UTHEP-95-1101
Nov., 1995

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi Evolution and the Renormalization Group Improved Yennie-Frautschi-Suura Theory in QCD[§]

B.F.L. Ward
Department of Physics and Astronomy,
The University of Tennessee, Knoxville, Tennessee 37996-1200
SLAC, Stanford University, Stanford, California 94309
and
CERN, Theory Division, CH-1211 Geneva 23, Switzerland
S. Jadach
Institute of Nuclear Physics, ul. Kawiory 26a, Kraków, Poland
CERN, Theory Division, CH-1211 Geneva 23, Switzerland,

Abstract

We show that the recently derived renormalization group improved Yennie-Frautschi-Suura (YFS) exponentiation of soft ($k_o \to 0$) gluons in QCD is fully compatible with the usual Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution of the structure functions of hadrons in the respective hadron-hadron hard interactions. We show how to implement the YFS exponentiation without double or over counting effects already implied by the DGLAP equation. In this way, we arrive at a theory which allows for the development of realistic, multiple gluon Monte Carlo event generators for hard hadron-hadron scattering processes in which the DGLAP evolved structure functions are correctly synthesized with the respective YFS exponentiated soft gluon effects in a rigorous way.

Submitted to Phys. Lett. B

§ Work supported in part by the US DoE contract DE-FG05-91ER40627, Polish Government grants KBN 2P30225206 and 2P03B17210 and Polish-French Collaboration within IN2P3.

UTHEP-95-1101
November 1995

An important step in the implementation of the recently derived [1] renormalization group improved soft Yennie-Frautschi-Suura (YFS)[2] gluon exponentiation in QCD is the proper synthesization of these new infrared (IR) effects (where by infrared we do mean the gluon energy $k_o \to 0$ limit as opposed to the soft gluon regime analyzed by Dokshitzer *et al.*[3] in which $k_\perp \geq \frac{1}{R}$, $R \simeq$ hadron radius $\simeq 1/(200MeV) \simeq \frac{1}{\Lambda_{QCD}}$) with the required Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [4] evolution of the respective hadron structure functions in computing any particular hard hadron-hadron scattering cross section, such as $p + \bar{p} \to t\bar{t} + X$, for example. Indeed, a naive use of the results in Ref. [1] would lead to double or over counting some of the same effects, rendering the precision of any such calculated cross section questionable at best. In what follows, we will show how one properly synthesizes the DGLAP structure function evolution with renormalization group improved soft gluon YFS QCD exponentiation derived in Ref. [1]. In this way, we develope the necessary remaining theoretical apparatus needed for the construction of realistic hadron-hadron hard scattering Monte Carlo event generators in which finite $P_\perp$ multiple soft gluon effects are taken into account at the level of the hard scattering subprocess amplitude while the proper structure function evolution is realized, on an event-by-event basis. Such a Monte Carlo event generator will appear elsewhere [5].

Specifically, let us recall the basic result from Ref. [1] for the YFS exponentiated differential cross section for the process $q + \overset{(}{\bar{q}}{}^{)\prime} \to q'' + \overset{(}{\bar{q}}{}^{)\prime\prime\prime} + n(G)$, where G is a soft gluon,

$$\begin{aligned} d\hat{\sigma}_{exp} &= exp[\mathrm{SUM_{IR}(QCD)}] \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3k_j}{k_j} \int \frac{d^4y}{(2\pi)^4} e^{+iy(p_1+p_2-q_1-q_2-\sum_j k_j)+D_{QCD}} \\ &\qquad \times \bar{\beta}_n(k_1,\ldots,k_n) \frac{d^3q_1 d^3q_2}{q_1^0 q_2^0} \end{aligned} \quad (1)$$

where

$$D_{QCD} = \int \frac{d^3k}{k_0} \tilde{S}_{\mathrm{QCD}} \left[e^{-iy\cdot k} - \theta(K_{max} - |\vec{k}|) \right] \quad . \quad (2)$$

Here, we have defined

$$\mathrm{SUM_{IR}(QCD)} \equiv 2\alpha_s \tilde{B}_{\mathrm{QCD}} + 2\alpha_s \Re B_{\mathrm{QCD}} \quad (3)$$

from Ref. [1] with the QCD YFS virtual and real genuinely infrared functions $\tilde{B}_{\mathrm{QCD}}$, B_{QCD}, and $\tilde{S}_{\mathrm{QCD}}$ as defined therein[1]. The hard gluon residuals $\bar{\beta}_n(k_1,\ldots,k_n)$ are also as defined in Ref. [1], for example. At issue is how one synthesizes a calculation such as that in (1) with the fundamental structure functions for the incoming partons which are themselves solutions of the DGLAP equation and hence in general may have in themselves some of the effects in (1) already included?

