

NLO QUARKONIUM PRODUCTION IN HADRONIC COLLISIONS*

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Abstract

We present some preliminary results on the next-to-leading order calculation in QCD of quarkonium production cross sections in hadronic collisions. We will show that the NLO total cross sections for P -wave states produced at high energy are not reliable, due to the appearance of very large and negative contributions. We also discuss some issues related to the structure of final states in colour-octet production and to high- p_t fragmentation.

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1 Introduction

The production of quarkonium states in hadronic collisions has recently attracted a lot of interest in the theoretical community, as the contributions to this Workshop confirm. Most of the studies done so far have concentrated on key issues such as whether the colour-octet mechanism [1] can indeed explain both Tevatron and fixed-target data, and on trying to identify the most direct and distinct signatures of it. Applications to production of quarkonium at LEP and HERA have also been considered. In general, I believe it is fair to say that the field is still in its infancy. The theoretical predictions require the introduction of several new nonperturbative parameters [2] to describe the values of colour-singlet and colour-octet operators on different states, and the available data can only in part fix these values. In absence of exactly known relations between the values of the different nonperturbative matrix elements, connecting for example their values on 1S states to their values on 2S or P states, it is difficult to reduce the number of independent parameters. This increases the number of separate measurements needed to fix them, and reduces the set of data available to test the predictions of the theory. While the use of nonperturbative matrix elements extracted from the Tevatron data [3] provides an acceptable description of the fixed-target data [4], the naive use of the Tevatron fits at HERA significantly overestimates the yield of observed Ψ in the region $z \rightarrow 1$ [5]. Whether this is a real problem of the current theory, or whether it is just a result of our incomplete understanding of it, is a question that still waits an answer.

With so many very basic open questions, I decided nevertheless to concentrate in this presentation on the subject of higher-order QCD corrections. Given the size of the current theoretical uncertainties, I consider this subject perhaps academic. For example [6], although there are arguments that the right mass parameter to be used in the perturbative evaluation of the short-distance matrix elements and in the phase space constraints should be $2m_Q$ (*i.e.* twice the heavy-quark mass), it is not clear that this prescription properly reflects the correct nonperturbative result, where one expects that, at least for the determination of the phase space boundary, the quarkonium mass should be used. The two choices lead to results which differ by large factors. Undertaking the task of calculating next-to-leading order (NLO) corrections to the LO results seems therefore a bit premature, and the hope that the inclusion of NLO effects could help making the predictions more accurate is in my view, today, not supported by solid evidence. The reason why I am interested in higher-order corrections is that hopefully they will help learning more about the structure of perturbation theory for quarkonium production in both the colour-singlet and colour-octet models. It is a well known fact that NLO corrections to charmonium decay widths are very large. It is important to remark that their size is not just a consequence of the large value of α_s evaluated at the charm mass scale: it is mostly the result of very large coefficients which multiply α_s in the radiative corrections. It is interesting to see what happens of these large coefficients when we study charmonium production at NLO. Should the size of NLO corrections be very large, the whole exercise of extracting the value of nonperturbative parameters from a comparison of LO matrix elements with data, and the use of these parameters to perform predictions for different observables, would clearly have less chances of producing reasonable results. This is because NLO corrections to different observables might be in principle very different in size. I would not be surprised if this were part of the solution to the puzzle uncovered by the application of the colour-octet model to the HERA data.

In the present talk I will first of all briefly illustrate the technique we used to evaluate the NLO cross sections. This technique makes use of dimensional regularization, but uses universal properties of soft and collinear singularities to avoid the need of calculating the real-emission cross sections in D dimensions. This technique was first introduced for hadronic processes in ref. [7], and in the specific context of heavy quark production in ref. [8]. The full details of the NLO quarkonium calculation, including explicit results for colour-singlet $^1S_0^{[1]}$ and $^3P_{0,2}^{[1]}$ states and for several colour-octet states ($^1S_0^{[8]}$, $^3S_1^{[8]}$, $^3P_{0,2}^{[8]}$), will be documented in a forthcoming publication [11]. The NLO cross sections for the $^1S_0^{[1]}$ state have already appeared in the literature, in the papers of Kühn and Mirkes [10] and Schuler [9]. Our results and theirs for this channel are in full agreement.

