

# CP Violating $B$ Decays in the Standard Model and Supersymmetry

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## Abstract

We study the uncertainties of the Standard Model (SM) predictions for CP violating  $B$  decays and investigate where and how supersymmetric (SUSY) contributions may be disentangled. The first task is accomplished by letting the relevant matrix elements of the effective Hamiltonian vary within certain ranges. The SUSY analysis makes use of a formalism which allows to obtain model-independent results. We show that in some cases it is possible a) to measure the CP  $B-\bar{B}$  mixing phase and b) to discriminate the SM and SUSY contributions to the CP decay phases. The golden-plated decays to this purpose are the  $B \rightarrow \phi K_S$  and  $B \rightarrow K_S \pi^0$  channels.

13.25.Hw, 11.30.Er, 12.15.Ji, 12.60.Jv

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Searches for CP violation in  $B$  decays represent the new frontier in the realm of Flavour Changing Neutral Current (FCNC) in the Standard Model (SM) and beyond. SM predictions, however, are plagued by large uncertainties which have to be taken into account in order to probe the SM itself and to disentangle SM effects from new physics. A critical assessment of these uncertainties constitutes a major goal of this work in which we discuss several possibilities of looking for signals of low-energy supersymmetry (SUSY) in CP violating  $B$  decays.

Early works on FCNC and CP violation were focused on a particular realization of SUSY denoted as the Minimal Supersymmetric Standard Model (MSSM). Remarkably enough, it was realized that the MSSM succeeds to pass all the challenging FCNC- and CP-tests unscathed. This statement holds true particularly for CP violation, as long as one puts to zero (or takes very small) the two CP violating phases that the MSSM exhibits in addition to the usual CKM one. For CP violation in  $B$  physics, under these conditions, the MSSM does not yield major deviations from what we expect in the SM .

Our view on low-energy SUSY has considerably changed in these last years in relation with new insights on the “parent”  $N=1$  supergravity theories. As soon as we move from the MSSM to SUSY-GUT’s, or to models without universality in the SUSY soft-breaking sector, we encounter major differences in FCNC and CP violating processes. In view of the large variety of low-energy SUSY models that can be obtained by varying the “initial conditions” at some superlarge scale, it is appropriate to study SUSY predictions as model-independently as possible. To this aim, following the early work of ref. [1], systematic analyses of FCNC phenomena in general SUSY models have been performed [2,3]. For CP violation these studies have focused so far on  $\varepsilon$ ,  $\varepsilon'/\varepsilon$  and the electron and neutron electric-dipole moments.

The impact of new physics on  $B-\bar{B}$  mixing has been widely explored (see ref. [4] for a review) and SUSY contributions to CP phases in  $B$ -decay amplitudes have recently been analyzed [5]. When considering SUSY as an example for new physics, these authors, however, rely on some specific SUSY realization. Our approach, instead, allows to draw conclusions which apply to any low-energy SUSY extension of the SM.

This letter addresses two basic and related questions: i) how large the uncertainties of the SM predictions for CP asymmetries in  $B$  decays are and ii) in which processes and how one can possibly distinguish SUSY from SM contributions without making any commitment to a particular model.

Concerning point i), we will work in the theoretical framework of ref. [6]. We use the effective Hamiltonian ( $\mathcal{H}_{eff}$ ) formalism, including LO QCD corrections; in the numerical analysis, we use the LO SM Wilson coefficients evaluated at  $\mu = 5$  GeV, as given in ref. [7]. In most of the cases, by choosing different scales (within a reasonable range) or by using NLO Wilson coefficients, the results vary by about 20 – 30% . This is true with the exception of some particular channels where uncertainties are larger. The matrix elements of the operators of  $\mathcal{H}_{eff}$  are given in terms of the following Wick contractions between hadronic states: Disconnected Emission ( $DE$ ), Connected Emission ( $CE$ ), Disconnected Annihilation ( $DA$ ), Connected Annihilation ( $CA$ ), Disconnected Penguin ( $DP$ ) and Connected Penguin ( $CP$ ) (either for left-left ( $LL$ ) or for left-right ( $LR$ ) current-current operators). Following ref. [6], where a detailed discussion can be found, instead of adopting a specific model for estimating the different diagrams, we let them vary within reasonable ranges. In order to illustrate the relative strength and variation of the different contributions, in table I we only