Indeed, one might think that the usual formula

$$\sigma(p\bar{p} \to t\bar{t} + X) = \int \sum_{i,j} D_i(x_i) \bar{D}_j(x_j) d\hat{\sigma}_{exp}(x_i x_j s) dx_i dx_j \quad (4)$$

could be used to leading log accuracy when the existing structure functions $D_i(x_i), \bar{D}_j(x_j)$ for partons i, j in $p, \bar{p}$, respectively, in the literature[6] are used. Here, x_i, x_j are the respective light-cone momentum fractions. But, this is not even true because the DGLAP equation is well known to contain all leading logs from the initial state radiation of gluons and all attendant collinear infrared effects. Thus, in using (4) naively one would double count some leading log and collinear infrared effects.

Specifically, in order to use the existing experience on the structure functions together with the YFS exponentiation improvement of perturbation theory given by (1), one has to identify those effects in the YFS formula which are already contained in the DGLAP equation for the structure functions and remove them. This we now do.

Since we have exponentiated the quantity $SUM_{IR}(QCD)$ in (1), we start by isolating from it and from all of its attendant parts in (1) that collinear infrared contribution which is generated by the DGLAP evolution of the structure functions. Viewing the DGLAP evolution as the complete LL series from the initial state radiative effects, we see that we must do the following to synthesize (1) with the existing structure functions [6] in (4):

- From the initial state part of $\tilde{S}_{QCD}$, we must remove that part which corresponds to the YFS limit of the DGLAP kernel, $\tilde{S}_{QCD}|_{DGLAP}$. This can be computed using the definitions in the original paper of Altarelli and Parisi[4] and we get the result that, if we wish to use (1) in (4), we should replace $\tilde{S}_{QCD}$ in (1) with $\tilde{S}^{nls}_{QCD}$ where

$$\tilde{S}^{nls}_{QCD} = \tilde{S}_{QCD} - \tilde{S}_{QCD}|_{DGLAP} \tag{5}$$

for

$$\tilde{S}_{QCD}|_{DGLAP}(k) = \frac{C_F\alpha_s}{4\pi^2}\Big[\frac{\theta(z)\ 4(1-zv)^2}{(k_\perp^2+z^2v^2m_q^2)}\left(1-\frac{m_q^2v^2z^2}{k_\perp^2+z^2v^2m_q^2}\right) + \frac{\theta(-z)\ 4(1+zv)^2}{(k_\perp^2+z^2v^2m_q^2)}\left(1-\frac{m_q^2v^2z^2}{k_\perp^2+z^2v^2m_q^2}\right)\Big] \tag{6}$$

