{F}\right)S + \text{Re}\left(\frac{M}{F}\right)P \right] F_{\mu\nu}\tilde{F}^{\mu\nu} \\
&- \frac{1}{4\sqrt{2}}\bar{G}[\gamma^\mu, \gamma^\nu]\lambda \left[\text{Re}\left(\frac{M}{F}\right)F_{\mu\nu} - \text{Im}\left(\frac{M}{F}\right)\tilde{F}_{\mu\nu} \right] \\
&- \frac{m^2}{|M|}\frac{M}{F} \left(\tilde{e}_L^* \bar{G}P_L e + \tilde{e}_R \bar{e}P_L \tilde{G} \right) - \frac{m^2}{|M|}\frac{M^*}{F^*} \left(\tilde{e}_R^* \bar{G}P_R e + \tilde{e}_L \bar{e}P_R \tilde{G} \right)
\end{aligned}$$

$$- \frac{m^2}{2|F|^2} \left(\bar{G} e \bar{e} \tilde{G} - \bar{G} \gamma_5 e \bar{e} \gamma_5 \tilde{G} \right) + \dots \quad (4)$$

In eq. (4), $P_L = (1 - \gamma_5)/2$, $P_R = (1 + \gamma_5)/2$ and $\tilde{F}_{\mu\nu} = 1/2 \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$, where $\epsilon_{0123} = -\epsilon^{0123} = 1$. The parameter $F = \langle \bar{w}_{\bar{z}} (K_{\bar{z}z})^{-1/2} \rangle$ (lower indices denote here derivatives) has the dimension of a mass squared and defines the supersymmetry-breaking scale, $\Lambda_S = |F|^{1/2}$. We remind the reader that, in our flat space-time, $|F|$ is linked to the gravitino mass $m_{3/2}$ by the universal relation

$$|F| = \sqrt{3} m_{3/2} M_{\text{P}}, \quad \left[M_{\text{P}} \equiv (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18} \text{ GeV} \right]. \quad (5)$$

The phases of M and F are assumed to be arbitrary. By means of field redefinitions, we can remove all the phases but one, assigned here to the ratio M/F . For fermions, we have used a four-component notation. The covariant derivatives are given by

$$D_\mu e = (\partial_\mu - i g A_\mu) e, \quad D_\mu \tilde{e} = (\partial_\mu - i g A_\mu) \tilde{e}, \quad D_\mu \tilde{e}^c = (\partial_\mu + i g A_\mu) \tilde{e}^c, \quad (6)$$

where g is the coupling constant of $U(1)_{em}$. Finally, the dots in eq. (4) stand for terms that do not contribute to the amplitude evaluated here¹.

3 Low-energy limit

The lagrangian of eq. (4) would allow the computation of the cross-section for $e^+e^- \rightarrow \tilde{G}\tilde{G}\gamma$ in a generic kinematical regime. Here we restrict ourselves to the case of a heavy superpartner spectrum, with photino, selectrons and the scalars S and P beyond the production threshold at some chosen value of \sqrt{s} . In this case, it is convenient to build an effective lagrangian for the light fields by integrating out the heavy particles. The crucial property of such an effective lagrangian will be its dependence only on the gauge coupling g and on the supersymmetry-breaking scale $|F|^{1/2}$, without any further reference to the supersymmetry-breaking masses ($m_{S,P}^2, m^2, M$). By standard techniques, and focussing only on the terms relevant for our calculation, we obtain:

$$\mathcal{L}_{eff} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{e} \not{D} e + \frac{i}{2} \bar{G} \not{\partial} \tilde{G} + \sum_{i=1}^4 \mathcal{O}_i. \quad (7)$$

The terms \mathcal{O}_i ($i = 1, \dots, 4$) are local operators obtained through the exchange of massive particles in the large mass limit, as shown in fig. 1. Their field-dependent parts have mass dimension $d = 8$, and the corresponding coefficients scale as $1/|F|^2$. The operator \mathcal{O}_1 involves two photons and two goldstinos, and is generated [2] by integrating over S , P and λ :

$$\mathcal{O}_1 = -\frac{i}{64|F|^2} \bar{G} [\gamma^\mu, \gamma^\nu] F_{\mu\nu} \not{\partial} [\gamma^\alpha, \gamma^\beta] F_{\alpha\beta} \tilde{G}. \quad (8)$$

¹There are interaction terms proportional to $\langle \tilde{K}_S \rangle$ and $\langle \tilde{K}_{\bar{S}} \rangle$, not explicitly listed here, that are in principle relevant. An explicit computation shows that their total contribution vanishes. This is in agreement with the possibility of choosing normal coordinates [13], where such terms are absent.

