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MEASUREMENT OF THE EXCITATION OF THE COUPLING RESONANCE $Q_h - Q_v = 0$

by

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Summary

To take advantage of the large resonance-free regions close to the diagonal $Q_h = Q_v$ in the tune diagram, the Intersecting Storage Rings (ISR) operate with nearly equal tunes. Thus, the excitation of the coupling resonance $Q_h - Q_v = 0$ is of importance and this has stimulated the study of its effects, the measurement of its excitation by axial and skew quadrupole fields and its compensation. A complex coupling coefficient C can be defined in terms of axial and skew quadrupole fields, and the unperturbed machine parameters. An electronic device has been built to measure $|C|$ by kicking a small beam and analysing the coherent oscillation. By combining different coupling vectors, phase measurements are also possible. Examples are given of coupling vectors measured in the ISR for magnet tilts, skew quadrupoles and solenoids. The outlines of two methods for directly measuring both amplitude and phase are also given. Some ideas are extended to higher order resonances.

1. Introduction

To stabilize the beams and for reasons of background, the ISR perform best on working lines with large tune spreads parallel to the diagonal (Fig. 1). The position of a working line depends on the relative importance of the surrounding resonances and it is in this context that the $Q_h - Q_v = 0$ linear coupling resonance is of particular interest. Since it is a low order resonance, it is liable to have a large bandwidth but it is stable and its effects are more annoying than catastrophic. In the ISR, this resonance causes the vertical emittances to be blown up with a consequent loss of luminosity. It also perturbs and often precludes tune measurements. Apart from being stable, this resonance has the advantage of being easily measured and compensated, so that it is in fact better to be close to this resonance than any other.

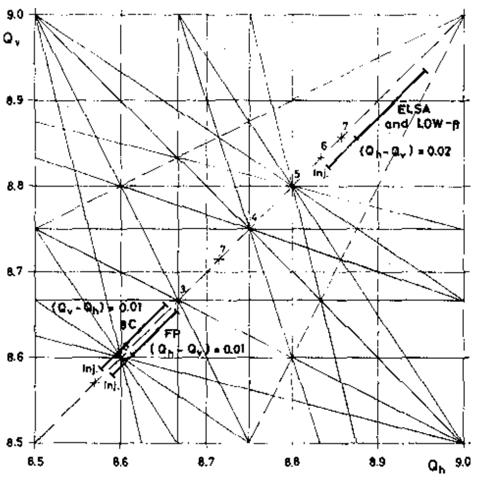

Fig. 1 ISR Working Lines

2. Coupling Theory

For simplicity, the formalism used in Ref. 1 will be applied only to the resonance $Q_h - Q_v = 0$.

It is easy to write the equations for the transverse motion of the protons for the unperturbed case and the associated Hamiltonian

$$x'' + K_1 x = 0 \quad \text{and} \quad z'' + K_2 z = 0 \ , \tag{1}$$

where $' \equiv d/d\theta$ and θ = axial distance(s)/average radius (R). The general solutions of (1) are

$$x = a_1\, u\, e^{iQ_h\theta} + \bar{a}_1\, \bar{u}\, e^{-iQ_h\theta}$$
$$z = a_2\, v\, e^{iQ_v\theta} + \bar{a}_2\, \bar{v}\, e^{-iQ_v\theta} \tag{2}$$

where u and v are the Floquet's functions $\sqrt{\beta/2R}\, \exp i(\mu - Q\theta)$ in the horizontal and vertical planes, respectively and $\bar{}$ means complex conjugate. μ and β have their usual meanings.

When perturbed by skew quadrupole and longitudinal fields, these equations and the perturbing part of the Hamiltonian become

$$x'' + K_1 x = -(K + \tfrac{1}{2} M')\, z - Mz' \ ,$$
$$z'' + K_2 z = -(K - \tfrac{1}{2} M')\, x + Mx' \ , \tag{3}$$
$$H_1 = Kxz - \frac{1}{2} Mzp_x + \frac{1}{2} Mxp_z + \frac{1}{8} M^2 z^2 + \frac{1}{8} M^2 x^2 \ ,$$

where:

$$K = \frac{1}{2} \frac{R^2}{B\rho}\left(\frac{\partial B_x}{\partial x} - \frac{\partial B_z}{\partial z}\right) \ ; \quad M = \frac{R}{B\rho} B_s \ ; \tag{4}$$

B_x, B_z and B_s are the three field components and $B\rho$ is the magnetic rigidity.