with the identifications $zk = k_z$, $k^o = v\sqrt{s}/2, \vec{k} = (k_z, \vec{k}_\perp)$ and we have written the RHS of (6) for incoming q and $\bar{q}$ respectively for definiteness; for the $q\bar{q}'$ incoming case, for example, one would need to substitute $m_{q'}$ for m_q in the coefficient of $\theta(-z)$ on the RHS of (6). The superscript nls on $\tilde{S}_{QCD}$ on the LHS of (5) denotes that it is non-leading in the s-channel relative to DGLAP evolution. Thus, $\tilde{S}^{nls}_{QCD}$ has the property that its integral over the gluon phase space has no collinear big logarithm terms proportional to $\frac{\alpha_s L}{\pi}$ or $\frac{\alpha_s L^2}{\pi}$, where $L = \ln s/m_q^2 - 1$ in the case of incoming $q\bar{q}$, for example. To obtain the result (6), one follows the derivation of the DGLAP kernel $P_{Gq}(\bar{z})$ in the paper of Altarelli and Parisi in Ref. [4] but one substitutes the YFS vertex $-ig\tau^a(2p+k)_\mu$ instead of the complete vertex $-ig\tau^a\gamma_\mu$ and one retains the quark mass corrections through order m_q^2 in the respective kinematics. Here, g is the QCD coupling constant and τ^a generate the quark (vector) representation of the QCD gauge group; p is the final 4-momentum of the respective radiating quark. We stress that $\tilde{S}^{nls}_{QCD}$ defined in this way is positive definite. The result (5), which corresponds to the replacement $\tilde{B}_{QCD}(K_{max}) \rightarrow \tilde{B}^{nls}_{QCD}(K_{max}) = \int \frac{d^3k}{k_o}\theta(K_{max} - |\vec{k}|)\tilde{S}^{nls}_{QCD}$, then avoids the double counting of real gluon radiation from the initial state that is already included in the DGLAP evolution of the structure functions.

- This brings us to the possible double counting of the virtual corrections represented by $\Re B_{QCD}$ in (1). We address this problem as follows. The DGLAP evolution equation generates the entire leading log series associated with initial state real and virtual gluon radiation. Thus, we should also remove from $\Re B_{QCD}$ all leading log effects associated with the initial state. Here, we stress that the definition of the big log L has to be the same as that used in the DGLAP evolution. In practice, this means that, in the formula for $SUM_{IR}(QCD)$ in eq.(9) of Ref. [1], for the initial state contribution corresponding to the case $A = s$ in the respective sum over channels in this formula, we should use the result

$$B_{tot}(s) = \frac{\pi^2}{3} - \frac{1}{2} \tag{7}$$

instead of the result given by eq.(10) in Ref. [1]. We denote this new result by $SUM^{nls}_{IR}(QCD)$. In addition, in the computation of the hard gluon residuals $\bar{\beta}_n$ in (1) one should remember again to remove from the real part of the integral over the first term in eq.(4) in Ref. [1] (this integral can be found in Refs. [7]), which is the initial state part of $\Re B_{\rm QCD}$, all terms involving either $\frac{\alpha_s L}{\pi}$ and $\frac{\alpha_s L^2}{\pi}$, where $L \equiv \ln s/m_q^2 - 1$ here in the $q\bar{q}$ incoming case, for example. The generalization to the $q\bar{q}'$ incoming case is immediate. Henceforth, we denote this new $\Re B_{QCD}$ with its initial state s-channel big logarithms removed as $\Re B^{nls}_{QCD}$. It is $\tilde{S}^{nls}_{QCD}$ and $\Re B^{nls}_{QCD}$ that should now be used in computing $\bar{\beta}_n$ in (1); we denote the resulting new hard gluon residual as $\bar{\beta}^{nls}_n$, where it is understood that the various finite order perturbative QCD differential cross sections in this definition of $\bar{\beta}^{nls}_n$ are the reduced, hard mass factorized cross sections.

When these steps are followed, we arrive at a new representation of the cross section in (1) in which $\tilde{S}^{nls}_{QCD}$, $SUM^{nls}_{IR}(QCD)$ and $\Re B^{nls}_{QCD}$ are substituted for $\tilde{S}_{QCD}$, $SUM_{IR}(QCD)$ and $\Re B_{QCD}$ respectively and we write this new version of the result in (1) as

$$\begin{aligned} d\hat{\sigma}'_{exp} &= exp[\mathrm{SUM}^{\mathrm{nls}}_{\mathrm{IR}}(\mathrm{QCD})] \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3k_j}{k_j} \int \frac{d^4y}{(2\pi)^4} e^{+iy(p_1+p_2-q_1-q_2-\sum_j k_j)+D^{nls}_{QCD}} \\ &\times \bar{\beta}^{nls}_n(k_1,\ldots,k_n) \frac{d^3q_1 d^3q_2}{q_1^0 q_2^0}. \end{aligned} \tag{8}$$

We may now use this result (8) in (4) with the existing DGLAP evolved structure functions [6] without double counting any effects whatsoever. Moreover, the result (8) is finite and has a smooth limit as the initial state light quark masses $m_q \rightarrow 0$.

