

NON-FACTORIZABLE CORRECTIONS TO W-PAIR PRODUCTION**W. Beenakker^{*}** , **A.P. Chapovsky[†]**

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ABSTRACT

In this paper we study the non-factorizable QED corrections to W -pair-mediated (charged-current) four-fermion production in electron-positron collisions. A brief account of the obtained analytical results is given. They turn out to be different from the ones published in the literature. For the first time numerical results are presented, in particular the effects on the W line-shape. These effects are of the order of a per cent. Applying the same methods to ZZ - or ZH -mediated four-fermion production, the non-factorizable $\mathcal{O}(\alpha)$ corrections to the Z or H line-shape vanish.

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1 Introduction

With the start of LEP2, quantitative knowledge of the radiative corrections to the four-fermion production process $e^+e^- \rightarrow 4f$ is needed [1]. The full calculation of all these corrections will be extremely involved and at present one relies on approximations [1], such as leading-log initial-state radiation and running couplings [2]. Another approach is to exploit the fact that in particular the corrections associated with the production of an intermediate W -boson pair are important. This (charged-current) production mechanism dominates at LEP2 energies and determines the LEP2 sensitivity to the mass of the W boson and to the non-Abelian triple gauge-boson interactions. As such, one could approximate the complete set of radiative corrections by considering only the leading terms in an expansion around the W poles. The double-pole residues thus obtained could be viewed as a gauge-invariant definition of corrections to “ W -pair production”. The sub-leading terms in this expansion are generically suppressed by powers of Γ_W/M_W , with M_W and Γ_W denoting the mass and width of the W boson. The quality of this double-pole approximation degrades in the vicinity of the W -pair production threshold, but a few Γ_W above threshold it is already quite reliable [3]. It is conceivable that in the near future a combination of the above-mentioned approximations will result in sufficiently accurate theoretical predictions for four-fermion production processes.

In the double-pole approximation the complete set of first-order radiative corrections to the charged-current four-fermion processes can be divided into so-called factorizable and non-factorizable corrections [1, 3], i.e. corrections that manifestly contain two resonant W propagators and those that do not. In view of gauge-invariance requirements, some care has to be taken with the precise definition of this split-up (see Sect. 2.3). In the factorizable corrections one can distinguish between corrections to W -pair production and W decay. In this letter we address the question of the size of the usually neglected non-factorizable corrections. From the complete set of electroweak Feynman diagrams that contribute to the full $\mathcal{O}(\alpha)$ correction, we will therefore only consider the non-factorizable ones, both for virtual corrections and real-photon bremsstrahlung. To be more precise, since we are only interested in the double-pole terms we are led to consider only non-factorizable QED diagrams in the soft-photon limit. Subsequently, the photon is treated inclusively, without imposing any limits on the photon phase space [4].

This is the same approach as adopted by the authors of Ref. [5], who were the first to calculate non-factorizable W -pair corrections. For the present calculations, we have used two different methods. One is an extension of the treatment in [5], the other is a modification of the standard methods, which involves a combination of the decomposition of multipoint scalar functions and the Feynman-parameter technique. The results obtained with our two methods are in complete mutual agreement. However, in contrast to [5] a clear separation between virtual and real photonic corrections has been made in both methods, which is essential to establish the cancellations of infrared and collinear divergences. This treatment reveals a significant difference between our results and those obtained by the authors of [5]. Our final results do not contain any logarithmic terms involving the final-state fermion masses, whereas in the results of [5] explicit logarithms of fermion-mass ratios occur (see discussion in Sect. 4.1 of Ref. [5]). This difference can be traced back to the fact that although the fermion masses can formally