Next I will show some numerical results. For simplicity, I will just confine myself to the case of $^3P_{0,2}^{[1]}$ states, studied at fixed-target and at collider energies. While the results at fixed target are extremely encouraging, displaying relatively small K -factors and a significant reduction of the scale dependence, the results at collider energies are very worrisome. In short, the radiative corrections turn out to be extremely large and negative, so as to leave us with negative total cross sections. The origin of this problem will be discussed in some detail.

To conclude, I will make a few remarks on the issue of understanding the full structure of the final states produced in conjunction with quarkonium, with particular reference to the production via fragmentation of a gluon jet.

The style of this written contribution is very informal, trying to convene to the reader who did not attend the wonderful spirit of this Workshop. I wish to thank the organizers, the session chairpersons and, most of all, the participants, for the great atmosphere in which the Workshop took place. I look forward to more opportunities like this to openly discuss future progress in the field!

2 The Structure of Quarkonium Total Cross Sections

We are interested here in the process $h_1 h_2 \rightarrow O + X$, where $h_{1,2}$ are arbitrary hadrons and O is a quarkonium state. We will limit ourselves to states for which the lowest-order process $gg \rightarrow O$ is non-zero. In this way the $\mathcal{O}(\alpha_s^3)$ contributions represent genuine NLO effects. These involve the evaluation of the virtual corrections to the ggO vertex, in addition to the real emission processes $gg \rightarrow Og$, $qg \rightarrow Oq$ and $q\bar{q} \rightarrow gO$. We will consider these processes separately.

The contribution of the gg process to the total cross section gives rise to collinear and soft singularities. These can be regulated by working in dimensional regularization, giving rise to poles in $1/\epsilon$: double poles for the leading soft singularities, and single poles for the sub-leading soft and for the collinear singularities. The soft singularities are absorbed by similar singularities present in the virtual corrections to the ggO vertex, leaving finite terms which contribute to the processes with Born-like kinematics $gg \rightarrow O$. The collinear singularities are absorbed into a redefinition of the initial-state parton densities, according to the standard procedure of factorization of the initial-state mass singularities. The residual finite contributions correspond to purely inelastic processes $gg \rightarrow Og$, regulated at the boundary of phase space (namely in the soft and collinear region) by an appropriate “+” prescription. Given the simplicity of the kinematics of the LO process, and given the universal character of soft and collinear emission,

it can be shown [11] that the structure of the NLO partonic cross section is given in general[§] by the following expression:

$$\begin{aligned} \sigma_{NLO}^{J,(gg)}(x) &= \sigma_{Born}^{J,(gg)} \delta(1-x) = \frac{\alpha_s}{\pi} \sigma_{Born}^{J,(gg)} x \\ &\times \left\{ \delta(1-x) \left[A_J - C_A \frac{\pi^2}{3} \right] + F_J(x) - P_{gg}(x) \log x \right. \\ &\left. + 4C_A \left[\left(\frac{1}{x} + x(1-x) - 2 \right) \log(1-x) + \left[\frac{\log(1-x)}{1-x} \right]_+ \right] \right\}, \end{aligned} \quad (1)$$

where $J = 0, 2$ is the total angular momentum of the P -wave state considered, $x = m^2/\hat{s}$, m is the quarkonium mass, $C_A = N_c = 3$ is the number of colours, and P_{gg} is the Altarelli-Parisi splitting kernel. For simplicity I have put the factorization (μ_F) and renormalization scales (μ_R) equal to m . This sets to zero some universal terms proportional to $\log(\mu_R/m)$ or $\log(\mu_F/m)$. The factorization of collinear singularities was performed in the \overline{MS} scheme. All of the dependence on the quarkonium state is included in the numbers A_J and in the functions $F_J(x)$. A_J is related to the finite part of the virtual corrections, defined by the equation:

$$\sigma_{virt}^{J,(gg)}(x) = \frac{\alpha_s}{\pi} \sigma_{Born,D}^{J,(gg)} \left(\frac{4\pi\mu^2}{\hat{s}} \right)^\epsilon \Gamma(1+\epsilon) \delta(1-x) \times \left(A_J + \frac{C_2}{\epsilon^2} + \frac{C_1}{\epsilon} \right), \quad (2)$$

where $\sigma_{Born,D}^{J,(gg)}$ is the Born cross section in D dimensions and C_1 and C_2 are numerical coefficients independent of ϵ . From an explicit calculation, we get:

$$A_0 = C_F \left(-\frac{7}{3} + \frac{\pi^2}{4} \right) + C_A \left(\frac{1}{3} + \frac{5}{12}\pi^2 \right) \quad (3)$$

$$A_2 = -4C_F + C_A \left(\frac{1}{3} + \frac{\pi^2}{6} + \frac{5}{3} \log 2 \right). \quad (4)$$