A related point is that, from the formulas for $\tilde{S}_{QCD}$ (eq.(5) in the first paper in Ref. [1]) and for $\tilde{S}_{QCD}|_{DGLAP}$ it is clear that the terms we are exponentiating in the factor $exp\{SUM_{IR}(QCD)\}$ in (1) are indeed the usual ones that all published calculations have treated to a finite order in α_s when they combine soft real radiation cross sections with virtual gluon radiation corrected ones to cancel the respective infrared singularities at

$k_0 \to 0$ to the respective order in α_s; here, we treat this cancelation to all orders in α_s. Referring to Refs. [8, 9, 10, 11, 12], we see that the value of α_s that should be used in $SUM_{IR}(QCD)$ is $\alpha_s(\mu)$ where μ is the respective hard renormalization scale∼factorization scale in the hard process- otherwise, the real gluon emission $k_o \to 0$ singularity will not cancel the respective virtual $k_o \to 0$ singularity: for illustration, we recall the result in Ref. [8] for the parton level Drell-Yan [13] process $q + \bar{q} \to \mu^+\mu^- + G$ for a muon pair mass M with the IR gluon regulator mass $m_G \to 0$, eq.(5.3.17) in Ref. [8] (here, $\hat{\sigma}_0$ is the respective Born cross section),

$$\hat{\sigma}_{DY}(real) = \frac{2\alpha_s(M)}{3\pi}\hat{\sigma}_0\{\ln^2(m_G^2/M^2) + 3\ln(m_G^2/M^2) + \pi^2\} \tag{9}$$

and its corresponding virtual correction, eq.(5.3.18) in Ref. [8],

$$\hat{\sigma}_{DY}(virtual) = \frac{2\alpha_s(M)}{3\pi}\hat{\sigma}_0\{-\ln^2(m_G^2/M^2) - 3\ln(m_G^2/M^2) - \frac{7}{2} - \frac{2\pi^2}{3} + \pi^2\}, \tag{10}$$

the two of which sum to the total result, eq.(5.3.19) in Ref. [8],

$$\hat{\sigma}_{DY}(real) + \hat{\sigma}_{DY}(virtual) = \frac{2\alpha_s(M)}{3\pi}\hat{\sigma}_0\left\{\frac{4\pi^2}{3} - \frac{7}{2}\right\}, \tag{11}$$

which is independent of the infrared ($k_0 \to 0$) gluon regulator mass m_G – the $k_0 \to 0$ gluon infrared singularities regulated by m_G in (9) and (10) are cancelled in (11) *at the hard scattering coupling* $\alpha_s(M)$. We refer the reader to Refs. [9, 10], to the second paper in Refs. [11], and to the second paper in Ref. [12] for further explicit examples of this choice of α_s in the cancelation of IR singularities at $k_o \to 0$ in finite order perturbative QCD calculations. More recently, the reader may also see the analysis in Ref. [14] for further illustrations of this point. Finally, we recall the fundamental analysis of Altarelli and Parisi in Ref. [4] in which the regularized DGLAP kernel, $P_{qq}(z)$, generates the differential change in the total qq splitting function given by

$$d\Gamma_{qq}(z,t) = \frac{\alpha_s(Q)}{2\pi}P_{qq}(z)dt \tag{12}$$

and

$$P_{qq}(z) = C_F\left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right) \tag{13}$$

where the +function has the standard definition

$$\int_0^1 dz\frac{f(z)}{(1-z)_+} \equiv \int_0^1 dz\frac{f(z)-f(1)}{(1-z)} \tag{14}$$

for any appropriate test function $f(z)$, $C_F = 4/3$ is the respective quark color Casimir invariant and $t = \ln Q^2/m_q^2 - 1$ for example. Q is the respective hard scale. We focus here on the $\delta(1-z)$ part of $d\Gamma_{qq}(z,t)$ which realizes the cancellation of the $k_0 \to 0$ real and virtual gluon singularities in $d\Gamma_{qq}(z,t)$ and which enters into it with the squared coupling constant of $\alpha_s(Q)$, the coupling constant of the respective hard scale Q, in complete analogy with the value of α_s which we use in $SUM_{IR}(QCD)$. The correctness of the value of α_s multiplying the delta function in $d\Gamma_{qq}(z,t)$ is proven by agreement between the predictions of the DGLAP equation and the Wilson expansion for the deep inelastic structure functions (see Ref. [4, 15, 16], for example) and by the agreement of these predictions with the available data [17]. Thus, the value of α_s which we use in $SUM_{IR}(QCD)$ is well-founded in the published literature.