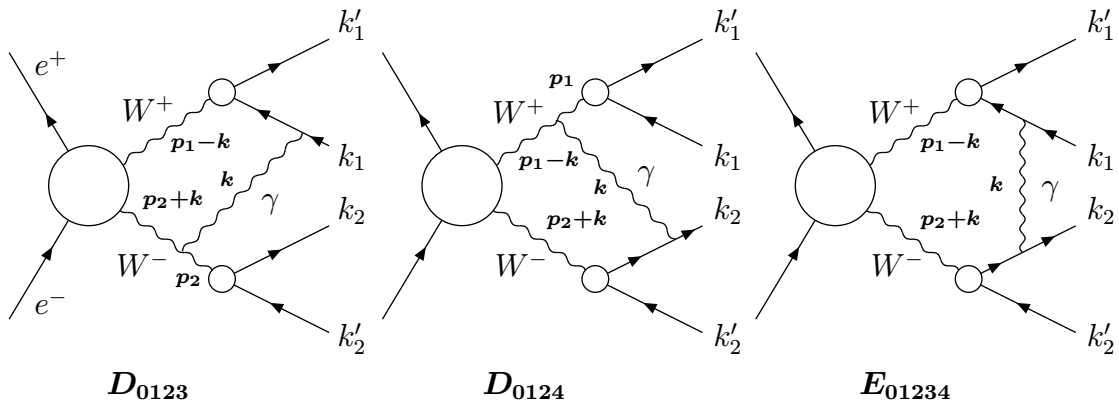


Figure 1: Virtual diagrams contributing to the manifestly non-factorizable W -pair corrections. The scalar functions corresponding to these diagrams are denoted by D_{0123} , D_{0124} , and E_{01234} .

be neglected in the absence of collinear divergences, they have to be introduced in intermediate results in order to regularize those divergences before dropping out from the final results.

In the next section we briefly focus on the analytical results as obtained with the modified standard technique. A detailed account of our study and a discussion of both calculational methods will be published elsewhere. In the last section we present numerical implications of the non-factorizable corrections. Our calculations confirm that non-factorizable corrections vanish in the special case of initial–final state interference, thereby making non-factorizable radiative corrections independent from the W production angle, and in all cases when the integrations over both invariant masses of the virtual W bosons are performed [4]. The practical consequence of the latter is that pure angular distributions are unaffected by non-factorizable $\mathcal{O}(\alpha)$ corrections. So, the studies of non-Abelian triple gauge-boson couplings at LEP2 [6] are not affected by these corrections. The completely new aspect that we have addressed in our analysis is the effect of non-factorizable corrections on invariant-mass distributions (W line-shapes). These distributions play a crucial role in extracting the W -boson mass from the data through direct reconstruction of the Breit–Wigner resonances. The non-factorizable corrections to the line-shapes turn out to be the same for quark and lepton final states, provided the integrations over the decay angles have been performed.

2 Non-factorizable corrections: analytical results

In this section we present the results calculated in the modified standard technique. Details of the calculation and the alternative treatment, which is an extension of the method of [5], will be published elsewhere.

2.1 Virtual corrections for purely leptonic final states

As a first step we consider the manifestly non-factorizable corrections to the simplest class of charged-current four-fermion processes, involving a purely leptonic final state:

$$e^+(q_1)e^-(q_2) \rightarrow W^+(p_1) + W^-(p_2) \rightarrow \nu_\ell(k'_1)\ell^+(k_1) + \ell'^-(k_2)\bar{\nu}_{\ell'}(k'_2). \quad (1)$$

All external fermions are taken to be massless whenever possible. The relevant contributions consist of the final-final and intermediate-final state photonic interactions displayed in Fig. 1. In principle also manifestly non-factorizable vertex corrections exist, which arise when the photon in Fig. 1 does not originate from a W -boson line but from the $\gamma WW/ZWW$ vertex (hidden in the central blob). Those contributions can be shown to vanish, using power-counting arguments [1]. Also the manifestly non-factorizable initial-final state interference effects disappear in our approach. This happens upon adding virtual and real corrections.

The double-pole contribution of the virtual corrections to the differential cross-section can be written in the form

$$d\sigma_{\text{virt}} = 32\pi\alpha \text{Re} \left[i(p_2 \cdot k_1)D_1D_{0123} + i(p_1 \cdot k_2)D_2D_{0124} + i(k_1 \cdot k_2)D_1D_2E_{01234} \right] d\sigma_{\text{Born}}, \quad (2)$$

where $D_{1,2} = p_{1,2}^2 - M_W^2 + iM_W\Gamma_W$ are the inverse (Breit-Wigner) W -boson propagators. The functions D_{0123} , D_{0124} , and E_{01234} are the scalar integrals corresponding to the diagrams shown in Fig. 1, with the integration measure defined as $d^4k/(2\pi)^4$. The propagators occurring in these integrals are labelled according to: 0 = photon, 1 = W^+ , 2 = W^- , 3 = ℓ^+ , and 4 = ℓ^- . Note that the factorization property exhibited in Eq. (2) is a direct consequence of the soft-photon approximation, which is inherent in our approach.