The function $F_J(x)$ is given instead by the following relation:

$$\frac{\alpha_s}{\pi} \sigma_{Born}^{J,(gg)} F_J^{(gg)}(x) = \frac{1}{32\pi\hat{s}} \left(\frac{1}{1-x} \right)_+ \int_{-1}^1 dy \left(\frac{1}{1-y} \right)_+ \overline{M^J}_{(4)}(x, y), \quad (5)$$

where $y = \cos\theta$ is the cosine of the scattering angle in the hard process CoM frame, and $\overline{M^J}_{(4)}(x, y)$ is related to the four-dimensional matrix element squared for the $gg \rightarrow Og$ process by the following relation:

$$\overline{M^J}_{(4)}(x, y) = (1-x)^2 (1-y^2) M_{(4)}^J(x, y). \quad (6)$$

The explicit expressions will be reported in [11].

The contribution of the qg process to the total cross section only appears at $\mathcal{O}(\alpha_s^3)$. It gives rise to singularities due to the collinear emission of the gluon entering the hard scattering from the initial-state quark. As before, these singularities are absorbed into a redefinition of

[§]Although I will only present here results for P-waves, the structure of the cross sections is exactly the same for the production of other states, regardless of the $^S L_J$ quantum numbers.

the parton densities, leaving a residual finite contribution corresponding to the purely inelastic process $qg \rightarrow Og$. Once more the universal character of collinear emission can be used to reduce the final result to a simple expression, given by:

$$\sigma_{NLO}^{J,(qq)}(x) = \frac{\alpha_s}{\pi} \sigma_{Born}^{J,(gg)} x \times \left[F_J^{(qq)}(x) + P_{gq}(x) \log(1-x) + \frac{x C_F}{2} \right]. \quad (7)$$

All of the dependence on the quarkonium state is included in the function $F_J^{(qq)}(x)$, which is defined by the analogous of eq. 5 and will be given explicitly for the various states in ref. [11].

The study of the qq channel is interesting from the theoretical point of view, since this channel exhibits a singularity related to the quarkonium binding energy. This is the analogous of the singularities found in the case of P-wave decays to $q\bar{q}g$. In dimensional regularization, we obtain:

$$\sigma_{NLO}^{J,(qq)}(x) = (2J+1) \left(-\frac{1}{\epsilon} \right) B \delta(1-x) + F_J^{(qq)}(x), \quad (8)$$

where B is a constant factor, independent of J . The $1/\epsilon$ pole can be removed by a renormalization of the ${}^3S_1^{[8]} \rightarrow {}^3P_J^{[1]}$ transition matrix element, and including the $q\bar{q} \rightarrow {}^3S_1^{[8]} \rightarrow {}^3P_J^{[1]} X$ contribution to the cross section. Any reasonable value of the renormalized coupling will however make the numerical impact of the qq channel totally negligible in all experimental configurations of interest, except for $\pi N \rightarrow \chi_b$ production at very low energy. We base this claim on a study of the qq production mechanism done using an IR cutoff in four dimensions. In this scheme, the divergence is proportional to the logarithm of the binding energy. Even with values as low as few MeV the qq channel contribution to 3P_J production is overwhelmed by the gg and qg channels.

3 Numerical Results

The partonic cross sections described in the previous section can be used to obtain total cross sections in hadronic collisions. For illustration, I will consider here the case of pp collisions. In fig. 1(a) I present the results for $\chi_{c,2}$ production at fixed-target energies, comparing the NLO to the LO predictions. I use the MRSA [12] PDF set, and show results for three different scale choices. As can be seen, the NLO calculation significantly reduces the scale dependence of the LO result. The size of the K-factor depends on the scale chosen, as well as on the beam energy. The same distributions, plotted as a function of \sqrt{S} in the energy domain of the Tevatron collider, are given in fig. 1(b). I chose here the MRSD0 [13] set of PDFs. The results are extremely disappointing: not only have the NLO cross sections a very strong scale dependence, but they also become negative for sufficiently large \sqrt{S} . What is the origin of this behaviour, which makes the perturbative evaluation of the cross sections totally unreliable?