show, for six different cases, results obtained by taking the extreme values of these ranges. In the first column only  $DE = DE_{LL} = DE_{LR}$  are assumed to be different from zero. For simplicity, unless stated otherwise, the same numerical values are used for diagrams corresponding to the insertion of  $LL$  or  $LR$  operators, i.e.  $DE = DE_{LL} = DE_{LR}$ ,  $CE = CE_{LL} = CE_{LR}$ , etc. We then consider, in addition to  $DE$ , the  $CE$  contribution by taking  $CE = DE/3$ . Annihilation diagrams are included in the third column, where we use  $DA = 0$  and  $CA = 1/2DE$  [6]. Inspired by kaon decays, we allow for some enhancement of the matrix elements of left-right (LR) operators and choose  $DE_{LR} = 2DE_{LL}$  and  $CE_{LR} = 2CE_{LL}$  (fourth column). Penguin contractions,  $CP$  and  $DP$ , can be interpreted as long-distance penguin contributions to the matrix elements and play an important role: if we take  $CP_{LL} = CE$  and  $DP_{LL} = DE$  (fifth column), in some decays these terms dominate the amplitude. Finally, in the sixth column, we allow for long distance effects which might differentiate penguin contractions with up and charm quarks in the loop, giving rise to incomplete GIM cancellations (we assume  $\overline{DP} = DP(c) - DP(u) = DE/3$  and  $\overline{CP} = CP(c) - CP(u) = CE/3$ ). For any given decay channel, whenever two terms with different CP phases contribute in the SM, we give in the first row of table I, the ratio  $r$  of the two amplitudes.

As for the SUSY contribution (point ii)), we make use of the parameterization of the SUSY FCNC and CP quantities in the framework of the so-called mass insertion approximation [1]. For the fermion and sfermion states, we choose a basis where all the couplings of these particles to neutral gauginos are flavour diagonal, while the FC arises from the non-diagonality of the sfermion propagators. These propagators can be expanded as a series in terms of the quantities  $\delta \equiv \Delta/m_{\tilde{q}}^2$ , where  $m_{\tilde{q}}$  is an average sfermion mass and  $\Delta$  denote off-diagonal terms in the sfermion mass matrices (i.e., the mass terms relating sfermions with the same electric charge, but different flavour). As long as  $\delta \lesssim 1$ , by taking the first term in this expansion the experimental information on the FCNC and CP violating phenomena translates into upper bounds on these  $\delta$ s. Even when the expansion fails, for  $\delta \sim 1$ , the mass-insertion approximation can still be used as an estimate of the SUSY effects.

Using this basis it is possible to account for both gluino- (or neutralino-) and chargino-mediated FCNC and CP violation. In view of the complexity of the analysis which includes chargino contributions, and given that the main features are already present with gluinos only [8], in this letter we limit ourselves to the SUSY source of CP violation arising from gluino exchanges.

Four different  $\Delta$  mass-insertions in the down-squark propagators give rise to  $b \rightarrow s$  or  $b \rightarrow d$  transitions:  $(\Delta_{i3})_{LL}$ ,  $(\Delta_{i3})_{RR}$ ,  $(\Delta_{i3})_{LR}$  and  $(\Delta_{i3})_{RL}$ . The indices  $L$  and  $R$  refer to the helicity of the fermion partners. The index  $i$  takes the value 1 or 2 for  $b \rightarrow d$  or  $b \rightarrow s$  transitions, respectively. In the present analysis, we make explicit use of  $\Delta_{LL}$  insertions only. While  $|(\delta_{23})_{LL}|$  is left essentially unconstrained by  $b \rightarrow s\gamma$ ,  $(\delta_{13})_{LL}$  has to satisfy the bound  $|\text{Re}(\delta_{13})_{LL}^2|^{1/2} < 0.1 m_{\tilde{q}}(\text{GeV})/500$  for degenerate squarks and gluino [2]. In the following, we will take  $|(\delta_{23})_{LL}| = 1$  (corresponding to  $x_s = (\Delta M/\Gamma)_{B_s} > 70$  for the same values of SUSY masses), with amplitudes scaling linearly with  $|(\delta_{23})_{LL}|$ .

