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OBSERVATION AND MODAL ANALYSIS OF COUPLED-BUNCH LONGITUDINAL INSTABILITIES VIA A DIGITAL FEEDBACK CONTROL SYSTEM☆

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A modal decomposition is used to study the coupled longitudinal motion of electron bunches in a storage ring. The decomposition allows the simultaneous measurement of growth rates and (feedback induced) damping rates of all unstable beam modes via a time-domain transient technique. The damping rates of naturally stable modes are measured via a related external transient technique. The measured rates are compared to projections based on the estimated cavity impedance. Measurements were made using a high-speed digital signal processing system at the Lawrence Berkeley National Laboratory Advanced Light Source.

Keywords: Longitudinal instability; Coupled-bunch; Feedback

1 INTRODUCTION

Coupled-bunch instabilities[1] are among the most important mechanisms limiting the operating current in high-current storage rings and particle accelerators. This paper uses a modal analysis technique to characterize longitudinal instabilities measured at the LBL Advanced Light Source (ALS).[2]

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The ALS is a 1.0–1.9 GeV electron storage ring which exhibits many of the collective effects which will be important in B Factories[3,4] and ϕ factories[5] under construction. It has demonstrated longitudinal coupled-bunch instabilities since commissioning in 1993.[6]

The eigenmodes of longitudinal motion of the bunch centers in the case of a symmetric beam with N evenly spaced bunches of equal charge consist of all bunches oscillating at the same amplitude. The N eigenmodes are characterized by the angular separation in phase space between successive bunches, which is a constant in any given mode. At a fixed azimuth in the ring mode l produces a beam signal at $l+Q_s$ times the revolution frequency f_{rev} (upper sideband) and at $(N-l-Q_s)f_{rev}$ (lower sideband). Q_s is the ratio of the longitudinal (synchrotron) oscillation frequency f_s to f_{rev}. For Gaussian bunches the growth rates of the modes can be calculated using the Robinson damping equations:[7]

$$1/\tau_l = \frac{I_0 f_{rf}\,\alpha}{2(E/e)Q_s}\mathrm{Real}(Z_l^{\mathrm{eff}}) - 1/\tau_{\mathrm{rad}}, \tag{1}$$

$$Z_l^{\mathrm{eff}} = \sum_{p=-\infty}^{p=+\infty} \frac{\omega_{p,l}}{\omega_{rf}}\exp(-\omega_{p,l}^2\sigma_\tau^2)Z(\omega_{p,l}), \tag{2}$$

$$\omega_{p,l} = (pN + l + Q_s)\omega_{\mathrm{rev}}, \tag{3}$$

where τ_l is the growth time of mode l, τ_{rad} is the radiation damping time, I_0 is the beam current, $Z(\omega)$ is the total ring impedance, E is the beam energy, e is the charge of an electron, σ_τ is the bunch length, f_{rf} is the frequency of the rf cavity holding voltage and α is the momentum compaction factor.

The modal structure of an unstable beam can be qualitatively studied by examining the beam spectrum once nonlinearities limit mode growth.[8,9] It is more useful though to quantify the instability in the linear small oscillation region, since this directly yields the impedance of external resonant structures. These measurements have been made by recording the output of narrowband filters tuned to the frequencies of the relevant modes while oscillations grow.[10] Unfortunately, a B factory machine with hundreds or thousands of bunches and external resonances driving a large number of unstable modes would pose practical problems for such an approach.