From the treatment of perturbations in analytical mechanics, it is possible to say that the complete solutions of equations (3) are still given by the analytical expressions (2) only the complex amplitudes a_1 and a_2 become functions of θ such that

$$a_1' = i\,\frac{\bar{C}}{2}\, a_2\, e^{-i\theta\Delta} \quad \text{and} \quad a_2' = i\,\frac{C}{2}\, a_1\, e^{i\theta\Delta} \ , \tag{5}$$

where:

$$\Delta = Q_h - Q_v \ ,$$
$$C = \frac{1}{2\pi R} \int_0^{2\pi} \sqrt{\beta_h \beta_v} \left[K + \frac{1}{2} MR\left(\frac{\alpha_h}{\beta_h} - \frac{\alpha_v}{\beta_v}\right) - \frac{i}{2} MR\left(\frac{1}{\beta_h} + \frac{1}{\beta_v}\right)\right] \exp\left\{i\left[(\mu_h - \mu_v) - (Q_h - Q_v)\theta\right]\right\} d\theta \ . \tag{6}$$

C characterizes the coupling and once known all effects can be calculated. The following shows how to measure this coefficient. Equations (5) can be decoupled, solved and re-introduced into equations (2). Considering only the boundary conditons for an inclined kick, i.e. $x_o = z_o = 0$, $x_o' \neq 0$, $z_o' \neq 0$, we get[2]:

$$z = \frac{\sqrt{\beta_v}}{\eta} A_z \cos\left(\mu_v + \frac{\Delta}{2}\theta - \phi\right) \ , \tag{7}$$

where:

$$A_z^2 = z_o'^2 \frac{\eta^2}{\beta_{h_o}} - \frac{x_o' z_o'}{\sqrt{\beta_{h_o}\beta_{v_o}}} 2\eta\, (\mathrm{Im.}C) \sin\frac{\eta}{2}\theta \cos\frac{\eta}{2}\theta + \left[|C|^2\left(\frac{x_o'^2}{\beta_{v_o}} - \frac{z_o'^2}{\beta_{h_o}}\right) - \frac{x_o' z_o'}{\sqrt{\beta_{h_o}\beta_{v_o}}} 2\Delta\, (\mathrm{Re.}C)\right] \sin^2\frac{\eta}{2}\theta \tag{8}$$

A_z is the slowly varying envelope of the motion, ϕ is a phase shift which is also a function of θ, $\eta = \sqrt{\Delta^2 + |C|^2}$.

The period of the slowly oscillating envelope A_z is[2,3]

$$T = \frac{1}{f_{rev}\, \eta} = \frac{1}{f_{rev} \sqrt{\Delta^2 + |C|^2}} \ , \tag{9}$$

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where f_{rev} is the revolution frequency.

Furthermore, the three important parameters (Re.C), (Im.C) and the modulus $|C|$ appear in the expression A_z. Depending on the choice of the initial conditions, the signal can be a function of the modulus $|C|$ only, or a function of (Re.C) and (Im.C) only. Let us first consider the former case[2,3], i.e. $x_0' = 0$:

$$A_z^2 = \frac{z_0'^2}{\beta_{h_0}}\left(\eta^2 - |C|^2 \sin^2 \frac{\eta}{2}\theta\right) . \tag{10}$$

It follows from (10) that the ratio S of the minimum to the maximum amplitudes of the modulation is[3]

$$S = \frac{|\Delta|}{\eta} = \frac{|\Delta|}{\sqrt{\Delta^2 + |C|^2}} . \tag{11}$$

Thus, it is possible to measure $|C|$ and $|\Delta|$ by kicking the beam vertically and by measuring S and T from the vertical signal from a pick-up. The forms of the signal envelope are shown in Fig. 2.

