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Systematics of quarkonium production

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Abstract

Quarkonium production in high-energy reactions is found to exhibit a behaviour more universal than that expected from velocity scaling. Total rates of quarkonia produced in hadronic interactions as well as Feynman-x and transverse momentum distributions can be described over the full range of accessible energies ($15 \lesssim \sqrt{s} \lesssim 1800\,\mathrm{GeV}$) by two-stage processes. The quarkonium production cross section factors into a process-dependent short-distance part and a single long-distance matrix element. The first part describing the production of a free quark–antiquark pair is the perturbatively calculated subthreshold cross section. The non-perturbative factor turns out to be universal, giving the model great predictive power. Furthermore we estimate the fraction of the heavy-quark cross section leading to quarkonium for both the charm and bottom systems. Finally, we comment on quarkonium photoproduction.

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Recently much effort has been devoted to explain quarkonium production in a new factorization approach [1]. Any quarkonium cross section is given by the sum of infinitely many terms, each of which factors into the product of two terms. The first one, calculable as a series in $\alpha_s(\mu)$ where μ is of the order of the relevant hard scale, is the cross section to produce a free quark–antiquark pair ($Q\bar{Q}$) in a particular angular momentum and colour state ${}^{2S+1}L_J^{(c)}$ (in the spectroscopic notation and $c = 1$ ($c = 8$) denotes a colour-singlet (colour-octet) state). The second factor determines the probability that such a $Q\bar{Q}$ pair binds to form a quarkonium $H(nJ^{PC})$ of given total spin J, parity P, and charge conjugation C. The factorization approach becomes meaningful through the velocity-scaling rules, which determine the relative importance of the various long-distance matrix elements (ME). At any order in v, the velocity of the heavy quark within the bound state, the quarkonium cross section is hence given by a finite number of contributions [2].

This results in an expansion of the quarkonium cross section in both $\alpha_s(\mu)$ and v. Although this "velocity-scaling model" (VSM) suggests an explanation of quarkonium production at the Tevatron [1], its weak point is the fact that currently the non-perturbative ME cannot be calculated in QCD[1]. A crucial test of the approach is therefore the determination of the MEs from as many different high-energy reactions as possible. This endeavour is, however, rendered more difficult by the fact that, in general, different combinations of MEs arise. Nonetheless, preliminary attempts indicate that the velocity-scaling is not perfect: J/ψ production at the Tevatron requires considerably larger $c = 8$ MEs [4] than J/ψ hadroproduction [5] or the z-distribution in J/ψ photoproduction [6]. Also the hadroproduction ratio χ_{c1}/χ_{c2} is too low compared to data since χ_{c1} production is clearly disfavoured by either a power of $\alpha_s(m_c)$ or a factor $(v^2)^2$ compared to χ_{c2}. Last but not least, the J/ψ (non-) polarization is difficult to account for in the VSM [5, 7].

Some time ago we [8] have shown that existing quarkonium production data at fixed-target energies are, in fact, compatible with the assumption that the non-perturbative transition of the $Q\bar{Q}$ pair to quarkonium is more universal than expected from the velocity-scaling rules. Indeed, low-energy data are well reproduced if the (infinite) sum of short-distance coefficients times long-distance MEs is truncated to a single term

$$\sigma\left[H\left(nJ^{PC}\right)\right] = F[nJ^{PC}]\,\tilde{\sigma}\left[Q\bar{Q}\right] \ . \tag{1}$$

In this letter we shall demonstrate that the colour-evaporation model (CEM) of eq. (1) also accounts for quarkonium production at the Tevatron and comment upon its application to photoproducton of quarkonia. We emphasize that in contrast to the VSM, the number of non-perturbative parameters is minimal, hence the model possesses great predictive power.

The short-distance part of eq. (1) is the perturbative subthreshold cross section expanded in powers of $\alpha_s(\mu)$ where $\mu \propto m_Q$. Specifying to charm, the cross section is

$$\tilde{\sigma}\left[\mathrm{c\bar{c}}\right] = \int_{2m_c}^{2m_D} \mathrm{dM_{c\bar{c}}}\,\frac{\mathrm{d}\sigma\left[\mathrm{c\bar{c}}\right]}{\mathrm{dM_{c\bar{c}}}} \tag{2}$$

where $\sigma[\mathrm{c\bar{c}}]$ is the *spin- and colour-averaged* open heavy-quark pair production cross section. The CEM is hence based on two ingredients. First, the quarkonium dynamics are assumed to be identical to those of low mass open $Q\bar{Q}$ pairs. All perturbative QCD corrections apply to the short-distance cross section and hence are identical for open

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[1] Attempts to calculate decay MEs on the lattice have just started [3].