To write down the analytical results we need to introduce some kinematic invariants:

$$m_{1,2}^2 = k_{1,2}^2, \quad s = (p_1 + p_2)^2, \quad s_{122'} = (k_1 + k_2 + k'_2)^2, \quad s_{12} = (k_1 + k_2)^2, \quad (3)$$

and some short-hand notations:

$$y_0 = \frac{D_1}{D_2}, \quad \zeta = 1 - \frac{s_{122'}}{M_W^2} - i0, \quad x_s = \frac{\beta - 1}{\beta + 1} + i0, \quad \beta = \sqrt{1 - 4M_W^2/s}. \quad (4)$$

Here $\pm i0$ denotes an infinitesimal imaginary part, the sign of which is determined by causality.

The scalar four-point function D_{0123} is infrared-finite, owing to the finite decay width of the W boson. In the soft-photon limit it takes the form

$$\begin{aligned} D_{0123} = & \frac{i}{16\pi^2 M_W^2} \frac{1}{[D_2 - \zeta D_1]} \left\{ 2 \mathcal{L}i_2\left(\frac{1}{y_0}; \frac{1}{\zeta}\right) - \mathcal{L}i_2\left(x_s; \frac{1}{y_0}\right) - \mathcal{L}i_2\left(\frac{1}{x_s}; \frac{1}{y_0}\right) \right. \\ & \left. + \mathcal{L}i_2\left(x_s; \zeta\right) + \mathcal{L}i_2\left(\frac{1}{x_s}; \zeta\right) + \left[\ln\left(\frac{M_W^2}{m_1^2}\right) + 2 \ln(\zeta) \right] \left[\ln(y_0) + \ln(\zeta) \right] \right\}. \quad (5) \end{aligned}$$

The function $\mathcal{L}i_2(x; y)$ is the continued dilogarithm

$$\mathcal{L}i_2(x; y) = \text{Li}_2(1 - xy) + \ln(1 - xy) [\ln(xy) - \ln(x) - \ln(y)], \quad (6)$$

with $\text{Li}_2(x)$ the usual dilogarithm and x, y lying on the first Riemann sheet. The answer for the second four-point function, D_{0124} , can be written in a similar way.

The five-point scalar function, E_{01234} , can be evaluated by means of a decomposition into a sum of four-point functions [7]. In the double-pole approximation this decomposition reads

$$w^2 E_{01234} = 2\Delta_4 D_{1234} + (w \cdot v_1) D_{0234} + (w \cdot v_2) D_{0134} + (w \cdot v_3) D_{0124} + (w \cdot v_4) D_{0123}, \quad (7)$$

with

$$\begin{aligned} v_{1\mu} &= -\varepsilon_{\mu\alpha\beta\gamma} p_2^\alpha k_1^\beta k_2^\gamma, & v_{2\mu} &= +\varepsilon_{\alpha\mu\beta\gamma} p_1^\alpha k_1^\beta k_2^\gamma, & v_{3\mu} &= -\varepsilon_{\alpha\beta\mu\gamma} p_1^\alpha p_2^\beta k_2^\gamma, \\ v_{4\mu} &= +\varepsilon_{\alpha\beta\gamma\mu} p_1^\alpha p_2^\beta k_1^\gamma, & w^\mu &= D_1 v_1^\mu + D_2 v_2^\mu, & \Delta_4 &= [\varepsilon_{\alpha\beta\gamma\delta} p_1^\alpha p_2^\beta k_1^\gamma k_2^\delta]^2, \end{aligned} \quad (8)$$