There are at least two problems[¶]. The first one is that the virtual corrections are very large and negative. The large universal term $-C_A \alpha_s \pi/3 \sim 1$, proportional to $\delta(1-x)$, is only in part cancelled by the state-dependent coefficient A (see eqs. (3,4)). This indicates that two-loop

[¶]Most of the remarks which follow have already been made by G. Schuler in his '94 review [9]. Schuler at the time had available the full NLO corrections to η production, as well as the leading small- x behaviour of the χ cross sections. It is a pity that those remarks have passed almost unnoticed in the community!

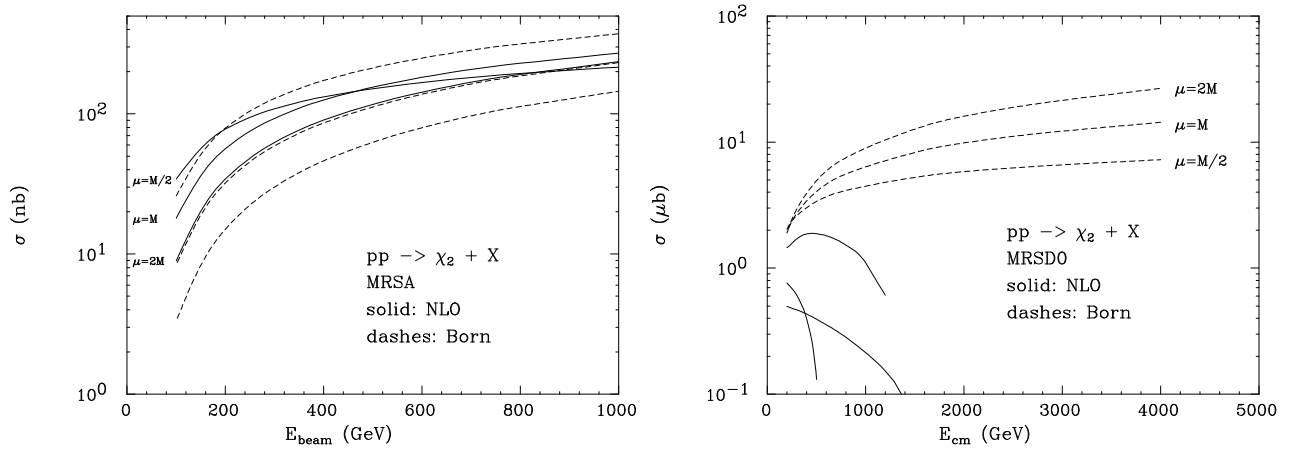


Figure 1: Total cross section for $pp \rightarrow \chi_2 X$ as a function of beam energy for fixed-target collisions (a), and as a function of \sqrt{S} for collider configurations (b).

corrections coming from the square of the 1-loop matrix elements are likely to be large^{||}. The second problem is that, after subtraction of the collinear singularities, the $\mathcal{O}(\alpha_s^3)$ corrections to both gg and qg processes tend to a negative constant in the $x \rightarrow 0$ limit:

$$\sigma_{NLO}^{J,(gg)}(x) \xrightarrow{x \rightarrow 0} 2C_A \frac{\alpha_s}{\pi} \sigma_{Born} \times \left(\log \frac{m^2}{\mu_F^2} - C_J \right), \quad (9)$$

$$\sigma_{NLO}^{J,(qg)}(x) \xrightarrow{x \rightarrow 0} \frac{C_F}{2C_A} \sigma_{NLO}^{J,(gg)}(x), \quad (10)$$

where $C_J = 43/27$ and $53/36$ for $J = 0$ and $J = 2$, respectively. There is nothing wrong in principle with these cross sections turning negative in the small- x region, as what is subtracted is partly returned to the gluon density via the evolution equations. However in this particular case two things happen:

- the standard DGLAP evolution might not be adequate, as $x \ll 1$ at collider energies.
- the factorization scale (of the order of the charmonium mass) is very close to the scale at which the input PDF is measured or parametrized, and there is therefore no room for evolution (*i.e.* resummation of large logs). The cross sections will therefore critically depend on the assumed shape of PDFs.