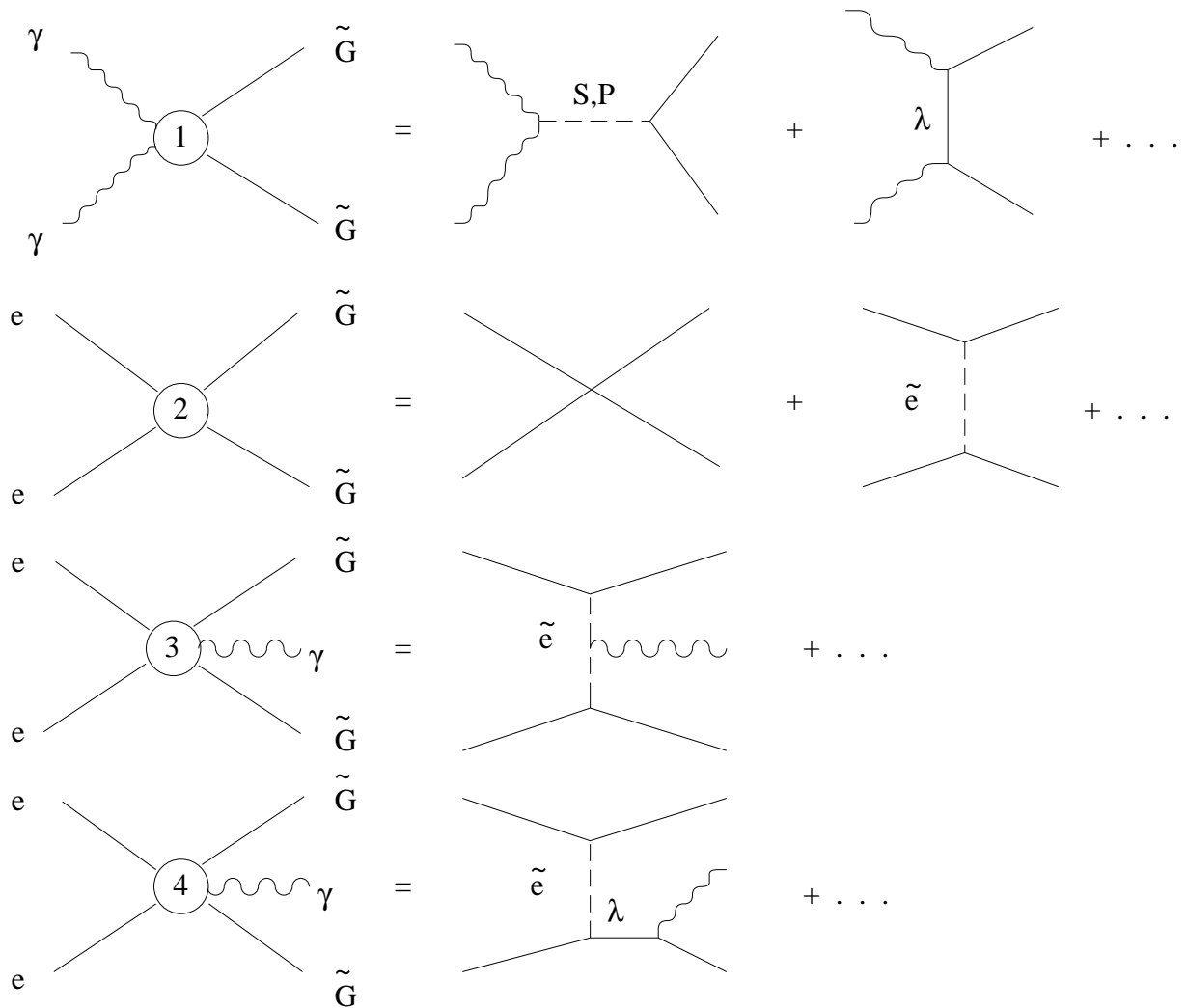


Figure 1: *Diagrammatic origin of the effective operators \mathcal{O}_{1-4} .*

Notice that the form of \mathcal{O}_1 is the result of a crucial cancellation among individual contributions, corresponding to $d = 7$ operators with coefficients scaling as M/F^2 . The operator \mathcal{O}_2 is a four-fermion interaction term among electrons and goldstinos recovered by combining the contact term in the last line of eq. (4) with contributions originating from selectron exchanges [9]:

$$\mathcal{O}_2 = -\frac{1}{2|F|^2} \left\{ \bar{e}\tilde{G}\square\left(\tilde{G}e\right) - \bar{e}\gamma_5\tilde{G}\square\left(\tilde{G}\gamma_5e\right) \right\}. \quad (9)$$

Also in this case, the form of \mathcal{O}_2 is the result of a crucial cancellation among individual $d = 6$ operators scaling as m^2/F^2 . The operator \mathcal{O}_3 is generated by attaching a photon to a selectron exchanged among electrons and goldstinos. Notice that, despite the presence of two selectron propagators, this contribution does not vanish in the large m limit, due

to a compensating factor m^4 from the electron-selectron-goldstino vertices:

$$\mathcal{O}_3 = \frac{igA^\mu}{|F|^2} \left\{ \bar{e}\tilde{G}\partial_\mu \left(\tilde{G}e \right) - \bar{e}\gamma_5\tilde{G}\partial_\mu \left(\tilde{G}\gamma_5e \right) \right\}. \quad (10)$$

The operators \mathcal{O}_2 and \mathcal{O}_3 are not separately gauge invariant. Only their sum is contained in a gauge-invariant combination, as can be seen by recasting it in the form:

$$\mathcal{O}_2 + \mathcal{O}_3 + \dots = \frac{1}{2|F|^2} \left\{ D^\mu \left(\bar{e}\tilde{G} \right) D_\mu \left(\tilde{G}e \right) - D^\mu \left(\bar{e}\gamma_5\tilde{G} \right) D_\mu \left(\tilde{G}\gamma_5e \right) \right\}, \quad (11)$$

where the dots denote terms quadratic in A_μ or vanishing on-shell. Finally, the operator \mathcal{O}_4 is a contact term that directly contributes (as \mathcal{O}_3) to the scattering amplitude under investigation. It originates from a combined selectron and photino exchange:

$$\begin{aligned} \mathcal{O}_4 = \frac{ig}{8|F|^2} & \left\{ \tilde{G} [\gamma^\mu, \gamma^\nu] F_{\mu\nu} \left(e\bar{e}\tilde{G} - \gamma_5e\bar{e}\gamma_5\tilde{G} \right) \right. \\ & \left. + \left(\tilde{G}e\bar{e} - \tilde{G}\gamma_5e\bar{e}\gamma_5 \right) F_{\mu\nu} [\gamma^\mu, \gamma^\nu] \tilde{G} \right\}. \end{aligned} \quad (12)$$

Notice that, despite the presence of a selectron and a photino propagator, this contribution does not vanish in the limit of large m and M , due to the compensating factors m^2 and M coming from the electron-selectron-goldstino and the photon-photino-goldstino vertices, respectively.