This paper presents results from the operation of a longitudinal instability damping system, which can digitally sample and record the oscillations of all bunches in the ring simultaneously.[11] This allows the simultaneous measurement of all unstable coupled-bunch modes, directly from a snapshot of growing oscillations in the linear region. We also demonstrate the use of externally excited transients to measure naturally stable modes. The modal growth rates are studied at various beam currents and accelerating (rf) cavity temperatures and compared to calculations based on the measured cavity impedance.[12]

2 SYSTEM DESCRIPTION

Feedback systems that damp coupled-bunch instabilities are necessary to achieve the design goals of B factories, ϕ factories and next generation light sources. We have developed a programmable longitudinal feedback system at the ALS that uses a scalable array of commercial digital signal processors (DSPs) to compute feedback signals (Figure 1). Table I lists the relevant parameters of the accelerator and the feedback system. Similar systems have been constructed for the SLAC PEP-II[3,13] and INFN-LNF DAFNE colliders. The high

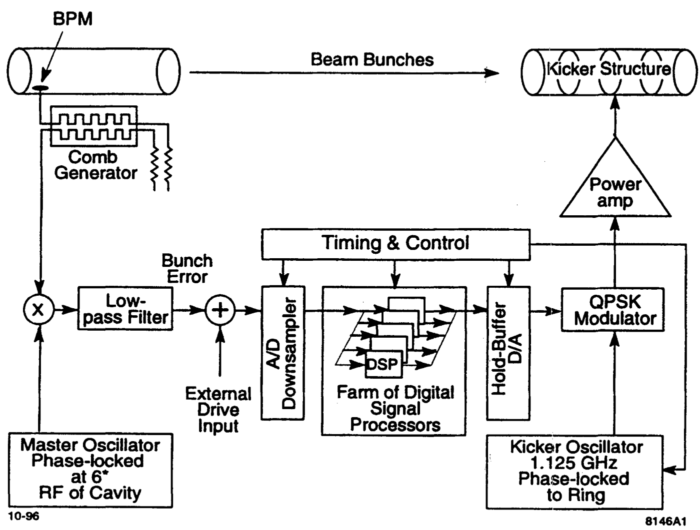

FIGURE 1 Block diagram of the longitudinal feedback system.

TABLE I Parameters of accelerator, feedback system

Parameter	*Description*	*Value*
E	Beam energy	1.5 GeV
f_{rf}	RF frequency	499.65 MHz
h	Harmonic number	328
f_{rev}	Revolution frequency	1.5233 MHz
α	Momentum compaction factor	1.594e − 3
I_0	Operating current	400 mA
f_s	Synchrotron frequency	11.3 kHz
—	Bunch sampling rate	499.65 MHz
—	Downsampling factor	21–31
—	Feedback loop gain	6–28 dB
—	Feedback output power	200 W
—	Output amplifier bandwidth	1–2 GHz

bandwidth required for simultaneous tracking of all beam modes is achieved by a 500 MHz bunch sampling rate at the ALS. After downsampling, the feedback signal is calculated in a digital processing array composed of 40 processors. The processors provide an aggregate multiply-accumulate rate of 1.6×10^9 operations/s.

The DSPs implement a bunch-by-bunch discrete-time feedback algorithm. For the results discussed in this paper, they were programmed to act as finite impulse response (FIR) filters:[14,15]

$$u_i(n) = \sum_{k=1}^{N} h(k)\phi_i(n-k). \tag{4}$$

This equation represents a discrete-time convolution in which the correction signal u_i of the ith bunch at turn n is computed using a weighted average of several past measurements, $\phi_i(n-N), \ldots, \phi_i(n-1)$, of the phase of only that bunch. The filter coefficients, $h(1), \ldots, h(N)$, are selected to provide zero DC response and maximum gain G_{fb} (V/rad) at f_s. Their phase shift is adjusted for net negative feedback at f_s.

The net growth rate $1/\tau_l^{\mathrm{net}}$ of a feedback stabilized mode is the difference between its natural growth rate $1/\tau_l$ and the feedback induced damping rate $1/\tau^{\mathrm{fb}}$. It is calculated as follows:

$$1/\tau_l^{\mathrm{net}} = 1/\tau_l - 1/\tau^{\mathrm{fb}}, \tag{5}$$

$$1/\tau^{\mathrm{fb}} = \frac{f_{\mathrm{rf}}\alpha}{2(E/e)Q_s} G_{\mathrm{fb}}. \tag{6}$$

It should be noted that τ^{fb} is ideally the same for all coupled-bunch modes, and depends only on G_{fb}.