using the convention $\varepsilon^{0123} = -\varepsilon_{0123} = 1$. The labelling of the scalar functions (D_{ijkl}) is defined below Eq. (2). Note that the scalar four-point function D_{1234} is purely a consequence of the decomposition (7). It does not involve the exchange of a photon and is therefore not affected by the soft-photon approximation. Since we are only interested in the double-pole residue, it should be calculated for on-shell W bosons. For the analytical expression, which is too involved to be presented here, we refer to the literature [8]. The other new scalar four-point functions, D_{0134} and D_{0234} , are infrared-divergent and should be calculated in the soft-photon approximation. Using a regulator mass λ for the photon we can write

$$\begin{aligned} D_{0234} &= -\frac{i}{16\pi^2 s_{12}} \frac{1}{D_2} \left[\text{Li}_2\left(1 + \frac{\zeta M_W^2}{s_{12}}\right) - 2 \ln\left(\frac{M_W \lambda}{-D_2}\right) \ln\left(\frac{m_1 m_2}{-s_{12} - i0}\right) \right. \\ &\quad \left. + \frac{\pi^2}{3} + \ln^2\left(\frac{M_W}{m_2}\right) + \ln^2\left(\frac{m_1}{\zeta M_W}\right) \right], \end{aligned} \quad (9)$$

with a similar expression for D_{0134} .

2.2 Real-photon radiation for purely leptonic final states

Only interferences of the real-photon diagrams can give contributions to the manifestly non-factorizable corrections. The relevant interferences can be read off from Fig. 1 by taking the exchanged photon to be on-shell. The infrared divergences contained in the virtual corrections will cancel against those present in the corresponding bremsstrahlung interferences.

It should be noted that it is more complicated to obtain the five-point radiative interference correction. This is because the decomposition that we used in the case of the virtual five-point function cannot be carried over to the real-photon case. However, it is still possible to derive another decomposition using similar, but less straightforward arguments. Denoting the radiative analogues of the virtual scalar functions by a superscript ‘R’, we find

$$\begin{aligned} w'^2 E_{01234}^{\text{R}} &= (w' \cdot v'_1) D_{0234}^{\text{R}} + (w' \cdot v'_2) D_{0134}^{\text{R}} + (w' \cdot v'_3) D_{0124}^{\text{R}} \\ &\quad + (w' \cdot v'_4) D_{0123}^{\text{R}} + 2i\Delta_4 D_{1234}^{\text{R}}. \end{aligned} \quad (10)$$

The four-vectors w' and v'_i are defined as before, but for real-photon emission. This is equivalent to the following substitutions: $p_1 \rightarrow -p_1$, $k_1 \rightarrow -k_1$ and $D_2 \rightarrow D_2^*$. The radiation function D_{1234}^R is an artefact of the decomposition (10) and does not involve the exchange of a photon. It can be obtained from D_{1234} by the substitutions $p_1 \rightarrow -p_1$ and $k_1 \rightarrow -k_1$, resulting in the relation $\text{Im } D_{1234}^R = \text{Im } D_{1234}$.

As will be explained in detail elsewhere, the radiative interferences can in fact be obtained from the virtual corrections by only considering the contribution from the photon pole to the complex k^0 integration and by making certain substitutions. The photon-pole part D_{ijkl}^γ of the scalar four-point function D_{ijkl} is obtained by subtracting the particle-pole part D_{ijkl}^{part} from D_{ijkl} :

$$D_{ijkl}^\gamma = D_{ijkl} - D_{ijkl}^{\text{part}}. \quad (11)$$

The particle-pole parts are found to be

$$D_{0123}^{\text{part}} = \frac{1}{8\pi M_W^2} \frac{1}{D_2 - \zeta D_1} \left[\ln(1 - y_0 x_s) - \ln(1 - x_s/\zeta) \right], \quad (12)$$

$$D_{0234}^{\text{part}} = \frac{1}{8\pi s_{12}} \frac{1}{D_2} \left[\ln\left(\frac{D_2}{iM_W^2}\right) - \ln(-\zeta) - \ln\left(\frac{\lambda}{m_1}\right) \right], \quad (13)$$

with similar expressions for D_{0124}^{part} and D_{0134}^{part} , respectively. The radiative interferences can be obtained from Eq. (2) by adding a minus sign, by inserting the decomposition given in Eq. (7), and by substituting

- in the D_{0123}, D_{0134} terms: $D_{0123}, D_{0134} \rightarrow D_{0123}^\gamma, D_{0134}^\gamma$ followed by $D_1 \rightarrow -D_1^*$,
- in the D_{0124}, D_{0234} terms: $D_{0124}, D_{0234} \rightarrow D_{0124}^\gamma, D_{0234}^\gamma$ followed by $D_2 \rightarrow -D_2^*$,
- in the D_{1234} terms: $D_{1234} \rightarrow D_{1234}^R$ followed by $D_2 \rightarrow -D_2^*$.

