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# Extended Time-Reversal Operator and the Symmetries of the Hyperon-Nucleon Interaction

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**Abstract**

An extended time-reversal operator incorporating the charge conjugation in the flavor $SU_3$ space is introduced in order to study the spin-flavor structure of the hyperon-nucleon interaction. The particle-hole conjugation in terms of this extended time reversal is found to be useful to characterize time-reversal odd components for the space-spin part of the hyperon-nucleon interaction, which yield non-zero contribution to the exchange Feynman diagram or to the transition amplitude between $\Lambda N$ and $\Sigma N$ systems when the flavor symmetry breaking is properly introduced. Some selection rules are also given for the spin-flavor factors appearing in the quark-model potentials of the hyperon-nucleon interaction.

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## §1. Introduction

Since the quantum chromodynamics (QCD) was found to be the fundamental theory of the strong interaction, a number of models have already been proposed to understand the nucleon-nucleon ($NN$) and hyperon-nucleon ($YN$) interactions from more basic elements of quarks and gluons. Among them the non-relativistic quark model is in a unique position to enable us to take full account of the dynamical motion of the two composite baryons though it has to describe confinement with a phenomenological potential and to employ the quark-quark ($qq$) residual interaction derived from the color analogue of the Fermi-Breit interaction. Thanks to this advantage it can give a realistic description of the $NN$ and $YN$ interaction, if the missing meson-exchange effect, which dominates at the long-range part of the interaction, is properly taken into account. Since the quark model can describe the short-range part of the $NN$ and $YN$ interactions within the unified framework of the resonating-group method (RGM), the symmetry properties of these interactions in the spin-flavor $SU_6$ space become one of the most important issues which indicate the quality of the model.

A simultaneous description of the $NN$ and $YN$ interactions has recently been achieved by two groups. One is a series of models, RGM-F,[1),2)] FSS[3),4)] and RGM-H,[4)] by Kyoto-Niigata group, and the other is the $SU_3$-chiral symmetry quark model by Beijing-Tübingen group,[5),6)] in which chiral-symmetric effective meson-exchange potentials (EMEP) generated from the scalar and pseudo-scalar meson exchange between quarks are incorporated. In both models it was found that the flavor-nonet scalar mesons play an important role in order to describe the $NN$ and $YN$ interactions in a single framework which employs a unique set of model parameters. We stress that a simultaneous and realistic description of the $NN$ and $YN$ interactions is very important, since the experimental data for the $YN$ interaction are at present very limited and thus one has to rely on the theoretical consistency of the framework in order to make best use of the rich experimental information on the $NN$ interaction.

It would be appropriate to mention that the purpose of the quark-model study of the hyperon-nucleon interaction is not only to reproduce the experimental data, but is also to clarify the rich behavior of various baryon-baryon interactions, which reflects dynamical processes taking place between the composite systems of quarks. In the present approach by the QCD-inspired quark-model, these processes are dominated by the effective $qq$ interaction as well as important kinematical requirements arising from the quark Pauli principle, the conservation laws of energy-momentum and other quantum numbers, and the tensorial properties related to the spin-flavor-color degrees of freedom. Although it would be difficult to justify the non-relativistic treatment of the many-quark systems in this type of approach, our experience obtained so far shows that the quark-model potentials for the $NN$ and $YN$ interactions have correct degrees of freedom with respect not only to the space-spin structure but also to the rich flavor dependence of the interaction.

The purpose of this paper is to study the symmetry properties of the $NN$ and $YN$ interactions commonly observed in the OBEP approach and the quark-model approach. In particular, we will extend the time reversal symmetry to incorporate the flavor degree of freedom, and discuss the $NN$ and $YN$ interactions unifiedly. Namely, we introduce an extended time-reversal operator incorporating the charge conjugation in the flavor $SU_3$ space. We also define the so-called particle-hole conjugation by using this extended time-reversal operator. This symmetry is found to be useful to characterize the time-reversal odd components for the space-spin part of the hyperon-nucleon interaction; i.e., the time-reversal odd tensor force $S_{12}(\boldsymbol{r}, \boldsymbol{p})$ and the parity-dependent antisymmetric spin-orbit force $LS^{(-)}\sigma$.[7] These forces do not appear in the $NN$ force and feature the $YN$ interaction involving the strangeness degree of freedom. It will be shown that the $S_{12}(\boldsymbol{r}, \boldsymbol{p})$ force gives non-zero contribution only to the transition amplitude between $\Lambda N$ and $\Sigma N$ systems, and that the $LS^{(-)}\sigma$ force is non-zero when the flavor symmetry breaking (FSB) is properly taken into account.

In the next section, we discuss the structure of the scattering amplitudes for the $NN$ and $YN$ systems by introducing the Pauli-spinor invariants. The Nijmegen hard-core models[8)-10)] are one of the standard OBEP models which are derived from the scattering amplitudes through the Fourier transformation. These potentials also share the same space-spin invariant structure as the original transition amplitudes possess, except that the energy conservation law is not automatically incorporated in the potentials. A brief discussion of the quark-model potentials is also given. In §3 we first start with a simple introduction of the time reversal invariance including the discussion of the phase convention of the state vectors and the tensor operators. By incorporating the $SU_3$ conjugation in the flavor space, we define an extended time-reversal operator and the particle-hole conjugation. Basic symmetries of the $YN$ interaction involving the exchange symmetry of the two baryons, hermiticity, and the extended time-reversal symmetry, are discussed in § 4. These are employed to find the symmetries of the flavor factors of the $YN$ interaction under the constraint of the space-spin structure given in § 2. One can then extract the operator form of these flavor factors both in the OBEP and in the quark-model potentials. Some examples are given with respect to the Nijmegen hard-core potentials. In § 5, the symmetry properties of the spin-flavor factors in the quark model are discussed with respect to the non-central forces. A brief summary is given in the last section.



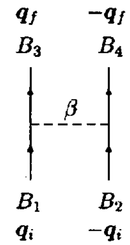
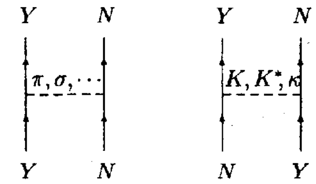



Fig. 1. The second-order Feynman diagram of $\beta$-meson exchange from $B_1$-$B_2$ to $B_3$-$B_4$.




(a) direct (b) exchange

Fig. 2. (a) direct and (b) exchange Feynman diagrams for the $YN$ elastic scattering.

## §2. Invariant amplitudes, OBEP and the quark-model potentials

The scattering (or transition) amplitudes of two spin-1/2 particles from $B_1$-$B_2$ to $B_3$-$B_4$ (see Fig. 1) should take the following general form required from the invariance of the interaction under the space rotation and the reflection of the coordinate frame:[11)]

$$M(\boldsymbol{q}_f, \boldsymbol{q}_i) = \sum_{i=1}^{8} M^{(i)}(\boldsymbol{q}_f, \boldsymbol{q}_i) P_i \quad , \tag{2·1}$$

where $\boldsymbol{q}_i$ and $\boldsymbol{q}_f$ are the incident and out-going relative momenta, by which one constructs three orthogonal vectors

$$\boldsymbol{q} = \frac{1}{2}(\boldsymbol{q}_f + \boldsymbol{q}_i) \quad , \qquad \boldsymbol{k} = \boldsymbol{q}_f - \boldsymbol{q}_i \quad , \qquad \boldsymbol{n} = \boldsymbol{q}_i \times \boldsymbol{q}_f = \boldsymbol{q} \times \boldsymbol{k} \quad . \tag{2·2}$$

Our choice of the Pauli-spinor invariants $P_i$ is given in Table I in terms of $\boldsymbol{k}$, $\boldsymbol{q}$, $\boldsymbol{n}$, $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$. Among these eight possible invariants, the last three from $P_6$ through $P_8$ are not allowed in the $NN$ scattering because of the identity of the two particles ($P_6$ and $P_8$) and because of the time reversal invariance ($P_7$ and $P_8$). The combination, $P_{5'} = (\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q})$, is also listed in Table I, although this combination is not independent of the others. The invariant amplitudes, $M^{(i)}(\boldsymbol{q}_f, \boldsymbol{q}_i)$ in Eq. (2·1), are in general functions of the three independent scalar quantities, $(\boldsymbol{q}_i)^2$, $(\boldsymbol{q}_f)^2$ and $(\boldsymbol{q}_i \cdot \boldsymbol{q}_f)$, which correspond to the energies $E_i$, $E_f$ of the initial and final channels and the scattering angle $\theta$ in the center-of-mass (c.m.) system, respectively. Alternatively, one can also take $\boldsymbol{q}^2$, $\boldsymbol{k}^2$ and $\boldsymbol{k} \cdot \boldsymbol{q}$ as independent variables. For the elastic $YN$ scattering in Fig. 2, the relative momenta in the initial and final states have the same magnitude, $(\boldsymbol{q}_i)^2 = (\boldsymbol{q}_f)^2$, because of the energy conservation. This condition is equivalent to $\boldsymbol{k} \cdot \boldsymbol{q} = 0$, and plays an important role in the following sections.

Table I. Pauli-spinor invariants $P_i$ and the space-spin invariants $\mathcal{O}_\Omega$ employed in this paper. A generalized tensor operator $S_{12}(\boldsymbol{a},\boldsymbol{b})$ is defined through $S_{12}(\boldsymbol{a},\boldsymbol{b}) = (3/2)[(\boldsymbol{\sigma}_1\cdot\boldsymbol{a})(\boldsymbol{\sigma}_2\cdot\boldsymbol{b})+(\boldsymbol{\sigma}_2\cdot\boldsymbol{a})(\boldsymbol{\sigma}_1\cdot\boldsymbol{b})] - (\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{a}\cdot\boldsymbol{b})$, and $\boldsymbol{S}^{(-)} = (1/2)(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)$ for the antisymmetric spin-orbit force. The quadratic $LS$ operator, $Q_{12}$, is defined in Eq. (B·12). The invariant $P_{5'}$ corresponds to the momentum (mom.) tensor $S_{12}(\boldsymbol{p},\boldsymbol{p})$, and is redundant if the other eight invariants are used. $\Omega$ for $i=7$ is the time-reversal odd ($\mathcal{T}$-odd) tensor.

| $i$ | $P_i$ | $\Omega$ | $\mathcal{O}_\Omega$ |
|---|---|---|---|
| 1 | 1 | central | 1 |
| 2 | $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ | spin-spin | $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ |
| 3 | $(\boldsymbol{\sigma}_1\cdot\boldsymbol{k})(\boldsymbol{\sigma}_2\cdot\boldsymbol{k})$ | tensor | $S_{12}\ (=S_{12}(\hat{\boldsymbol{r}},\hat{\boldsymbol{r}}))$ |
| 4 | $i\boldsymbol{n}\cdot\boldsymbol{S}$ | $LS$ | $\boldsymbol{L}\cdot\boldsymbol{S}$ |
| 5 | $(\boldsymbol{\sigma}_1\cdot\boldsymbol{n})(\boldsymbol{\sigma}_2\cdot\boldsymbol{n})$ | $QLS$ | $Q_{12}$ |
| 5′ | $(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})$ | mom. tensor | $S_{12}(\boldsymbol{p},\boldsymbol{p})$ |
| 6 | $i\boldsymbol{n}\cdot\boldsymbol{S}^{(-)}$ | $LS^{(-)}$ | $\boldsymbol{L}\cdot\boldsymbol{S}^{(-)}$ |
| 7 | $(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{k})+(\boldsymbol{\sigma}_1\cdot\boldsymbol{k})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})$ | $\mathcal{T}$-odd tensor | $i\,S_{12}(\boldsymbol{r},\boldsymbol{p})$ |
| 8 | $(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{k})-(\boldsymbol{\sigma}_1\cdot\boldsymbol{k})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})$ | $LS^{(-)}\sigma$ | $(\boldsymbol{L}\cdot\boldsymbol{S}^{(-)})P_\sigma$ |

One of the reasons we prefer to use the variables, $\boldsymbol{k}$ and $\boldsymbol{q}$, in $M^{(i)}$ is that they show a simple transformation property under the time reversal operation. Namely, the transformation, $\boldsymbol{q}_i \to -\boldsymbol{q}_f$ and $\boldsymbol{q}_f \to -\boldsymbol{q}_i$, implies $\boldsymbol{q}\to-\boldsymbol{q}$, $\boldsymbol{k}\to\boldsymbol{k}$ and $\boldsymbol{n}\to-\boldsymbol{n}$. This property together with $\boldsymbol{\sigma}_i\to-\boldsymbol{\sigma}_i$ $(i=1,2)$ clearly shows that $P_1 \sim P_6$ in Table I do not change the sign, while $P_7$ and $P_8$ do. We call the former case time-reversal even, and the latter odd.

The standard procedure to derive OBEP consists of a couple of steps involving various approximations. In the Nijmegen hard-core models,[8)-10)] one first writes down the second-order Feynman diagram in Fig. 1 for the exchange of the scalar, pseudo-scalar and vector mesons between $Y$ and $N$. After some approximations on the mass factors and the propagator in the exchange diagram, the non-relativistic reduction of the relativistic amplitude for a particular meson species $\beta$ becomes

$$M_\beta^{OBEP}(\boldsymbol{q}_f,\boldsymbol{q}_i) = \frac{4\pi}{\boldsymbol{k}^2+m^2}\sum_{i=1}^{8}\Omega_i(\boldsymbol{k}^2,\boldsymbol{q}^2)P_i \ , \tag{2·3}$$

where $m$ is the meson (or effective meson) mass and $\Omega_i(\boldsymbol{k}^2,\boldsymbol{q}^2)$ are in quadratic form of the baryon-meson coupling constants. The explicit expressions of $\Omega_i(\boldsymbol{k}^2,\boldsymbol{q}^2)$ are given in Eqs. (23) $\sim$ (26) of Ref. 12) except for $\Omega_8(\boldsymbol{k}^2,\boldsymbol{q}^2)$, which is given in Appendix A. These involve no $\boldsymbol{k}\cdot\boldsymbol{q}$ term and the $\boldsymbol{q}^2$ dependence is at least linear. In the Nijmegen hard-core potentials in Refs. 8) $\sim$ 10), the $\boldsymbol{q}^2$ dependence in $\Omega_i(\boldsymbol{k}^2,\boldsymbol{q}^2)$ is neglected and the energy dependence of the potential is controlled by choosing the core radius in some appropriate



way. The $\boldsymbol{k}^2$ dependence is also ignored by setting $\boldsymbol{k}^2=-m^2$, since the higher order terms simply yield the delta function of the relative coordinate and its derivatives in the coordinate representation. After these approximations one constructs a local expression of the OBEP in such a way that the Born amplitude of this potential coincides with Eq. (2·3). This process is conveniently carried out by using the Wigner transform technique for non-local kernels as discussed in Appendix B. The resultant OBEP is given by

$$V_\beta^{OBEP} = \sum_{i=1}^{8}\Omega_i(-m^2,0)\ \mathcal{O}_i^{op}(\boldsymbol{r}) \ , \tag{2·4}$$

where the invariant factors $\mathcal{O}_i^{op}(\boldsymbol{r})$ in the operator form are recovered from the corresponding Wigner transform

$$\mathcal{O}_i^{W}(\boldsymbol{r},\boldsymbol{q}) = \frac{4\pi}{(2\pi)^3}\int d\boldsymbol{k}\ e^{i\boldsymbol{k}\boldsymbol{r}}\frac{1}{\boldsymbol{k}^2+m^2}P_i \tag{2·5}$$

through

$$\mathcal{O}_i^{op}(\boldsymbol{r}) = \int d\boldsymbol{s}\ e^{\frac{1}{2}\boldsymbol{s}\nabla}\left\{\frac{1}{(2\pi)^3}\int d\boldsymbol{q}\ e^{-i\boldsymbol{q}\boldsymbol{s}}\ \mathcal{O}_i^{W}(\boldsymbol{r},\boldsymbol{q})\right\}e^{\frac{1}{2}\boldsymbol{s}\nabla} \ . \tag{2·6}$$

Here $\nabla = \partial/\partial\boldsymbol{r}$. Since the $\boldsymbol{q}$ dependence of $\mathcal{O}_i^W(\boldsymbol{r},\boldsymbol{q})$ is simple, the transformation in Eq. (2·6) is actually very easy. The final expression of $\mathcal{O}_i^{op}(\boldsymbol{r})$, shown in Appendix B, contains the full spin-dependence of the central and non-central potentials together with the standard Yukawa functions $Y(x)$, $Z^{(1)}(x)$ and $Z(x)$ with $x = m\,|\boldsymbol{r}|$. Finally one can express the OBEP somewhat symbolically as

$$V_\beta^{OBEP} = \sum_\Omega X_\beta^\Omega\ U^\Omega(x)\ \mathcal{O}_\Omega \ , \tag{2·7}$$

where $\Omega$ is used to specify the interaction types of the space-spin invariants in the coordinate representation. Refer to Table I for the common names of $\Omega$ and the explicit form of $\mathcal{O}_\Omega$. In Eq. (2·7), $U^\Omega(x)$ denotes some appropriate Yukawa function corresponding to the interaction type $\Omega$, and $X_\beta^\Omega$ are expressed in quadratic form of the baryon-meson coupling constants involving mass factors.