3 EXPERIMENTAL RESULTS

The modal analysis presented here uses data from measurements made by switching off the feedback, allowing unstable modes to grow spontaneously from the noise floor and then damping them back down in a few milliseconds. Naturally stable modes are studied by exciting them through the external drive input of the feedback system and observing the resulting decay transients. The bunch oscillations stay linear throughout.

The results presented in this paper are for a 320-bunch beam followed by an eight bucket gap. Charge variation within the populated rf buckets is at least 15%. We will however project the recorded beam motion onto the symmetric beam eigenmodes, since they are well known and simple. This approach is borne out by the results shown in this section.

A typical data set consists of around 1000 samples (one every 22 turns) of the phase of each of the 320 bunches. The projection onto symmetric beam modes is achieved by taking the FFT (Fast Fourier Transform) of all the bunch phases on one turn. The strength of the symmetric beam modes is tracked by observing the magnitude of the FFTs over time. The frequency resolution Δf of such a plot is the inverse of the time-length T_{d} of the data. In this case, $T_{\mathrm{d}} = T_{\mathrm{rev}}$ (one turn), so $\Delta f = f_{\mathrm{rev}} = 1.5\,\mathrm{MHz}$. Each frequency bin in this FFT contains the sum of the contributions of a pair of counter-rotating modes (one upper and one lower sideband). We look at only half the magnitude spectrum (0–250 MHz), since it is symmetric about the midpoint.

Figure 2 illustrates the measurement of growth rates of unstable modes with and without feedback, using the grow/damp technique: The beam is initially stable under the action of negative feedback. At $t = 0\,\mathrm{ms}$ the feedback system is turned off under software control, and the exponential growth of unstable modes begins. At $t = 7\,\mathrm{ms}$ the feedback is turned on again, and the oscillation amplitudes damp back to their initial steady state level. The bunch motion is recorded