2.3 Gauge-invariant definition of non-factorizable corrections

The set of manifestly non-factorizable QED diagrams displayed in Fig. 1 is not gauge invariant. In order to achieve a gauge-invariant definition of the non-factorizable corrections, all (soft) photonic interactions between the positively (e^+, W^+, ℓ^+) and negatively (e^-, W^-, ℓ^-) charged particles should be taken into account. Looking at Fig. 1, this is equivalent to the set of all up-down QED interferences. In the soft-photon, double-pole approximation only the ‘‘Coulomb’’ interaction between the off-shell W bosons survives as an extra contribution to the differential cross-section:

$$d\sigma_{\text{virt}}^C(p_1|p_2) = 32\pi\alpha \text{Re} \left[i(p_1 \cdot p_2) C_{012} \right] d\sigma_{\text{Born}}. \quad (14)$$

The scalar three-point function C_{012} is defined according to the notation of Sect. 2.1. In our approximation it is artificially ultraviolet-divergent. Introducing an upper bound Λ for the energy of the photon, this scalar function reads

$$C_{012} = \frac{i}{16\pi^2 s \beta} \left\{ \mathcal{L}i_2\left(y_0; \frac{1}{x_s}\right) + \mathcal{L}i_2\left(\frac{1}{y_0}; \frac{1}{x_s}\right) - 2 \text{Li}_2\left(1 - \frac{1}{x_s}\right) + \frac{1}{2} \ln^2(y_0) \right\}$$

$$+ \ln(x_s) \left[\ln\left(\frac{-iD_1}{2M_W\Lambda}\right) + \ln\left(\frac{-iD_2}{2M_W\Lambda}\right) \right] - 2i\pi \ln\left(\frac{1+x_s}{2}\right) \Big\}. \quad (15)$$

The corresponding radiative interference can again be related to the virtual correction (14) by adding a minus sign and by substituting $C_{012} \rightarrow C_{012}^\gamma$ followed by $D_1 \rightarrow -D_1^*$. The photon-pole part $C_{012}^\gamma = C_{012} - C_{012}^{\text{part}}$ can be derived from Eq. (15) and

$$C_{012}^{\text{part}} = \frac{1}{8\pi s\beta} \left\{ \ln(1-x_s) + \ln(1+x_s) - \ln(1-y_0x_s) - \ln\left(\frac{-iD_2}{M_W\Lambda}\right) \right\}. \quad (16)$$

If the virtual and real corrections are added, the dependence on the cut-off parameter Λ vanishes. When we mention *non-factorizable* corrections in the following, we implicitly refer to the gauge-invariant sum of the manifestly non-factorizable corrections and the above-mentioned ‘‘Coulomb’’ contribution.

2.4 Semi-leptonic and purely hadronic final states

For the purely hadronic final states there are many more diagrams, as the photon can interact with all four final-state fermions. In order to make efficient use of the results presented in the previous subsections, we first introduce some short-hand notations based on the results for the purely leptonic (LL) final states. These short-hand notations involve the summation of virtual and real corrections to the differential cross-section. For instance, the virtual corrections originating from the first diagram of Fig. 1 can be combined with the corresponding real-photon correction into the contribution $d\sigma_{LL}^{(4)}(k_1; k'_1|p_2)$. In a similar way virtual and real five-point corrections can be combined into $d\sigma_{LL}^{(5)}(k_1; k'_1|k_2; k'_2)$. The gauge-restoring ‘‘Coulomb’’ contribution will be indicated by $d\sigma^C(p_1|p_2)$. In terms of this notation the non-factorizable differential cross-section for purely leptonic final states becomes

$$d\sigma_{LL}(k_1; k'_1|k_2; k'_2) = d\sigma_{LL}^{(4)}(k_1; k'_1|p_2) + d\sigma_{LL}^{(4)}(k_2; k'_2|p_1) + d\sigma_{LL}^{(5)}(k_1; k'_1|k_2; k'_2) + d\sigma^C(p_1|p_2). \quad (17)$$