It is now possible to list the independent amplitudes contributing, in the low-energy regime, to the scattering process $e^+e^- \rightarrow \tilde{G}\tilde{G}\gamma$. They are shown in fig. 2. The contribution of the operator \mathcal{O}_1 is obtained by attaching an electron-positron line to one of the photons. The operator \mathcal{O}_2 contributes via initial state radiation, and the remaining operators are by themselves local contributions to the process. The total amplitude is therefore the sum of four terms:

$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \mathcal{M}_4, \quad (13)$$

with the following expressions in momentum space:

$$\mathcal{M}_1 = \frac{ig}{16|F|^2} \frac{1}{(p_1 + p_2)^2} [\bar{v}(p_2)\gamma_\mu u(p_1)] [\bar{u}_G(q_1)\mathcal{A}_1^\mu v_G(q_2) - \bar{u}_G(q_2)\mathcal{A}_2^\mu v_G(q_1)], \quad (14)$$

$$\begin{aligned} \mathcal{M}_2 = -\frac{ig}{|F|^2} \bar{v}(p_2) & \left\{ \not{\epsilon} \frac{1}{\not{k} - \not{p}_2} \mathcal{A}_3 + \mathcal{A}_4 \frac{1}{\not{p}_1 - \not{k}} \not{\epsilon} \right. \\ & \left. - \not{\epsilon} \frac{1}{\not{k} - \not{p}_2} \gamma_5 \mathcal{A}_3 \gamma_5 - \gamma_5 \mathcal{A}_4 \gamma_5 \frac{1}{\not{p}_1 - \not{k}} \not{\epsilon} \right\} u(p_1), \end{aligned} \quad (15)$$

$$\mathcal{M}_3 = -\frac{ig}{|F|^2} \epsilon_\mu \bar{v}(p_2) \{ \mathcal{A}_5^\mu - \gamma_5 \mathcal{A}_5^\mu \gamma_5 \} u(p_1), \quad (16)$$

$$\mathcal{M}_4 = \frac{ig}{4|F|^2} \bar{v}(p_2) \{ \mathcal{A}_6 - \gamma_5 \mathcal{A}_6 \gamma_5 \} u(p_1), \quad (17)$$

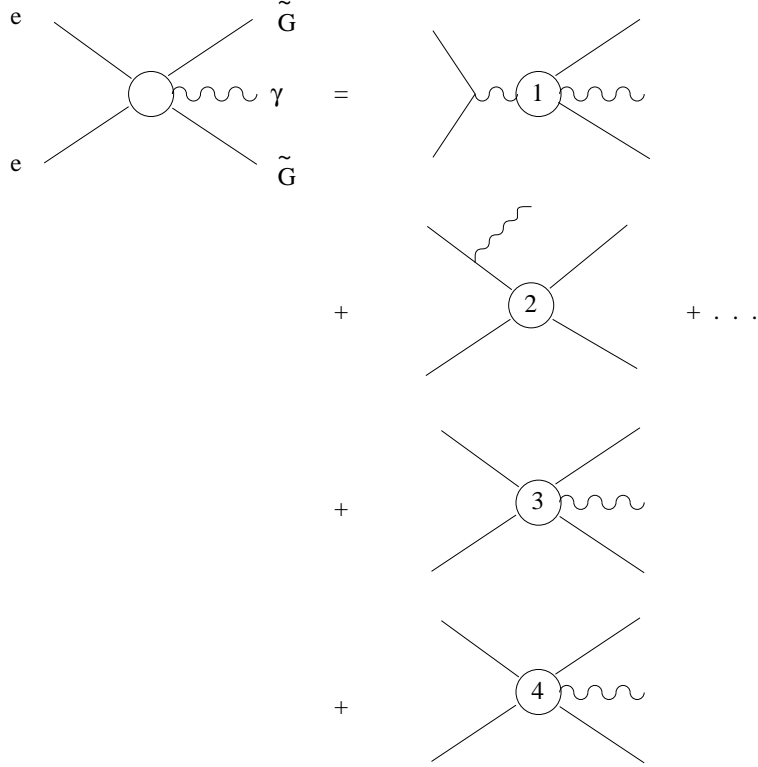


Figure 2: *Feynman diagrams contributing to $e^+e^- \rightarrow \tilde{G}\tilde{G}\gamma$ in the low-energy effective theory.*

where

$$\mathcal{A}_1 = [\not{p}_1 + \not{p}_2, \gamma^\mu](\not{k} + \not{q}_2)[\not{k}, \not{\epsilon}] - [\not{k}, \not{\epsilon}](\not{k} + \not{q}_1)[\not{p}_1 + \not{p}_2, \gamma^\mu], \quad (18)$$

$$\mathcal{A}_2 = [\not{p}_1 + \not{p}_2, \gamma^\mu](\not{k} + \not{q}_1)[\not{k}, \not{\epsilon}] - [\not{k}, \not{\epsilon}](\not{k} + \not{q}_2)[\not{p}_1 + \not{p}_2, \gamma^\mu], \quad (19)$$

$$\mathcal{A}_3 = q_1 \cdot p_1 v_G(q_2) \bar{u}_G(q_1) - q_2 \cdot p_1 v_G(q_1) \bar{u}_G(q_2), \quad (20)$$

$$\mathcal{A}_4 = q_2 \cdot p_2 v_G(q_2) \bar{u}_G(q_1) - q_1 \cdot p_2 v_G(q_1) \bar{u}_G(q_2), \quad (21)$$

$$\mathcal{A}_5^\mu = (p_1 - q_1)^\mu v_G(q_2) \bar{u}_G(q_1) - (p_1 - q_2)^\mu v_G(q_1) \bar{u}_G(q_2), \quad (22)$$

$$\mathcal{A}_6 = [v_G(q_2) \bar{u}_G(q_1) - v_G(q_1) \bar{u}_G(q_2)][\not{k}, \not{\epsilon}] + [\not{k}, \not{\epsilon}][v_G(q_2) \bar{u}_G(q_1) - v_G(q_1) \bar{u}_G(q_2)]. \quad (23)$$

We have denoted by p_1 , p_2 , q_1 , q_2 and k the four-momenta of the incoming electron and positron and of the outgoing goldstinos and photon, respectively. Notice that, due to the structure of the fermionic currents in eqs. (14)–(17) and to the Majorana nature of the goldstino, the only non-vanishing amplitudes are those where electron and positron have opposite helicities, and the two goldstinos have opposite helicities. Taking into account the two possible photon polarizations, we conclude that eight out of sixteen helicity configurations do not contribute to the process.

