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OVERLAP KNOCK-OUT RESONANCES IN THE CERN INTERSECTING STORAGE RINGS (ISR)

by

J.P. Gourber, H.G. Hereward, S. Myers

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J.P. Gourber, H.G. Hereward, S. Myers
CERN
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Summary

Overlap knock-out arises from an overlap between frequencies present in a bunched beam and the betatron frequencies in a stack. The "single ring" effect is the interaction of a bunched beam with a stack in the same ring. Here the coupling forces are fairly linear and are transmitted by machine elements. The "two-ring" effect is the interaction of a bunched beam with a stack in the other ring. Here the coupling forces are non-linear since they are produced by the beam-beam interaction. A brief outline of the general theory of these effects is given. The single ring and two-ring dipole effects have been observed and shown to cause a large increase in the transverse size of the stacked beam. These observations are explained well by theoretical considerations. A special stacking program has greatly reduced the transverse blow-up, giving a smaller beam effective height and hence higher luminosity. Two-ring higher order effects have also been identified and explained by theory. The effects are small for the present ISR but can be an important consideration in the design of future machines.

1. Introduction

When the ELSA working line, with tune values close to integral resonances, was put into operation in 1974, transverse blow-up and some loss of protons in the top half of the stack were observed during stacking in the same (single ring effect) or in the other ring (two-ring effect). The name of "overlap knock-out" has been given to this phenomenon by which the stack is subjected to transverse kicks from the bunches. This produces blow-up when the longitudinal frequency spectrum of the bunches overlaps with the betatron frequency spectrum of the coasting stacked beam.

2. Elements of Theory

A general theory has been developed and will soon be published[2]. Here a more simple approach is presented for first order effects in which the perturbing fields are assumed constant over the beam aperture. The resulting expressions are later generalized in order to apply to higher order effects.

Consider N bunches (revolution frequency $\Omega_b/2\pi$) acting on a particle P (revolution frequency $\Omega_p/2\pi$, tune value Q). Figure 1 gives the geometry. The equation for the perturbed motion of the particle is

$$\frac{d^2\eta}{d\Phi^2} + Q^2\eta = \frac{1}{B\rho}\, Q^2\, \beta^{3/2}\, B(\theta_p, t)\ , \qquad (1)$$

with the usual Courant and Snyder[1] notations.

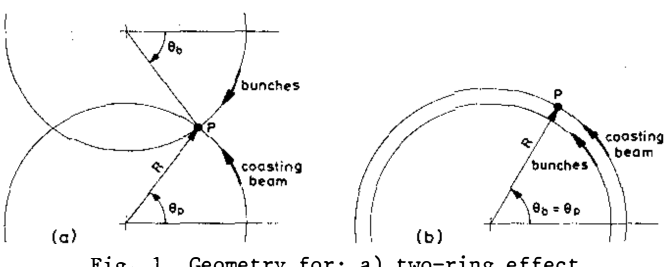

Fig. 1 Geometry for: a) two-ring effect
b) single ring effect

The perturbing field $B(\theta_p,t)$ can be written as the product $B^*(\theta_p) \times f(\theta_p,t)$ where $B^*(\theta_p)$ is the field corresponding to the peak current of the bunches and $f(\theta_p,t)$ is the modulation produced by the bunches at the azimuth (θ_p,θ_b) (Fig. 2). The dependence of f on θ_p comes from the time delay θ_b/Ω_b for bunch number 1 to reach the azimuth (θ_p,θ_b). The right hand side of equation (1) can be treated in the following way:

a) expand f in a Fourier series $\Sigma_m \ldots e^{jmN\Omega_b t}$ and change t in $\theta_p = \Omega_p t$ and then in $\Phi = \int_0^s \frac{ds}{Q\beta}$;

b) isolate the terms $e^{jmN\,\Omega_b/\Omega_p \Phi}$ and expand the remaining part, which has the period 2π in Φ, in a Fourier series $\Sigma_p \ldots e^{jp\Phi}$ which gives:

$$\frac{d^2\eta}{d\Phi^2} + Q^2\eta = \sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} a_{m,p}\; e^{j(mN\,\Omega_b/\Omega_p + p)\Phi}$$

with:

$$a_{m,p} = \frac{Rc_m}{2\pi B\rho}\int_0^{2\pi} \beta^{\frac{1}{2}}\, e^{j[Q\Phi - mN(\theta_p\Omega_b/\Omega_p - \theta_b)]}\, B^*\, d\theta_p\ . \qquad (2)$$

c_m is the amplitude of the m^{th} harmonic of the bunch frequency normalized to the peak value

$$c_m = \frac{N\Omega_b}{2\pi}\int_0^{\frac{2\pi}{N\Omega_b}} f(0,t)\, e^{-jmN\Omega_b t}\, dt\ . \qquad (3)$$

The forced oscillation amplitude η grows to infinity at resonance:

$$Q = mN\frac{\Omega_b}{\Omega_p} + p \quad \text{(m and p being positive or negative integers).} \qquad (4)$$

A particle crossing a resonance at a constant speed from an initial to a final state far from it gets a finite amplitude blow-up:

$$|\eta| = \frac{|a_{m,p}|}{2Q}\left[\frac{2\pi}{\frac{d}{d\Phi}\left(mN\frac{\Omega_b}{\Omega_p} + p - Q\right)}\right]^{\frac{1}{2}} \simeq \frac{|a_{m,p}|}{2Q}\left[\frac{2\pi\,\Omega_p}{\frac{d}{dt}\left(mN\frac{\Omega_b}{\Omega_p} + p - Q\right)}\right]^{\frac{1}{2}} \qquad (5)$$

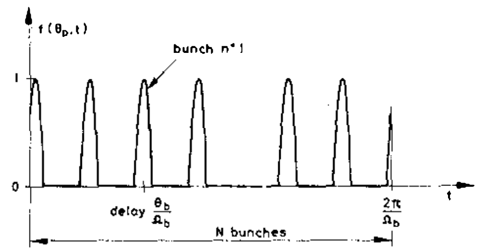

Fig. 2 Modulation by the bunches at azimuth (θ_p,θ_b)

For the two-ring effect, the interaction takes place in the intersections. For large horizontal crossing angle machines, such as the ISR, the horizontal forces vanish and only vertical resonances can be observed. The excitation amplitude for a single intersection region and a one-dimensional model of beams having a gaussian charge distribution is

$$|a_{m,p}| = \sqrt{2\pi}\; Q_v\, \frac{z_o}{\beta_v^{\frac{1}{2}}}\, c_m\, \Delta Q_v^*\, \mathrm{erf}\left(\frac{z_s}{\sqrt{2}\, z_o}\right), \qquad (6)$$

where z_s is the beam separation, z_o the r.m.s. beam height of the bunched beam and ΔQ_v^* the linear tune shift for the bunch peak current. Figure 3 shows the variation of $a_{m,p}$ with z. For the single ring effect, the direct or image bunch space charge forces are too small to explain the observed blow-up. This effect is better explained[3] by electromagnetic coupling caused by any electrostatic plates such as clearing electrodes (vertically) or electrostatic pick-ups (horizontally and vertically). The single ring effect is smaller in the vertical plane because the beam is usually well centred between any such vertical plates and hence the induced difference voltage is smaller.

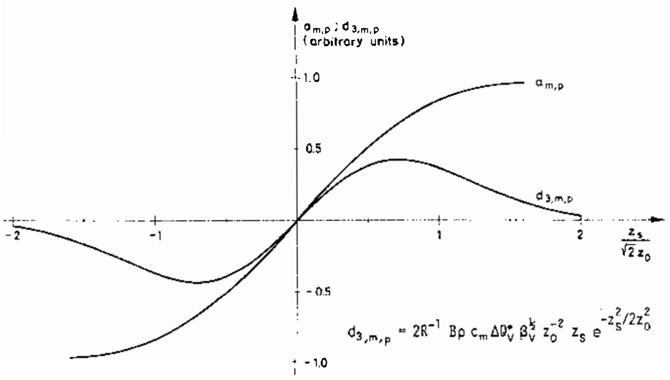

Fig. 3 Variation of $a_{m,p}$ and $d_{3,m,p}$ with the beam separation in one intersection