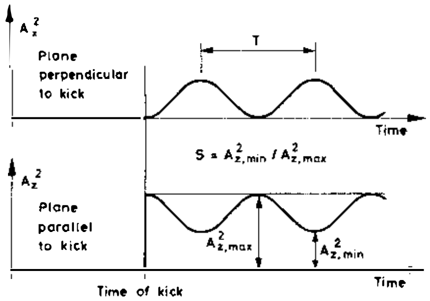

Fig. 2 Envelopes of Pick-up Signals

Considering the second case where A_z is only a function of (Re.C) and (Im.C), i.e. when $x_0'/\sqrt{\beta_{v_0}} = z_0'/\sqrt{\beta_{h_0}}$, the envelope becomes[2]

$$A_z^2 = \frac{z_0'^2}{\beta_{h_0}}\left\{\eta^2 - 2\left[(\text{Re.C})\ \Delta \sin^2 \frac{\eta}{2}\theta + (\text{Im.C})\ \eta \sin\frac{\eta}{2}\theta \cos\frac{\eta}{2}\theta\right]\right\}. \tag{12}$$

The minima and maxima of the function (12) appear at the angles

$$\theta_n = n\frac{\pi}{\eta} - \frac{1}{\eta}\ \text{arctg}\ \frac{\eta(\text{Im.C})}{\Delta(\text{Re.C})} , \tag{13}$$

while the extrema of function (10) were at $\theta_n = n\pi/\eta$.

Thus, the signal following an inclined kick is shifted in time by[2]

$$\delta_t = \frac{T}{2\pi}\ \text{arctg}\ \frac{\eta(\text{Im.C})}{\Delta(\text{Re.C})} . \tag{14}$$

This time shift, together with the relations (9) and (11), gives the possibility of measuring the ratio (Im.C)/(Re.C) as well as $|C|$.

The Hamiltonian approach gives the detailed form of the coupling coefficient C for a machine with any form factor. The other results can also be obtained with a simplified analysis of a smooth approximation on the direct solution of the equations of motion. If the pick-up signals from one observation point are normalized by $\sqrt{\beta_x}$ and $\sqrt{\beta_z}$, it is not possible to distinguish between a machine with any form factor[1] and an equivalent sinusoidally smooth machine[4]. The basic equations for a smooth machine are given in (3), but with the assumption that K_1, K_2, K and M are constants

$$K_1 = RQ_h\ ; \quad K_2 = RQ_v\ ; \quad M' = 0 .$$

By putting $\sqrt{K_1} = \sqrt{K_0} + \delta/2$ and $\sqrt{K_2} = \sqrt{K_0} - \delta/2$, and by making a transformation to an inclined coordinate system (U,V), the dependence on K can be removed by choosing the inclination α as follows:

$$\text{tg}\ \alpha = \frac{-\Delta^2 \pm \sqrt{\Delta^2 + (\text{Re.C})^2}}{(\text{Re.C})} . \tag{15}$$

The equations of motion for the normal modes U and V then become

$$U'' + U(K_0 + y) = -M\ V' \quad \text{and} \quad V'' + V(K_0 - y) = M\ U' , \tag{16}$$

where $y = \delta\sqrt{K_0}\cos 2\alpha + K_0 \sin 2\alpha$.

Only the axial field coupling is apparent in (16) and this is equivalent to Im.C in (6).

This result gives an alternative way of measuring the phase of C. By putting both a kicker and a pick-up at the same position and making them both rotating about the beam axis, it is possible to kick the beam at an angle and to observe the normalized coherent oscillations. At the angle α, given by equation (15), $|C|$ will be a minimum, so giving Im.C directly.

3. Coupling Meter

So far, only the measurement of $|C|$ has been realized practically, although relative phases can be measured indirectly by the addition of known vectors. The coupling meter used in the ISR measures the modulation S and the period T (equations (9) and (11)) of the coherent oscillation following a kick in the following range:

Carrier frequency : 30 - 300 kHz
Modulation frequency : 0.5 - 10 kHz .

The pick-up signal is taken via the tune meter filter[5] and passes through a rectifier followed by a sharp edge low-pass filter (70 dB/decade attenuation above 10 kHz) (see Fig. 3). It is then differentiated and the zero crossings used for period measurement and triggering two sample hold units which store separately maxima and minima of the detected signal. Each sample hold output is integrated for a preselected number of periods (1-9) and then displayed on a DVM. The integration is started after a delay of 400 μs after the kick.