Up to this point in our discussion, we have focussed on how one uses the existing DGLAP evolved structure functions in conjunction with the renormalization group improved YFS theory to compute the rigorous hadron level cross section. It is also possible to go one step further, to use the explicit application of the YFS theory to the one-loop and single bremsstrahlung correction illustrated in Ref. [18] to exponentiate the IR singularities in the DGLAP equation itself. This calculation, which together with its application will be reported elswehere [19], yields an entirely new approach to DGLAP evolution in which the splitting functions themselves are actually YFS exponentiated.

A final important issue that can be addressed is the relationship between our YFS exponentiated result (1) and the soft gluon resummation theory of Sterman [20] and of Catani and Trentadue [21]. We now turn to this relationship. We note that this latter resummation theory has recently been used in Refs. [22, 23, 24] at the respective leading log level to discuss the effects of soft gluons in the $p+\bar{p} \to t+\bar{t}+X$ cross section normalization.

Specifically, referring to the notation of Refs. [20, 21, 22, 23, 24] we can identify the resummed soft gluon cross section of the theory of Refs. [20, 21] with the cross section in (1) via

$$\exp[\mathrm{SUM_{IR}(QCD)}]\sum_{n=0}^{\infty}\int\prod_{j=1}^{n}\frac{d^3k_j}{k_j}\int\frac{d^4y}{(2\pi)^4}e^{iy\cdot(p_1+p_2-q_1-q_2-\sum k_j)+D_{\mathrm{QCD}}}$$

$$*\bar{\beta}_n(k_1,\ldots,k_n)\frac{d^3q_1}{q_1^0}\frac{d^3q_2}{q_2^0}/d\Phi \quad \underset{1-z\to 0}{\Longrightarrow} \quad e^{E(1-z,\alpha_s)}d\hat{\sigma}_B/d\Phi, \tag{15}$$

where we note the limit $1-z\to 0$ is the limit studied by Sterman, Catani and Trentadue in which the hard scattering invariant mass squared fraction z, defined by s'/s, approaches its maximum value 1, and where $E(1-z,\alpha_s)$ is the resummation exponent estimated in Ref. [20] at the leading-log level and in Ref. [21] at the next-leading log level and *used* in Refs. [22, 23, 24] at the leading log level for the normalization of $\sigma(t\bar{t})$ at FNAL, $d\Phi$ is the respective Lorentz invariant phase space considered in Refs. [20, 21] and $d\hat{\sigma}_B$ is the respective Born level cross section. For example, the well-known double integral over $\alpha_s(\bar{k}_\perp^2)$ in E is easily recovered from the lefthand side of (15) by approximating

$$D_{QCD}(y) \;\approx\; D_{QCD}(0) = \int\frac{d^3k}{k}\tilde{S}_{QCD}(k)(1-\theta(K_{max}-k))$$

$$= \int \frac{dz}{1-z} \int \frac{dk_\perp^2}{k_\perp^2} \frac{\alpha_s(\bar{k}_\perp^2)}{\pi} C_F(1-\theta((1-z)_{max}-(1-z)))\{1+\cdots\}, \quad (16)$$

so that for the leading log initial state part of $SUM_{IR}(QCD) + D_{QCD}(0)$ we recover exactly the double integral over $\alpha_s(\bar{k}_\perp^2) = \alpha_s((1-z)k_\perp^2) \simeq \alpha_s((1-z)Q^2)$ proposed for E in Refs. [20, 21] (the $\cdots$ in (16) stand for the remainder of $D_{QCD}(0)$ [1]). Here, in our infinite momentum frame (light-cone) kinematics , we identified $(1-z)_{max} = K_{max}/p_1^0$ and $Q^2 = s$. We note that, as the authors in Refs. [20, 21] have pointed-out, in allowing α_s to run in (16) one resums a certain class of leading logs which in our exact series in $\alpha_s(Q^2)$ in (1) are generated by the infinite sum on n in (1). We note further that the remainder of E corresponds to the next leading log terms generated by the infinite sum on n in (1) over the $\bar{\beta}_n$ when one makes the approximation