To illustrate the second point, let us assume for simplicity that we can approximate the inelastic part of the quarkonium cross section by its small- x behaviour, in such a way that:

$$\sigma_{NLO}(x) \sim A \delta(1-x) - \frac{\alpha_s}{\pi} C, \quad (11)$$

^{||} Although the term $-\alpha_s/\pi C_A/3\pi^2$ is universal and directly linked to the IR behaviour of the real emission diagrams in a $gg \rightarrow X^{[1]}$ process, with $X^{[1]}$ an arbitrary colour-singlet state, we have no argument suggesting that it should be exponentiated. Other π^2 terms arise from the virtual corrections, not all of them universal. Understanding which (if any) of them exponentiates requires more work, which we have not done so far.

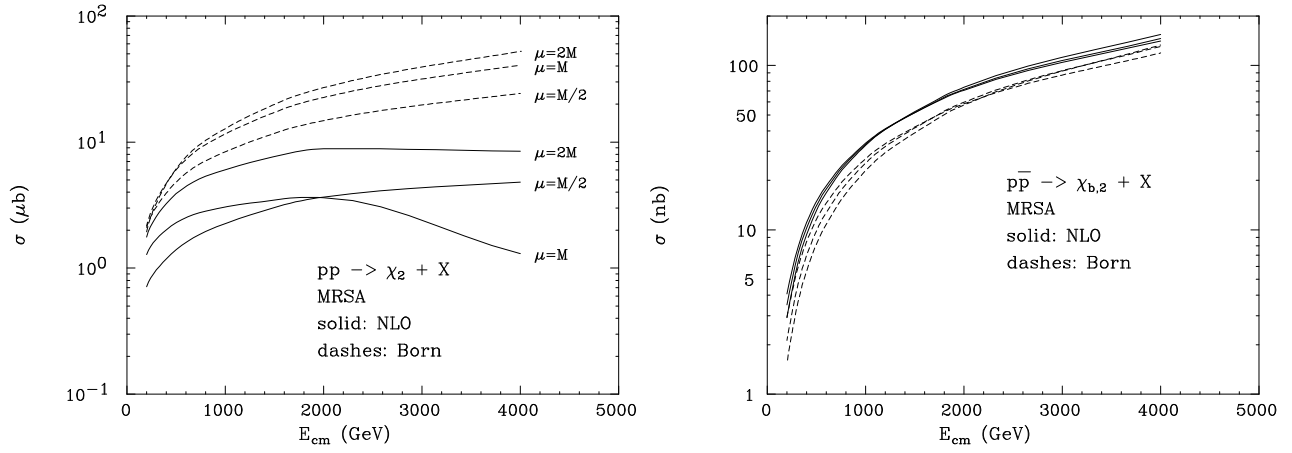


Figure 2: *Left: total cross section for $pp \rightarrow \chi_{c,2} X$ as a function of \sqrt{S} with PDF set MRSA. Total cross section for $pp \rightarrow \chi_{b,2} X$ as a function of \sqrt{S} with PDF set MRSA.*

(we left out irrelevant overall constants) and let us assume that we can parametrize the gluon density with the following form:

$$G(x) = \frac{1}{x^{1+\delta}}, \quad (12)$$

with $0 < \delta < 1$. It is then easy to show that the total cross section has the following behaviours, depending on the value of the parameter δ :

$$\sigma_{NLO} = \sigma_{Born} \times \begin{cases} A - C \frac{\alpha_s}{\pi} \log \frac{S}{m^2} & \text{if } \delta \log S/m^2 \ll 1 \\ A - C \frac{\alpha_s}{\pi} \frac{1+\delta}{\delta} & \text{if } \delta \log S/m^2 \gg 1 \end{cases} \quad (13)$$

If the input gluon density $G(x)$ is not sufficiently steep (*i.e.* if $\delta \log S/m^2 \ll 1$), very large logarithms $[\alpha_s \log S/m^2]^n$ will appear at all orders of perturbation theory, and will need to be resummed [14], or accounted for by corrections to the DGLAP evolution. If δ instead has a value of the order of 0.3–0.5, typical of the most recent fits to HERA data, no large logarithmic corrections appear.

As an example of how a different choice of PDF can change things, I present in fig. 2(a) the $\chi_{c,2}$ cross sections obtained using the PDF set MRSA, for which the input gluon density is steeper than $1/x$. The NLO cross section remains now positive over a much larger range of \sqrt{S} . Nevertheless the scale dependence is still so large that I would sadly conclude that no predictive power is available at NLO for total χ_c production cross sections at energies \sqrt{S} larger than few hundred GeV. The situation is significantly better in the case of bottomonium states, χ_b , shown in fig. 2(b). In this case the inclusion of NLO corrections significantly reduces the scale dependence of the LO result.