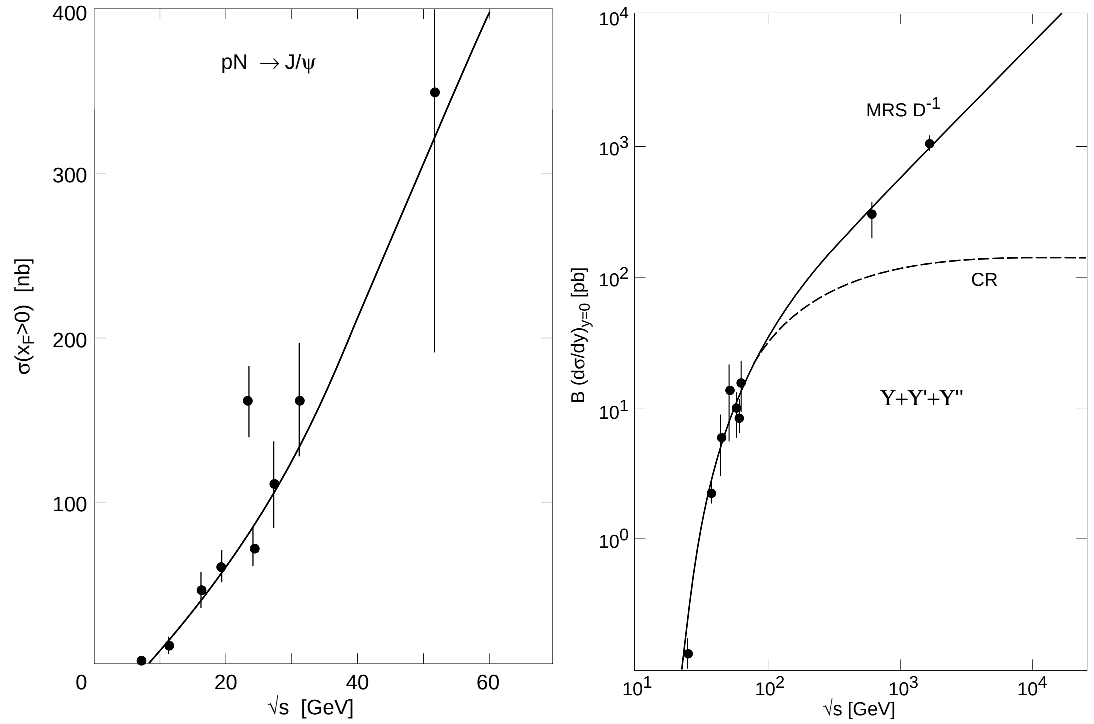

Figure 1: Left: The J/ψ production cross section $\sigma^{\mathrm{pN}}[J/\psi]$ for $x_F > 0$, calculated with MRS D-' PDF, compared to data [9]. Right: Energy dependence of Υ production $\sum_n \mathrm{Br}[\Upsilon(\mathrm{nS}) \to \mu^+\mu^-]\mathrm{d}\sigma[\Upsilon(\mathrm{nS})]^{\mathrm{pN}}/\mathrm{dy}$ at $y = 0$ compared to data [10, 11]; the predictions with MRS D-' and GRV HO PDF essentially coincide. Also shown (CR) is the phenomenological fit of [12]; from [8].