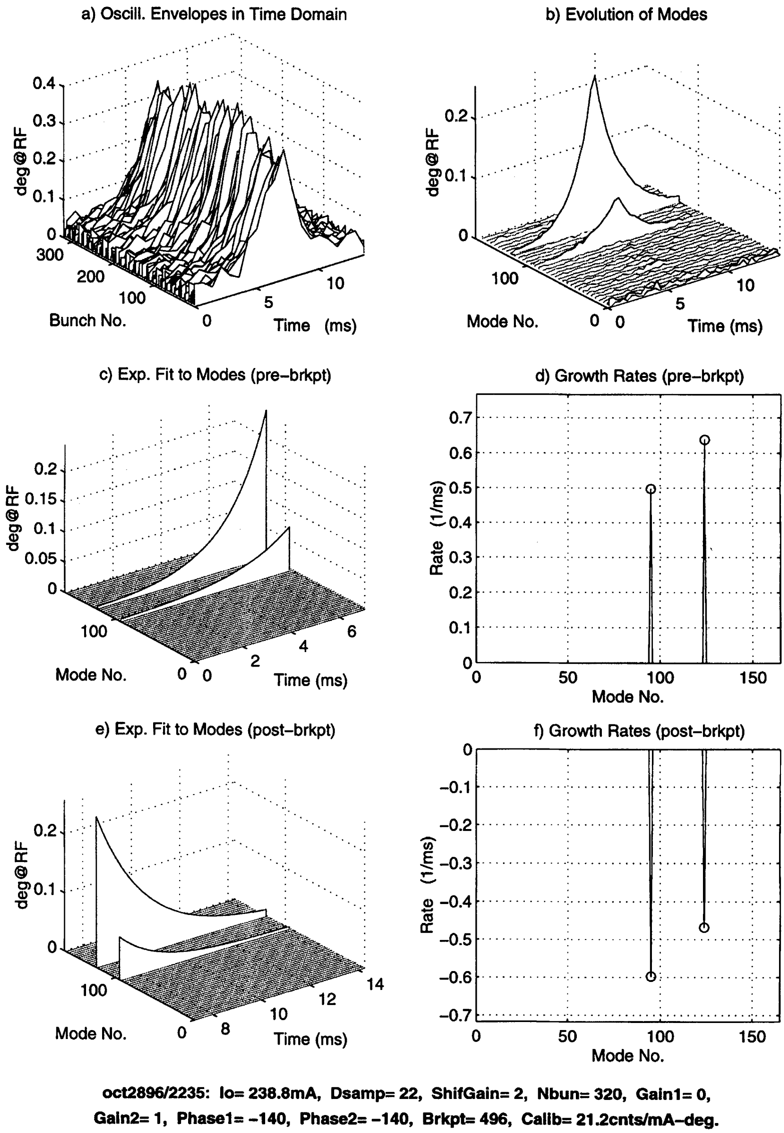

FIGURE 2 Example of grow/damp technique for identifying unstable modes and measuring their growth rates with and without feedback. (a) Oscillations grow from the noise floor when feedback is turned off, and damp down when feedback is reapplied at $t = 7\,\mathrm{ms}$. (b) FFT magnitude vs time reveals the exponential evolution of unstable modes at 94 f_{rev} and 125 f_{rev}. Exponential fits yield growth rates without feedback (c, d) and with feedback (e, f). The measurement is from the Advanced Light Source at the Lawrence Berkeley Laboratory.

in a dual-port memory which is read by an external processor. The data is then processed offline. After the data is read, the DSP processors can be triggered again to record another transient.

Figure 2(a) shows the envelopes of the longitudinal oscillations of the 320 bunches for such a transient. We see growth of unstable oscillations up to $t=7\,\mathrm{ms}$, followed by damping of the motion. The envelopes do not contain information about the phase relationship between individual bunch oscillations. The unstable modes of oscillation can be revealed by taking turn-by-turn FFTs of the bunch phases, as described earlier. Figure 2(b) traces the evolution over time of the 328 modes (folded at mode 164) during a grow/damp transient. The figure shows that the beam motion is the result of unstable growth of modes at $95 f_{\mathrm{rev}}$ and $124 f_{\mathrm{rev}}$. It will be shown at the end of this section that these are modes 233 (328 − 95) and 204 (328 − 124) respectively. The behavior is similar to that of a perfectly symmetric beam.

The growth rates of the modes were found by curve fitting to be $0.5\,\mathrm{ms}^{-1}$ and $0.64\,\mathrm{ms}^{-1}$ respectively (Figure 2(c), (d)). Exponential fits to the tails of the transients in Figure 2(b) revealed damping rates of $0.6\,\mathrm{ms}^{-1}$ and $0.47\,\mathrm{ms}^{-1}$ respectively (Figure 2(e), (f)). We see here the action of the feedback system in turning the net growth rate from positive to negative. The growth rates correspond to effective cavity impedances of $67\,\mathrm{k\Omega}$ and $84\,\mathrm{k\Omega}$ respectively (see Eq. (1)). We can see that the action of the feedback shifts the two growth rates down by the same amount (approximately $1.1\,\mathrm{ms}^{-1}$). This is consistent with the expectation that bunch-by-bunch feedback should damp all symmetric beam modes equally (Eq. (5)). The feedback system therefore has a damping impedance of $130\,\mathrm{k\Omega}$ (Eq. (1)) and a gain of 31 V/mrad (Eq. (6)).