New physics changes SM predictions on CP asymmetries in  $B$  decays in two ways: by shifting the phase of the  $B_d-\bar{B}_d$  mixing amplitude and by modifying both phases and absolute values of the decay ones. The generic SUSY extension of the SM considered here affects all

these quantities.

In the SUSY case, by using for the Wilson coefficients in eq. (12) the results of ref. [3] and by parameterizing the matrix elements as we did for the SM case discussed above, we obtain the ratios of SUSY to SM amplitudes given in table I. For each decay channel we give results for squark and gluino masses of 250 and 500 GeV (second and third row, respectively). From the table, one concludes that the inclusion of the various terms in the amplitudes,  $DE$ ,  $DA$ , etc., can modify the ratio  $r$  of SUSY to SM contributions up to one order of magnitude.

In terms of the decay amplitude  $A$ , the CP asymmetry reads

$$\mathcal{A}(t) = \frac{(1 - |\lambda|^2) \cos(\Delta M_{dt}) - 2\text{Im}\lambda \sin(\Delta M_{dt})}{1 + |\lambda|^2} \quad (1)$$

with  $\lambda = e^{-2i\phi^M} \bar{A}/A$ . In order to be able to discuss the results model-independently, we have labelled as  $\phi^M$  the generic mixing phase. The ideal case occurs when one decay amplitude only appears in (or dominates) a decay process: the CP violating asymmetry is then determined by the total phase  $\phi^T = \phi^M + \phi^D$ , where  $\phi^D$  is the weak phase of the decay. This ideal situation is spoiled by the presence of several interfering amplitudes. If the ratios  $r$  in table I are small, then the uncertainty on the sine of the CP phase is  $\lesssim r$ , while if  $r$  is  $\mathcal{O}(1)$   $\phi^T$  receives, in general, large corrections.

The results of our analysis are summarized in table II. In the third column, for each channel, we give the possible SM decay phases when one or two decay amplitudes contribute, and the range of variation of their ratio,  $r_{SM}$ , as deduced from table I. A few comments are necessary at this point: a) for  $B \rightarrow K_S \pi^0$  the penguin contributions (with a vanishing phase) dominate over the tree-level amplitude because the latter is Cabibbo suppressed; b) for the channel  $b \rightarrow s \bar{s} d$  only penguin operators or penguin contractions of current-current operators contribute; c) the phase  $\gamma$  is present in the penguin contractions of the  $(\bar{b}u)(\bar{u}d)$  operator, denoted as  $u$ -penguin  $\gamma$  in table II [9]; d)  $\bar{b}d \rightarrow \bar{q}q$  indicates processes occurring via annihilation diagrams which can be measured from the last two channels of table II; e) in the case  $B \rightarrow K^+ K^-$  both current-current and penguin operators contribute; f) in  $B \rightarrow D^0 \bar{D}^0$  the contributions from the  $(\bar{b}u)(\bar{u}d)$  and the  $(\bar{b}c)(\bar{c}d)$  current-current operators (proportional to the phase  $\gamma$ ) tend to cancel out.

SUSY contributes to the decay amplitudes with phases induced by  $\delta_{13}$  and  $\delta_{23}$  which we denote as  $\phi_{13}$  and  $\phi_{23}$ . The ratios of  $A_{SUSY}/A_{SM}$  for SUSY masses of 250 and 500 GeV as obtained from table I are reported in the  $r_{250}$  and  $r_{500}$  columns of table II.

We now draw some conclusions from the results of table II. In the SM, the first six decays measure directly the mixing phase  $\beta$ , up to corrections which, in most of the cases, are expected to be small. These corrections, due to the presence of two amplitudes contributing with different phases, produce uncertainties of  $\sim 10\%$  in  $B \rightarrow K_S \pi^0$ , and of  $\sim 30\%$  in  $B \rightarrow D^+ D^-$  and  $B \rightarrow J/\psi \pi^0$ . In spite of the uncertainties, however, there are cases where the SUSY contribution gives rise to significant changes. For example, for SUSY masses of  $\mathcal{O}(250)$  GeV, SUSY corrections can shift the measured value of the sine of the phase in  $B \rightarrow \phi K_S$  and in  $B \rightarrow K_S \pi^0$  decays by an amount of about 70%. For these decays SUSY effects are sizeable even for masses of 500 GeV. In  $B \rightarrow J/\psi K_S$  and  $B \rightarrow \phi \pi^0$  decays, SUSY effects are only about 10% but SM uncertainties are negligible. In  $B \rightarrow K^0 \bar{K}^0$  the larger effect,  $\sim 20\%$ , is partially covered by the indetermination of about 10% already existing in