Needless to say, no conclusion on the behaviour of the charmonium cross sections at large p_t can be reached from the previous study, since at large p_t additional higher-order diagrams need to be calculated to achieve a NLO accuracy, and since the range of x and the scales probed are significantly different than those explored in the total cross section calculation. No full NLO calculation is currently available for quarkonium production in hadronic collisions at non-zero

p_t . Even the simplest case of 1S_0 production, although irrelevant phenomenologically given that no data exist, might provide an interesting theoretical insight if it were available. I would put this calculation very high on the list of things to be done!

4 Concluding Remarks

In this final section I would like to address a few additional issues related to the understanding of quarkonium production at higher orders in perturbation theory:

- the exclusive structure of final states in quarkonium production via the colour-octet mechanism and
- the approximations involved in the use of the fragmentation functions for production at large p_t .

As will appear from the following discussion, the two issues are not entirely separated.

It is generally accepted by now that the proper treatment of the hierarchy of higher Fock states is most likely the solution to a large fraction of the puzzles present in quarkonium production data**. It is also clear, however, that a complete understanding of the full dynamics of the interactions involving colour-octet states (or, more generally, higher Fock states) is still missing. To which extent this ignorance can affect our capability to perform quantitative predictions for production rates is, in my view, unclear. To give an example, let me consider charmonium production via the colour-octet mechanism at large p_t . In this case, we believe that production is dominated by fragmentation contributions, with a high- p_t gluon turning into some colour-octet state (O), that will then undergo a nonperturbative transition to a given, colour-singlet, onium state (S). This final step is usually parametrized by assigning a probability, proportional to a well defined nonperturbative matrix element, and by assuming that S will carry all of the energy of the parent O . The nonperturbative transition $O \rightarrow S$ should however be thought of as an inclusive process, $O \rightarrow S + X$, since one (or more) gluons need to be radiated. In the standard approach, these gluons are assigned zero energy. In practice, however, we know that these gluons cannot carry zero energy, because they will have to materialize into some hadron, say into pions. In the rest frame of O , it is reasonable to assume that the energy of these gluons will at least be of the order of Λ_{QCD} , *i.e.* a number of the order of, say, 300 MeV. In we consider as an example the transition $^3S_1^{[8]} \rightarrow ^3S_1^{[1]}$, believed to be responsible for the large J/Ψ rate measured at the Tevatron, there should be at least two gluons emitted. In the laboratory frame, and for production at large p_t , the ratio of the energy carried by these gluons and the energy carried by the $^3S_1^{[1]}$ will be equal on average to the ratio of their masses, namely:

$$\frac{E_g}{E_\psi} = \frac{2\Lambda_{QCD}}{M_\psi} \sim 0.2. \quad (14)$$

As a result, the actual energy fraction z of the colour-octet state carried by the final J/Ψ will be around 0.8. So the fragmentation function instead of peaking at $z = 1$ will peak at $z = 0.8$. Once convoluted with the p_t spectrum of prompt gluons, assuming a behaviour like

**That this is possibly true for other, more exotic, aspects of quarkonium physics as well, has been argued in the past and during this meeting by Brodsky [15]

$d\sigma/dp_t \sim 1/p_t^n$, this change will induce a change in the production rate at a given p_t of the order of $-n \times 0.2$. For a typical value of $n = 4$, this is a -80% correction. It is important to point out that this is not a higher-twist phenomenon, in the sense that the effect is not reduced by going to larger p_t . In order to make a more accurate prediction of the high- p_t production rate, it is therefore essential to improve the understanding of the colour-bleaching mechanism^{††} Notice also that this problem is not totally unrelated to the problem of connecting nonperturbative matrix elements “measured” in quarkonium decay via colour-octet states to the nonperturbative matrix elements needed in the evaluation of production cross sections.