For the measurement of naturally stable modes, a narrowband excitation at the desired mode frequency is injected into the feedback system at the external drive input (see Figure 1). This excitation is impressed on the beam through the power amplifier and kicker, and bunch motion at the desired frequency is excited (a single longitudinal mode). When the excitation and feedback are turned off, the excited mode decays at its natural rate. The growth rate of the mode becomes more negative when feedback is turned on again. The transient is recorded and processed as before. Figure 3 shows the measurement of mode 61, which is naturally stable. The natural growth

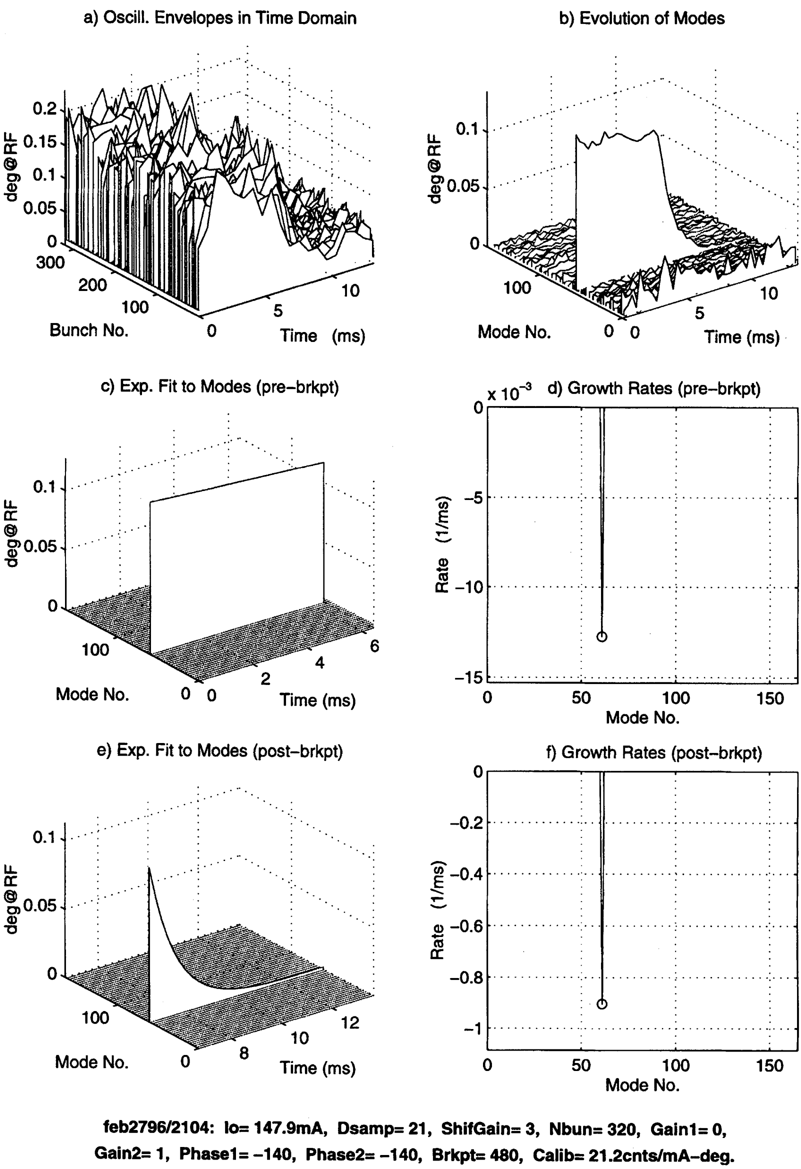

FIGURE 3 Example of external transient technique for measuring growth rates of naturally stable modes with and without feedback. Mode 61 is externally excited and then allowed to decay naturally until $t = 7\,\text{ms}$, at which point feedback is turned on and the mode is rapidly damped.

rate of the mode is $-0.01\,\mathrm{ms}^{-1}$, and the feedback induced growth rate is $-0.9\,\mathrm{ms}^{-1}$. The feedback system in this case has a damping impedance of $172\,\mathrm{k}\Omega$.†

If the above measurement is repeated for several modes (or an excitation is applied which excites several modes) the gain of the feedback system can be measured as a function of frequency. This is a useful system check, since it can be used to examine the combined frequency response of the power amplifier and kicker.