$$\sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_j^n \frac{d^3k_j}{k_j} \delta(p_1+p_2-q_1-q_2-\sum k_j)\bar{\beta}'_n(k_1,\cdots,k_n) \frac{d^3q_1}{q_1^0}\frac{d^3q_2}{q_2^0}/d\hat{\sigma}_B \equiv \mathcal{S}(\bar{\beta}')$$
$$\approx \sum_{n=0}^{\infty} \frac{1}{n!} (E^{nll})^n \quad (17)$$

where we define E^{nll} to be the next leading log part of E and to be consistent we have defined $\bar{\beta}'_n$ to be that part of the $\bar{\beta}_n$ in our exact result (1) whose leading log content is not already resummed by integral over the running α_s in (16). We see that in our approach, the results of Refs. [20, 21] for E^{nll} correspond to the approximation

$$(E^{nll})^n = \int \prod_j^n \frac{d^3k_j}{k_j} \delta(p_1+p_2-q_1-q_2-\sum k_j)\bar{\beta}'_n(k_1,\cdots,k_n) \frac{d^3q_1}{q_1^0}\frac{d^3q_2}{q_2^0}/d\hat{\sigma}_B. \quad (18)$$

We conclude that, at the parton level, the formulas in Refs. [20, 21] are entirely contained in our formula (1) and that they represent an approximation to our result (1) in the limit that $1-z \to 0$.

One of the important consequences of the identification of the Sterman-Catani-Trentadue exponent E in (15) is that we can address the issue of the presence or absence of renormalon [25] behavior associated with the leading log and next-leading log truncation of our exact result (1) as it is reperesented by E. Specifically, as the authors in Refs. [20, 21, 22, 23, 24] have noted, as $1-z \to 0$ in (16), the argument of the running α_s would appear to approach the Landau pole, creating an ambiguity in the non-leading terms in the exponent, for example, of the famous renormalon type [25]*if one expands α_s in terms of this one-loop result*– we stress that the two and three loop formulas are known and they do not seem to support this pole in α_s [26]. On the other hand, we have argued in Ref. [1], by the uncertainty principle, that the truly long wavelength gluons , with wavelengths much larger than $1/\Lambda_{QCD}$, should decouple from the hard process under discussion here. Does this decoupling really happen or not? We look at the exact initial state formula for the sum

$$
\begin{aligned}
SUM_{IR}(QCD)|_{initial\ state} &= \{2\alpha_s ReB_{QCD} + 2\alpha_s \tilde{B}_{QCD}(K_{\max})\}|_{initial\ state} \\
&= -\frac{C_F}{4\pi^2}\int \frac{d^3k}{k^0}\alpha_s(\bar{k}_\perp^2)(\frac{p_1}{p_1k}-\frac{p_2}{p_2k})^2\theta(K_{\max}-k) \\
&+ \Re\frac{iC_F}{4\pi^2}\int\frac{d^4k}{\pi}(\frac{1}{k^2-m_G^2+i\epsilon})\Big(\frac{2p_1+k}{k^2+2p_1k+i\epsilon} \\
&\quad + \frac{2p_1+k}{k^2+2p_1k+i\epsilon}\Big)^2\alpha_s(\bar{k}_\perp^2)
\end{aligned}
\tag{19}
$$

where we have restored the running ansatz of Refs. [20, 21] in their estimate of E. We stress that for consistency the same definition of α_s has to be used for both soft real and soft virtual gluons in order for the infrared singularities to cancel. Carrying out the virtual k^0 integral by standard contour methods in the upper complex k^0 plane, we get, for the residue of the gluon propagator $\equiv Res(gluon\ propagator)$, the contribution