So far the scattering amplitude for a specific combination of $Y$ and $N$ is obtained for the exchange of a meson $\beta$ with some definite internal quantum numbers in the flavor degree of freedom. The full scattering amplitude is obtained by summing up all the meson contributions allowed for the direct and exchange Feynman diagrams. The needed calculation in this process is most efficiently carried out by using the operator formalism for the coupling constants, in which flavor-singlet and octet $SU_3$ tensor operators, $f_{(\lambda\lambda)\beta}$, (which we call coupling-constant operators) are postulated in the flavor $SU_3$ space. In this formalism, $Y$ and $N$ are considered to be two different states of an identical particle, namely, the octet baryon $B_8$, and the generalized Pauli principle

$$(-1)^L\,(-1)^{1-S}\,\mathcal{P} = -1 \tag{2·8}$$

is always assumed if one uses the spin-flavor wave function

$$\begin{aligned} \eta_\alpha^{SF} &\equiv \chi_{SS_z}[B_1B_2]^{\mathcal{P}}_{II_z} \\ &= \chi_{SS_z}\frac{1}{\sqrt{2(1+\delta_{B_1\,B_2})}}\left\{[B_1B_2]_{II_z}+\mathcal{P}(-1)^{I_1+I_2-I}[B_2B_1]_{II_z}\right\} \ . \end{aligned} \tag{2·9}$$

In Eq. (2·8), $S$ denotes the total spin, $L$ is the orbital angular momentum of the relative motion, and the parity is given by $\pi=(-1)^L$. The flavor exchange phase $\mathcal{P}$ is the eigen-value of the flavor exchange operator $P^F_{12}$ with the eigen-vector $[B_1B_2]^{\mathcal{P}}_{II_z}$. In Eq. (2·9), $B_i \equiv B_{(11)a_i}$ with $a_i=Y_iI_i$, and the subscript $\alpha$ of $\eta_\alpha^{SF}$ specifies a set of spin-flavor quantum numbers: $\alpha \equiv [1/2(11)a_1, 1/2(11)a_2]SS_zYII_z;\mathcal{P}$. We use the convention that the first $B_i$ always refers to particle 1 and the second to particle 2. The matrix elements of the two coupling-constant operators with respect to $[B_1B_2]^{\mathcal{P}}_{II_z}$ are expressed by the isospin-standard coupling constants multiplied with the isospin factors related to the baryon channels and the isospin $I_\beta$ of the exchanged mesons. By using these notations and the baryon-exchange operator, $P_{12}=P^r_{12}\,P^\sigma_{12}\,P^F_{12}$, the total scattering amplitude by the OBEP, $\mathcal{V}^{OBEP}=\sum_\beta \mathcal{V}^{OBEP}_\beta$ in Eq. (2·7), is given by

$$\begin{aligned} &M^{\text{total}}_{\alpha\alpha'}(\boldsymbol{q}_f,\boldsymbol{q}_i) \\ &= \left\langle \frac{1}{\sqrt{2}}(1-P_{12})e^{i\boldsymbol{q}_f\boldsymbol{r}}\eta^{SF}_\alpha \;\middle|\; \mathcal{V}^{OBEP} \;\middle|\; \frac{1}{\sqrt{2}}(1-P_{12})e^{i\boldsymbol{q}_i\boldsymbol{r}}\eta^{SF}_{\alpha'}\right\rangle \\ &= M_{\alpha\alpha'}(\boldsymbol{q}_f,\boldsymbol{q}_i)+(-1)^{S'}\mathcal{P}'\,M_{\alpha\alpha'}(\boldsymbol{q}_f,-\boldsymbol{q}_i) \end{aligned} \tag{2·10}$$

with

$$\begin{aligned} \alpha' &= [1/2(11)a_1, 1/2(11)a_2]S'S'_zYII_z;\mathcal{P}' \ , \\ \alpha &= [1/2(11)a_3, 1/2(11)a_4]SS_zYII_z;\mathcal{P} \ , \\ \pi &= (-1)^{S'}\mathcal{P}' = (-1)^S\mathcal{P} \ . \end{aligned} \tag{2·11}$$

Here, $M_{\alpha\alpha'}(\boldsymbol{q}_f,\boldsymbol{q}_i)$ is the spin-flavor matrix element of Eq. (2·1), which should be given by

$$M_{\alpha\alpha'}(\boldsymbol{q}_f,\boldsymbol{q}_i)=\langle e^{i\boldsymbol{q}_f\boldsymbol{r}}\eta^{SF}_\alpha|\mathcal{V}^{OBEP}|e^{i\boldsymbol{q}_i\boldsymbol{r}}\eta^{SF}_{\alpha'}\rangle \ . \tag{2·12}$$

Care is needed for the treatment of the quark-model potential because the RGM equation for the $NN$ and $YN$ interactions is not straightforwardly reduced to the simple equation of Schrödinger type. We start from the RGM equation for the parity-projected relative-motion wave function, $\chi^\pi_\alpha(\boldsymbol{r})=(1/2)(\chi_\alpha(\boldsymbol{r})+\pi\chi_\alpha(-\boldsymbol{r}))$:

$$\left[\varepsilon_\alpha+\frac{\hbar^2}{2\mu_\alpha}\left(\frac{\partial}{\partial\boldsymbol{r}}\right)^2\right]\chi^\pi_\alpha(\boldsymbol{r})=\sum_{\alpha'}\int d\boldsymbol{r}'\;G_{\alpha\alpha'}(\boldsymbol{r},\boldsymbol{r}')\;\chi^\pi_{\alpha'}(\boldsymbol{r}') \ . \tag{2·13}$$



The parity non-projected $\chi_\alpha(\boldsymbol{r})$ satisfies Eq. (2·13) with the exchange kernel $G_{\alpha\alpha'}(\boldsymbol{r},\boldsymbol{r}')$ replaced by

$$\frac{1}{2}\left[G_{\alpha\alpha'}(\boldsymbol{r},\boldsymbol{r}')+(-1)^{S'}\mathcal{P}'G_{\alpha\alpha'}(\boldsymbol{r},-\boldsymbol{r}')\right] \ , \tag{2·14}$$

which corresponds to $\mathcal{V}^{OBEP}_{\alpha\alpha'}(\boldsymbol{r})$.*) The Born amplitude of the RGM kernel also takes the form of Eq. (2·10) with $M_{\alpha\alpha'}(\boldsymbol{q}_f,\boldsymbol{q}_i)$ given by

$$M_{\alpha\alpha'}(\boldsymbol{q}_f,\boldsymbol{q}_i)=\langle\, e^{i\boldsymbol{q}_f\boldsymbol{r}}\,|\,G_{\alpha\alpha'}(\boldsymbol{r},\boldsymbol{r}')\,|\,e^{i\boldsymbol{q}_i\boldsymbol{r}'}\rangle \ . \tag{2·15}$$

The exchange kernel $G_{\alpha\alpha'}(\boldsymbol{r},\boldsymbol{r}')$ is calculated as the matrix element of the quark-model Hamiltonian with respect to the spin-flavor $SU_6$ wave functions, $\xi^{SF}_\alpha$, defined through Eqs. (2.6) and (2.8) of Ref. 1). We can find a simple transformation rule to convert the spin-flavor matrix elements of a single baryon into those of some flavor operators at the baryon level. We can thus postulate that $G(\boldsymbol{r},\boldsymbol{r}')$ contains the spin and flavor operators at the baryon level, whose matrix element with respect to $\eta^{SF}_\alpha$ and $\eta^{SF}_{\alpha'}$ becomes $G_{\alpha\alpha'}(\boldsymbol{r},\boldsymbol{r}')$.

The WKB-RGM method enables us to convert the nonlocal exchange kernel to the momentum-dependent local potential.[13] The Wigner transform of $G(\boldsymbol{r},\boldsymbol{r}')$ is given in a form similar to Eq. (2·7):

$$G_W(\boldsymbol{r},\boldsymbol{q})=\sum_\Omega\sum_T X^\Omega_T\;G^\Omega_{WT}(\boldsymbol{r},\boldsymbol{q})\;O^W_\Omega \ , \tag{2·16}$$

where $T$ specifies five different interaction types, $E$, $S$, $S'$, $D_+$ and $D_-$, for a particular piece of the $qq$ interaction.[15] Since the spin-flavor factor $X^\Omega_T$ and the spatial function $G^\Omega_{WT}(\boldsymbol{r},\boldsymbol{q})$ depend on each type of the $qq$ interaction, the summation over $\Omega$ in Eq. (2·16) implies not only for the interaction types at the baryon level, but also for those at the quark.

The Wigner-transformed version of the space-spin invariants, $O^W_\Omega$, is obtained by replacing the momentum-operator $\boldsymbol{p}=(i/\hbar)(\partial/\partial\boldsymbol{r})$ in $O_\Omega$ with its classical counterpart and thus neglecting the operator ordering. (See the discussion in Appendix B.) An energy-dependent term like $-\varepsilon_\alpha(X_N)_{\alpha\alpha'}G^N_W(\boldsymbol{r},\boldsymbol{q})$ is included in the central component as a part of the $E$-type interaction, where $N$ stands for the quantity related to the exchange normalization kernel.

Actually the Wigner transform of Eq. (2·16) well approximates the original RGM kernel only when the local momentum $\boldsymbol{q}^2$ is determined by solving the self-consistent transcendental equation.[1] The use of the Wigner transform technique is possible so far only for the

*) The first term of Eq. (2·14) corresponds to the $(-9)P_{36}$-type single-quark exchange term, while the second to the two-quark exchange term expressed as the single-quark exchange followed by the $(3q)$ exchange. In the WKB-RGM technique, these two terms give the same contribution to the Wigner transform since the second term should be transformed as a Majorana-type non-local kernel.[13] However, this is not true for the total Born amplitude in Eq. (2·10). This fact is first pointed out by C. W. Wong and his co-workers in their quark-model study of the $LS$ amplitudes for the $YN$ interaction.[14]

elastic scattering ($\alpha = \alpha'$) with the central force. Even in this case, the solution of the transcendental equation sometimes does not exist, if the Pauli repulsion is very strong, e.g., in the odd-parity case of the $NN$ interaction. Nevertheless, it would be useful to examine the Wigner transform by simply assuming $\boldsymbol{q} = 0$ in the spatial Gaussian functions $G^{\Omega}_{WT}(\boldsymbol{r}, \boldsymbol{q})$ of Eq. (2·16), since we are interested in the low-energy scattering. The effective quark-model potential in this approximation can be given by

$$\mathcal{V}^{\rm eff} = \sum_{\Omega} \sum_{\mathcal{T}} X^{\Omega}_{\mathcal{T}} \, G^{\Omega}_{WT}(\boldsymbol{r}, 0) \, O_{\Omega} \quad . \tag{2·17}$$

The $LS$ potentials thus derived from the Fermi-Breit interaction were found to have a good correspondence to the OBEP (especially to the Nijmegen model-F $LS$ forces) for the $NN$ and $YN$ single-channel systems.[7] The same technique is also applied to the $(q\bar{q})$ exchange potentials in the $NN$ system.[16),17] They show very good correspondence to the OBEP in the medium- and long-range region, except for the OPEP-type $(q\bar{q})$ exchange potential.

Before discussing symmetries of the flavor operators, $X^{\Omega}_{\beta}$ (in Eq. (2·7)) and $X^{\Omega}_{\mathcal{T}}$ (in Eqs. (2·16) and (2·17)), we will discuss some simple properties of the time reversal operator and their extension to the flavor $SU_3$ space in the next section.

## §3. Time reversal invariance and the extended time-reversal operator

For the basic properties of the time reversal symmetry, we refer to Refs. 18) and 19). The time reversal symmetry is usually employed to determine the phase of state vectors and operators. The invariance under this symmetry is therefore intimately related to the reality of the matrix elements.

In the simplest system composed of a single spin-1/2 particle, the time reversal operator is defined by*)

$$\mathcal{T} = \mathcal{R}^{-1} K \quad , \tag{3·1}$$

where $\mathcal{R} = \mathcal{R}_y(\pi) = e^{-i(\pi/2)\sigma_y}$ implies the rotation of the coordinate frame through an angle $\pi$ about the $y$ axis, and $K$ taking the complex conjugate of all $c$-numbers. The time-reversal odd character of the spin operator

$$\mathcal{T} \boldsymbol{\sigma} \mathcal{T}^{-1} = -\boldsymbol{\sigma} \tag{3·2}$$

*) Here we use the standard notation $\mathcal{T}$ for the time reversal operator, which should not be confused with the subscript $\mathcal{T}$ in $X^{\Omega}_{\mathcal{T}}$ etc., specifying the interaction types of the quark exchange diagrams (see Eq. (2·16) and the subsequent discussion.)



can be easily proved by using the explicit expression of the Pauli matrices. One of the most important properties of $\mathcal{T}$ is the commutation relations

$$[\mathcal{RT}, \boldsymbol{S}^2] = [\mathcal{RT}, S_z] = [\boldsymbol{S}^2, S_z] = 0 \tag{3·3}$$

for the spin operator $\boldsymbol{S} = \boldsymbol{\sigma}/2$, which allows us a simultaneous eigen-state of $\boldsymbol{S}^2$, $S_z$ and $\mathcal{RT}$. The phase of the spin eigen-state is usually taken as

$$\mathcal{RT} |SS_z\rangle = |SS_z\rangle \quad . \tag{3·4}$$

This relation gives us the well-known transformation rule of the spin angular-momentum states under time reversal:

$$\mathcal{T} |SS_z\rangle = \mathcal{R}^{-1} |SS_z\rangle = e^{i\pi S_y} |SS_z\rangle = (-1)^{S+S_z} |S - S_z\rangle \quad . \tag{3·5}$$

It should be noted that these relations are easily extended to the many-particle systems involving the spatial degree of freedom, since the Clebsch-Gordan (C-G) coefficients are chosen to be real. For the orbital angular-momentum state, the well-known $i^\ell$ factor should be included for the spherical harmonics in the coordinate representation.