4 Cross-section for $e^+e^- \rightarrow \tilde{G}\tilde{G}\gamma$

Starting from the amplitudes of eqs. (13)–(23), we computed the double differential cross-section $d\sigma/(dx_\gamma d\cos\theta_\gamma)$, where x_γ is the fraction of the beam energy carried by the photon, and θ_γ is the scattering angle of the photon with respect to the direction of the incoming electron, in the centre-of-mass frame. To cross-check our computation, we used two independent methods. One method proceeded through the evaluation of the helicity amplitudes in the centre-of-mass frame, with explicit expression for spinors, four-momenta and polarization vectors. The second method consisted in the direct computation of the unpolarized cross-section, by means of standard trace techniques, with the help of the program Tracer [14]. After integrating over part of the phase space, we obtained the following result:

$$\frac{d^2\sigma}{dx_\gamma d\cos\theta_\gamma} = \frac{\alpha s^3}{320\pi^2|F|^4} \cdot f(x_\gamma, \cos\theta_\gamma), \quad (24)$$

where

$$f(x_\gamma, \cos\theta_\gamma) = 2(1-x_\gamma)^2 \left[\frac{(1-x_\gamma)(2-2x_\gamma+x_\gamma^2)}{x_\gamma \sin^2\theta_\gamma} + \frac{x_\gamma(-6+6x_\gamma+x_\gamma^2)}{16} - \frac{x_\gamma^3 \sin^2\theta_\gamma}{32} \right]. \quad (25)$$

Notice that the dependence on \sqrt{s} and on the supersymmetry-breaking scale $|F|^{1/2}$ can be factorized out of the function f , which carries instead all the dependence on the photon fractional energy x_γ and on the photon angle θ_γ . For these reasons, it is convenient to illustrate our results in terms of the function f . Figs. 3 and 4 show the dependence of $f(x_\gamma, \cos\theta_\gamma)$ on each of its two variables, keeping the other one fixed to some representative value. Contours corresponding to constant values of f in the $(x_\gamma, |\cos\theta_\gamma|)$ plane are displayed in fig. 5.

It is interesting to observe that, for the soft/collinear part of the photon spectrum, $x_\gamma \ll 1$ and/or $\sin^2\theta_\gamma \ll 1$, our result could have been derived by considering the total cross-section σ_0 for the process $e^+e^- \rightarrow \tilde{G}\tilde{G}$ and applying a standard approximate formula for photon radiation [15]:

$$\frac{d\sigma}{dx_\gamma d\cos\theta_\gamma} \simeq \sigma_0[\hat{s}] \cdot \frac{\alpha}{\pi} \frac{1+(1-x_\gamma)^2}{x_\gamma \sin^2\theta_\gamma}, \quad (26)$$

where σ_0 is evaluated for $\hat{s} \equiv (1-x_\gamma)s$. Computing σ_0 directly from the effective operator of eq. (9), we found [9]:

$$\sigma_0 \equiv \sigma(e^+e^- \rightarrow \tilde{G}\tilde{G}) = \frac{s^3}{160\pi|F|^4}. \quad (27)$$

Plugging this result into (26) does indeed reproduce the leading term of the above result (24)–(25). As expected, for relatively soft/collinear photons the cross-section is dominated by initial-state radiation, and the signal must compete with the irreducible Standard Model background coming from $e^+e^- \rightarrow \nu\bar{\nu}\gamma$. This suggests that, in the absence of an

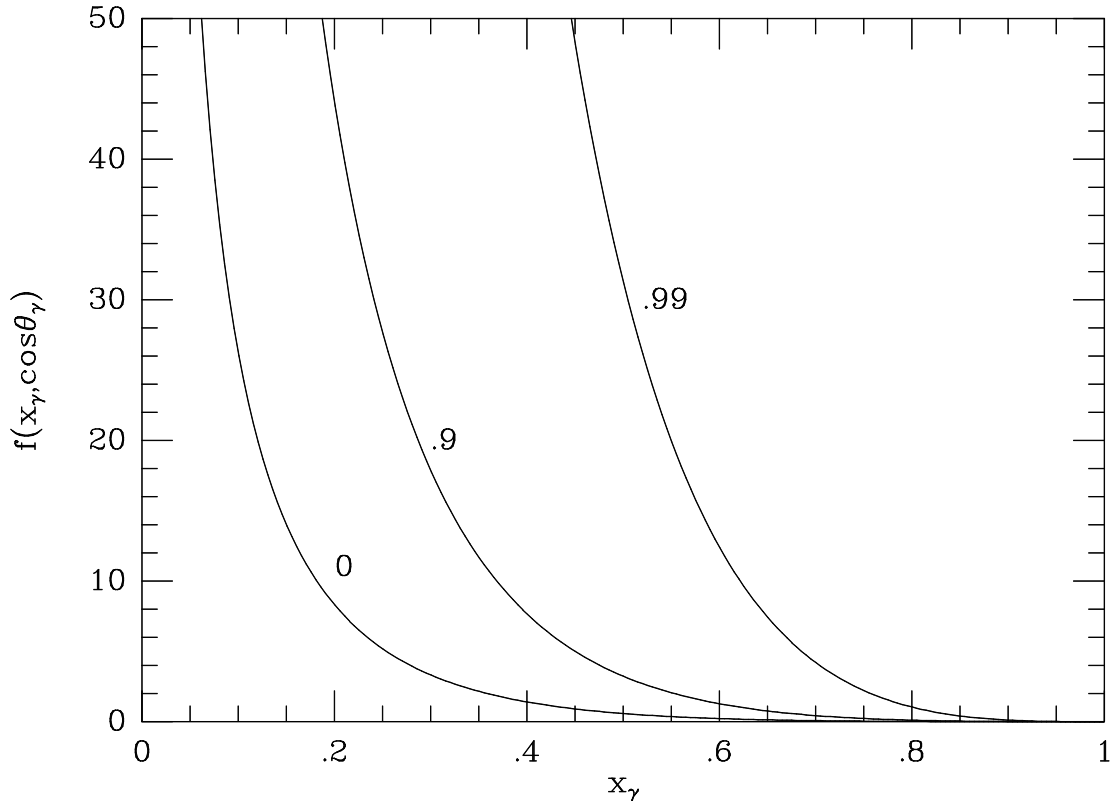


Figure 3: *The function f of eq. (25), plotted versus x_γ , for the indicated representative values of $|\cos \theta_\gamma|$.*