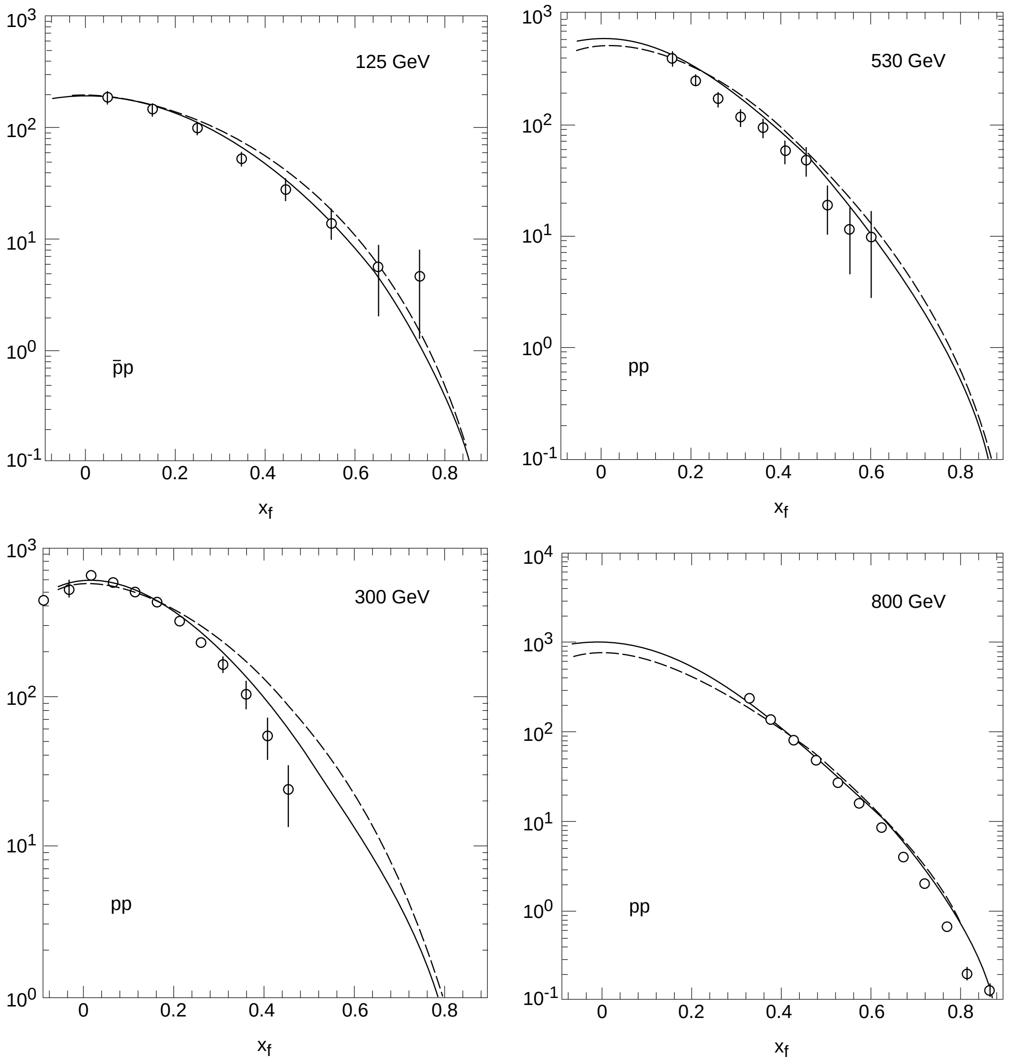

Figure 2: The J/ψ longitudinal momentum distributions compared to $\overline{\mathrm{p}}N$ and pN data [13], with $x_F = p_L[J/\psi]/p_{max}[J/\psi]$; results obtained with the MRS D-' (GRV) PDF are denoted by a solid (dashed) line; from [8].

and bound heavy-quark production. Second, although the $Q\overline{Q}$ pair is produced at short distances in different states (distinguished by colour, angular momentum, relative momentum) and their relative production rates may (and will) be different for different high-energy collisions, it is only the average over many long-distance matrix elements, combined in the long-distance factor $F[nJ^{PC}]$, that determines the probability to form a specific bound state. Necessarily, the factor F needs to be universal, i.e. process- and kinematics-independent.

To illustrate the success of the CEM, in Fig. 1 we compare the prediction for the total J/ψ and Υ production rates with data. Note that the model uniquely predicts the shape of the energy dependence while the absolute normalization at low energies fixes the non-perturbative factor F. Fig. 2 shows the prediction of fixed-target x_F distributions. There is remarkable agreement over a wide energy range, from low-energy $\mathrm{p\overline{p}}$ collisions where valence $q\overline{q}$ fusion dominates up to high-energy pp collisions dominated by gluon-gluon fusion. Note that both the shape and normalization of the x_F distributions are obtained from the model once F has been fixed by the energy dependence.