Analogously the non-factorizable differential cross-section for a purely hadronic final state (HH) can be written in the following way

$$\begin{aligned} d\sigma_{HH}(k_1; k'_1|k_2; k'_2) = & 3 \times 3 \left[\frac{1}{3} d\sigma_{LL}^{(4)}(k_1; k'_1|p_2) + \frac{2}{3} d\sigma_{LL}^{(4)}(k'_1; k_1|p_2) + \frac{1}{3} d\sigma_{LL}^{(4)}(k_2; k'_2|p_1) \right. \\ & + \frac{2}{3} d\sigma_{LL}^{(4)}(k'_2; k_2|p_1) + \frac{1}{3} \cdot \frac{1}{3} d\sigma_{LL}^{(5)}(k_1; k'_1|k_2; k'_2) + \frac{2}{3} \cdot \frac{1}{3} d\sigma_{LL}^{(5)}(k'_1; k_1|k_2; k'_2) \\ & \left. + \frac{1}{3} \cdot \frac{2}{3} d\sigma_{LL}^{(5)}(k_1; k'_1|k'_2; k_2) + \frac{2}{3} \cdot \frac{2}{3} d\sigma_{LL}^{(5)}(k'_1; k_1|k'_2; k_2) + d\sigma^C(p_1|p_2) \right]. \quad (18) \end{aligned}$$

In order to keep the notation as uniform as possible, the momenta of the final-state quarks are defined along the lines of the purely leptonic case with k_i (k'_i) corresponding to down (up) type quarks. If one would like to take into account quark-mixing effects, it suffices to add the appropriate squared quark-mixing matrix elements ($|V_{ij}|^2$) to the overall factor. Note that top

quarks do not contribute to the double-pole residues, since the on-shell decay $W \rightarrow tb$ is not allowed. Therefore the approximation of massless final-state fermions is still justified.

For a semi-leptonic final state (say HL), when the W^+ decays hadronically and the W^- leptonically, one can write

$$d\sigma_{HL}(k_1; k'_1 | k_2; k'_2) = 3 \left[\frac{1}{3} d\sigma_{LL}^{(4)}(k_1; k'_1 | p_2) + \frac{2}{3} d\sigma_{LL}^{(4)}(k'_1; k_1 | p_2) + d\sigma_{LL}^{(4)}(k_2; k'_2 | p_1) + \frac{1}{3} d\sigma_{LL}^{(5)}(k_1; k'_1 | k_2; k'_2) + \frac{2}{3} d\sigma_{LL}^{(5)}(k'_1; k_1 | k_2; k'_2) + d\sigma^C(p_1 | p_2) \right]. \quad (19)$$

Upon integration over the decay angles, the functions $d\sigma_{LL}^{(5)}$ and $d\sigma_{LL}^{(4)}$ become symmetric under $k_i \leftrightarrow k'_i$. As a result, the expressions (18) and (19) take on the form of (17) multiplied by the colour factors 9 and 3, respectively. These are precisely the colour factors that also arise in the Born cross-section. Therefore, after integration over the decay angles, the relative non-factorizable correction is the same for all final states. This universality property holds for all situations that exhibit the $k_i \leftrightarrow k'_i$ symmetry.