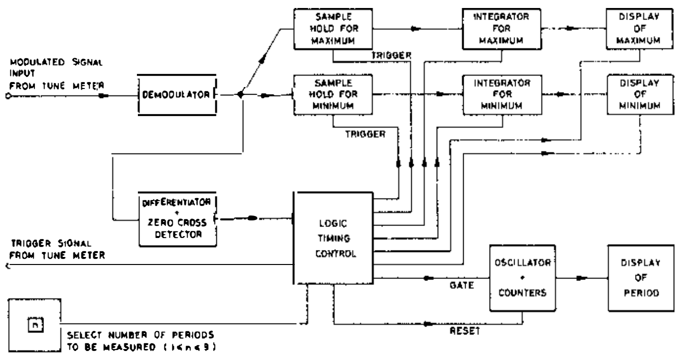

Fig. 3 Block Diagram of Coupling Meter

The precision is ± 3 % for the maximum and minimum values and ± 1 % for the period. As the modulation decreases, it becomes more difficult to separate it from the carrier and in practice, the results are not consistent below 5 % modulation.

4. Measurements Made in the ISR

4.1 Skew Quadrupole Excitation

At present, the ISR are equipped with two chains of skew quadrupoles per ring. Since this scheme was originally intended for compensating the coupling from random magnet tilts, the phases associated with these quadrupole strings are also close to the average of $(\mu_h - \mu_v)$. The coupling meter was used to calibrate these quadrupoles and deduce the relative phase of the two chains for comparison with the theoretical values (see Table 1).

TABLE 1
Calibration of the skew quadrupole chains at 11.78 Gev/c

	Measured*	Calculated	Error
Q1	$\|C\| = 0.411$	$\|C\| = 0.426$	-3.5 %
Q2	$\|C\| = 0.536$	$\|C\| = 0.559$	-4.1 %
Relative phase	$2.6^{\circ} \pm 1^{\circ}$	1.8°	---

* Values are scaled to maximum current.

4.2 Solenoid Excitation

Figure 4 shows the relative sizes and positions of the vectors excited by the basic ISR and a recently installed superconducting solenoid. The coupling effect of the solenoid is small since it has end-plate slots which partially compensate the main field. By using the theoretical phase angle, $C_{solenoid}$ was calculated to be 2.6×10^{-3}, which is within 8 % of the theoretical value of 3.33×10^{-3}. At the time of these measurements, the coupling from the basic ISR was approximately three times stronger than is usually measured after a machine re-alignment.

With the present layout of skew quadrupoles, it is difficult to produce a vector for correcting the solenoid. However, this was done as an academic exercise according to the scheme shown in Fig. 4.

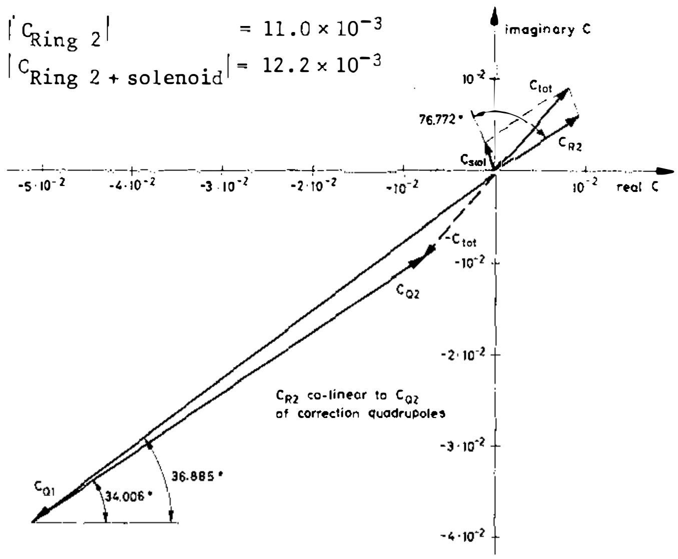

Fig. 4 Coupling Vectors in the ISR

5. Higher Order Difference Resonances

From the Hamiltonian approach (Section 2), it is possible to find two invariants of the motion. These can be used as a basis for measuring the parameter κ, which is equal to C/2 in the linear coupling case.