The long-distance factors determined from the low-energy total cross sections in Figs. 1 are

$$\begin{aligned} F_{tot}[J/\psi] &= 2.5\% \\ \sum_{n=1}^{3} \mathrm{Br}[\Upsilon(nS) \to \mu^+\mu^-]\, F_{tot}[\Upsilon(nS)] &= 1.6 \times 10^{-3} \ . \end{aligned} \tag{3}$$

Here the subthreshold cross sections were calculated in next-to-leading order (NLO) using the MRS D-' parametrization [14] of the parton-distribution functions (PDF) with renormalization and factorization scales μ_R and μ_F chosen to be $\mu_R = \mu_F = 2m_c = 2.4\,\mathrm{GeV}$ and $\mu_R = \mu_F = m_b = 4.75\,\mathrm{GeV}$, respectively[2]. The results in eq. (3) are rather insensitive to variations of the parameters in the open heavy-quark cross section, if they are tuned to the open heavy-flavour total cross section data. For instance, the GRV HO parametrization [15] with $\mu_R = \mu_F = m_c = 1.3\,\mathrm{GeV}$ leads to very similar results: the smaller subthreshold region is basically compensated by the larger two-loop α_s value, 0.298 for GRV HO ($\Lambda_4 = 0.2\,\mathrm{GeV}$) compared to 0.243 for MRS D-' ($\Lambda_4 = 0.23\,\mathrm{GeV}$). Note, however, that the long-distance factors will be considerably larger if the open heavy-quark cross section is calculated to leading order (LO) only, a factor 2.2 larger for J/ψ and a factor 1.9 larger for Υ.

In Fig. 3 we show the transverse momentum distributions of prompt charmonium production (i.e. not originating from B decays) at the Tevatron energy $\sqrt{s} = 1.8\,\mathrm{TeV}$. The bottomonium transverse momentum distributions are given in Fig. 4. The normalizations for the various states are given in table 1, obtained from eq. (3) using the measured cross section ratios [8] and the branching ratio to muon pairs [18]. Good agreement with data is found, typically better than 50%. Note that the CEM prediction shown in figs. 3 and 4 is based on the subthreshold cross section calculated to $O(\alpha_s^3)$, which is NLO for the p_T integrated cross section but LO only for the p_T distribution[3]. In the absence of the NLO corrections to the p_T spectrum one might apply a K factor to account for the unknown higher-order corrections or simply use F-values extracted in LO (two such examples are

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[2] To calculate the NLO subthreshold cross section, we use the program of Mangano, Nason, and Ridolfi [16], restricting the $Q\overline{Q}$ mass range.

[3] In the calculation of the p_T distribution, we used $\mu_R^2 = \mu_F^2 = n^2[m_Q^2 + (p_{T,Q}^2 + p_{T,\overline{Q}}^2)/2]$ with $n = 2$ for charm and 1 for bottom consistent with the scale advocated for open $Q\overline{Q}$ production [16].