$$
2\alpha_s ReB_{QCD}|_{initial\ state, Res(gluon\ propagator)} = \frac{C_F}{4\pi^2}\int\frac{d^3k}{k^0}\alpha_s(\bar{k}_\perp^2)(\frac{p_1}{p_1k}-\frac{p_2}{p_2k})^2
\tag{20}
$$

so that, when we combine this result with the real emission term in (19), we get finally the representation

$$
\begin{aligned}
SUM_{IR}(QCD)|_{initial\ state} &= \frac{C_F}{4\pi^2}\int_{k^0\geq K_{max}}\frac{d^3k}{k^0}\alpha_s(\bar{k}_\perp^2)(\frac{p_1}{p_1k}-\frac{p_2}{p_2k})^2 \\
&- \frac{C_F}{2\pi}\Re\int\frac{d^3k}{\pi}\sum_{Res(fermion\ propagators)}(\frac{1}{k^2-m_G^2+i\epsilon}) \\
&\quad\times\Big(\frac{2p_1+k}{k^2+2p_1k+i\epsilon}+\frac{2p_1+k}{k^2+2p_1k+i\epsilon}\Big)^2\alpha_s(\bar{k}'^2_\perp),
\end{aligned}
\tag{21}
$$

where we have introduced the scale $\bar{k}'^2_\perp$ for α_s in the second term in this last equation to take into account that, as this term is not infrared divergent and is dominated by gluon momenta $\mathcal{O}(\sqrt{s}/2)$, $\bar{k}'_\perp$ should also be of this order for consistency. In the second term in (21), only the fermion propagator residues , $Res(a), a = fermion\ propagators$, enter into the respective sum over residues, as we indicate explicitly. We see in the first term in (21) that only gluons with energies exceeding the dummy parameter K_{max} (our result (1) is independent of K_{max} and $1-z_{max}$) are actually involved in $SUM_{IR}(QCD)$ so that, if we use the resummation ansatz of Refs. [20, 21] we do not encounter the regime $(1-z)s = \Lambda^2_{QCD}$ in our result for the LL part of E as we may take $(1-z_{max}) \gg \frac{\Lambda^2_{QCD}}{s}$ without loss of content or predictive power. This is consistent with the earlier observation that (1), which is a series in $\alpha_s(Q^2)$, is consistent with the uncertainty principle and hence in fact is insensitive to the behavior of α_s at scales $\mathcal{O}(\Lambda_{QCD})$ when $Q^2 \gg \Lambda^2_{QCD}$.

In summary, we have shown how one rigorously synthesizes the powerful results of DGLAP evolution and renormalization group improved YFS exponentiation in QCD without double counting. Explicit Monte Carlo event generator data based on our prescription will appear elsewhere [5].

Acknowledgments We would thank Profs. G. Veneziano and G. Altarelli for the support and kind hospitality of the CERN Theory Division, where a part of this work was performed. One of us (B.F.L. W.) thanks Prof. C. Prescott of SLAC for the kind hospitality of SLAC Group A while this work was completed.

References

[1] D.B. DeLaney, S. Jadach, C. Shio, G. Siopsis, B. F. L. Ward, UTHEP-95-0102; Phys. Lett. **B342** (1995) 239; Phys. Rev. **D52** (1995) 108; see also F. A. Berends and W. T. Giele, Nucl. Phys. **B313**, 595(1989).

[2] D. R. Yennie, S. C. Frautschi, and H. Suura, Ann. Phys. **13** (1961) 379.

[3] Yu. L. Dokshitzer *et al.*, Basics of perturbative QCD,(Editions Frontieres, Gif-Sur-Yvette, 1991).

[4] G. Altarelli and G. Parisi, Nucl. Phys. **B126** (1977) 298; Yu. L. Dokshitzer, Sov. Phys. JETP **46** (1977) 641; L. N. Lipatov, Yad. Fiz. **20** (1974) 181; V. Gribov and L. Lipatov, Sov. J. Nucl. Phys. **15** (1972) 675, 938.

[5] S. Jadach *et al.*, in preparation.