Let us consider the matrix element of an operator $T$ with the basis set of Eq. (3·4):

$$\begin{aligned} \langle SS_z | T | S'S'_z \rangle^* &= \langle K\{SS_z\} | K\{T|S'S'_z\rangle\} \\ &= \langle \mathcal{RT}\{SS_z\} | (\mathcal{RT}) T (\mathcal{RT})^{-1} | \mathcal{RT}\{S'S'_z\} \rangle \\ &= \langle SS_z | (\mathcal{RT}) T (\mathcal{RT})^{-1} | S'S'_z \rangle \quad . \end{aligned} \tag{3·6}$$

The matrix element of the operator $T$ becomes real if the operator $T$ is invariant under the transformation by $\mathcal{RT}$. If $T$ changes a sign under $\mathcal{RT}$, the matrix element is purely imaginary. Thus the transformation property of operators under the time reversal characterizes the phase of their matrix elements. An operator which satisfies the relationship

$$(\mathcal{RT}) T (\mathcal{RT})^{-1} = T \tag{3·7}$$

is called real. For the tensor operators $T_{\lambda\mu}$, the transformation above by $\mathcal{RT}$ usually reduces each $\lambda\mu$ component to itself, except for a phase factor:

$$(\mathcal{RT}) T_{\lambda\mu} (\mathcal{RT})^{-1} = c_{\mathcal{T}} T_{\lambda\mu} \quad . \tag{3·8}$$

Then, the Wigner-Eckert theorem and the reality of the C-G coefficients yield

$$\langle S || T_\lambda || S' \rangle^* = c_{\mathcal{T}} \langle S || T_\lambda || S' \rangle \quad . \tag{3·9}$$

Namely, if $c_{\mathcal{T}} = 1$ and the operator $T_\lambda$ is real, then the reduced matrix element in Eq. (3·9) is real, so as to all the matrix elements. Note that the phase factor $c_{\mathcal{T}}$ is not an intrinsic property of $T_\lambda$, since it depends on the phase choice of the operator. Thus, by multiplying $T_\lambda$ with a suitable phase factor, we can always achieve $c_{\mathcal{T}} = 1$. For examples, the operators, $T_{\frac{1}{2}\mu} = \sigma_\mu$ and $T_{\lambda\mu} = i^\lambda Y_{\lambda\mu}(\hat{\boldsymbol{r}})$, are real but the spherical vector $r_\mu$ is not real:

$$(\mathcal{RT})r_\mu(\mathcal{RT})^{-1} = -r_\mu \quad . \tag{3·10}$$

On the other hand, the momentum spherical vector, $p_\mu = (-1)^\mu(\hbar/i)(\partial/\partial r_{-\mu})$, is real because it includes the imaginary unit $i$. By using these, we can easily show that all the space-spin invariants $\mathcal{O}_\Omega$ given in Table I are real operators.

So far we have considered only the space-spin degrees of freedom. When the above formalism is applied to the $YN$ interaction, it is convenient to incorporate further the flavor degree of freedom and to introduce an extended time-reversal operator as shown below. It should be noted that the flavor operators are also affected by the time reversal operation $\mathcal{T}$, since it contains $K$. We need to modify the transformation property of the tensor operators in the flavor space such that it becomes compatible with the effect of $K$. This can be done by using a technique similar to the one employed for the isospin-dependent operators in the nuclear many-body problems.[19] We first introduce an operator for the $SU_3$ conjugation, $\mathcal{C}$, through

$$\mathcal{C}\mathcal{O}_{(\lambda\mu)\alpha}\mathcal{C}^{-1} = (-1)^{\phi((\lambda\mu)\alpha)}\mathcal{O}_{(\mu\lambda)\alpha_c} \tag{3·11}$$

for the $SU_3$ tensor operators $\mathcal{O}_{(\lambda\mu)\alpha}$. Here, $\phi((\lambda\mu)\alpha)$ is the conjugation phase given by

$$\phi((\lambda\mu)YII_z) = \frac{1}{3}(\lambda-\mu) + \frac{1}{2}Y + I_z \quad , \tag{3·12}$$

following the Draayer and Akiyama's phase convention,[20] and the conjugate quantum number $\alpha_c$ of $\alpha = YII_z$ stands for $\alpha_c = (-Y)I(-I_z)$. Since $\phi((\lambda\mu)\alpha) + \phi((\mu\lambda)\alpha_c) = 0$, one can assume $\mathcal{C}^2 = 1$ or $\mathcal{C}^{-1} = \mathcal{C}$. The extended time-reversal operator is defined by

$$\mathcal{F} \equiv \mathcal{CT} = \mathcal{CR}^{-1}K \quad . \tag{3·13}$$

Using $\mathcal{RC}^{-1}\mathcal{F} = \mathcal{RT} = K$, we can easily show that

$$\begin{aligned}
&[\mathcal{RC}^{-1}\mathcal{F}, S^2] = [\mathcal{RC}^{-1}\mathcal{F}, S_z] = 0 \quad , \\
&[\mathcal{RC}^{-1}\mathcal{F}, C_2] = [\mathcal{RC}^{-1}\mathcal{F}, C_3] = [\mathcal{RC}^{-1}\mathcal{F}, Y] = [\mathcal{RC}^{-1}\mathcal{F}, \boldsymbol{I}^2] \\
&= [\mathcal{RC}^{-1}\mathcal{F}, I_z] = 0 \quad ,
\end{aligned} \tag{3·14}$$

where $C_2$ and $C_3$ are the $SU_3$ Casimir operators of the second and third ranks, respectively. To prove Eq. (3·14), we have used the fact that the $SU_3$ tensor generators $\lambda_{(11)YII_z}$ are all expressed by $3 \times 3$ matrix units as is shown in Ref. 20). We can construct such spin-flavor $SU_6$ basis state that simultaneously diagonalizes $\mathcal{RC}^{-1}\mathcal{F}$, $S^2$, $S_z$, $C_2$, $C_3$, $Y$, $\boldsymbol{I}^2$ and $I_z$. Choosing the phase of the basis state as

$$\mathcal{RC}^{-1}\mathcal{F}|SS_z(\lambda\mu)\alpha\rangle = |SS_z(\lambda\mu)\alpha\rangle \quad , \tag{3·15}$$

we find that

$$\begin{aligned}
\mathcal{F}|SS_z(\lambda\mu)\alpha\rangle &= \mathcal{CR}^{-1}|SS_z(\lambda\mu)\alpha\rangle \\
&= (-1)^{S+S_z}(-1)^{\phi((\lambda\mu)\alpha)}|S\; -S_z(\mu\lambda)\alpha_c\rangle \quad .
\end{aligned} \tag{3·16}$$

The reality of the $SU_6$ tensor operators, $\mathcal{O}_{\tau\tau_z(\lambda\mu)\alpha}$, is also defined similarly. (We use the notation, $\tau\tau_z$, to specify the spin quantum numbers of the spin operators.) Suppose that

$$(\mathcal{RC}^{-1}\mathcal{F})\mathcal{O}_{\tau\tau_z(\lambda\mu)\alpha}(\mathcal{RC}^{-1}\mathcal{F})^{-1} = c_{\mathcal{F}}\mathcal{O}_{\tau\tau_z(\lambda\mu)\alpha} \tag{3·17}$$

be satisfied. We can choose the phase of $\mathcal{O}_{\tau\tau_z(\lambda\mu)\alpha}$ such that $c_{\mathcal{F}} = 1$, and then all the matrix elements become real. As an example, let us consider coupling-constant operators; a set of $SU_3$ operators $f_{(\lambda\lambda)\beta}$ ($\lambda = 0,\, 1$) for the baryon-meson coupling constants. By choosing the phase of the operator as

$$(\mathcal{RC}^{-1}\mathcal{F})f_{(\lambda\lambda)\beta}(\mathcal{RC}^{-1}\mathcal{F})^{-1} = f_{(\lambda\lambda)\beta} \quad , \tag{3·18}$$

it follows that

$$\begin{aligned}
\mathcal{F}f_{(\lambda\lambda)\beta}\mathcal{F}^{-1} &= \mathcal{C}f_{(\lambda\lambda)\beta}\mathcal{C}^{-1} \\
&= (-1)^{\phi((\lambda\lambda)\beta)}f_{(\lambda\lambda)\beta_c} \equiv f^C_{(\lambda\lambda)\beta} \quad .
\end{aligned} \tag{3·19}$$

Namely, the effect of $\mathcal{F}$ in the flavor space is simply the $SU_3$ conjugation for the real operators. Under the phase convention of Eqs. (3·15) and (3·18), all the coupling constants become real.

Since the extended time-reversal operator $\mathcal{F}$ contains $K$, $c_{\mathcal{F}}$ in Eq. (3·17) depends on the phase choice of the operators. To avoid this ambiguity, we use an extra operation of the hermitian conjugation and define the so-called particle-hole conjugation. In order to realize this, we should recall that the hermite conjugate of $T_{\lambda\mu}$ (in the spin space for example) should be defined through

$$T^H_{\lambda\mu} \equiv (-1)^{\lambda+\mu}T^\dagger_{\lambda\,-\mu} \quad . \tag{3·20}$$

Then $T^H_{\lambda\mu}$ also becomes a tensor operator of rank $\lambda$ with the $z$-components $\mu$. Similarly, we can show that

$$f^H_{(\lambda\lambda)\beta} \equiv (-1)^{\phi((\lambda\lambda)\beta)}f^\dagger_{(\lambda\lambda)\beta_c} \tag{3·21}$$

is also the $SU_3$ tensor operator with the same irreducible representation label $(\lambda\lambda)\beta$. (See, for example, p. 218 of Ref. 21).) It is not always true, however, that $f^H_{(\lambda\lambda)\beta}$ is proportional to the original $f_{(\lambda\lambda)\beta}$. Nevertheless, the coupling-constant operators $f_{(\lambda\lambda)\beta}$ in the OBEP usually satisfy

$$f^\dagger_{(\lambda\lambda)\beta} = f^C_{(\lambda\lambda)\beta} \quad , \tag{3·22}$$

just like the $SU_3$ generators $\lambda_{(11)\alpha}$. We write this as $f^H_{(\lambda\lambda)\beta} = c_H f_{(\lambda\lambda)\beta}$ with $c_H = 1$, and call the operator satisfying Eq. (3·22) self-conjugate.

Let us now define the particle-hole conjugation operation through

$$\mathcal{O}^{\mathrm{phc}}_{\tau\tau_z(\lambda\mu)\alpha} \equiv -(\mathcal{F}\mathcal{O}_{\tau\tau_z(\lambda\mu)\alpha}\mathcal{F}^{-1})^\dagger \quad . \tag{3·23}$$

The relationship in Eq. (3·17) and the hermitian phase $c_H$ yield

$$\begin{aligned}\mathcal{O}^{\mathrm{phc}}_{\tau\tau_z(\lambda\mu)\alpha} &= -c_{\mathcal{F}}(-1)^{\tau+\tau_z}(-1)^{\phi((\lambda\mu)\alpha)}\mathcal{O}^\dagger_{\tau-\tau_z(\mu\lambda)\alpha_c}\\ &= -c_{\mathcal{F}}c_H\,\mathcal{O}_{\tau\tau_z(\lambda\mu)\alpha}\\ &= c\,\mathcal{O}_{\tau\tau_z(\lambda\mu)\alpha} \quad ,\end{aligned} \tag{3·24}$$

where $c = -c_{\mathcal{F}}c_H$ does not depend on the phase choice of the operator. It should be noted that combining two operators in the particle-hole conjugation is useful only when these two operators are for different degrees of freedom. Suppose that $A$ and $B$ be such operators as satisfying, $[A, B] = 0$, and $A^{\mathrm{phc}} = c_A A$, $B^{\mathrm{phc}} = c_B B$. Then the particle-hole conjugation phase $c_{AB}$ is obtained through $(AB)^{\mathrm{phc}} = c_{AB}AB$ with $c_{AB} = -c_A c_B$. Therefore, both of the $A$ and $B$ should be particle-hole conjugation odd or even, if we want to have $AB$ odd.

For the real flavor operators $f_{(\lambda\lambda)\beta}$ which satisfy Eq. (3·18), the particle-hole conjugation Eq. (3·23) is reduced to $f^{\mathrm{phc}}_{(\lambda\lambda)\beta} = -(f^C_{(\lambda\lambda)\beta})^\dagger$. Thus the self-conjugateness property means that $f^{\mathrm{phc}}_{(\lambda\lambda)\beta} = -f_{(\lambda\lambda)\beta}$. Namely, $f_{(\lambda\lambda)\beta}$ is particle-hole conjugation odd; $c = -1$.

The real self-conjugate operators $f_{(\lambda\lambda)\beta}$ satisfy a couple of specific relations. First, the flavor-singlet operator $f_{(00)}$ is always hermite; $f^\dagger_{(00)} = f_{(00)}$. If we combine two such real self-conjugate operators, $f_{(\lambda\lambda)\beta}$ and $g_{(\lambda\lambda)\beta}$, into flavor-singlet form, the product is always hermite. In fact, we find that

$$Z^{(\lambda\lambda)} = \sum_\beta f^C_{(\lambda\lambda)\beta}(1)g_{(\lambda\lambda)\beta}(2) = \sum_\beta f_{(\lambda\lambda)\beta}(1)g^C_{(\lambda\lambda)\beta}(2) \tag{3·25}$$

satisfies $Z^{(\lambda\lambda)\dagger} = Z^{(\lambda\lambda)}$. We use a notation

$$Z = \sum_{(\lambda\lambda)=(00),(11)} Z^{(\lambda\lambda)} = fg^\dagger \quad , \tag{3·26}$$



by assuming that particle 1 for the first operator and particle 2 for the second operator. Then the hermiticity of $Z$ is expressed as

$$fg^\dagger = (fg^\dagger)^\dagger = f^\dagger g \quad . \tag{3·27}$$

This product of the coupling constant operators is always real, since $f$ and $g$ are real operators. Under the particle-hole conjugation, it is odd since $f$ and $g$ are both odd.

## §4. Basic symmetries of the $YN$ interaction

In this section, we discuss some basic symmetries of the $YN$ interaction and apply them to the OBEP potential in Eq. (2·7) and to the quark-model potential in Eq. (2·17). As in the study of the $NN$ interaction, these symmetries turn out to give strong constraints on the models for the $YN$ interaction. Since the space-spin invariants $\mathcal{O}_\Omega$ satisfy some specific symmetries, the symmetry properties of the flavor factors, $X^\Omega_{\mathcal{B}}$ and $X^\Omega_{\mathcal{T}}$, are consequently uniquely determined from these constraints. One can then use these to recover the operator form of the flavor factors.