the SM. Moreover the rate for this channel is expected to be rather small. In  $B \rightarrow D^+D^-$  and  $B \rightarrow K^+K^-$ , SUSY effects are completely obscured by the errors in the estimates of the SM amplitudes. In  $B^0 \rightarrow D_{CP}^0\pi^0$  the asymmetry is sensitive to the mixing angle  $\phi_M$  only because the decay amplitude is unaffected by SUSY. This result can be used in connection with  $B^0 \rightarrow K_S\pi^0$ , since a difference in the measure of the phase is a manifestation of SUSY effects.

Turning to  $B \rightarrow \pi\pi$  decays, both the uncertainties in the SM and the SUSY contributions are very large. Here we witness the presence of three independent amplitudes with different phases and of comparable size. The observation of SUSY effects in the  $\pi^0\pi^0$  case is hopeless. The possibility of separating SM and SUSY contributions by using the isospin analysis remains an open possibility which deserves further investigation. For a thorough discussion of the SM uncertainties in  $B \rightarrow \pi\pi$  see ref. [6].

In conclusion, our analysis shows that measurements of CP asymmetries in several channels may allow the extraction of the CP mixing phase and to disentangle SM and SUSY contributions to the CP decay phase. The golden-plated decays in this respect are  $B \rightarrow \phi K_S$  and  $B \rightarrow K_S\pi^0$  channels. The size of the SUSY effects is clearly controlled by the non-diagonal SUSY mass insertions  $\delta_{ij}$ , which for illustration we have assumed to have the maximal value compatible with the present experimental limits on  $B_d^0-\bar{B}_d^0$  mixing.

A.M. acknowledges partial support from the EU contract ERBFMRX CT96 0090; M.C., E.F. and G.M. acknowledge partial support from EU contract CHRX-CT93-0132.

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TABLES

Process	$DE$	$DE + CE$	$DE + CE +$ $CA$	$DE + CE +$ $CA + DE_{LR} + CE_{LR}$	$DE + CE +$ $DP + CP$	$DE + CE +$ $\overline{DP} + \overline{CP}$
$B_d^0 \rightarrow J/\psi K_S$	-	-	-	-	-	-
	-0.03	0.1	0.1	0.1	0.1	0.1
	-0.008	0.02	0.02	0.04	0.02	0.02
$B_d^0 \rightarrow \phi K_S$	-	-	-	-	-	-
	0.7	0.7	0.7	0.6	0.4	0.4
	0.2	0.2	0.2	0.1	0.1	0.09
$B_d^0 \rightarrow K_S \pi^0$	0.08	-0.06	-0.05	-0.02	-0.009	-0.01
	0.7	0.7	0.6	0.6	0.4	0.4
	0.2	0.2	0.2	0.1	0.1	0.09
$B_d^0 \rightarrow D_{CP}^0 \pi^0$	0.02	0.02	0.02	0.02	0.02	0.02
$B_d^0 \rightarrow \pi^0 \pi^0$	-0.6	0.9	-0.7	-2	6	4
	0.3	-0.07	0.4	-0.4	-0.07	-0.06
	0.06	-0.02	0.09	-0.1	-0.02	-0.02
$B_d^0 \rightarrow \pi^+ \pi^-$	-0.09	-0.1	-0.1	-0.3	-0.9	-0.8
	0.02	0.02	0.03	0.09	0.8	0.4
	0.005	0.006	0.008	0.02	0.2	0.1
$B_d^0 \rightarrow D^+ D^-$	0.03	0.04	0.05	0.1	0.3	0.2
	-0.007	-0.008	-0.01	-0.02	-0.02	-0.02
	-0.002	-0.002	-0.002	-0.005	-0.006	-0.005
$B_d^0 \rightarrow K^0 \bar{K}^0$	0	0	0	0	0	0.07
	-0.2	-0.2	-0.2	-0.2	-0.09	-0.08
	-0.06	-0.05	-0.05	-0.04	-0.02	-0.02
$B_d^0 \rightarrow K^+ K^-$	-	-	-0.2	-0.4	-	-
	-	-	0.04	0.1	-	-
	-	-	0.01	0.03	-	-
$B_d^0 \rightarrow D^0 \bar{D}^0$	-	-	-	-	-	-
	-	-	-0.01	-0.03	-	-
	-	-	-0.003	-0.006	-	-
$B_d^0 \rightarrow J/\psi \pi^0$	-0.04	0.1	0.1	0.3	0.1	0.1
	0.007	-0.02	-0.02	-0.03	-0.02	-0.02
	0.002	-0.005	-0.005	-0.008	-0.005	-0.005
$B_d^0 \rightarrow \phi \pi^0$	-	-	-	-	-	-
	-0.06	-0.1	-0.1	-0.1	-0.1	-0.1
	-0.01	-0.03	-0.03	-0.03	-0.03	-0.03