anomaly with respect to the Standard Model predictions and applying rather loose cuts on the photon spectrum, the approximation of eq. (26) should be sufficient to obtain a lower bound on the gravitino mass. On the other hand, in the presence of an anomaly the specific features of the spectrum may be useful in establishing the significance of its interpretation in terms of a superlight gravitino.

From the double differential cross-section of eqs. (24) and (25), the total cross-section can be easily computed. We recall that, because of the QED infrared and collinear singularities, we need to specify some cuts on the photon spectrum. To illustrate our results, we specify our cuts in terms of a minimum value for x_γ and a maximum value for $|\cos \theta_\gamma|$, to be denoted by $x_{\gamma,min}$ and $|\cos \theta_\gamma|_{max}$, respectively². The integrated cross-section is then given by

$$\sigma = \frac{\alpha s^3}{320\pi^2 |F|^4} \cdot I(x_{\gamma,min}, |\cos \theta_\gamma|_{max}), \quad (28)$$

where the integral

$$I(x_{\gamma,min}, |\cos \theta_\gamma|_{max}) = \int_{x_{\gamma,min}}^1 dx_\gamma \int_{-|\cos \theta_\gamma|_{max}}^{|\cos \theta_\gamma|_{max}} d\cos \theta_\gamma f(x_\gamma, \cos \theta_\gamma) \quad (29)$$

²Alternatively, we could have chosen to cut in $(x_\gamma)_T \equiv x_\gamma \sin \theta_\gamma$ rather than in x_γ .

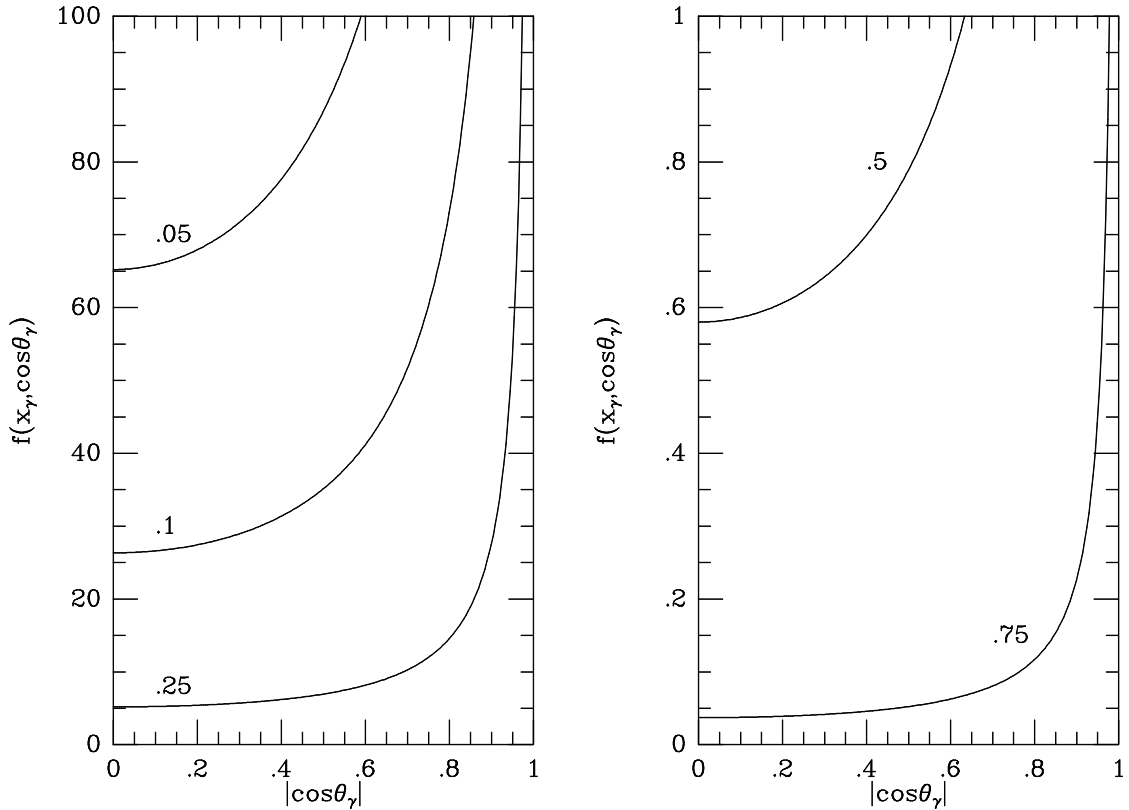


Figure 4: *The function f of eq. (25), plotted versus $|\cos \theta_\gamma|$, for the indicated representative values of x_γ (notice the different scale in the two windows).*

can be evaluated analytically by elementary methods. Table 1 gives the values of the integral I , defined in eq. (29), for some representative choices of the cuts $x_{\gamma,min}$ and $|\cos \theta_\gamma|_{max}$.