Here we discuss the following four types of symmetries:

(P$_{12}$) : $1 \leftrightarrow 2$ (or $(3q)$-cluster) exchange, $P_{12} = P_r P_\sigma P_F$ ,

(H) : hermiticity ,

(T) : time reversal, $\mathcal{T} = \mathcal{R}^{-1}K$, or the extended time reversal, $\mathcal{F} = \mathcal{C}\mathcal{T}$ ,

(PH) : particle-hole conjugation with $\mathcal{F}$, $\mathcal{O}^{\mathrm{phc}} = -(\mathcal{F}\mathcal{O}\mathcal{F}^{-1})^\dagger$ . (4·1)

In addition to the parity conservation and the rotational invariance, the full potential $\mathcal{V}^{\mathrm{pot}}$ for the $NN$ and $YN$ interactions should satisfy

$$\begin{aligned}&1)\quad P_{12}\,\mathcal{V}^{\mathrm{pot}}\,P_{12} = \mathcal{V}^{\mathrm{pot}} \quad ,\\ &2)\quad (\mathcal{V}^{\mathrm{pot}})^\dagger = \mathcal{V}^{\mathrm{pot}} \quad ,\\ &3)\quad \mathcal{F}\,\mathcal{V}^{\mathrm{pot}}\,\mathcal{F}^{-1} = \mathcal{V}^{\mathrm{pot}} \quad .\end{aligned} \tag{4·2}$$

The last condition in Eq. (4·2) indicates the invariance under the extended time reversal.*) This condition with the hermiticity makes the full potential PH conjugation odd; $c = -1$.

First, let us consider the symmetries of the space-spin part. Since the space-spin invariants, $P_i$ ($i = 1 \sim 8$ and $5'$) and $\mathcal{O}_\Omega$ in Table I, have one-to-one correspondence, we will use a notation $\mathcal{O}_i$ in order to indicate $\mathcal{O}_\Omega$. For the exchange symmetry in 1) of Eq. (4·2), we can

*) Actually this condition is equivalent to the time-reversal invariance, since the full potential is usually invariant under the $SU_3$ conjugation; $\mathcal{C}\,\mathcal{V}^{\mathrm{pot}}\,\mathcal{C}^{-1} = \mathcal{V}^{\mathrm{pot}}$. The effect of FSB is always expressed by octet tensor operators with the internal quantum numbers $YII_z = 000$.

Table II. Symmetries of the space-spin factors $\mathcal{O}_i$ or $\mathcal{O}_\Omega$. The operators $\mathcal{O}_i$ with $i = 1 \sim 6$ and $5'$ are hermite ($\mathcal{O}_i^\dagger = \mathcal{O}_i$), and those with $i = 7$ and $8$ are anti-hermite ($\mathcal{O}_i^\dagger = -\mathcal{O}_i$). The spin exchange operator is denoted by $P_\sigma = (1 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)/2$. For $i = 7$ and $8$, a proper phase convention for the time-reversal odd ($\mathcal{T}$-odd) operators is given in the parentheses.

| | hermite | anti-hermite ($\mathcal{T}$-odd) |
|---|---|---|
| | $\mathcal{O}_1$ central | |
| | $\mathcal{O}_2$ spin-spin | |
| $P_\sigma \mathcal{O}_i P_\sigma = \mathcal{O}_i$ | $\mathcal{O}_3$ tensor | $\mathcal{O}_7$ $\mathcal{T}$-odd tensor |
| | $\mathcal{O}_4$ $LS$ | $(-i\,\mathcal{O}_7)$ |
| | $\mathcal{O}_5$ $QLS$ | |
| | $\mathcal{O}_{5'}$ mom. tensor | |
| $P_\sigma \mathcal{O}_i P_\sigma = -\mathcal{O}_i$ | $\mathcal{O}_6$ $LS^{(-)}$ | $\mathcal{O}_8$ $LS^{(-)}\sigma$ |
| | | $(i\,\mathcal{O}_8)$ |

consider $P_\sigma \mathcal{O}_i P_\sigma$ instead of $(P_r P_\sigma)\mathcal{O}_i(P_r P_\sigma)$, since all the $\mathcal{O}_i$ are parity conserving. This symmetry, together with the hermitian property, gives that $\mathcal{O}_i$ are classified into four groups as is shown in Table II. On the other hand, the effect of the extended time-reversal operator $\mathcal{F}$ is reduced into $\mathcal{T}$ for $\mathcal{O}_i$. However, the time reversal property is usually referred to the hermite operators. Thus we consider

$$-i\,\mathcal{O}_7 = S_{12}(\boldsymbol{r}, \boldsymbol{p}) \ ,$$
$$i\,\mathcal{O}_8 = i\,(\boldsymbol{L} \cdot \boldsymbol{S}^{(-)})\,P_\sigma = \frac{1}{2}([\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2] \cdot \boldsymbol{L}) \ , \qquad (4{\cdot}3)$$

instead of the anti-hermite operators, $\mathcal{O}_7$ and $\mathcal{O}_8$, themselves. In the phase independent terminology, these are PH conjugation even operators. (On the other hand, the other operators, $\mathcal{O}_1 \sim \mathcal{O}_6$, are PH conjugation odd.)

Before proceeding to the discussion on the symmetries of the flavor factors, we need some comments on the spatial functions in each model. The spatial functions, $U^\Omega(x)$, of the OBEP in Eq. (2·7) are the simple Yukawa functions which are real and depend only on $r = |\boldsymbol{r}|$. In this particular case, the symmetries of the flavor factors, $X^\Omega = X^\Omega_\beta$ in Eq. (2·7) or $\Omega_i$ in Eq. (2·4), are uniquely specified from the combinations of Eq. (4·2) and the symmetries in Table II. Here we assume the isospin invariance and the subscript $\beta$ is supposed to indicate the meson species of some particular isospin supermultiplet. On the other hand, the spatial wave functions of the quark model, $G^\Omega_{W\mathcal{T}}(\boldsymbol{r}, \boldsymbol{q})$ in Eq. (2·16), are the functions of $\boldsymbol{r}^2$, $\boldsymbol{q}^2$, and $i\,\boldsymbol{r} \cdot \boldsymbol{q}$. We need the factor $i$ for $\boldsymbol{r} \cdot \boldsymbol{q}$ to make this combination a real operator in the coordinate space. Since the original quark-model Hamiltonian is a real operator, $G^\Omega_{W\mathcal{T}}(\boldsymbol{r}, \boldsymbol{q})$



Table III. Symmetries of the flavor factors $\Omega_i$ or $X^\Omega$. The operators $\Omega_i$ with $i = 1 \sim 6$ and $5'$ are hermite ($\Omega_i^\dagger = \Omega_i$), and those with $i = 7$ and $8$ are anti-hermite ($\Omega_i^\dagger = -\Omega_i$). The flavor exchange operator is denoted by $P_F$. For $i = 7$ and $8$, a proper phase convention for the extended time-reversal odd ($\mathcal{F}$-odd) operators is given in the parentheses. The selection rules for $i = 6 \sim 8$ components are for the diagonal baryon channels.

| | hermite | anti-hermite ( $\mathcal{F}$-odd ) |
|---|---|---|
| | $\Omega_1$ central | |
| | $\Omega_2$ spin-spin | |
| $P_F \Omega_i P_F = \Omega_i$ | $\Omega_3$ tensor | $\Omega_7$ $\mathcal{T}$-odd tensor |
| | $\Omega_4$ $LS$ | $(i\,\Omega_7)$ |
| | $\Omega_5$ $QLS$ | no direct |
| | $\Omega_{5'}$ mom. tensor | and no exchange |
| $P_F \Omega_i P_F = -\Omega_i$ | $\Omega_6$ $LS^{(-)}$ | $\Omega_8$ $LS^{(-)}\sigma$ |
| | no exchange | $(-i\,\Omega_8)$ no direct |

are real functions of these three variables. A detailed analysis of the quark exchange kernels shows that they are hermite for the interaction types, $\mathcal{T} = E$, $D_+$, and $D_-$, while the kernels of the $\mathcal{T} = S$ and $S'$ types are transformed to each other by the hermitian conjugation.[15] This situation makes the symmetry discussion of the full Wigner transform in Eq. (2·16) a little complicated. Here we rather dwell on a simpler version Eq. (2·17) with the $\boldsymbol{q} = 0$ Wigner transform, and discuss the symmetries of the flavor factors, $X^\Omega = X^\Omega_E$, $X^\Omega_{D_+}$ and $X^\Omega_{D_-}$. The $S$ and $S'$ factors appear only in the combination of $X^\Omega_S + X^\Omega_{S'}$ in Eq. (2·17) because of the property, $G^\Omega_{WS}(\boldsymbol{r}, 0) = G^\Omega_{WS'}(\boldsymbol{r}, 0)$. Therefore, this combination should be considered to be $X^\Omega$. More complete discussion on each of the quark-model flavor factors, $X^\Omega_\mathcal{T}$, is made in the next section, after the explicit definition of these factors is given.

Table III shows the obtained symmetries of the flavor factors $\Omega_i = \Omega_i(-m^2, 0)$ in Eq. (2·4). Although these are shown by using the $\Omega_i$, the factors $X^\Omega$ discussed above also satisfy these symmetries. First, the condition 1) of Eq. (4·2) yields the flavor exchange symmetry with respect to $P_F$. For example, the relationship, $P_F \Omega_i P_F = -\Omega_i$ ($i = 6,\ 8$) implies that the flavor factors for the $LS^{(-)}$ and $LS^{(-)}\sigma$ forces are antisymmetric with respect to the interchange of the first and second particles. On the other hand, the flavor factors, $\Omega_7$ and $\Omega_8$, are characterized by the anti-hermiticity; $(\Omega_i)^\dagger = -\Omega_i$ ($i = 7,\ 8$). Here the type $i = 7$ corresponds to the time-reversal odd tensor in $X^\Omega$. These non-hermite operators are again converted to the hermite ones by taking $i\,\Omega_7$ and $-i\,\Omega_8$, which are $\mathcal{F}$-odd. In either representation, these are even under the PH conjugation.

It should be noted that all the flavor factors, $\Omega_i$ and $X_\Omega$ in Table III, are real operators, since the original Hamiltonian is real. This condition gives some interesting features of the matrix elements of $\Omega_i$ for the elastic channel ($B_1B_2 \to B_1B_2$ or $B_2B_1 \to B_1B_2$). By using the symmetry properties of $\Omega_i$ in Table III and the reality of the matrix elements, we can easily show

$$\langle B_1\, B_2|\Omega_7|B_1\, B_2\rangle = \langle B_1\, B_2|\Omega_8|B_1\, B_2\rangle = 0 \ ,$$
$$\langle B_1\, B_2|\Omega_6|B_2\, B_1\rangle = \langle B_1\, B_2|\Omega_7|B_2\, B_1\rangle = 0 \ , \qquad (4\cdot4)$$

where the matrix elements are evaluated in the particle basis. The relationship in Eq. (4·4) implies that the $LS^{(-)}$ force has no exchange-term contribution, while the $LS^{(-)}\sigma$ force has no direct-term contribution. For the elastic $YN$ interaction, the non-strange mesons contribute to the direct Feynman diagram (Fig. 2(a)), while the strange mesons only to the exchange diagram (Fig. 2(b)). Consequently, the $LS^{(-)}$ force gets the contribution only from the non-strange mesons, while the $LS^{(-)}\sigma$ force only from the strange mesons. Note that these selection rules are useful only for the elastic (or diagonal) potentials. In particular, $\Omega_7$ piece or the time-reversal odd tensor exists only for the transition potentials like $\langle \Lambda N|\mathcal{V}^{\rm pot}|\Sigma N\rangle$.

Another important selection rule emerges from Table III, if FSB is neglected. In this case, all the coupling constant operators are self-conjugate. Since $\Omega_i$ are simple flavor-singlet products of the real self-conjugate operators, all the $\Omega_i$ in this case are hermite as is discussed at the end of the preceding section. Thus there is no $\Omega_7$ and $\Omega_8$ contributions to the $SU_3$-symmetric OBEP (and also to the quark-model potential Eq. (2·17) in the $SU_3$ limit.) Furthermore, $\Omega_6$ is not allowed either for the $NN$ system, owing to the identity of the two particles. This leads us to the well-known fact that only $\Omega_1 \sim \Omega_5$ (or $\Omega_{5'}$) are enough for the $NN$ interaction.

As an example of showing how these symmetries and selection rules are realized in the OBEP, let us reexamine the procedure to derive the Nijmegen hard-core potentials. The full relativistic expression of the one boson exchange amplitude does not have $i = 7$ piece for the elastic process which involves two octet baryons $B_1$ and $B_2$. Except for the standard pieces of interaction, $i = 1$ (central), 2 (spin-spin), 3 (tensor), 4 ($LS$), and 5 ($QLS$), we only have $i = 6$ ($LS^{(-)}$) type contribution for the direct diagram, which comes from the scalar and vector meson exchanges. For the exchange diagram, $i = 5'$ (momentum tensor) and $i = 8$ ($LS^{(-)}\sigma$) come from the pseudo-scalar and vector mesons, while $i = 6$ ($LS^{(-)}$) comes only from the vector mesons. However, $i = 5'$ term disappears through the non-relativistic approximation and through some extra approximations of the mass factors which we call the



Nijmegen approximation:[12]

i) $\dfrac{1}{{M_Y}^2} + \dfrac{1}{{M_N}^2} \sim \dfrac{2}{M_Y M_N}$ ,

ii) for $\Lambda N$-$\Sigma N$ transition potentials, use $M_Y = \dfrac{1}{2}(M_\Lambda + M_\Sigma)$ . (4·5)

The condition i) implies

$$\left(\frac{1}{M_Y} - \frac{1}{M_N}\right)^2 \sim 0 \ ,$$
$$\left(\frac{1}{M_Y} + \frac{1}{M_N}\right)^2 \sim \frac{4}{M_Y M_N} \ , \quad \text{or} \quad \frac{1}{M_Y} + \frac{1}{M_N} \sim \frac{2}{\sqrt{M_Y M_N}} \ . \qquad (4\cdot6)$$

Namely, we neglect the second order of the hyperon-nucleon mass difference, which causes the error of about 5 %. The exchange $\Omega_6$ factor for the vector mesons is given by

$$\Omega_6 = (g_{13}f_{42} - f_{13}g_{42})\frac{1}{2\mathcal{M}}\left(\frac{1}{M_Y} + \frac{1}{M_N}\right)$$
$$-\frac{f_{13}}{2\mathcal{M}}\frac{f_{42}}{2\mathcal{M}}\frac{1}{4}(M_Y - M_N)\left(\frac{1}{M_Y} + \frac{1}{M_N}\right)^3 (\boldsymbol{k}\cdot\boldsymbol{q}) \ , \qquad (4\cdot7)$$

where $g$ and $f$ are the vector-type and tensor-type coupling constants, respectively, and $\mathcal{M}$ is a some standard mass usually taken as the proton mass, $\mathcal{M} = M_p$. Here, $\boldsymbol{k}\cdot\boldsymbol{q}$ term is actually absent owing to the energy conservation. The coupling constants for the exchange diagram are

$$g_{13}f_{42} - f_{13}g_{42} = g_{NY}f_{NY} - f_{NY}g_{NY} = 0 \ . \qquad (4\cdot8)$$

(On the other hand, a similar combination appearing in the direct $\Omega_6$ factor of the vector mesons is $g_{13}f_{42} - f_{13}g_{42} = g_{YY}f_{NN} - f_{YY}g_{NN} \neq 0$.) After all, we get the following simple structure of the Nijmegen potentials with respect to the non-zero contributions of the $i = 5'$, 6, 7, and 8 pieces; the non-strange scalar and vector mesons contribute to the $LS^{(-)}$ force in the direct diagram, and the strange pseudo-scalar and vector mesons contribute to the $LS^{(-)}\sigma$ force in the exchange diagram.