Growth and damping rates of unstable modes were measured at currents ranging from 60 to 250 mA at a variety of cavity temperatures. The growth rates varied by upto a factor of 3 due to temperature changes alone. Small changes in the beam current over a few minutes occasionally caused significant changes in growth rates, possibly due to movement of the cavity tuners. Damping rates of stable modes were measured by exciting them to a measurable amplitude and tracking their decay.

It is instructive to compare measured growth rates to predictions based on the measured impedance $Z(\omega)$ of a model rf cavity and Eqs. (1)–(3). We expect to see unstable modes where the measured cavity resonances land on upper sidebands of revolution harmonics. It must be kept in mind, however, that the cavities installed in the ring could have slightly different resonant frequencies due to minor variations in geometry, temperature and tuner position. Another potential source of deviations from the expected modal structure is the difference between the exact resonance frequencies of the two installed rf cavities, which could blur the effective impedance seen by the beam.

The comparison between measured and predicted growth rates (at 100 mA, without feedback) is shown in Figure 4. The cavity induced growth rate is antisymmetric about 250 MHz, but the addition of radiation damping breaks this symmetry. Of the cavity resonances in the figure that could potentially drive instabilities at currents up to 300 mA, only two have been seen to do so at the nominal cavity temperature and filling pattern. These resonances drive modes 204 and 233, which appear as 'x's at 311 MHz and 355 MHz respectively in

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† In this experiment the system was configured slightly differently. The $172\,\mathrm{k}\Omega$ effective impedance is consistent with a DSP gain which is twice that of the previous case, and with a lower gain in the power amplifier.

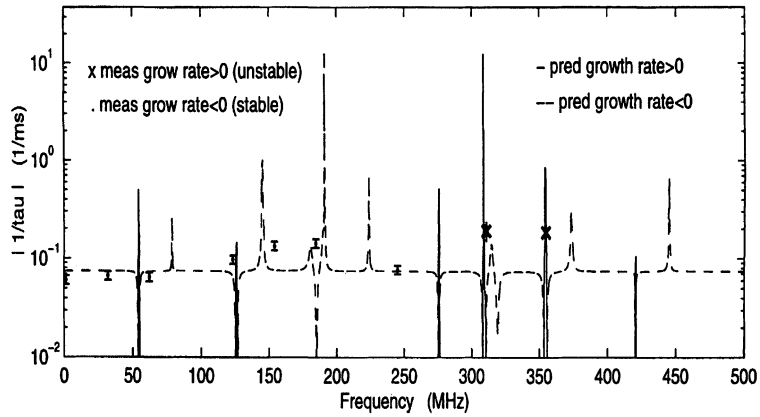

FIGURE 4 Log plot of experimentally measured and predicted growth rates as a function of modal frequency, with error bars around measured points. The predicted rates are based on the radiation damping rate and measurements of the impedance of a model rf cavity (Eqs. (1)–(3)). Ideally, the 'x's should coincide with the solid lines and the '.'s with the dashed lines. However, there are slight discrepancies due to differences between the model cavity and the cavities installed in the ring.

Figure 4. Their growth rates have been measured at 15 different beam currents and normalized to a current of 100 mA. Error bars for the measured positive growth rates are one standard deviation wide on each side. The sharpest resonance is the TM-011 mode at 808 MHz, which is aliased down to 308 MHz. This resonance correlates fairly well with the instability at mode 204, although an exact calculation places it between modes 202 and 203. The correspondence is greater between mode 233 and the resonances at 355 MHz (aliased from 2.3 and 2.8 GHz). Neither of the unstable modes shows the worst case growth rate, implying that the cavity resonances do not land exactly on a mode frequency.