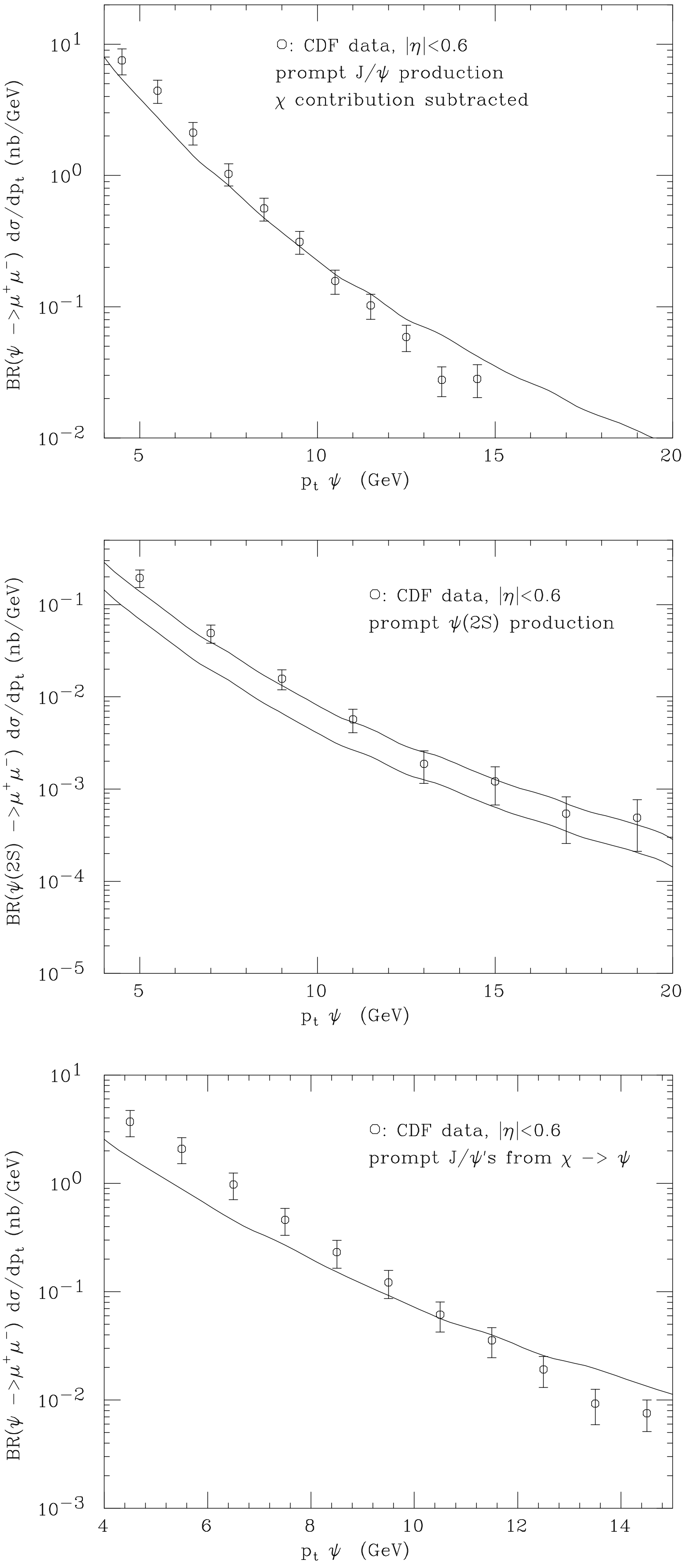

Figure 3: Transverse momentum distributions of charmonia compared to CDF data [17]. The upper curve for $\psi(2S)$ contains an extra K-factor of 2.