In the Nijmegen potentials, the exchange contribution is obtained from the direct contribution by just replacing both $M_Y$ and $M_N$ with $\sqrt{M_Y M_N}$, and by adding an extra minus sign. An exception is $\Omega_6 = 0$ for the exchange contribution. In this case, $\Omega_8$ is given by Eqs. (A·2) and (A·3). Note that these $LS^{(-)}\sigma$ contributions are zero, if we set $M_Y = M_N$ by neglecting FSB. An extra exchange term originating from the second part of the vector-meson propagator ($k_\mu k_\nu/m^2$) for the $K^*$ meson does not contribute to $\Omega_8$. (See Eq. (26) of Ref. 12).)

## §5. Symmetry properties of the spin-flavor factors in the quark model

In the quark-model calculation of the $NN$ and $YN$ interactions, FSB of the Fermi-Breit interaction is explicitly introduced through the quark-mass dependence by using the formula

$$\frac{m_{ud}}{m_i} = \frac{1}{3}\left(2+\frac{1}{\lambda}\right) + \left(1-\frac{1}{\lambda}\right) Y(i) \ . \tag{5·1}$$

Here $Y(i)$ is the hyperchage operator of particle $i$, $\lambda \equiv m_s/m_{ud}$, and the $SU_3$ limit is realized through $\lambda \to 1$. Various terms of the quark-model potential are classified into three groups of the central, spin-orbit and tensor components, which depend on the spin variables in the zeroth, linear and quadratic powers, respectively. Each of these groups includes the space-spin invariants in Table I as follows: the spin-independent central and spin-spin terms for the central component; $LS$, $LS^{(-)}$ and $LS^{(-)}\sigma$ terms for the spin-orbit component; and the tensor, momentum tensor (or quadratic $LS$) and time-reversal odd tensor terms for the tensor component. Here we only discuss the non-central spin-orbit and tensor components, since these have the most complicated symmetry structure.

The spin-flavor operators for the non-central pieces of the Fermi-Breit interaction are defined by

$$\begin{aligned}
\boldsymbol{w}^{(sLS)}(i,j) &= \frac{1}{3}\left(\frac{m_{ud}}{m_i}\right)^2 \boldsymbol{\sigma}_i + \frac{1}{3}\left(\frac{m_{ud}}{m_j}\right)^2 \boldsymbol{\sigma}_j + \frac{2}{3}\frac{(m_{ud})^2}{m_i m_j}(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \ , \\
\boldsymbol{w}^{(aLS)}(i,j) &= -\left(\frac{m_{ud}}{m_i}\right)^2 \boldsymbol{\sigma}_i + \left(\frac{m_{ud}}{m_j}\right)^2 \boldsymbol{\sigma}_j + 2\,\frac{(m_{ud})^2}{m_i m_j}(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j) \ , \\
w^{(T)}_\mu(i,j) &= \frac{(m_{ud})^2}{m_i m_j}[\,\boldsymbol{\sigma}_i \boldsymbol{\sigma}_j\,]^{(2)}_\mu \ ,
\end{aligned} \tag{5·2}$$

which correspond to the spatial operators $[\boldsymbol{r} \times (\boldsymbol{p}_i - \boldsymbol{p}_j)]$ for $sLS$, $[\boldsymbol{r} \times (\boldsymbol{p}_i + \boldsymbol{p}_j)]$ for $aLS$ and $[\,\hat{\boldsymbol{r}}_i\,\hat{\boldsymbol{r}}_j\,]^{(2)}_\mu$ for $T$, respectively. Note that the definitions in Eq. (5·2) are such that they are reduced to the simple combinations, $\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j$ ($sLS$) and $\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j$ ($aLS$), in the $SU_3$ limit $\lambda = 1$. From the spatial integrals, we find that only the interaction types $T = D_+$ and $D_-$ survive for $sLS$, while $S$ and $S'$ types for $aLS$. In the tensor component, all the types but $E$ are possible. In order to calculate $\langle \xi^{SF}_\alpha | w^{(\Omega)}(i,j) P^{SF}_{36} | \xi^{SF}_{\alpha'} \rangle$, we first express the exchange operator $P^{SF}_{36}$ by the $SU_6$ generators as in Eq. (C.1) of Ref. 22) and separate the quark operators from $w^{(\Omega)}(i,j)$ and $P^{SF}_{36}$ into two groups corresponding to each $(3q)$ baryon. Each matrix element of the $(3q)$ system is expressed as that of some abstract operators at the baryon level. In the $SU_3$ limit, the flavor part of these operators is given by the electric- and magnetic-type $SU_6$ unit vectors, $e^e_{(\lambda\lambda)}$ and $e^m_{(\lambda\lambda)}$ ($\lambda = 0, 1$), for the $SU_6$ coupling $[3] \times [2^2 1111] \to [3]$, introduced in Refs. 22) and 23). When the FSB is introduced, we need to extend these into several types



of flavor operators. These flavor operators play a role of the coupling-constant operators in the OBEP approach. The original matrix element of the $(3q)$-$(3q)$ system is then obtained by recombining two of these and by evaluating the matrix element at the baryon level. For the $LS$ interaction of Eq. (5·2), we should note that $\boldsymbol{w}^{(sLS)}(i,j)$ and $\boldsymbol{w}^{(aLS)}(i,j)$ are spin vectors. We can therefore express the full matrix elements as those of $\eta^{SF}_\alpha$ in Eq. (2·9), by employing the spin operators $\boldsymbol{\sigma}_{B_1}$, $\boldsymbol{\sigma}_{B_2}$ and $i\,[\boldsymbol{\sigma}_{B_1} \times \boldsymbol{\sigma}_{B_2}]$. Alternatively, we can also use $\boldsymbol{S} = (1/2)(\boldsymbol{\sigma}_{B_1} + \boldsymbol{\sigma}_{B_2})$, $\boldsymbol{S}^{(-)} = (1/2)(\boldsymbol{\sigma}_{B_1} - \boldsymbol{\sigma}_{B_2})$ and $\boldsymbol{S}^{(-)}P_\sigma = (-i/2)[\boldsymbol{\sigma}_{B_1} \times \boldsymbol{\sigma}_{B_2}]$. The spin-orbit type spin-flavor factors, $X^{LS}_T$, $X^{LS^{(-)}}_T$ and $X^{LS^{(-)}\sigma}_T$, are thus defined through

$$\begin{aligned}
\langle \xi^{SF}_\alpha | \boldsymbol{w}^{(sLS)}(3,6) P^{SF}_{36} | \xi^{SF}_{\alpha'} \rangle &= \langle \eta^{SF}_\alpha | X^{LS}_{D_-} \boldsymbol{S} + X^{LS^{(-)}}_{D_-} \boldsymbol{S}^{(-)} + X^{LS^{(-)}\sigma}_{D_-} \boldsymbol{S}^{(-)} P_\sigma | \eta^{SF}_{\alpha'} \rangle \ , \\
\langle \xi^{SF}_\alpha | \boldsymbol{w}^{(sLS)}(2,5) P^{SF}_{36} | \xi^{SF}_{\alpha'} \rangle &= -\langle \eta^{SF}_\alpha | X^{LS}_{D_+} \boldsymbol{S} + X^{LS^{(-)}}_{D_+} \boldsymbol{S}^{(-)} + X^{LS^{(-)}\sigma}_{D_+} \boldsymbol{S}^{(-)} P_\sigma | \eta^{SF}_{\alpha'} \rangle \ , \\
\langle \xi^{SF}_\alpha | \boldsymbol{w}^{(aLS)}(2,6) P^{SF}_{36} | \xi^{SF}_{\alpha'} \rangle &= -3\,\langle \eta^{SF}_\alpha | X^{LS}_{S} \boldsymbol{S} + X^{LS^{(-)}}_{S} \boldsymbol{S}^{(-)} + X^{LS^{(-)}\sigma}_{S} \boldsymbol{S}^{(-)} P_\sigma | \eta^{SF}_{\alpha'} \rangle .
\end{aligned} \tag{5·3}$$

The factors, $(-1)$ for $D_+$ type and $(-3)$ for $S$ type, are just for convenience. Similarly, the tensor factors are defined through

$$\begin{aligned}
\langle \xi^{SF}_\alpha | w^{(T)}_\mu(3,6) P^{SF}_{36} | \xi^{SF}_{\alpha'} \rangle &= 3\,\langle \eta^{SF}_\alpha | X^{T}_{D_-} [\boldsymbol{\sigma}_{B_1} \boldsymbol{\sigma}_{B_2}]^{(2)}_\mu | \eta^{SF}_{\alpha'} \rangle \ , \\
\langle \xi^{SF}_\alpha | w^{(T)}_\mu(2,5) P^{SF}_{36} | \xi^{SF}_{\alpha'} \rangle &= 3\,\langle \eta^{SF}_\alpha | X^{T}_{D_+} [\boldsymbol{\sigma}_{B_1} \boldsymbol{\sigma}_{B_2}]^{(2)}_\mu | \eta^{SF}_{\alpha'} \rangle \ , \\
\langle \xi^{SF}_\alpha | w^{(T)}_\mu(2,6) P^{SF}_{36} | \xi^{SF}_{\alpha'} \rangle &= -\frac{3}{2}\,\langle \eta^{SF}_\alpha | X^{T}_{S} [\boldsymbol{\sigma}_{B_1} \boldsymbol{\sigma}_{B_2}]^{(2)}_\mu | \eta^{SF}_{\alpha'} \rangle \ .
\end{aligned} \tag{5·4}$$

The $S'$-type factors are defined from the $S$-type ones through the replacement $w^{(\Omega)}(2,6) \to w^{(\Omega)}(2,3)$.

For the present discussion of the $YN$ interaction with $B_2 = B_4 = N$ and $B_1$, $B_3 = N$, $\Lambda$, $\Sigma$ and $\Xi$, we also use the notation

$$(X^{\Omega}_T)_{B_3 B_1} = \langle [B_3 N]^{\mathcal{P}}_{I I_z} | X^{\Omega}_T | [B_1 N]^{\mathcal{P}'}_{I I_z} \rangle \ , \tag{5·5}$$

which is always real since all the spin and flavor operators we are dealing with are real. The spin-flavor factor for the particular channel, $(X^{\Omega}_T)_{B_3 B_1}$, is generally expressed as a linear combination of the iso-scalar term, the iso-vector term with $(\boldsymbol{\tau}_{B_1} \cdot \boldsymbol{\tau}_{B_2})$, and the strangeness exchange term with $P_F$. The eigen-value of $P_F$ should be considered to be $\mathcal{P}'$ in Eq. (5·5). In the diagonal channel with $B_1 = B_3 = B$, this $P_F$ term does not exist for $B = N$ and $\Xi$, but appears for $B = \Lambda$, and also for $\Sigma$ in the combination of $(1 + \boldsymbol{\tau}_\Sigma \cdot \boldsymbol{\tau}_N)\, P_F$. In the off-diagonal channel with $B_3 B_1 = \Lambda\, \Sigma$ or $\Sigma\, \Lambda$, the total isospin is $I = 1/2$, and the isospin factor of the iso-vector term $(-\sqrt{3})$ and that of the strangeness exchange term $(\sqrt{3})$ are included in $(X^{\Omega}_T)_{B_3 B_1}$. These factors for the spin-orbit and tensor components are explicitly given in Appendix C.

The symmetries of the spin-flavor factors, $X^{\Omega}_{\mathcal{T}}$ and $(X^{\Omega}_{\mathcal{T}})_{B_3B_1}$, are discussed from the above definitions in Eqs. (5·3) ~ (5·5). We consider the following three cases.

i) $\mathcal{P} = \mathcal{P}'$ case for $\Omega =$ central, $LS$, and $T$

Let us consider the type $\Omega = LS$, for example. The hermiticity of $X^{LS}_{\mathcal{T}}\boldsymbol{S}$ yields $(X^{LS}_{\mathcal{T}}\boldsymbol{S})^{\dagger} = X^{LS}_{\mathcal{T}}\boldsymbol{S}$ for $\mathcal{T} = D_-$ and $D_+$, and $(X^{LS}_{S}\boldsymbol{S})^{\dagger} = X^{LS}_{S'}\boldsymbol{S}$. Thus we find, for $\Omega =$ central, $LS$ and $T$,

$$(X^{\Omega}_{\mathcal{T}})^{\dagger} = X^{\Omega}_{\mathcal{T}} \quad \text{for} \quad \mathcal{T} \neq S,\ S' \ ,$$
$$(X^{\Omega}_{S})^{\dagger} = X^{\Omega}_{S'} \ . \tag{5·6}$$

Since $\mathcal{P} = \mathcal{P}'$ in the present case, an alternative expression in terms of the spin-flavor factors in Eq. (5·5) is

$$(X^{\Omega}_{\mathcal{T}})_{B_3B_1} = (X^{\Omega}_{\mathcal{T}})_{B_1B_3} \quad \text{for} \quad \mathcal{T} \neq S,\ S' \ ,$$
$$(X^{\Omega}_{S})_{B_3B_1} = (X^{\Omega}_{S'})_{B_1B_3} \ . \tag{5·7}$$

For $B_3 = B_1 = B$, this implies

$$(X^{\Omega}_{S})_{BB} = (X^{\Omega}_{S'})_{BB} \quad \text{for} \quad \Omega = \text{ central, } LS \text{ and } T \ . \tag{5·8}$$

In the case of $B_3 \neq B_1$, we find that $X^{\Omega}_{\mathcal{T}}$ is symmetric for $\mathcal{T} \neq S$ or $S'$, while $X^{\Omega}_{S}$ and $X^{\Omega}_{S'}$ are interchanged if $B_3$ and $B_1$ are exchanged. For example, the result in Eq. (C·4) yields

$$(X^{T}_{S})_{\Lambda\Sigma} = -\frac{1}{6\cdot 54}\left[\left(12 - \frac{5}{\lambda}\right) - \frac{6}{\lambda}P_F\right] \ ,$$
$$(X^{T}_{S'})_{\Lambda\Sigma} = -\frac{1}{6\cdot 54}\left[\left(2 + \frac{5}{\lambda}\right) - 6\,P_F\right] \ . \tag{5·9}$$

For $B_3B_1 = \Sigma\Lambda$ matrix elements, $S$ and $S'$ in Eq. (5·9) should be interchanged. Note that, for $\lambda = 1$, the two expressions in Eq. (5·9) are equal:

$$(X^{T}_{S})_{\Lambda\Sigma} = (X^{T}_{S'})_{\Lambda\Sigma} = (X^{T}_{S})_{\Sigma\Lambda} = (X^{T}_{S'})_{\Sigma\Lambda} = -\frac{1}{6\cdot 54}\left(7 - 6\,P_F\right) \qquad \text{for} \quad \lambda = 1 \ . \tag{5·10}$$

Also, note the equality of the $LS$ factors $(X^{LS}_{S})_{\Lambda\Sigma} = (X^{LS}_{S'})_{\Lambda\Sigma} = (X^{LS}_{S})_{\Sigma\Lambda} = (X^{LS}_{S'})_{\Sigma\Lambda}$ for an arbitrary $\lambda$, which seems to be accidental.