3 Numerical results

In this section some numerical results will be presented. The quantity of interest is the relative non-factorizable correction δ_{nf} , defined as

$$\frac{d\sigma}{d\xi} = \frac{d\sigma_{\text{Born}}}{d\xi} [1 + \delta_{nf}(\xi)], \quad (20)$$

where ξ represents some set of variables. Here we consider consecutively the distributions $d\sigma/[dM_1 dM_2 d\cos\theta_1]$, $d\sigma/[dM_1 dM_2]$, $d\sigma/dM_1$ and $d\sigma/dM_{av}$, with $M_i = \sqrt{p_i^2}$, $M_{av} = \frac{1}{2}(M_1 + M_2)$ and θ_1 is the decay angle between \vec{k}_1 and \vec{p}_1 in the lab system. The results are shown in Fig. 2 for the angular distribution, and in Table 1 and Fig. 3 for the invariant-mass distributions. The pure invariant-mass distributions play an important role in the extraction of the W -boson mass from the data through direct reconstruction of the Breit–Wigner resonances. In this context especially the position of the maximum of these Breit–Wigner curves is of importance.

All results in this section are presented for the following set of input parameters:

$$M_W = 80.22 \text{ GeV}, \quad \Gamma_W = 2.08 \text{ GeV}, \quad M_Z = 91.187 \text{ GeV}, \quad \Gamma_Z = 2.49 \text{ GeV},$$

$$\alpha = 1/137.0359895, \quad \sin^2 \theta_W = 0.226074.$$

From Fig. 2 it is clear that corrections of a few per cent could arise for angular distributions. They should, however, vanish after integration over M_1 and M_2 , as was mentioned before. The non-factorizable corrections $\delta_{nf}(M_1, M_2)$ to the double invariant-mass distribution are presented in Table 1. From those results one can expect that δ_{nf} will be less steep for the M_1 distribution

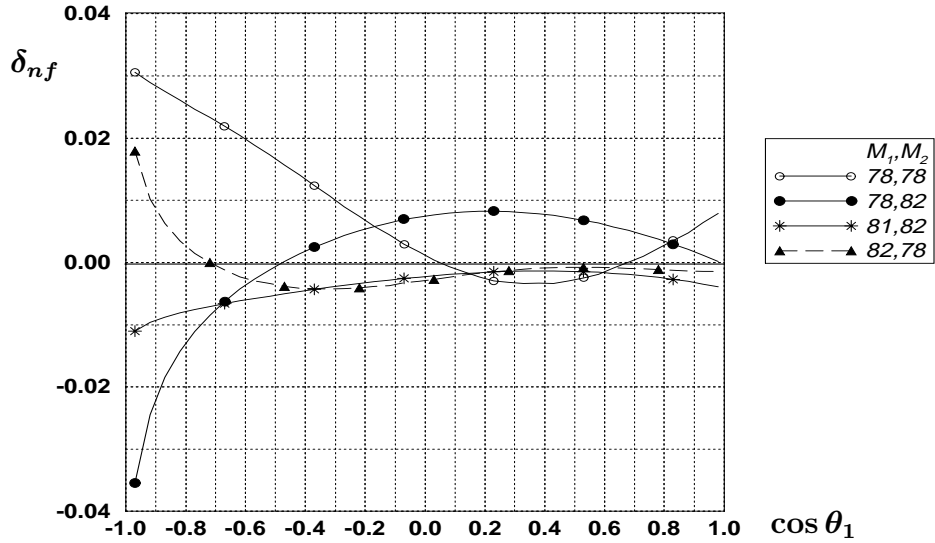


Figure 2: The relative non-factorizable correction $\delta_{nf}(M_1, M_2, \cos \theta_1)$ to the decay angular distribution $d\sigma/[dM_1 dM_2 d \cos \theta_1]$ for fixed values of the invariant masses $M_{1,2}$ [in GeV]. Centre-of-mass energy: $\sqrt{s} = 184$ GeV.

than for the M_{av} distribution. This is confirmed by Fig. 3. The corrections shown in Fig. 3 lead to a shift in the position of the maximum of the Breit–Wigner curves of the order of 1–2 MeV. These results have been obtained for the centre-of-mass energy $\sqrt{s} = 184$ GeV. On the interval 170–190 GeV the largest corrections are observed for 170 GeV, where the corrections are about a factor of two larger than those at 184 GeV. At 190 GeV the corrections are slightly smaller than those at 184 GeV.