The (negative) growth rates of a few naturally stable modes have also been measured. They are mostly close to the radiation damping rate (Figure 4). These rates are less sensitive to variations in cavity temperature and tuner position, since the modal frequencies do not correspond to large cavity resonances. The corresponding error bars are conservatively estimated at around 10%.

Another way of analyzing the transient data is to concatenate the sampled bunch phases over a few ms and take the FFT of the resulting vector. The distance of the sidebands from the revolution harmonics gives us the frequency shifts of the modes, and the width of the sidebands gives us an equivalent way of calculating their growth rates. The beam pseudospectrum resulting from a single 4–8 ms transient covers the entire 250 MHz range of the modes with a resolution of 250–125 Hz. A heterodyned spectrum analyzer would take at least a few minutes to perform the 328 narrowband sweeps required to produce an equivalent spectrum, by which time the oscillations would have reached a damped or saturated steady state.

Figure 5(a) shows the magnitude of the FFT of the bunch-phase signal over the last 4 ms of mode growth ($\Delta f = 250$ Hz). This is compared to the real part of the effective impedance of the rf cavity (Figure 5(b)). The two plots show a good degree of agreement. The three largest impedance peaks between 100 and 200 MHz all drive longitudinal motion at or close to their aliased resonant frequencies. Two of them correspond to the two prominent modes in Figure 2. The impedance plot suggests that at least two more cavity resonances should poke up above radiation damping, but this has never been observed, possibly for the same reasons mentioned above.

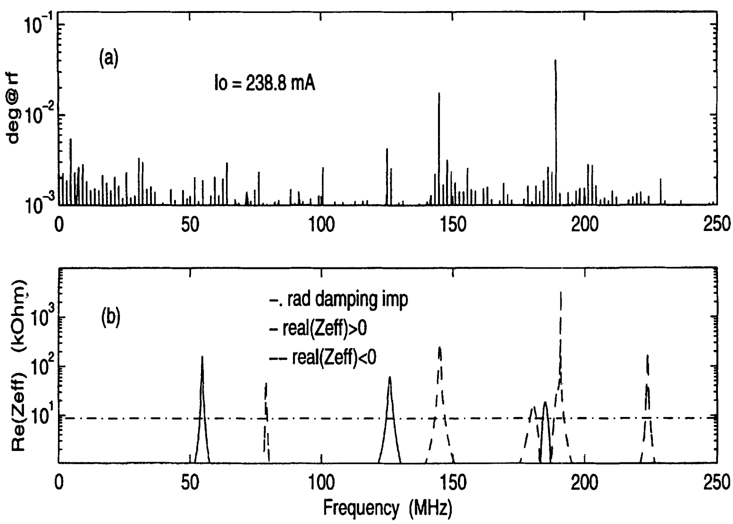

FIGURE 5 (a) Magnitude of high-resolution FFT of growing transient in Figure 2, showing all the spectral components over a 250-MHz bandwidth. (b) Real part of the effective cavity impedance. Radiation damping is converted to an effective impedance using Eq. (1).

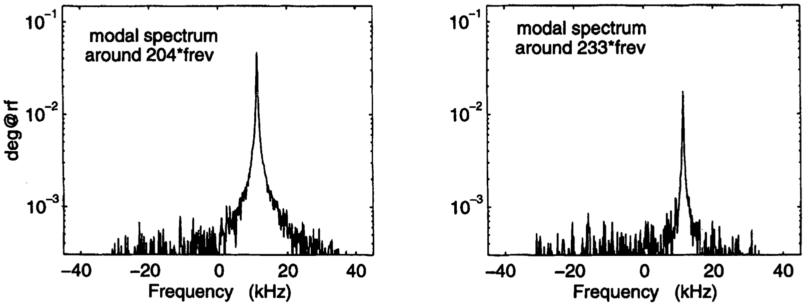

FIGURE 6 90 kHz sections of the high resolution spectrum in Figure 5(a), showing an unstable upper sideband and a damped lower sideband at 204 f_{rev} and also at 233 f_{rev}. The section of the spectrum of the growing transient corresponding to the mode at 204 f_{rev} shows a broader linewidth, which is consistent with a faster growth rate.