The equality, $(X^{\Omega}_{S})_{B_3B_1} = (X^{\Omega}_{S'})_{B_3B_1}$, in the $SU_3$ limit with $\lambda = 1$ can be understood from the discussion similar to the one given at the end of the preceding section for the OBEP. We consider the combination $X^{\Omega}_{S} - X^{\Omega}_{S'}$, which is real and anti-hermite because of the second equation of Eq. (5·6). In the $SU_3$ limit, all the spin-flavor factors of the quark model are expressed by the $SU_3$ scalar combinations of $e^{\epsilon}_{(\lambda\lambda)}$ and $e^{m}_{(\lambda\lambda)}$. Since these $SU_6$ unit vectors are all real self-conjugate operators, these $SU_3$ scalar combinations cannot be anti-hermite, leading to $X^{\Omega}_{S} - X^{\Omega}_{S'} = 0$.

ii) $\mathcal{P} \neq \mathcal{P}'$ case for $\Omega = LS^{(-)}$

In this case, Eq. (5·6) is also valid since $\boldsymbol{S}^{(-)}$ is hermite. However, Eq. (5·7) should be modified since we now have $\mathcal{P}\mathcal{P}' = -1$. Instead, we find

$$(X^{LS^{(-)}}_{\mathcal{T}})_{B_3B_1} = (X^{LS^{(-)}}_{\mathcal{T}})_{B_1B_3} \quad \text{with} \quad P_F \to -P_F \quad \text{for} \quad \mathcal{T} \neq S,\ S' \ ,$$
$$(X^{LS^{(-)}}_{S})_{B_3B_1} = (X^{LS^{(-)}}_{S'})_{B_1B_3} \quad \text{with} \quad P_F \to -P_F \ . \tag{5·11}$$

In particular, the diagonal case yields

$$(X^{LS^{(-)}}_{D_-})_{BB} \quad \text{and} \quad (X^{LS^{(-)}}_{D_+})_{BB} \quad \text{do not have } P_F \text{ term} \ ,$$
$$(X^{LS^{(-)}}_{S})_{BB} = (X^{LS^{(-)}}_{S'})_{BB} \quad \text{with} \quad P_F \to -P_F \ . \tag{5·12}$$

The first equality is indeed satisfied in Eqs. (C·1), (C·2) and (C·3). For $B_3B_1 = \Lambda\Sigma$ and $\Sigma\Lambda$ factors given in Eq. (C·4), we find that the $D_-$ and $D_+$ factors have the opposite sign in the $P_F$ term, while for $S'$ we have, for example,

$$(X^{LS^{(-)}}_{S})_{\Lambda\Sigma} = -\frac{1}{2\cdot 108}\left[\frac{1}{3}\left(7 + \frac{16}{\lambda} - \frac{5}{\lambda^2}\right) + \frac{4}{\lambda}\left(4 - \frac{1}{\lambda}\right)P_F\right] \ ,$$
$$(X^{LS^{(-)}}_{S'})_{\Lambda\Sigma} = -\frac{1}{2\cdot 108}\left[\frac{1}{3}\left(7 + \frac{16}{\lambda} - \frac{5}{\lambda^2}\right) + 4\left(2 + \frac{2}{\lambda} - \frac{1}{\lambda^2}\right)P_F\right] . \tag{5·13}$$

Again, these coincide with each other if $\lambda = 1$. The rule Eq. (5·11) can be reproduced if we assume $(P_F)^{\dagger} = -P_F$ in $(X^{LS^{(-)}}_{\mathcal{T}})^{\dagger}_{B_3B_1}$ (and also for $LS^{(-)}\sigma$ below).

iii) $\mathcal{P} \neq \mathcal{P}'$ case for $\Omega = LS^{(-)}\sigma$

Since $\boldsymbol{S}^{(-)}P_{\sigma}$ is anti-hermite, the relationship in Eq. (5·6) involves an extra minus sign:

$$(X^{LS^{(-)}\sigma}_{\mathcal{T}})^{\dagger} = -X^{LS^{(-)}\sigma}_{\mathcal{T}} \quad \text{for} \quad \mathcal{T} \neq S,\ S' \ ,$$
$$(X^{LS^{(-)}\sigma}_{S})^{\dagger} = -X^{LS^{(-)}\sigma}_{S'} \ . \tag{5·14}$$

Accordingly, Eq. (5·11) also involves an extra minus sign:

$$(X^{LS^{(-)}\sigma}_{\mathcal{T}})_{B_3B_1} = -\left\{(X^{LS^{(-)}\sigma}_{\mathcal{T}})_{B_1B_3} \quad \text{with} \quad P_F \to -P_F\right\} \quad \text{for} \quad \mathcal{T} \neq S,\ S' \ ,$$
$$(X^{LS^{(-)}\sigma}_{S})_{B_3B_1} = -\left\{(X^{LS^{(-)}\sigma}_{S'})_{B_1B_3} \quad \text{with} \quad P_F \to -P_F\right\} \ . \tag{5·15}$$

The diagonal case yields

$$(X^{LS^{(-)}\sigma}_{D_-})_{BB} \text{ and } (X^{LS^{(-)}\sigma}_{D_+})_{BB} \text{ do not have } P_F\text{-independent term} \ ,$$
$$(X^{LS^{(-)}\sigma}_{S})_{BB} = -\left\{(X^{LS^{(-)}\sigma}_{S'})_{BB} \quad \text{with} \quad P_F \to -P_F\right\} \ . \tag{5·16}$$

The first condition is actually satisfied in Eqs. (C·1), (C·2) and (C·3). In particular, $B_3B_1 = \Xi\Xi$ factors give $(X_{D_-}^{LS^{(-)}\sigma})_{\Xi\Xi} = (X_{D_+}^{LS^{(-)}\sigma})_{\Xi\Xi} = 0$, since there is no $P_F$ term permissible (i.e., the single-quark exchange of the strange quark can not restore the $\Xi N$ system to itself). For $B_3B_1 = \Lambda\Sigma$ channel, Eq. (C·4) gives

$$(X_S^{LS^{(-)}\sigma})_{\Lambda\Sigma} = -\frac{1}{108}\left(1-\frac{1}{\lambda}\right)\left[\frac{5}{3}\left(3-\frac{1}{\lambda}\right)+\left(1-\frac{1}{\lambda}\right)P_F\right] ,$$
$$(X_{S'}^{LS^{(-)}\sigma})_{\Lambda\Sigma} = \frac{1}{108}\left(1-\frac{1}{\lambda}\right)^2\left(\frac{5}{3}-P_F\right) , \qquad (5\cdot17)$$

which are both zero if $\lambda = 1$.

Note that the $LS^{(-)}\sigma$ factors are all zero if we assume the $SU_3$ limit with $\lambda = 1$. In fact, $X_{D_-}^{LS^{(-)}\sigma}$, $X_{D_+}^{LS^{(-)}\sigma}$ and $X_S^{LS^{(-)}\sigma} + X_{S'}^{LS^{(-)}\sigma}$ are anti-hermite from Eq. (5·14) and therefore zero through the same discussion as before. One can easily prove that $X_S^{LS^{(-)}\sigma} = X_{S'}^{LS^{(-)}\sigma}$ for $\lambda = 1$ by employing $w^{(aLS)}(2,6) - w^{(aLS)}(2,3) = \boldsymbol{\sigma}_3 - \boldsymbol{\sigma}_6$, which changes the sign for the $(3q)$-cluster exchange. *)

The spatial integral of the quark exchange kernel and its Wigner transform $G^{\Omega}_{W\mathcal{T}}(\boldsymbol{r},\boldsymbol{q})$ in Eq. (2·16) are easily calculated by using the standard RGM technique. It is convenient to factor out a common Gaussian factor of the exchange normalization kernel and the strength of the Fermi-Breit interaction. For the spin-orbit components with $\Omega = LS$, $LS^{(-)}$ and $LS^{(-)}\sigma$, $G^{\Omega}_W(\boldsymbol{r},\boldsymbol{q}) \equiv \sum_{\mathcal{T}} X^{\Omega}_{\mathcal{T}} G^{\Omega}_{W\mathcal{T}}(\boldsymbol{r},\boldsymbol{q})$ in Eq. (2·16) is given by

$$G^{\Omega}_W(\boldsymbol{r},\boldsymbol{q}) = G^N_W(\boldsymbol{r},\boldsymbol{q})\sqrt{\frac{2}{\pi}}\alpha_S x^3 m_{ud}c^2\left(-\frac{3}{4}\right)$$
$$\times\left\{X^{\Omega}_{D_-}\left(\frac{2}{3}\right)^{\frac{3}{2}} h_1\left(-\frac{1}{3}\left(\frac{bq}{\hbar}\right)^2\right) + X^{\Omega}_{D_+}2^{\frac{3}{2}} h_1\left(\frac{9}{4}\left(\frac{r}{b}\right)^2\right)\right.$$
$$+\left(X^{\Omega}_S + X^{\Omega}_{S'}\right) h^C_1\left(\frac{9}{32}\left(\frac{r}{b}\right)^2 - \frac{1}{8}\left(\frac{bq}{\hbar}\right)^2 ; \frac{3}{8}\left(\frac{\boldsymbol{r}\cdot\boldsymbol{q}}{\hbar}\right)\right)$$
$$\left.+\left(X^{\Omega}_S - X^{\Omega}_{S'}\right) i\, h^S_1\left(\frac{9}{32}\left(\frac{r}{b}\right)^2 - \frac{1}{8}\left(\frac{bq}{\hbar}\right)^2 ; \frac{3}{8}\left(\frac{\boldsymbol{r}\cdot\boldsymbol{q}}{\hbar}\right)\right)\right\} , \qquad (5\cdot18)$$

where $b$ is the harmonic oscillator constant of the $(3q)$ clusters, $x = (\hbar/m_{ud}cb)$ and

$$G^N_W(\boldsymbol{r},\boldsymbol{q}) = \left(\frac{3}{2}\right)^2 \exp\left\{-\frac{3}{4}\left(\frac{r}{b}\right)^2 - \frac{1}{3}\left(\frac{bq}{\hbar}\right)^2\right\} . \qquad (5\cdot19)$$

*) This fact is also shown by observing that all the flavor operators involved in $X_{\mathcal{T}}^{LS^{(-)}\sigma}$ are reduced to the flavor singlet combination of the type $e^{\dagger}_m(1)e_m(2)$ for $\lambda \to 1$, and that it can not be antisymmetric with respect to the exchange between the first and second particles. On the other hand, $LS^{(-)}$ factors are non-zero, since we can make an antisymmetric combination from $e_m$ and $e_e$;[23] $e^{\dagger}_m(1)e_e(2) - e^{\dagger}_e(1)e_m(2)$.



The functions $h_1(a)$, $h^C_1(a,b)$ and $h^S_1(a,b)$ in Eq. (5·18) are defined by

$$\left.\begin{array}{l} h^C_n(a;b) \\ h^S_n(a;b) \end{array}\right\} = (2n+1)\int_0^1 dt\, e^{-at^2}\, t^{2n} \left\{\begin{array}{l} \cos(bt^2) \\ \sin(bt^2) \end{array}\right. , \qquad (5\cdot20)$$

and $h_n(a) = h^C_n(a,0)$ is normalized such that $h_n(0) = 1$ $(n = 0,\ 1,\ 2)$. For the elastic scattering, the last term of Eq. (5·18) does not contribute since the odd power of $i\boldsymbol{r}\cdot\boldsymbol{q}$ implies the odd power of $\boldsymbol{k}\cdot\boldsymbol{q}$ in the scattering amplitude. A similar discussion of the other interaction types leads us to the conclusion that the quark-model potential for the elastic scattering is real. In particular, the symmetry properties of the spin-flavor factors discussed above implies that there is no $P_F$ term in $LS^{(-)}$ and no $P_F$-independent term in $LS^{(-)}\sigma$, as long as the diagonal potential is concerned. For the off-diagonal potential with $B_3B_1 = \Lambda\Sigma$ or $\Sigma\Lambda$, the imaginary component like the last term in Eq. (5·18) is possible in general, but this term vanishes in the $SU_3$ limit with $\lambda = 1$ or in the low-energy limit with $\boldsymbol{q} = 0$. The $LS^{(-)}\sigma$ component is always zero in $\lambda = 1$.

Finally, we briefly discuss the $\mathcal{T}$-odd tensor term of the quark-model potentials. This term comes from the expansion of the tensor operators $S_{12}(\boldsymbol{a},\boldsymbol{b})$ of the $S$ and $S'$ types. The invariant part of the $\mathcal{T}$-odd tensor term in Eq. (2·16) (except for the factor $i\,S_{12}(\boldsymbol{r},\boldsymbol{p})$) is given by

$$G^{\mathcal{T}\text{-odd tensor}}_W(\boldsymbol{r},\boldsymbol{q}) = G^N_W(\boldsymbol{r},\boldsymbol{q})\sqrt{\frac{2}{\pi}}\alpha_S x^3 m_{ud}c^2\left(-\frac{3}{20}\right)$$
$$\times\left\{\left(X^T_S - X^T_{S'}\right) h^C_2\left(\frac{9}{32}\left(\frac{r}{b}\right)^2 - \frac{1}{8}\left(\frac{bq}{\hbar}\right)^2 ; \frac{3}{8}\left(\frac{\boldsymbol{r}\cdot\boldsymbol{q}}{\hbar}\right)\right)\right.$$
$$\left.+\left(X^T_S + X^T_{S'}\right) i\, h^S_2\left(\frac{9}{32}\left(\frac{r}{b}\right)^2 - \frac{1}{8}\left(\frac{bq}{\hbar}\right)^2 ; \frac{3}{8}\left(\frac{\boldsymbol{r}\cdot\boldsymbol{q}}{\hbar}\right)\right)\right\} . \qquad (5\cdot21)$$

For the elastic scattering, the first term $(X^T_S - X^T_{S'})\, h^C_2$ term in Eq. (5·21) is zero because of Eq. (5·8), while the second term does not contribute to the scattering amplitude because of the energy conservation. For the transition potential, however, both terms are not zero in general, in contrast to the OBEP example given in the preceding section. Even in $\lambda = 1$, the imaginary term, $2X^T_S\, i\, h^S_2$, survives, which is PH conjugation even. This term, however, should be small in the low-energy scattering. It should also be noted that the approximation in Eq. (2·17) is too crude to study the momentum tensor and $\mathcal{T}$-odd tensor components contained in the quark-model potentials.

## §6. Summary

In this investigation, we have studied the symmetry aspects of the hyperon-nucleon ($YN$) interaction, by taking some examples from the Nijmegen hard-core models and the quark-model potentials. The new feature of the $YN$ interaction is the exchange symmetry of two baryons and the rich flavor contents which can be conveniently described by the $SU_3$ algebra related to the up-down and strange quarks. Since the strange quark has a heavier mass than the up-down quarks, the correct treatment of the flavor symmetry breaking (FSB) is very important for a realistic description of the $YN$ interaction. Nevertheless, the correct treatment of FSB is still not easy, since it is related to the dynamical aspect of the quark motion, which is apparently relativistic. In this paper, we have assumed that the octet baryons hold the exact $SU_6$ symmetry for the spin-flavor degree of freedom, and that their spatial motion is described by a simple $(0s)^3$ harmonic-oscillator wave function with a common width parameter. On the other hand, FSB originating from the quark-mass dependence in the residual Fermi-Breit interaction is exactly taken into account without any approximation.