For a general difference resonance $n_1 Q_h + n_2 Q_v = p$, where n_1 and n_2 have opposite signs, these invariants have the form[1] below for no damping:

$$r_1^2 - \frac{n_1}{n_2} r_2^2 = A$$
$$r_2^2 \Delta + 2n_2 \, |\kappa| \, r_1^{|n_1|} \, r_2^{|n_2|} \cos \psi = B \, , \qquad (17)$$

where:

r_1 and r_2 are real horizontal and vertical amplitudes, related to equation (2) by $r_j^2 = a_j \bar{a}_j$,

ψ is a phase depending on the azimuthal angle θ,

A and B are constants depending on the initial conditions,

Δ is the distance from the resonance $(n_1 Q_h + n_2 Q_v - p)$,

κ is a complex parameter defined for two-dimensional fields by[1]

$$\kappa = \frac{1}{2\pi (2R)^{N/2} \, n_1! n_2!} \int_0^{2\pi} \beta_h^{|n_1|/2} \beta_v^{|n_2|/2} \frac{R^2}{B\rho} \frac{\partial^{(N-1)} B_{\substack{z\\x}}}{\partial x^{(N-1)}}$$
$$\exp \left\{ i [n_1 \mu_h + n_2 \mu_v - (n_1 Q_h + n_2 Q_v - p)\theta] \right\} d\theta \, , \qquad (18)$$

$B_{\substack{z\\x}}$ is taken as B_z when n_2 is even and as B_x when n_2 is odd.

Considering the coherent oscillations of a vertically kicked beam, the initial conditions are $r_{10} = 0$ and $r_{20} = r_{2,max}$ and the constants A and B become equal to $-n_1 r_{2,max}/n_2$ and $r_{2,max}^2 \Delta$, respectively. With these boundary conditions, it is possible to eliminate r_1 from (17). Choosing the extreme values of cos ψ, the resulting equation relates the minimum amplitude $r_{2,min}$ to the maximum $r_{2,max}$

$$\Delta (r_{2,min}^2 - r_{2,max}^2) \pm 2n_2 |\kappa| \left[\frac{n_1}{n_2} (r_{2,min}^2 - r_{2,max}^2) \right]^{|n_1|/2} r_{2,min}^{|n_2|} = 0 \qquad (19)$$

In the case where $\Delta \neq 0$ and $|\kappa| \neq 0$, equation (19) can be solved with respect to the vertical beating factor $S = r_{2,min}/r_{2,max}$, keeping in mind that $0 \leq S \leq 1$. Thus, the measurement of the beating of the vertical signal gives a possibility of determining $|\kappa|$.

Equation (19) has been solved in three specific cases:

a) Sextupole resonance $2Q_v - Q_h = 9$

$$S = \frac{\Delta}{2\sqrt{2} |\kappa| r_{2,max}} \sqrt{-\frac{1}{2} + \left(\frac{1}{4} + \frac{8 |\kappa|^2 r_{2,max}^2}{\Delta^2} \right)^{\frac{1}{2}}} \qquad (20)$$

$S \leq 1$ only implies $r_{2,max} \geq 0$.

b) Skew sextupole resonance $2Q_h - Q_v = 9$

$$S = \frac{\Delta}{4 |\kappa| r_{2,max}} \; ; \quad S \leq 1 \text{ implies } r_{2,max} \geq \frac{\Delta}{4 |\kappa|} \, . \qquad (21)$$

c) Octupole resonance $2Q_h - 2Q_v = 0$

$$S = \sqrt{\frac{\Delta}{4 |\kappa| r_{2,max}^2}} \; ; \quad S \leq 1 \text{ implies } r_{2,max} \geq \sqrt{\frac{\Delta}{4 |\kappa|}} \, . \qquad (22)$$

These relations show the existence of a threshold in the kick amplitude for the resonances in which $|n_2| \leq |n_1|$ (for a vertical kick). In the ISR, these particular resonances have been observed after having excited them with the corresponding multipoles.

Acknowledgements

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