TABLE I. Ratios of amplitudes for exclusive  $B$  decays. For each channel, whenever two terms with different  $CP$  phases contribute in the  $SM$ , we give the ratio  $r$  of the two amplitudes. For each channel, the second and third lines, where present, contain the ratios of  $SUSY$  to  $SM$  contributions for  $SUSY$  masses of 250 and 500 GeV respectively.

Incl.	Excl.	$\phi_{SM}^D$	$r_{SM}$	$\phi_{SUSY}^D$	$r_{250}$	$r_{500}$
$b \rightarrow c\bar{c}s$	$B \rightarrow J/\psi K_S$	0	–	$\phi_{23}$	0.03 – 0.1	0.008 – 0.04
$b \rightarrow s\bar{s}s$	$B \rightarrow \phi K_S$	0	–	$\phi_{23}$	0.4 – 0.7	0.09 – 0.2
$b \rightarrow u\bar{u}s$		Penguin 0				
	$B \rightarrow \pi^0 K_S$		0.009 – 0.08	$\phi_{23}$	0.4 – 0.7	0.09 – 0.2
$b \rightarrow d\bar{d}s$		Tree $\gamma$				
$b \rightarrow c\bar{u}d$		0				
	$B \rightarrow D_{CP}^0 \pi^0$		0.02	–	–	–
$b \rightarrow u\bar{c}d$		$\gamma$				
	$B \rightarrow D^+ D^-$	Tree 0	0.03 – 0.3		0.007 – 0.02	0.002 – 0.006
$b \rightarrow c\bar{c}d$				$\phi_{13}$		
	$B \rightarrow J/\psi \pi^0$	Penguin $\beta$	0.04 – 0.3		0.007 – 0.03	0.002 – 0.008
	$B \rightarrow \phi \pi^0$	Penguin $\beta$	–		0.06 – 0.1	0.01 – 0.03
$b \rightarrow s\bar{s}d$				$\phi_{13}$		
	$B \rightarrow K^0 \bar{K}^0$	$u$ -Penguin $\gamma$	0-0.07		0.08 – 0.2	0.02 – 0.06
$b \rightarrow u\bar{u}d$	$B \rightarrow \pi^+ \pi^-$	Tree $\gamma$	0.09 – 0.9	$\phi_{13}$	0.02 – 0.8	0.005 – 0.2
$b \rightarrow d\bar{d}d$	$B \rightarrow \pi^0 \pi^0$	Penguin $\beta$	0.6 – 6	$\phi_{13}$	0.06 – 0.4	0.02 – 0.1
	$B \rightarrow K^+ K^-$	Tree $\gamma$	0.2 – 0.4		0.04 – 0.1	0.01 – 0.03
$b\bar{d} \rightarrow q\bar{q}$				$\phi_{13}$		
	$B \rightarrow D^0 \bar{D}^0$	Penguin $\beta$	only $\beta$		0.01 – 0.03	0.003 – 0.006

TABLE II. *CP* phases for *B* decays.  $\phi_{SM}^D$  denotes the decay phase in the SM; for each channel, when two amplitudes with different weak phases are present, one is given in the first row, the other in the last one and the ratio of the two in the  $r_{SM}$  column.  $\phi_{SUSY}^D$  denotes the phase of the SUSY amplitude, and the ratio of the SUSY to SM contributions is given in the  $r_{250}$  and  $r_{500}$  columns for the corresponding SUSY masses.