With this restriction on the treatment of the FSB, one can carry out an extensive study on the symmetry properties of the spin-flavor factors appearing in the quark model. We find that the time reversal operator is conveniently extended to include the $SU_3$ conjugation in the flavor space, which makes it possible to discuss the reality of the spin-flavor factors in the standard phase convention of the Clebsch-Gordan coefficients in the spin-flavor space. We can assume the invariance of the $YN$ interaction under this extended time reversal just as the $NN$ interaction is invariant under time reversal. The particle-hole conjugation with respect to this extended time-reversal symmetry is found to be convenient to characterize the time-reversal odd components for the space-spin part of the $YN$ interaction. These involve the $LS^{(-)}\sigma$ force acting between channels with different spins and the time-reversal odd tensor, $S_{12}(\boldsymbol{r},\boldsymbol{p})$, which is possible only for the transition potentials between $\Lambda N$ and $\Sigma N$ channels. When the FSB is neglected, the $LS^{(-)}\sigma$ force entirely vanishes, while the $S_{12}(\boldsymbol{r},\boldsymbol{p})$ component remains in the quark model. The reason why this term is missing in the Nijmegen hard-core models is simply because the transition potentials are derived from the diagonal ones by some mass-averaging procedure in these models. It is also found that the usual $LS^{(-)}$ force gets the contribution from the exchange process of up-down quarks, while the $LS^{(-)}\sigma$ force from the exchange of the strange quarks. This is the same feature as is observed in the OBEP.

It should be noted that the spin-orbit and tensor components of the quark-model potentials discussed in this paper have different levels of reality for describing the experimental data on the $NN$ and $YN$ interactions. The spin-orbit force is essentially short ranged and



is expected to be described reasonably well in the quark model. Although the absolute strength is still controversial, the $LS$, $LS^{(-)}$ and $LS^{(-)}\sigma$ potentials predicted by the quark model have correct flavor dependence corresponding to the result of very successful OBEP approaches.[7),24)] On the other hand, the tensor force is a long-range force which can get a large contribution from the meson-exchange processes, especially, from the OPEP. The quark-model tensor force from the Fermi-Breit interaction is supposed to describe only the short-range part of the whole tensor force. In this respect, it is interesting to examine the quadratic $LS$ ($QLS$) force which constitutes the dominant part of the momentum tensor force $S_{12}(\boldsymbol{p},\boldsymbol{p})$. This component is rather short ranged, since it does not get the contribution from the pions in the lowest approximation. A preliminary result shows that the $QLS$ force predicted from the Fermi-Breit tensor term has correct flavor dependence corresponding to that of the Nijmegen hard-core models, although the magnitude is too small.

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## Appendix A

—— *The OBEP factors $\Omega_i$ for the Nijmegen hard-core potentials* ——

The explicit expression of $\Omega_i(\boldsymbol{k}^2,\boldsymbol{q}^2)$ of Eq. (2·3) in the Nijmegen approximation is given in Eqs. (23) $\sim$ (26) of Ref. 12). Our result agrees with theirs except for some misprints (an extra minus sign in $\Omega_2^{(P)}$ and $4M_YM_N \rightarrow 4{M_Y}^2{M_N}^2$ in $\Omega_6^{(S)}$) and the $\boldsymbol{q}^2$ term in the vector-meson $\Omega_3$; i.e., we have obtained

$$\Omega_3 = \left[\left(g_{13}+f_{13}\frac{M_Y}{\mathcal{M}}\right)\left(g_{42}+f_{42}\frac{M_N}{\mathcal{M}}\right) - f_{13}f_{42}\frac{1}{2\mathcal{M}^2}\left(\boldsymbol{q}^2+\frac{\boldsymbol{k}^2}{4}\right)\right]\left(\frac{1}{4M_YM_N}\right) , \qquad \text{(A·1)}$$

while the $\boldsymbol{q}^2$ term is missing in Ref. 12). However, this term is usually neglected in the tensor expression. Note that their definition of $P_3$ is different from ours. Since they did not give the expression of $\Omega_8$ for the exchange term, we will give it here. For pseudo-scalar mesons, it is given by

$$\Omega_8 = -g_{YN}{}^2 \frac{1}{8}\left(\frac{1}{M_N{}^2} - \frac{1}{M_Y{}^2}\right) \quad , \tag{A·2}$$

while, for vector mesons,

$$\Omega_8 = -\left[\left(g_{YN} + f_{YN}\frac{\sqrt{M_Y M_N}}{\mathcal{M}}\right)^2 - f_{YN}{}^2 \frac{1}{\mathcal{M}^2}\left(q^2 + \frac{k^2}{4}\right)\right] \times \frac{1}{8}\left(\frac{1}{M_N{}^2} - \frac{1}{M_Y{}^2}\right) \quad . \tag{A·3}$$

For this factor, we should also refer to the result given by Dover and Gal.[25] Their result in Eq. (2.6) of Ref. 25) for the $K$ meson contribution to the $LS^{(-)}\sigma$ force is recovered, if we employ $\mathcal{O}_8$ in Eq. (B·10) and note that

$$\frac{1}{8}\left(\frac{1}{M_N{}^2} - \frac{1}{M_Y{}^2}\right) \sim \frac{1}{2M_Y M_N}\left(\frac{M_Y - M_N}{M_Y + M_N}\right) \tag{A·4}$$

in the Nijmegen approximation. For the $K^*$ meson contribution, their result is

$$\Omega_8 = -\left(g_{YN} + f_{YN}\frac{(M_Y + M_N)}{2\mathcal{M}}\right)^2 \left(1 + \frac{m^2}{16 M_Y M_N}\right) \times \frac{1}{2M_Y M_N}\left(\frac{M_Y - M_N}{M_Y + M_N}\right) \quad . \tag{A·5}$$

## Appendix B

*—— Fourier transformation of the Pauli-spinor invariants ——*

In this appendix, we will outline the procedure to derive the OBEP of Eq. (2·4) from Eq. (2·3), and show the final result for $\mathcal{O}_i^{op}(\boldsymbol{r})$. For any plane-wave amplitude $\mathcal{O}(\boldsymbol{q}_f, \boldsymbol{q}_i)$, we can postulate a non-local kernel $\mathcal{O}(\boldsymbol{r}, \boldsymbol{r}')$, its Wigner transform $\mathcal{O}^W(\boldsymbol{R}, \boldsymbol{P})$, and a local operator $\mathcal{O}^{op}(\boldsymbol{r})$ which generally involves the momentum operator $\boldsymbol{p} = (1/i)(\partial/\partial \boldsymbol{r})$ in it. (We use the unit of $\hbar = 1$ for simplicity.) Here we assume that the non-local kernel $\mathcal{O}(\boldsymbol{r}, \boldsymbol{r}')$ is of the Wigner type and the locality is stronger with respect to $\boldsymbol{r} - \boldsymbol{r}'$ than to $\boldsymbol{r} + \boldsymbol{r}'$.[13] We start from

$$\begin{aligned}\mathcal{O}(\boldsymbol{q}_f, \boldsymbol{q}_i) &\equiv \langle \boldsymbol{q}_f | \mathcal{O} | \boldsymbol{q}_i \rangle \\ &= \int d\boldsymbol{r}\, d\boldsymbol{r}'\, e^{-i\boldsymbol{q}_f \boldsymbol{r}}\, e^{i\boldsymbol{q}_i \boldsymbol{r}'}\, \mathcal{O}(\boldsymbol{r}, \boldsymbol{r}') \\ &= \int d\boldsymbol{R}\, e^{-i\boldsymbol{k}\boldsymbol{R}}\, \mathcal{O}^W(\boldsymbol{R}, \boldsymbol{q}) \quad ,\end{aligned} \tag{B·1}$$



where we have used the transformation of variables Eq. (2·2) and $\boldsymbol{R} = (\boldsymbol{r} + \boldsymbol{r}')/2$, $\boldsymbol{s} = \boldsymbol{r}' - \boldsymbol{r}$. The Wigner transform $\mathcal{O}^W(\boldsymbol{R}, \boldsymbol{q})$ in Eq. (B·1) is defined by

$$\mathcal{O}^W(\boldsymbol{R}, \boldsymbol{q}) \equiv \int d\boldsymbol{s}\, e^{i\boldsymbol{q}\boldsymbol{s}}\, \mathcal{O}\left(\boldsymbol{R} - \frac{\boldsymbol{s}}{2}, \boldsymbol{R} + \frac{\boldsymbol{s}}{2}\right) \quad , \tag{B·2}$$

and is related to the plane-wave amplitude through the inverse transformation of Eq. (B·1):

$$\mathcal{O}^W(\boldsymbol{r}, \boldsymbol{q}) \equiv \frac{1}{(2\pi)^3}\int d\boldsymbol{k}\, e^{i\boldsymbol{k}\boldsymbol{r}}\, \mathcal{O}(\boldsymbol{q}_f, \boldsymbol{q}_i) \quad . \tag{B·3}$$

Note that $\boldsymbol{q} = (\boldsymbol{q}_f + \boldsymbol{q}_i)/2$ in Eq. (B·3) is nothing but the classical local momentum of the non-local kernel $\mathcal{O}(\boldsymbol{r}, \boldsymbol{r}')$. In order to calculate the local operator form $\mathcal{O}^{op}$, we proceed as ($\nabla = \partial/\partial \boldsymbol{r}$)

$$\begin{aligned}\langle \boldsymbol{r} | \mathcal{O} | \Psi \rangle &= \int d\boldsymbol{r}'\, \langle \boldsymbol{r} | \mathcal{O} | \boldsymbol{r}' \rangle \langle \boldsymbol{r}' | \Psi \rangle \\ &= \int d\boldsymbol{s}\, \langle \boldsymbol{r} | \mathcal{O} | \boldsymbol{r} + \boldsymbol{s} \rangle\, \Psi(\boldsymbol{r} + \boldsymbol{s}) \\ &= \int d\boldsymbol{s}\, \langle \boldsymbol{r} | \mathcal{O} | \boldsymbol{r} + \boldsymbol{s} \rangle\, e^{\boldsymbol{s}\nabla}\, \Psi(\boldsymbol{r}) \\ &= \int d\boldsymbol{s}\, e^{\frac{1}{2}\boldsymbol{s}\nabla} \left\langle \boldsymbol{r} - \frac{\boldsymbol{s}}{2} \middle| \mathcal{O} \middle| \boldsymbol{r} + \frac{\boldsymbol{s}}{2} \right\rangle e^{\frac{1}{2}\boldsymbol{s}\nabla}\, \Psi(\boldsymbol{r}) \quad .\end{aligned} \tag{B·4}$$

Here we use the inverse Fourier transformation of Eq. (B·2) and define $\mathcal{O}^{op}$ through

$$\langle \boldsymbol{r} | \mathcal{O} | \Psi \rangle = \mathcal{O}^{op}(\boldsymbol{r})\, \Psi(\boldsymbol{r}) \quad . \tag{B·5}$$

Then we get the result in Eq. (2·6).

In order to simplify Eq. (2·6) further, let us assume

$$\mathcal{O}^W(\boldsymbol{r}, \boldsymbol{q}) = v(\boldsymbol{r}) f(\boldsymbol{q}) \quad , \tag{B·6}$$

and use

$$\frac{1}{(2\pi)^3}\int d\boldsymbol{q}\, e^{-i\boldsymbol{q}\boldsymbol{s}}\, f(\boldsymbol{q}) = \left\{ f\left(i\frac{\partial}{\partial \boldsymbol{s}}\right) \delta(\boldsymbol{s}) \right\} \quad , \tag{B·7}$$

where the differential operator $(\partial/\partial \boldsymbol{s})$ in $f(i(\partial/\partial \boldsymbol{s}))$ operates only on $\delta(\boldsymbol{s})$. Then, by using the integration by part. we can easily derive

$$\mathcal{O}^{op}(\boldsymbol{r}) = f\left(\frac{\partial}{\partial \boldsymbol{s}}\right) e^{\frac{1}{2}\boldsymbol{s}\boldsymbol{p}}\, v(\boldsymbol{r})\, e^{\frac{1}{2}\boldsymbol{s}\boldsymbol{p}} \Big|_{\boldsymbol{s}=0} \quad . \tag{B·8}$$

Here we first take the derivative of $\boldsymbol{s}$, and then take $\boldsymbol{s} = 0$. Note that the derivative in the left $e^{\frac{1}{2}\boldsymbol{s}\boldsymbol{p}}$ factor operates not only $v(\boldsymbol{r})$, but also all the wave functions in the right-hand side. If we approximate the momentum operator $\boldsymbol{p}$ by the $c$-number $\boldsymbol{q}$, the two exponential factors are combined into $e^{\boldsymbol{s}\boldsymbol{q}}$ and we recover the original Wigner transform in Eq. (B·6).

The explicit expression of $\mathcal{O}_i \equiv \mathcal{O}_i^{op}(\boldsymbol{r})$ in Eq. (2·6) is now straightforwardly obtained by using the above techniques. First, the central, spin-spin and tensor factors are simply given by

$$\begin{aligned} \mathcal{O}_1 &= m\, Y(x) \ , \\ \mathcal{O}_2 &= m\, Y(x)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \ , \\ \mathcal{O}_3 &= -\frac{m^3}{3} [\, Y(x)\,(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + Z(x)\, S_{12}\,] \ , \end{aligned} \tag{B·9}$$

where $Y(x) = e^x/x$ and $Z(x) = (1 + 3/x + 3/x^2)Y(x)$ are the standard Yukawa functions of $x = m|\boldsymbol{r}|$, and $S_{12}$ is the tensor operator $S_{12} = 3(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{r}})(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{r}}) - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$. The $\boldsymbol{q}$ dependence starts to appear from the $LS$ force, which has three different types:

$$\begin{aligned} \mathcal{O}_4 &= m^3\, Z^{(1)}(x)\, (\boldsymbol{L} \cdot \boldsymbol{S}) \ , \\ \mathcal{O}_6 &= m^3\, Z^{(1)}(x)\, (\boldsymbol{L} \cdot \boldsymbol{S}^{(-)}) \ , \\ \mathcal{O}_8 &= 2m^3\, Z^{(1)}(x)\, (\boldsymbol{L} \cdot \boldsymbol{S}^{(-)})\, P_\sigma \ , \end{aligned} \tag{B·10}$$

where the $LS$-type Yukawa function is defined through $Z^{(1)}(x) = (1/x + 1/x^2)Y(x)$. The Wigner transform of the above $LS$ operators, $\mathcal{O}_i^W$, are obtained by using the classical angular momentum, $\boldsymbol{L}^W = [\boldsymbol{r} \times \boldsymbol{q}]$, instead of $\boldsymbol{L}$. The factor for $i = 5$ is given by

$$\begin{aligned} \mathcal{O}_5 &= -m^5\, \frac{1}{x^2} Z(x)\, Q_{12} \\ &\quad + m^3\, \frac{1}{2}\left\{ [\boldsymbol{\sigma}_1 \times \boldsymbol{p}]\, Z^{(1)}(x)\, [\boldsymbol{\sigma}_2 \times \boldsymbol{p}] + [\boldsymbol{\sigma}_2 \times \boldsymbol{p}]\, Z^{(1)}(x)\, [\boldsymbol{\sigma}_1 \times \boldsymbol{p}] \right\} \ , \\ \mathcal{O}_5^W &= -m^5\, \frac{1}{x^2} Z(x)\, Q_{12}^W \\ &\quad + m^3\, Z^{(1)}(x)\, [\boldsymbol{\sigma}_1 \times \boldsymbol{q}]\, [\boldsymbol{\sigma}_2 \times \boldsymbol{q}] \ , \end{aligned} \tag{B·11}$$

where the quadratic $LS$ operator and its Wigner transform are defined through

$$\begin{aligned} Q_{12} &= \frac{1}{2} [(\boldsymbol{\sigma}_1 \cdot \boldsymbol{L})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{L}) + (\boldsymbol{\sigma}_2 \cdot \boldsymbol{L})(\boldsymbol{\sigma}_1 \cdot \boldsymbol{L})] \ , \\ Q_{12}^W &= (\boldsymbol{\sigma}_1 \cdot \boldsymbol{L}^W)(\boldsymbol{\sigma}_2 \cdot \boldsymbol{L}^W) \ . \end{aligned} \tag{B·12}$$