Let me now touch on one more issue related to fragmentation. The standard approach consists in determining the perturbative boundary condition for the evolution of the fragmentation function by using the following relation [16]:

$$D_0(z, m^2) = \int_{m^2}^{\infty} \frac{ds}{s} d(z, s), \quad (15)$$

where $d(z, s)$ is the probability that a gluon of virtuality s decays to a J/Ψ carrying longitudinal momentum fraction z in the infinite-momentum frame. The evolution of the fragmentation function $D(z, q^2)$ is then given by the standard DGLAP evolution equation:

$$\frac{\partial}{\partial \log \mu^2} D(z, \mu^2) = \frac{\alpha_s}{\pi} \int_z^1 \frac{dy}{y} P_{gg}(z/y) D(z, \mu^2). \quad (16)$$

The problem with this equation [17] is that it does not respect the phase-space constraint $D(z, \mu^2) = 0$ for $z < m^2/\mu^2$. The implementation of this constraint would slow down the evolution of the fragmentation function by delaying the depletion of the large- z fragmentation region. Since the spectrum of gluons falls rapidly with p_T/z , a proper treatment of the large- z region can have a significant effect on the cross section. A more accurate treatment would be to use the following equation:

$$\mu^2 \frac{\partial}{\partial \mu^2} D(z, \mu^2) = d(z, \mu^2) + \frac{\alpha_s}{2\pi} \int_z^1 \frac{dy}{y} P_{gg}(y) D(z/y, y\mu^2), \quad (17)$$

together with the boundary condition $D(z, \mu^2 = m^2) = 0$. This evolution equation respects the phase space constraint, as can be easily checked. A consistent calculation done using NLO fragmentation functions should use this evolution, instead of the naive one.

As a final point, I would like to present a simple proposal for how to describe the exclusive structure of the final state in quarkonium production via fragmentation through a colour-octet state. After generation of a hard final-state gluon, let the gluon shower evolve, until it generates a $c\bar{c}$ pair. Allow then the $c\bar{c}$ pair to evolve. If additional gluons are emitted, we can assume that no quarkonium state should be produced. This would be in fact a $1/N_c$ suppressed process. Given that the emitted gluons are perturbative, the only way to correctly calculate the probability that the colours will recombine into a singlet state after gluon emission is by using the colour-singlet matrix elements. If no additional gluons are emitted, then consider the invariant mass of the pair. If it is below a given value (to be parametrized by the BBL

^{††} As an aside, we remark here that this understanding could also lead to some specific and testable predictions. For example, the interplay between quantum numbers and phase space might lead to interesting selection rules on the possible sets of light hadrons produced in the $O \rightarrow S$ transition.

factorization scale, the scale at which NRQCD matrix elements are separated from the perturbative ones) then we can assume that the $c\bar{c}$ pair will be converted into a colour-singlet J/Ψ plus two gluons, with a probability proportional to the NRQCD matrix element (this matrix element depends on the factorization scale, so at the end the factorization scale dependence should cancel between the choice of the phase space boundary and the transition probability). This transition is equivalent to what is done in the cluster model for hadronization [19], where at the end of the perturbative evolution each gluon is split into a $q\bar{q}$ pair. The energy and angular distribution of the two gluons can just be taken to be given by the 3-body phase space for transition of the $c\bar{c}$ pair into the $J/\Psi gg$ final state. Colour lines can be drawn between these two gluons and the rest of the shower, so that hadronization can take place (say via the cluster model itself). From the point of view of the algorithm efficiency, one could use the forced evolution a' la Mike Seymour [18] to always get a $c\bar{c}$ pair at the end of each shower. All of the MC inefficiency is then related to the invariant mass distribution of the pair, which will often be above the factorization scale threshold.

The first attempt to produce J/Ψ states via the colour-octet mechanism in a full shower MC, using however a different approach than what suggested above, has recently been presented by Ernström and Lönnblad [20]. This calculation allows to make definite predictions for the structure of the gluon-jet which accompanies the J/Ψ . Comparisons of these predictions with data will certainly help improving our understanding of this important aspect of the production dynamics.

It is worth keeping in mind that all the effects discussed in this section can lead to significant changes in the shape of the p_t distribution of J/Ψ s observed at the Tevatron. In addition to the factors described here, one should consider of course the effect of higher-order corrections due to multiple-gluon emission from the initial-state, and the systematic uncertainties due to the choice of the input gluon densities and due to the chosen value of α_s . All of these effects will also induce a smearing of the p_t spectrum, and could therefore influence dramatically the extraction of the nonperturbative parameters for the different channels which contribute to J/Ψ production at the Tevatron.

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