The process under consideration can give rise to an observable signal at LEP, characterized by a single photon plus missing energy. From eq. (28), we see that an experimental upper bound σ_{exp} on the signal cross-section, obtained at the energy \sqrt{s} and for a given set of cuts, corresponding to a definite value of the integral I , translates into a lower limit on the supersymmetry breaking scale $|F|^{\frac{1}{2}}$:

$$|F|^{\frac{1}{2}} > 125 \text{ GeV} \left[\frac{\sqrt{s} \text{ (GeV)}}{200} \right]^{\frac{3}{4}} \left[\frac{I}{\sigma_{exp} \text{ (pb)}} \right]^{\frac{1}{8}}, \quad (30)$$

or, in terms of the gravitino mass and remembering eq. (5):

$$m_{3/2} > 3.8 \cdot 10^{-6} \text{ eV} \left[\frac{\sqrt{s} \text{ (GeV)}}{200} \right]^{\frac{3}{2}} \left[\frac{I}{\sigma_{exp} \text{ (pb)}} \right]^{\frac{1}{4}}. \quad (31)$$

Notice that these limits are mainly sensitive to the energy \sqrt{s} , while the dependence on σ_{exp} is quite mild. For example, a factor 10 reduction in σ_{exp} would increase the lower bounds

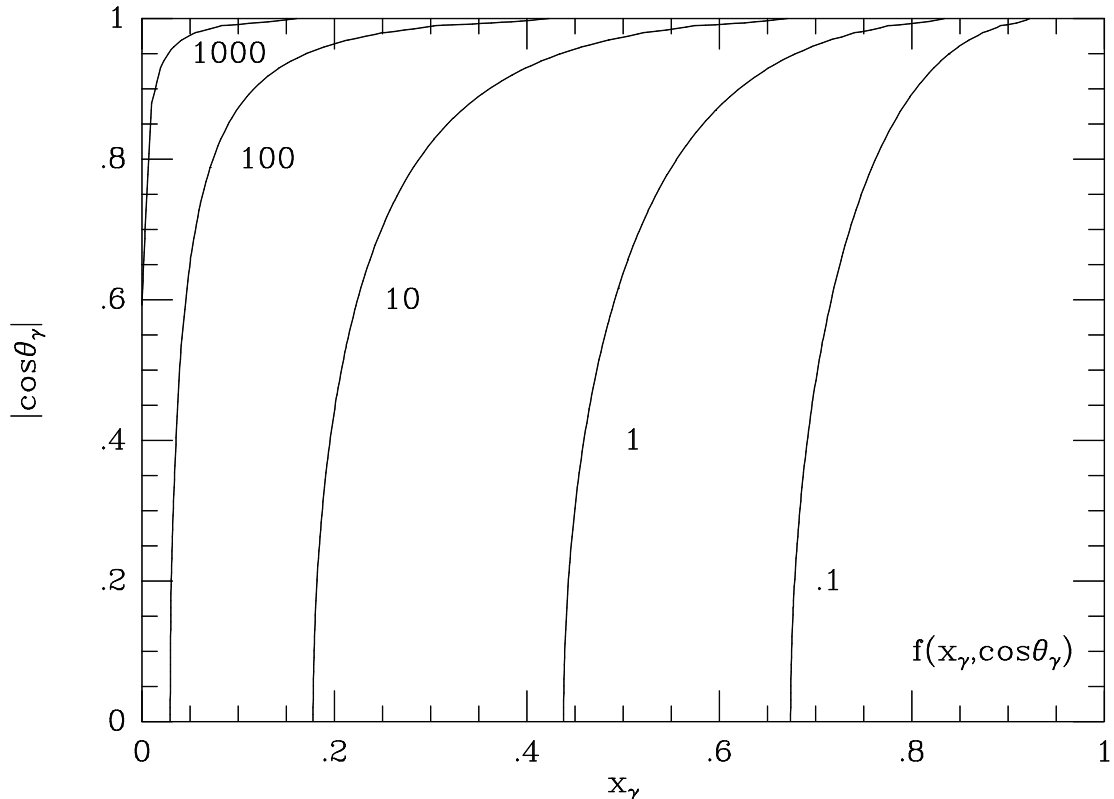


Figure 5: *Contours of the function f of eq. (25) in the $(x_\gamma, |\cos\theta_\gamma|)$ -plane.*

on $|F|^{\frac{1}{2}}$ and $m_{3/2}$ by a factor 1.3 and 1.8, respectively. Similarly, the limits do not depend strongly on the precise choice of the cuts, within the set displayed in table 1. Up to now, the highest energy reached by LEP is 183 GeV, where each of the four experiments has collected an integrated luminosity of about 60 pb^{-1} . Both at $\sqrt{s} = 183 \text{ GeV}$ and at lower energies, no anomalies have been reported so far [16] in the photon plus missing energy channel, which, in the Standard Model, is dominated³ by the process $e^+e^- \rightarrow \nu\bar{\nu}\gamma$. We leave to our experimental colleagues the detailed analysis of the resulting bounds. In the meantime, we can tentatively assume an upper bound of 0.2 pb on the signal cross-section σ_{exp} , integrated over $|x_\gamma| > 0.05$ and $|\cos\theta_\gamma| < 0.95$. Taking into account that, for our representative cuts, the Standard Model cross-section for $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ is approximately 5 pb, with a pronounced peak of the photon spectrum around the ‘radiative return to the Z ’, we obtain the lower bound

$$|F|^{\frac{1}{2}} > 200 \text{ GeV}, \quad (32)$$

corresponding to the following lower bound on the gravitino mass:

$$m_{3/2} > 10^{-5} \text{ eV}. \quad (33)$$

³At small angle, one should also consider the reducible background due to $e^+e^- \rightarrow (e^+e^-)\gamma$ and $e^+e^- \rightarrow (\gamma\gamma)\gamma$, where the particles in brackets are undetected.

$x_{\gamma,min}$	$ \cos \theta_{\gamma} _{max}$	0.7	0.8	0.85	0.9	0.95	0.975
0.2		1.59	2.02	2.31	2.72	3.39	4.05
0.15		2.54	3.22	3.68	4.32	5.39	6.44
0.1		4.21	5.34	6.11	7.17	8.93	10.7
0.05		7.79	9.87	11.3	13.2	16.5	19.7

Table 1: Values of the integral I , defined in eq. (29), for some representative choices of the cuts $x_{\gamma,min}$ and $|\cos \theta_{\gamma}|_{max}$.