[6] A. D. Martin, R. G. Roberts and W. J. Stirling, DTP-96-44, 1996; A. D. Martin, W. J. Stirling and R. G. Roberts, Int. J. Mod. Phys. **A10** (1995) 2885; M. Gluck, E. Reya and M. Stratmann, Phys. Rev. D**51** (1995) 3220; W. Tung, MSUHEP-60701, 1996; H. L. Lai *et al.*, Phys. Rev. D**51** (1995) 4763; andd references therein.

[7] S. Jadach, W. Placzek and B.F.L. Ward, UTHEP-95-0801, 1995; Phys. Rev. D (1996) in press; B. F. L. Ward, *ibid.***36** (1987) 939; and references therein.

[8] R. D. Field, Applications of Perturbative QCD ,(Addison Wesley Publ. Co., Redwood City, 1989).

[9] R.K. Ellis *et al.*, Nucl. Phys. **B173** (1980) 397.

[10] R. K. Ellis and J. Sexton, Nucl. Phys. **B269** (1986) 445; and references therein.

[11] P. Nason, S. Dawson, and R. K. Ellis, Nucl. Phys. **B303** (1988) 607; *ibid.* **B327** (1989) 49; *ibid.* **B335** (1990) 260.

[12] W. Beenakker *et al.*, Phys. Rev. **D40** (1989) 54; W. Beenakker *et al.*, Nucl. Phys. **B351** (1991) 507.

[13] S.D. Drell and T.M. Yan, Ann. Phys. **66**, 578 (1991).

[14] S. Catani, in Proc. 1996 Cracow International Symposium on Radiative Corrections, ed. S. Jadach, Acta Phys. Pol., to be published.

[15] K. G. Wilson, Phys. Rev. **D2**, 1473 (1970); *ibid.* **D2**, 1478 (1970); *ibid.* **D3**, 1818 (1971).

[16] D.J. Gross and F. Wilczek, Phys. Rev. **D9**, 980 (1974); *ibid.***D8**, 3633 (1973); Phys. Rev. Lett. **30**, 1343 (1973); H.D. Politzer, Phys. Rev. Lett. **30**, 1346 (1973); Phys. Rep. **14**, 335 (1974); H. Georgi and H. D. Politzer, Phys. Rev. **D9**, 416 (1974).

[17] See ,for example, J. Feltesse, in Proceedings of the XXVII International Conference on High Energy Physics, v.1, eds. I. G. Knowles and P. J. Bussey, (IOP Publ. Ltd., Bristol, 1995) p. 65; and, references therein.

[18] S. Jadach, E. Richter-Was, B.F.L. Ward and Z. Was, Phys. Rev. **D44**, 2669(1991).

[19] S. Jadach and B. F. L Ward, to appear.

[20] G. Sterman, *Nucl. Phys.* **B281** (1987) 310.

[21] S. Catani and L. Trentadue, *Nucl. Phys.* **B327** (1989) 323; *ibid.* **B353** (1991) 183.

[22] E. Laenen, J. Smith, and W. van Neerven, Phys. Lett. **B321** (1994) 254; Nucl. Phys. **B369** (1992) 543.

[23] E. Berger and H. Contopanagos, *Phys. Rev.* **D54** (1996) 3085.

[24] S. Catani *et al.*, preprint CERN-TH/96-21, 1996.

[25] G. 't Hooft, in The Whys of Subnuclear Physics, Erice, 1977, ed. A. Zichichi (Plenum, New York, 1979); Y. Firshman and A. R. White, Nucl. Phys. **B158** (1979) 221; J. C. Le Guillou and J. Zinn-Justin, eds., Large-order Behavior in Perturbation Theory, Current Physics – Sources and Comments, Vol. 7(North-Holland, Amsterdam, 1990); G. B. West, Phys. Rev. Lett. **67** (1991) 1388; L. S. Brown *et al.*, Phys. Rev. D**46** (1992) 4712; V. I. Zakharov, Nucl. Phys. **B385** (1992) 452; M. Beneke and V. I. Zakharov, Phys. Rev. Lett. **69** (1992) 2472; A. H. Mueller, QCD – 20 Years Later, Aachen, 1992, P. M. Zerwas and H. A. Kastrup, eds.,(World Scientific, Singapore,1993).

[26] S. J. Brodsky, comment at the 1996 CTEQ QCD Symp., FNAL, 1996.