The assumption of constant fields across the aperture is reasonably accurate for single ring effects. However, for the two-ring effect, the beam-beam forces are strongly nonlinear and, therefore, it is necessary to expand the theory to higher order resonances of the overlap knock-out type. The global theory[2] follows and generalizes Guignard's theory[4] of classical nonlinear resonances. The concepts of invariants, stop bands, etc. remain the same. In the formulae, only the resonance condition and the excitation term are different:

$$nQ_v = mN \frac{\Omega_b}{\Omega_p} + p \ , \qquad (7)$$

$$d_{n,m,p} = \frac{c_m}{2\pi} \int_0^{2\pi} \beta_v^{n/2}\, e^{j[nQ_v\Phi_v - mN(\theta_p\Omega_b/\Omega_p - \theta_b)]}\, \frac{\partial^{n-1}B_x^*}{\partial z^{n-1}}\, d\theta . \qquad (8)$$

The variation of $d_{n,m,p}$ with the beam separation in one intersection is also indicated in Fig. 3 for third order resonances.

3. First Order Effects in the ISR

Since m is limited due to the bunch length and since Ω_b is close to Ω_p, first order effects predominate for tune values close to integers. With the ELSA working line (Fig. 4), the positions of the relevant ($m < 7$) overlap resonances have been drawn for bunches at injection. These positions have been calculated in terms of momentum difference ($\Delta p/p$) with respect to the central orbit by introducing in (4) $Q = Q_0 + Q' \Delta p/p$ and $\Omega = \Omega_0(1-|\eta|\Delta p/p)$ and by solving for ($\Delta p/p$), i.e.

$$(\Delta p/p)_p = (\Delta p/p)_b + \frac{p + mN \pm (Q_0 + Q'(\Delta p/p)_b}{mN|\eta| \mp Q'} \ , \qquad (9)$$

(Q_0 and Ω_0 being taken at central orbit).

During acceleration, the resonances move towards the top of the stack which is, therefore, subjected to excitation by all resonances, including those with low value of m.

The influence of various relevant parameters was measured using a stack of 7.5 A placed in the resonance region (Fig. 4) and tightly aperture restricted both horizontally and vertically. In this way, a blow-up was seen as a current loss. The results are given in Table 1 for the case where single pulses were injected and kept

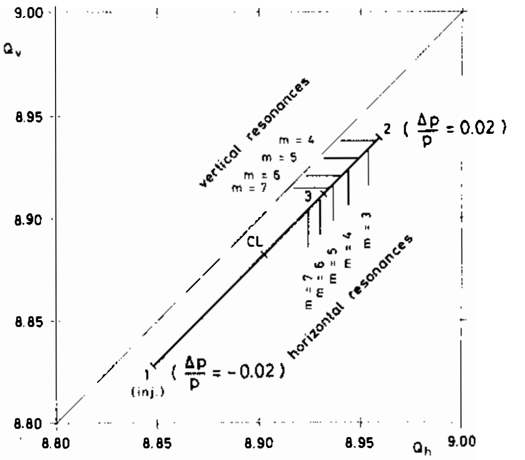

Fig. 4 ELSA Working Line (Points 2 and 3 indicate the limits of the 7.5 A stack)

bunched for a few seconds. Removal of the horizontal aperture restriction had no effect on the losses provoked by the two-ring effect whereas the measured single ring losses were reduced by about a factor 1.5. The dependence on the bunch length and the beam separations in the intersections reflect the variations of $a_{m,p}$ (2).