5 Discussion

As a first comment, we would like to remark that, even if our results were obtained in a simplified model, they remain valid when we include the full gauge group and matter content of the MSSM, as long as all supersymmetric particles but the gravitino cannot be produced on-shell.

We also recall that our computation was based on the explicit integration of the heavy superpartners in the low-energy limit, starting from a generic theory where supersymmetry is linearly realized but spontaneously broken by an F -term, with negligible Fayet-Iliopoulos and higher-derivative terms. This led to the non-linear realization of global supersymmetry associated with the effective lagrangian (7), and finally to explicit expressions for the differential and integrated cross-sections. However, as recently discussed in [9], such non-linear realization is not unique. For example, the authors of [10] computed the same cross-sections in a different non-linear realization [11] (believed then to be unique), and found a different result, although with the same dependence on F and s as ours. To relax the assumptions of the present calculation, and provide a framework encompassing the present case, the case of [10] and additional possibilities, we would need a general parametrization of the possible non-linear realizations, at least for the terms that affect the process under consideration. Unfortunately, such a general formulation is not yet available. Neglecting the electron mass and the mixing in the selectron sector, however, we know from [9] the most general form of the local four-fermion effective interaction that

replaces \mathcal{O}_2 [see eq. (9)] in an arbitrary non-linear realization. It depends on two free parameters, α_L and α_R , associated with the contributions of $e_L \equiv P_L e$ and $e_R \equiv P_R e$, respectively. Taking advantage of the fact that, for realistic experimental situations, the bulk of the cross-section is well reproduced by the approximate formula of eq. (26), we give here [9] the corresponding general unpolarized cross-section σ_0^{GEN} :

$$\sigma_0^{GEN}(e^+e^- \rightarrow \tilde{G}\tilde{G}) = \frac{s^3}{15360\pi|F|^4} \left[(8 + 10\alpha_L + 5\alpha_L^2) + (8 + 10\alpha_R + 5\alpha_R^2) \right]. \quad (34)$$

The results of the present paper are then obtained, in the approximation of eq. (26), for $\alpha_L = \alpha_R = -4$, those of [10] for $\alpha_L = \alpha_R = 0$. The minimum value of the cross-section is obtained for $\alpha_L = \alpha_R = -1$:

$$\sigma_0^{min}(e^+e^- \rightarrow \tilde{G}\tilde{G}) = \frac{s^3}{2560\pi|F|^4}. \quad (35)$$

In the absence of experimental anomalies, the combination of eq. (35) and eq. (26) can be safely used to establish a model-independent lower bound on the gravitino mass. Because of the strong and universal power-law behaviour of the cross-section, always proportional to $s^3/|F|^4$, this bound is rather stable with respect to variations of the parameters α_L and α_R over plausible ranges. However, should a signal show up at LEP or at future linear colliders, having the full expression of the cross-section for the most general class of models would be very important, since a detailed analysis of the photon spectrum would offer the unique opportunity of distinguishing among possible fundamental theories.

In the present paper, we have assumed that all exotic particles besides the gravitino are far from the production threshold at the available energy, so that the local operators \mathcal{O}_1 – \mathcal{O}_4 provide a good approximation of the dynamics. An advantage of our approach, with respect to the use of non-linear lagrangians, is that, by relaxing the kinematical assumption about superparticle masses, we could take into account also propagator effects, which might be important if a signal were detected.

Concerning the energy dependence of the cross-sections analyzed here, we recall that the authors of [12] have recently proposed an interaction term among two goldstinos and a photon that reads:

$$\delta\mathcal{L} = \frac{\mu^2}{F^2} \left(\partial^\mu \tilde{G} \right) \gamma^\nu \tilde{G} F_{\mu\nu}, \quad (36)$$

where μ^2 is an independent mass parameter. If present, and in the absence of cancellation mechanisms, such a term would induce a local four-fermion operator, involving two goldstinos and an electron-positron pair, with dimension $d = 6$, and characterized by a dimensionful coupling $\mathcal{O}(\mu^2/|F|^2)$. Such a four-fermion operator would contribute to the cross-section for $e^+e^- \rightarrow \tilde{G}\tilde{G}$ with terms scaling as $s\mu^4/|F|^4$. As a consequence, also the scaling properties of the cross-section for $e^+e^- \rightarrow \tilde{G}\tilde{G}\gamma$ would be modified. Depending on the value of μ^2 and on the typical energy of the process, the gravitino phenomenology could be more or less heavily affected. However, the term of eq. (36) does not arise in our

calculational framework. More importantly, it was shown in full generality [9] that local four-fermion operators such as those mentioned above are not allowed by supersymmetry. This leads us to the belief that the energy dependence of the cross-section derived here will survive in the most general case.

The processes considered here, or other processes controlled by the same effective interactions, could in principle be relevant for nucleosynthesis or stellar cooling [17, 2, 12]. In general, however, the bounds coming from high-energy colliders leave little room for interesting effects in astrophysics and cosmology [2], since the relevant cross-sections have a strong positive power-dependence on the energy, and the typical energies involved are much smaller than the present collider energies.

Bounds coming from the finite goldstino-smuon loop contribution to the anomalous magnetic moment of the muon [18] are not very stringent either. For smuon masses up to 1 TeV, they do not constrain $m_{3/2}$ more strongly than the processes discussed in this paper. Moreover, ambiguities similar to the ones discussed for our result should be taken into account, since the operator associated with $(g-2)_\mu$, when supersymmetrized, corresponds to a higher-derivative term, to which we may associate an arbitrary counterterm if we do not have additional informations on the fundamental theory.

In summary, high-energy colliders are by far the best environment to test the possible existence of a superlight gravitino. The present LEP data, analysed as discussed in the present paper, should allow to establish an absolute lower bound $m_{3/2} \gtrsim 10^{-5}$ eV. The prospects for the present Tevatron data are at least as good, since the higher available energy should be more than enough to compensate for the more difficult experimental environment [6].