Note that the operator $\mathcal{O}_5$ contains a component of the momentum tensor, $S_{12}(\boldsymbol{p}, \boldsymbol{p}) = 3(\boldsymbol{\sigma}_1 \cdot \boldsymbol{p})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{p}) - \boldsymbol{p}^2(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$. This extra part (i.e., the second term of Eq. (B·11)) is usually neglected and only the $Q_{12}$ part is retained: $\mathcal{O}_5 \sim -(m^5/x^2)\, Z(x)\, Q_{12}$. We can also calculate $\mathcal{O}_{5'}$ and $\mathcal{O}_7$, although these are not necessary for the application to the OBEP in the Nijmegen approximation. These pieces correspond to the momentum tensor and the time-reversal odd tensor, respectively, and are given by

$$\begin{aligned} \mathcal{O}_{5'} &= m\, \frac{1}{2} [\, (\boldsymbol{\sigma}_1 \cdot \boldsymbol{p})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{p})\, Y(x) + Y(x)\, (\boldsymbol{\sigma}_1 \cdot \boldsymbol{p})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{p})\,] - \frac{1}{4} \mathcal{O}_3 \ , \\ \mathcal{O}_{5'}^W &= m\, Y(x)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}) \ , \end{aligned} \tag{B·13}$$



and

$$\begin{aligned} \mathcal{O}_7 &= i\, m^3\, \frac{1}{3} \Big\{ [\, S_{12}(\boldsymbol{r}, \boldsymbol{p}) + (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{r} \cdot \boldsymbol{p})\,]^\dagger\, Z^{(1)}(x) \\ &\qquad + Z^{(1)}(x)\, [\, S_{12}(\boldsymbol{r}, \boldsymbol{p}) + (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{r} \cdot \boldsymbol{p})\,] \Big\} \ , \\ \mathcal{O}_7^W &= i\, m^3\, Z^{(1)}(x)\, \frac{2}{3} [\, S_{12}(\boldsymbol{r}, \boldsymbol{q}) + (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{r} \cdot \boldsymbol{q})\,] \ . \end{aligned} \tag{B·14}$$

Here, $S_{12}(\boldsymbol{r}, \boldsymbol{p})$ is defined by

$$S_{12}(\boldsymbol{r}, \boldsymbol{p}) = \frac{3}{2} [\, (\boldsymbol{\sigma}_1 \cdot \boldsymbol{r})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{p}) + (\boldsymbol{\sigma}_2 \cdot \boldsymbol{r})(\boldsymbol{\sigma}_1 \cdot \boldsymbol{p})\,] - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{r} \cdot \boldsymbol{p}) \ . \tag{B·15}$$

## Appendix C

### —— *Spin-flavor factors for the spin-orbit and tensor components of the $YN$ interactions in the quark model* ——

Here we list up the explicit expressions of $(X_{\mathcal{T}}^{\Omega})_{B_3 B_1}$ in Eq. (5·5) with respect to the spin-orbit and tensor components. The factors of the central component and the detailed derivation of these factors will be published elsewhere. The $NN$ factors are given in Eq. (B.6) of Ref. 17). The diagonal spin-orbit components for the $\Lambda N$ and $\Sigma N$ systems are already published in Ref. 7), but are included here for convenience. In the following, the parameter $\lambda\ (= m_{ud}/m_s)$ controls FSB of the Fermi-Breit interaction. The spin and isospin operators, $\boldsymbol{\sigma}_i$ and $\boldsymbol{\tau}_i$, are with respect to the two baryons $B_i$ with $i = 1$ or 2, and the flavor exchange operator $P_F$ is supposed to operate on the ket state. The factors of the interaction type $\mathcal{T} = S'$ are discussed in § 5.

[ $B_3 B_1 = \Lambda\Lambda$ ]

$$\begin{aligned} X_{D_-}^{LS} &= \frac{1}{18}\left[2 + \left(1 + \frac{4}{\lambda} + \frac{1}{\lambda^2}\right) P_F\right] \ , & X_{D_-}^{T} &= \frac{1}{18\lambda} P_F \ , \\ X_{D_+}^{LS} &= -\frac{1}{108}\left(1 + \frac{8}{\lambda} + \frac{3}{\lambda^2}\right) \ , & X_{D_+}^{T} &= \frac{1}{108\lambda} \ , \\ X_{S}^{LS} &= \frac{1}{72}\left[\frac{1}{3}\left(1 - \frac{4}{\lambda} + \frac{3}{\lambda^2}\right) + 2\left(1 + \frac{2}{\lambda} - \frac{1}{\lambda^2}\right) P_F\right] \ , & X_{S}^{T} &= -\frac{1}{108\lambda} \ , \\ X_{D_-}^{LS^{(-)}} &= -\frac{1}{9} \ , & X_{D_-}^{LS^{(-)}\sigma} &= -\frac{1}{18}\left(1 - \frac{1}{\lambda^2}\right) P_F \ , \\ X_{D_+}^{LS^{(-)}} &= \frac{1}{108}\left(1 - \frac{4}{\lambda} - \frac{3}{\lambda^2}\right) \ , & X_{D_+}^{LS^{(-)}\sigma} &= 0 \ , \\ X_{S}^{LS^{(-)}} &= -\frac{1}{2 \cdot 108}\left(1 + \frac{8}{\lambda} - \frac{3}{\lambda^2}\right) \ , & X_{S}^{LS^{(-)}\sigma} &= -\frac{1}{36}\left(1 - \frac{1}{\lambda}\right)^2 P_F \ . \end{aligned} \tag{C·1}$$

[ $B_3B_1 = \Sigma\Sigma$ ]

$$X_{D_-}^{LS} = \frac{1}{3}\left[1+\frac{7}{9}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 - \frac{1}{54}\left(1+\frac{4}{\lambda}+\frac{1}{\lambda^2}\right)(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)P_F\right] \quad ,$$

$$X_{D_+}^{LS} = -\frac{1}{3\cdot 108}\left[\left(25-\frac{8}{\lambda}-\frac{5}{\lambda^2}\right)+\frac{1}{3}\left(29-\frac{28}{\lambda}-\frac{13}{\lambda^2}\right)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + 16(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)P_F\right] \quad ,$$

$$X_S^{LS} = \frac{1}{6\cdot 108}\left[\left(9+\frac{28}{\lambda}-\frac{5}{\lambda^2}\right)+\left(11+\frac{68}{3\lambda}-\frac{13}{3\lambda^2}\right)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \right.$$
$$\left. -2\left(1+\frac{14}{3\lambda}-\frac{1}{\lambda^2}\right)(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)P_F\right] ,$$

$$X_{D_-}^{LS^{(-)}} = \frac{1}{9}(1-\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \quad ,$$

$$X_{D_+}^{LS^{(-)}} = \frac{1}{3\cdot 108}\left[\left(5+\frac{12}{\lambda}+\frac{1}{\lambda^2}\right)-\frac{1}{3}\left(23+\frac{24}{\lambda}+\frac{7}{\lambda^2}\right)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right] \quad ,$$

$$X_S^{LS^{(-)}} = \frac{1}{6\cdot 108}\left[\left(11+\frac{8}{\lambda}-\frac{1}{\lambda^2}\right)-\frac{1}{3}\left(29+\frac{32}{\lambda}-\frac{7}{\lambda^2}\right)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \right.$$
$$\left. -\frac{8}{3}\left(1-\frac{1}{\lambda}\right)(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)P_F\right] ,$$

$$X_{D_-}^{LS^{(-)}\sigma} = -\frac{1}{9\cdot 54}\left(1-\frac{1}{\lambda^2}\right)(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)P_F \quad , \qquad X_{D_+}^{LS^{(-)}\sigma} = 0 \quad ,$$

$$X_S^{LS^{(-)}\sigma} = \frac{1}{81}\left(1-\frac{1}{\lambda}\right)\left[1+\frac{5}{3}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 - \frac{1}{12}\left(1-\frac{1}{\lambda}\right)(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)P_F\right] \quad ,$$

$$X_{D_-}^{T} = \frac{2}{81}\left[1+\frac{5}{3}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + \frac{1}{12\lambda}(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)P_F\right] \quad ,$$

$$X_{D_+}^{T} = \frac{1}{3\cdot 108}\left[\left(2-\frac{1}{\lambda}\right)\left(1-\frac{1}{3}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right)+\frac{4}{3}(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)P_F\right] \quad ,$$

$$X_S^{T} = -\frac{1}{3\cdot 108}\left[\left(6-\frac{1}{\lambda}\right)+\frac{1}{3}\left(6-\frac{5}{\lambda}\right)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 - \frac{2}{3}\left(1+\frac{1}{\lambda}\right)(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)P_F\right] . \qquad (C{\cdot}2)$$

[ $B_3B_1 = \Xi\Xi$ ]

$$X_{D_-}^{LS} = \frac{2}{27}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \quad , \qquad X_{D_+}^{LS} = -\frac{1}{81}\left(1+\frac{4}{\lambda}+\frac{1}{\lambda^2}\right)\left(1-\frac{1}{3}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right) \quad ,$$

$$X_S^{LS} = \frac{1}{3\cdot 54}\left[-\left(1+\frac{2}{\lambda}-\frac{1}{\lambda^2}\right)+\frac{1}{3}\left(1+\frac{8}{\lambda}-\frac{1}{\lambda^2}\right)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right] \quad ,$$

$$X_{D_-}^{LS^{(-)}} = -\frac{1}{9}(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \quad ,$$

$$X_{D_+}^{LS^{(-)}} = \frac{1}{2\cdot 81}\left[\left(1-\frac{6}{\lambda}-\frac{4}{\lambda^2}\right)-\frac{1}{3}\left(1+\frac{18}{\lambda}+\frac{8}{\lambda^2}\right)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right] \quad ,$$

$$X_S^{LS^{(-)}} = \frac{1}{3\cdot 108}\left[\left(1-\frac{14}{\lambda}+\frac{4}{\lambda^2}\right)-\frac{1}{3}\left(1+\frac{34}{\lambda}-\frac{8}{\lambda^2}\right)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right] \quad ,$$



$$X_{D_-}^{LS^{(-)}\sigma} = X_{D_+}^{LS^{(-)}\sigma} = 0 \quad ,$$

$$X_S^{LS^{(-)}\sigma} = -\frac{1}{2\cdot 81}\left(1-\frac{1}{\lambda}\right)\left(1+\frac{5}{3}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right) \quad ,$$

$$X_{D_-}^{T} = -\frac{1}{2\cdot 81}\left(1+\frac{5}{3}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right) \quad , \qquad X_{D_+}^{T} = \frac{1}{2\cdot 81\lambda}\left(1-\frac{1}{3}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right) \quad ,$$

$$X_S^{T} = -\frac{1}{4\cdot 81}\left[\left(\frac{2}{\lambda}-1\right)+\frac{1}{3}\left(1+\frac{10}{\lambda}\right)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right] \quad . \qquad (C{\cdot}3)$$

[ $B_3B_1 = \Lambda\Sigma$ and $\Sigma\Lambda$ ]

$$X_{D_-}^{LS} = -\frac{1}{9}\left[1+\frac{1}{6}\left(1+\frac{4}{\lambda}+\frac{1}{\lambda^2}\right)P_F\right] \quad ,$$

$$X_{D_+}^{LS} = \frac{5}{3\cdot 108}\left(-1+\frac{12}{5\lambda}+\frac{1}{\lambda^2}\right) \quad ,$$

$$X_S^{LS} = -\frac{1}{108}\left[\frac{1}{6}\left(7-\frac{4}{\lambda}+\frac{5}{\lambda^2}\right)+\left(1+\frac{2}{\lambda}-\frac{1}{\lambda^2}\right)P_F\right] \quad ,$$

$$X_{D_-}^{LS^{(-)}} = \begin{cases} -\frac{1}{9}\left[1+\frac{1}{3}\left(1+\frac{4}{\lambda}+\frac{1}{\lambda^2}\right)P_F\right] & \text{for} \quad \Lambda\Sigma \ , \\ -\frac{1}{9}\left[1-\frac{1}{3}\left(1+\frac{4}{\lambda}+\frac{1}{\lambda^2}\right)P_F\right] & \text{for} \quad \Sigma\Lambda \ , \end{cases}$$

$$X_{D_+}^{LS^{(-)}} = \begin{cases} -\frac{1}{108}\left[\frac{1}{3}\left(5+\frac{8}{\lambda}+\frac{5}{\lambda^2}\right)+12\,P_F\right] & \text{for} \quad \Lambda\Sigma \ , \\ -\frac{1}{108}\left[\frac{1}{3}\left(5+\frac{8}{\lambda}+\frac{5}{\lambda^2}\right)-12\,P_F\right] & \text{for} \quad \Sigma\Lambda \ , \end{cases}$$

$$X_S^{LS^{(-)}} = \begin{cases} -\frac{1}{2\cdot 108}\left[\frac{1}{3}\left(7+\frac{16}{\lambda}-\frac{5}{\lambda^2}\right)+\frac{4}{\lambda}\left(4-\frac{1}{\lambda}\right)\,P_F\right] & \text{for} \quad \Lambda\Sigma \ , \\ -\frac{1}{2\cdot 108}\left[\frac{1}{3}\left(7+\frac{16}{\lambda}-\frac{5}{\lambda^2}\right)-4\left(2+\frac{2}{\lambda}-\frac{1}{\lambda^2}\right)\,P_F\right] & \text{for} \quad \Sigma\Lambda \ , \end{cases}$$

$$X_{D_-}^{LS^{(-)}\sigma} = -\frac{1}{54}\left(1-\frac{1}{\lambda^2}\right)\,P_F \quad ,$$

$$X_{D_+}^{LS^{(-)}\sigma} = \begin{cases} -\frac{5}{3\cdot 54}\left(1-\frac{1}{\lambda}\right)\left(3+\frac{1}{\lambda}\right) & \text{for} \quad \Lambda\Sigma \ , \\ \frac{5}{3\cdot 54}\left(1-\frac{1}{\lambda}\right)\left(3+\frac{1}{\lambda}\right) & \text{for} \quad \Sigma\Lambda \ , \end{cases}$$

$$X_S^{LS^{(-)}\sigma} = \begin{cases} -\frac{1}{108}\left(1-\frac{1}{\lambda}\right)\left[\frac{5}{3}\left(3-\frac{1}{\lambda}\right)+\left(1-\frac{1}{\lambda}\right)\,P_F\right] & \text{for} \quad \Lambda\Sigma \ , \\ -\frac{1}{108}\left(1-\frac{1}{\lambda}\right)^2\left(\frac{5}{3}+P_F\right) & \text{for} \quad \Sigma\Lambda \ , \end{cases}$$

$$X_{D_-}^{T} = -\frac{1}{54}\left(\frac{10}{3}-\frac{1}{\lambda}\,P_F\right) \quad , \qquad X_{D_+}^{T} = -\frac{1}{3\cdot 108} \quad ,$$

$$X_S^{T} = \begin{cases} -\frac{1}{6\cdot 54}\left[\left(12-\frac{5}{\lambda}\right)-\frac{6}{\lambda}\,P_F\right] & \text{for} \quad \Lambda\Sigma \ , \\ -\frac{1}{6\cdot 54}\left[\left(2+\frac{5}{\lambda}\right)-6\,P_F\right] & \text{for} \quad \Sigma\Lambda \ . \end{cases} \qquad (C{\cdot}4)$$

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