Table 1
Influence of the various parameters

Case number	Parameter changes	Current losses* single ring effect	Current losses* two-ring effect
1 (Reference)	16 kV beam sep.②	1	1
2 } beam separation	zero beam separat.		0.05
3 } beam separation	beam separation ①		0.005
4 } bunch length	V_{RF} = 8 kV	0.7	0.5
5 } bunch length	V_{RF} = 6 kV	0.5	0.5
6 } bunch length	V_{RF} = 4 kV	0.25	0.05
7 } bunch length	V_{RF} = 2 kV	0.06	0.03
8 } $\Delta Q_h = \Delta Q_v$	ΔQ = -0.01	0.8	0.7
9 } $\Delta Q_h = \Delta Q_v$	ΔQ = -0.02	0.3	0.55
10 } $\Delta Q_h = \Delta Q_v$	ΔQ = -0.05	0.3	0.2
11 } $\Delta Q_h = \Delta Q_v$	ΔQ = +0.015	1.75	2.0

* in arbitrary units.

(For cases 2 to 11, only the changes in parameters with respect to case 1 are indicated. Beam separations 1 and 2 mean 8 mm beam separation in all intersections with the same sign ① or opposite signs ② in diametrically opposed intersections.)

For example, in case 3 the contribution of each intersection is maximum and rather independent of small orbit distortions (Fig. 3), but the global effect is null because of the π-phase shift between two diametrically opposed intersections. The losses may be substantially reduced by increasing the bunch length which reduces the high frequency components in the bunch spectrum (Fig. 5). The losses are smaller when the Q-values are decreased, i.e. when the most dangerous resonances (with a low m) are eliminated from the stack(equation 9).

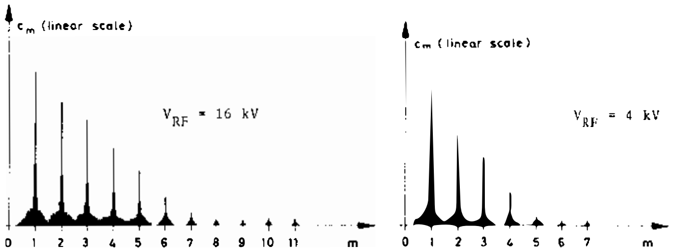

Fig. 5 Bunch Frequency Spectrum

In another experiment, a small and vertically aperture limited beam of 100 mA was left circulating at the top of ELSA while moving bunches in the other ring. Figure 6 shows localized enhancements of current loss when overlap knock-out resonances are crossed, i.e. for $(\Delta p/p)_b$ given by equation (9). The resonance widths appear quite large mainly because of the momentum spread $<(\Delta p/p)_p> \simeq 1$ °/oo of the test beam, but also because of the use of 20 bunches occupying only 2/3 of the ring circumference (the main harmonics of the bunch frequency have side bands spaced by the revolution frequency).