Δ_1	Δ_2						
	-1	-1/2	-1/4	0	1/4	1/2	1
-1	+0.84	+0.65	+0.51	+0.37	+0.21	+0.06	-0.17
-1/2	+0.65	+0.52	+0.39	+0.24	+0.07	-0.07	-0.26
-1/4	+0.51	+0.39	+0.28	+0.13	-0.02	-0.15	-0.31
0	+0.37	+0.24	+0.13	+0.00	-0.13	-0.24	-0.37
1/4	+0.21	+0.07	-0.02	-0.13	-0.24	-0.32	-0.43
1/2	+0.06	-0.07	-0.15	-0.24	-0.32	-0.39	-0.47
1	-0.17	-0.26	-0.31	-0.37	-0.43	-0.47	-0.53

Table 1: The relative non-factorizable correction $\delta_{nf}(M_1, M_2)$ [in %] to the double invariant-mass distribution $d\sigma/[dM_1 dM_2]$ for some particular values of $M_{1,2}$. The invariant masses $M_{1,2}$ are specified in terms of their distance from M_W in units of Γ_W , i.e. $\Delta_{1,2} = [M_{1,2} - M_W]/\Gamma_W$. Centre-of-mass energy: $\sqrt{s} = 184$ GeV.

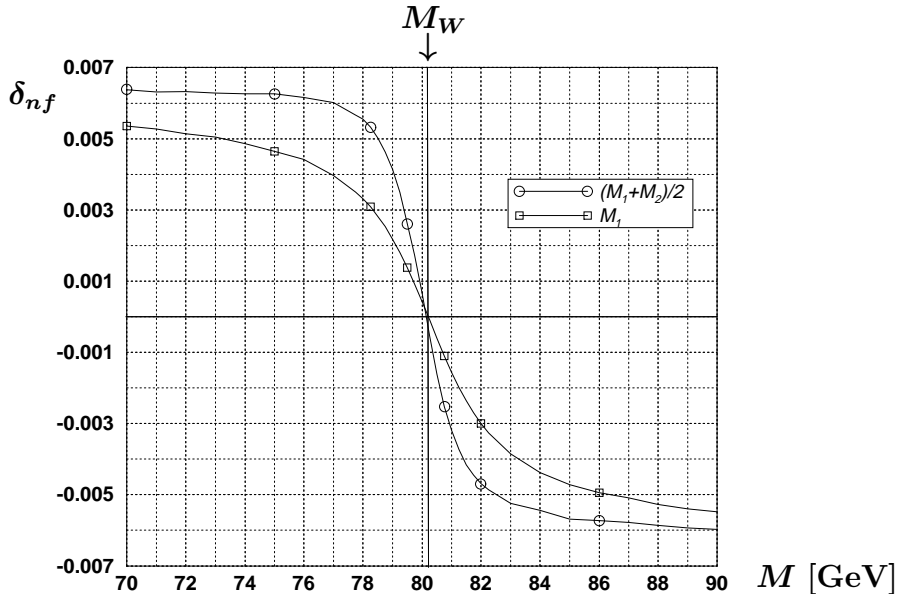


Figure 3: The relative non-factorizable correction $\delta_{nf}(M)$ to the single invariant-mass distributions $d\sigma/dM$. Shown are the corrections to the distributions with respect to M_1 and M_{av} . Centre-of-mass energy: $\sqrt{s} = 184$ GeV.

4 Conclusions

In this letter some analytical and numerical results are presented for non-factorizable corrections to W -pair-mediated four-fermion production. In principle these corrections could be relevant for tests of triple gauge-boson couplings and for the determination of the W -boson mass. For the latter the corrections are of $\mathcal{O}(\alpha)$ and change the W line-shape by about 1%. For the former they vanish at the $\mathcal{O}(\alpha)$ level. In view of the present experimental accuracy, the common practice of neglecting non-factorizable corrections is justified.

One may wonder how non-factorizable corrections affect Z -pair-mediated and ZH -mediated four-fermion final states. In those cases, only five-point functions contribute, of which there are four contributions, as in Eq. (18). However, in contrast to Eq. (18), the charge factors are pair-wise opposite, such that integration over the decay angles leads to a vanishing result. Thus $\mathcal{O}(\alpha)$ non-factorizable corrections to invariant-mass distributions in Z -pair-mediated or ZH -mediated four-fermion processes vanish.

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