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References

- [1] H.-P. Nilles, Phys. Rep. 110 (1984) 1;
H.E. Haber and G.L. Kane, Phys. Rep. 117 (1985) 75;
S. Ferrara, ed., *Supersymmetry* (North-Holland, Amsterdam, 1987).
- [2] A. Brignole, F. Feruglio and F. Zwirner, Nucl. Phys. B501 (1997) 332 [hep-ph/9703286].
- [3] P. Fayet, Phys. Lett. B84 (1979) 421, B86 (1979) 272 and B175 (1986) 471.
- [4] P. Fayet, Phys. Lett. B117 (1982) 460.
- [5] T. Bhattacharya and P. Roy, Phys. Rev. Lett. 59 (1987) 1517 [**E**: 60 (1988) 1455] and Phys. Rev. D38 (1988) 2284;
D.A. Dicus, S. Nandi and J. Woodside, Phys. Rev. D41 (1990) 2347, D43 (1991) 2951 and Phys. Lett. B258 (1991) 231;
D.A. Dicus and P. Roy, Phys. Rev. D42 (1990) 938;
J.L. Lopez, D.V. Nanopoulos and A. Zichichi, Phys. Rev. Lett. 77 (1996) 5168;
D.A. Dicus and S. Nandi, Phys. Rev. D56 (1997) 4166;
J.L. Lopez, D.V. Nanopoulos and A. Zichichi, Phys. Rev. D55 (1997) 5813;
J. Kim, J.L. Lopez, D.V. Nanopoulos, R. Rangarajan and A. Zichichi, Phys. Rev. D57 (1998) 373.
- [6] A. Brignole, F. Feruglio, M.L. Mangano and F. Zwirner, preprint hep-ph/9801329.
- [7] P. Fayet, Phys. Lett. B70 (1977) 461 and B86 (1979) 272;
R. Casalbuoni, S. De Curtis, D. Dominici, F. Feruglio and R. Gatto, Phys. Lett. B215 (1988) 313 and Phys. Rev. D39 (1989) 2281.
- [8] D.V. Volkov and V.P. Akulov, Phys. Lett. B46 (1973) 109;
E.A. Ivanov and A.A. Kapustnikov, J. Phys. A11 (1978) 2375;
T. Uematsu and C.K. Zachos, Nucl. Phys. B201 (1982) 250;
S. Samuel and J. Wess, Nucl. Phys. B221 (1983) 153.
- [9] A. Brignole, F. Feruglio and F. Zwirner, JHEP11 (1997) 001 [hep-th/9709111].
- [10] O. Nachtmann, A. Reiter and M. Wirbel, Z. Phys. C27 (1985) 577.
- [11] J. Wess, in *Quantum Theory of Particles and Fields* (B. Jancewicz and J. Lukierski eds.), World Scientific, 1983, p. 223.
- [12] M. A. Luty and E. Ponton, preprint hep-ph/9706268.
- [13] M.T. Grisaru, M. Rocek and A. Karlhede, Phys. Lett. B120 (1983) 110.

- [14] M. Jamin and M.E. Lautenbacher, *Comput. Phys. Commun.* 74 (1993) 265.
- [15] E.A. Kuraev and V.S. Fadin, *Sov. J. Nucl. Phys.* 41 (1985) 466;
 G. Altarelli and G. Martinelli, *Physics at LEP*, CERN Yellow Report 86-02 (J. Ellis and R. Peccei eds.), Vol. 1, p. 47;
 O. Nicrosini and L. Trentadue, *Phys. Lett.* B196 (1987) 551, *Z. Phys.* C39 (1988) 479 and *Nucl. Phys.* B318 (1989) 1.
- [16] The Aleph Collaboration, preprint CERN-PPE/97-122;
 the Delphi Collaboration, *Phys. Lett.* B380 (1996) 471;
 the L3 Collaboration, *Phys. Lett.* B415 (1998) 299 and B412 (1998) 201;
 the OPAL Collaboration, preprint CERN-PPE/97-132;
 P.J. Dornan (ALEPH), P. Charpentier (DELPHI), M. Pohl (L3) and A.K. Honma (OPAL), talks given at the open session of the LEP Experiments Committee, CERN, November 1997.
- [17] P. Fayet, *Phys. Lett.* B69 (1977) 489;
 M. Fukugita and N. Sakai, *Phys. Lett.* B114 (1982) 23;
 A. Bouquet and C.E. Vayonakis, *Phys. Lett.* B116 (1982) 219;
 T. Moroi, H. Murayama and M. Yamaguchi, *Phys. Lett.* B303 (1993) 289;
 M. Nowakowski and S.D. Rindani, *Phys. Lett.* B348 (1995) 115;
 T. Gherghetta, *Nucl. Phys.* B485 (1997) 25;
 J.A. Grifols, R. Mohapatra and A. Riotto, *Phys. Lett.* B400 (1997) 124 and B401 (1997) 283;
 J.A. Grifols, E. Massó and R. Toldra, *Phys. Rev.* D57 (1998) 614;
 D.A. Dicus, R.N. Mohapatra and V.L. Teplitz, *Phys. Rev.* D57 (1998) 578.
- [18] S. Ferrara and E. Remiddi, *Phys. Lett.* B53 (1974) 347;
 P. Fayet, *Phys. Lett.* B70 (1977) 461 and in *Unification of particle interactions* (S. Ferrara, J. Ellis and P. van Nieuwenhuizen eds., Plenum, New York, 1980), p. 587;
 R. Barbieri and L. Maiani, *Phys. Lett.* B117 (1982) 203;
 A. Georges and P. Le Doussal, *Nucl. Phys.* B255 (1985) 532;
 F. del Aguila, *Phys. Lett.* B160 (1985) 87;
 A. Méndez and F.X. Orteu, *Nucl. Phys.* B256 (1985) 181 and *Phys. Lett.* B163 (1985) 167.