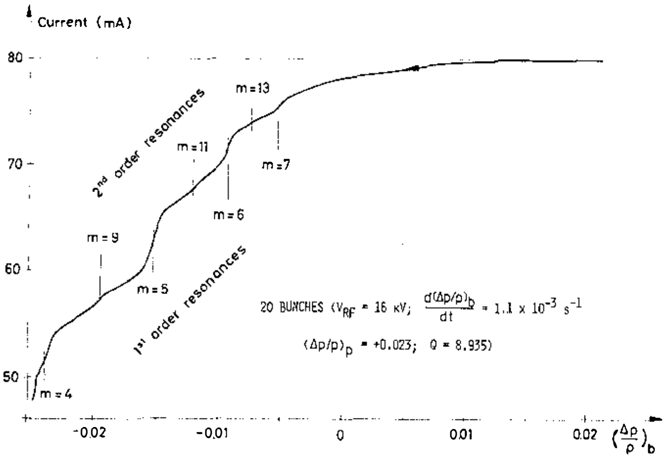

Fig. 6 1st and 2nd Order Overlap Knock-out Resonances

For normal machine operation, a special "double-decay" RF acceleration program has been developed in order to reduce the transverse blow-up produced during stacking. With this program, the full RF cavity voltage (16 kV) is applied in order to trap the injected bunches; as soon as the bunches are trapped, the cavity voltage is reduced (4 kV) in order to lengthen the bunches and thereby eliminate the harmful bunch harmonics. The pulse is then accelerated across the aperture in these intermediate buckets until the stack is approached. At this point, the RF voltage is again reduced so that the buckets fit tightly around the bunches for maximum longitudinal phase plane density. This technique used in conjunction with a slight reduction in Q during stacking (0.015) has been successful in eliminating the vertical beam blow-up and, therefore, allowed higher luminosities due to reduced effective beam height. In the horizontal plane, due to the higher Q-values and the increased acceleration time (due to the reduced acceleration voltage), the effect of overlap knock-out has not been totally eliminated but has been significantly reduced. The resulting slight increase in the horizontal emittance at the stack top is not harmful for ISR physics beams since the stacks are later centred in the aperture after removal of the injection system. Consequently, for protons, the ELSA working line may be used for high intensity stacking without any harmful effects on the beam quality.

However, overlap knock-out still remains a severe limitation when deutrons and protons are stacked together. For the same momentum, the ratio $\Omega_{deutrons}/\Omega_{protons}$ is such that the bunch frequency m = 1 excites a resonance when using the ELSA working line which, therefore, must be replaced by the 8C line (Q_o = 8.63).

4. High Order Resonances

According to (7), two-ring, n^{th} order resonances must be visible for tune values close to nQ_v = integer. In Fig. 6, the intermediate losses correspond indeed to crossing 2nd order overlap resonances whose positions are still given by equation (9) in which Q_o and Q' are replaced by nQ_o and nQ', respectively. An experiment,

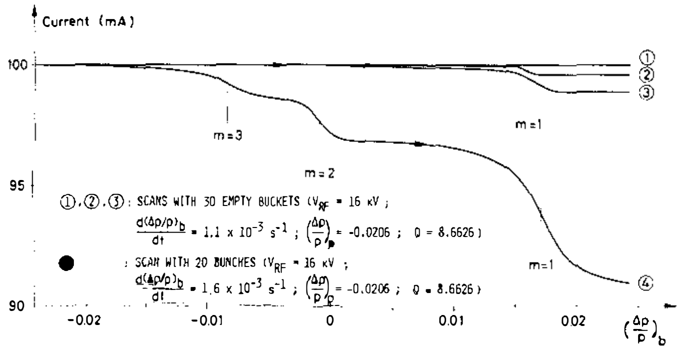

Fig. 7 3rd Order Overlap Knock-out Resonances

similar to that of Fig. 6, has been made to investigate 3rd order overlap knock-out resonances near the classical resonance $3Q_v = 26$. A zero Q' was used to reduce the resonance widening due to $<(\Delta p/p)_p>$. Current losses corresponding to resonances with m = 1, 2, 3 are visible from curve 4 in Fig. 7. Smaller losses have also been seen for "negative bunches", i.e. when moving empty buckets in a large stack (curves 1, 2, 3 in Fig. 7). The low harmonic content explains the presence of only m = 1 resonances. The variation with beam separation in two diametrically opposed intersections is in agreement with the phase of $d_{3,m,p}$ (equation 8). The amount of vertical blow-up corresponding to a certain current loss ΔI can be calculated if the loss ΔI_o due to the vertical aperture restriction z_a is known $\frac{\Delta z}{z_a} = \frac{\Delta I}{\Delta I_o} \frac{1}{2 \ln (\Delta I_o/I_o)}$.

For curve 4, the total loss corresponds to a blow-up of 0.1 at the limiting aperture $z_a \simeq 2\ z_o$.

Conclusion

In the ISR, the blow-up due to first order overlap knock-out resonances has been eliminated for proton-proton operation. However, they remain a severe limitation when stacking deutrons and protons with the standard ELSA working line. Higher order effects which have been studied for very special tune conditions can also cause serious difficulties when different particles or momenta are considered. For future large proton rings, the topology of these resonances can be even worse and serious problems of background may occur when bunched electron beams collide with coasting proton beams.

Acknowledgements

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