Figure 6 zooms in on two 90 kHz sections of the 250 MHz spectrum in Figure 5(a), which was obtained from the 4 ms transient mentioned earlier. It shows that the most unstable mode is an upper sideband at 204 f_{rev} (and therefore a lower sideband at $328-204=124\ f_{\mathrm{rev}}$), with a linewidth corresponding to the previously measured growth rate. The lower sideband is damped to the noise floor by the TM-011 cavity mode. The next most unstable mode is an upper sideband at 233 f_{rev} (and therefore a lower sideband at 95 f_{rev}). The smaller linewidth of this mode is consistent with its smaller growth rate.

4 SUMMARY

We have demonstrated that, in addition to controlling the beam, the DSP-based feedback system facilitates analysis of longitudinal coupled-bunch beam motion via measurement of millisecond-scale transients. Modal decomposition of the transient data was used to simultaneously measure the growth rates and (feedback induced) damping rates of all growing modes, over a range of beam currents and cavity temperatures. We also measured the damping rates of some naturally stable modes. These rates were fairly consistent with the estimated cavity impedance.

The time-domain based technique described in this paper is complementary to narrowband detection using tuned filters, in that both methods allow the identification of modes and quantification of growth rates. The latter however requires a filter for each expected mode, and becomes impractical if there are tens or hundreds of simultaneously unstable modes. Spectrum analyzers cover all modes, but they currently offer a choice between high resolution spectra in minutes or hours and 10 ms scans with poor frequency resolution. For a system with transients that last 1–10 ms, several potentially unstable modes and parameters such as cavity temperature or tuning that could drift unpredictably, the speed advantage makes the time-domain transient analysis a valuable accelerator diagnostic.

The modal analysis technique is currently being used to study the eigenstructure of asymmetric beams at the ALS. Initial results suggest that uneven charge distributions could have a significant effect on the shape and stability of coupled-bunch modes.

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References

[1] F. Sacherer, *IEEE Trans. Nucl. Sci.*, **NS-20-3** (1973).
[2] Lawrence Berkeley Lab. Publication No. LBL-PUB-643-Rev. 2, 1989.
[3] Stanford Linear Accelerator Center Report No. SLAC-372, 1993.
[4] KEK Report No. 95-7, 1995.
[5] Frascati INFN-LNF-90-031(R), 1990.
[6] A. Jackson *et al.*, in *Proc. Particle Accelerator Conference*, Washington, D.C., p. 1432 (1993).
[7] J. Laclare, in *Proc. 11th International Conference on High Energy Accelerators*, Geneva (Birkhauser, 1980).
[8] R. Kohaupt, *IEEE Trans. Nucl. Sci.*, **NS-22-3** (1975).
[9] R. Stiening and J. Griffin, *IEEE Trans. Nucl. Sci.*, **NS-22-3** (1975).
[10] F. Pedersen and F. Sacherer, *IEEE Trans. Nucl. Sci.*, **NS-24-3** (1977).
[11] J. Fox *et al.*, in *Proc. Particle Accelerator Conference*, Washington, D.C., p. 2076 (1993).
[12] J. Corlett and J. Byrd, in *Proc. Particle Accelerator Conference*, Washington, D.C., p. 3318 (1993).
[13] R. Claus *et al.*, in *Proc. Particle Accelerator Conference*, Dallas, 1995.
[14] A. Oppenheim and R. Schafer, *Discrete-Time Signal Processing*, Prentice Hall (1989).
[15] Franklin, Powell and Workman, *Digital Control of Dynamic Systems*, Addison Wesley (1990).