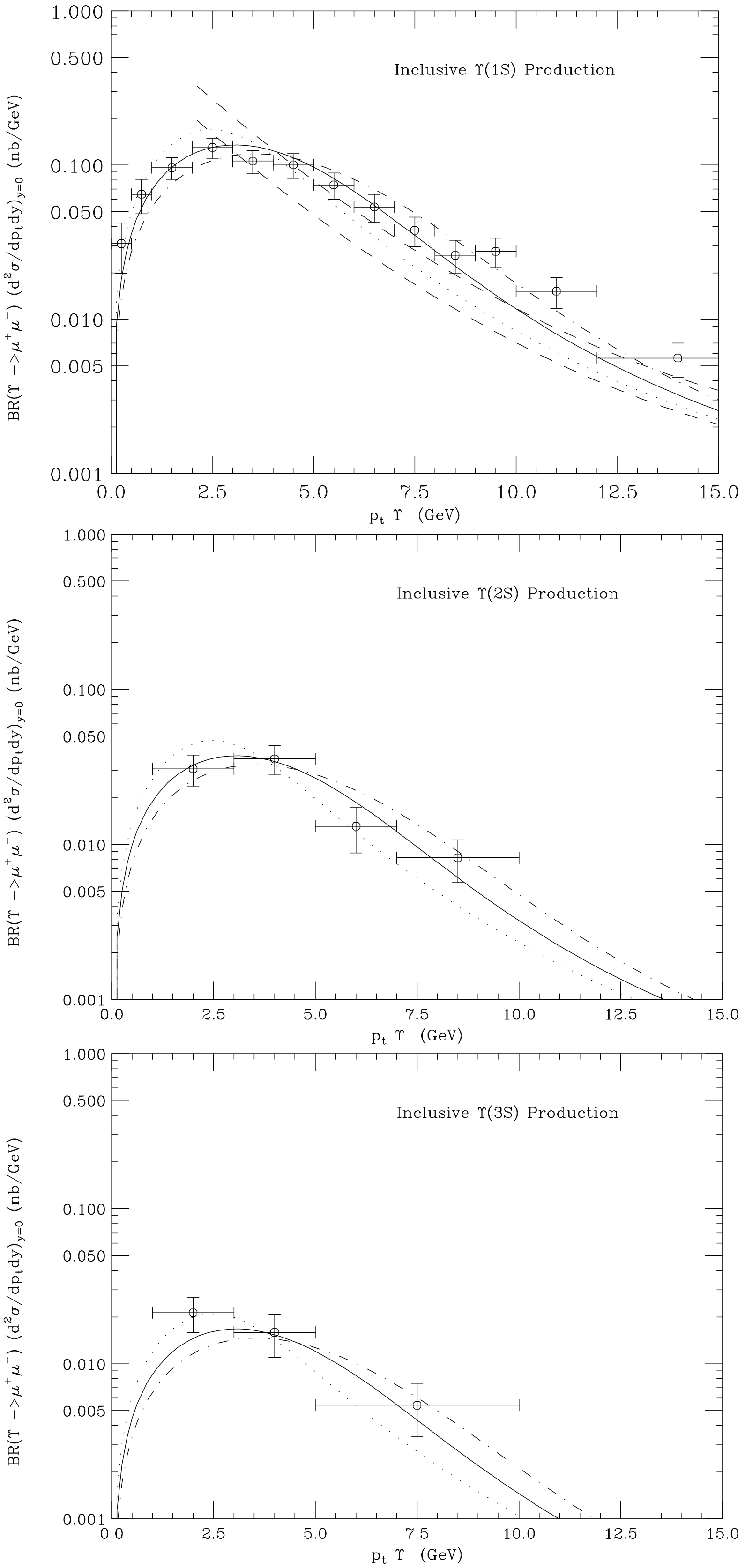

Figure 4: Transverse momentum distributions of bottomonia compared to CDF data [11] for various values of the intrinsic transverse momentum with F as in table 1: $\langle k_T \rangle = 1.25\,\mathrm{GeV}$ (dotted), $2.0\,\mathrm{GeV}$ (solid), $2.5\,\mathrm{GeV}$ (dot-dashed). The $\Upsilon(1S)$ prediction without smearing $\langle k_T \rangle = 0\,\mathrm{GeV}$ is shown as dashed lines, the upper curve containing an extra K-factor of 1.9.

J/ψ	$\psi(2S)$	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	$\sum_J \mathrm{Br}[\chi_{\rm cJ} \to J/\psi\, X]\, F[\chi_{\rm cJ}]$
2.5	0.35	4.6	2.4	0.78	1.0

Table 1: Long-distance factors $F_{\rm tot}$ from eq. (1) expressed in percent for 1^{--} states and the sum of the inclusive $\chi_{\rm cJ}$ production rates (i.e. including cascade decays). The factors which multiply the NLO subthreshold cross sections do not include the branching ratio into lepton pairs. For LO calculations the above numbers should be multiplied by 2.2 for charmonia and 1.9 for bottomonia.

given in Figs. 3 and 4). In either case one expects larger cross sections so that our estimates are rather conservative.

The CEM prediction for $\psi(2S)$ is about a factor of two lower than the data. This is a simple consequence of the fact that at high p_T the $\psi(2S)$-to-J/ψ ratio measured at the Tevatron is about twice as large as that observed at fixed-target energies for the p_T-integrated cross section. Multiplying the CEM prediction by a factor of two produces very good agreement. More precise data will show whether this is a systematic effect that would require refinements of the CEM.

The CEM prediction for the p_T distributions based on fixed-order perturbation theory cannot be trusted for $p_T \lesssim m_Q$. A correct treatment of the low-p_T region requires soft-gluon resummation and the inclusion of intrinsic transverse momenta, analogous to the Drell–Yan case. The effect of soft-gluon resummation can be mimicked through an effective, larger value of the average intrinsic transverse momentum $\langle k_T \rangle$. Fig. 4 shows that inclusion of $\langle k_T \rangle$ smearing results in good agreement with data down to very low p_T.

The information of table 1 can be used to estimate the total bound-state probability. In the case of charmonium this requires assumptions about the $\eta_c(nS)$ and $h_c(1P)$ cross sections. In the case of bottomonium, we also need assumptions about the ratios of the $\chi_{\rm bJ}(nP)$ to $\Upsilon(nS)$ cross sections. Taking the latter to be equal to that measured in the charmonium system and assuming that both the S- and P-wave state cross sections are proportional to $2J+1$ and disregarding possible $\mathrm{b\bar{b}}$ D-wave states we estimate

$$\begin{aligned} \sum_i F[i] &\approx (8-10)\% \ \text{charm} \\ &\approx (17-32)\% \ \text{bottom} \ . \end{aligned} \tag{4}$$

The dominant part of the subthreshold charm cross section produces open charm. This fraction is considerably reduced in the bottom system: We observe a significant increase of the bound state fraction with increasing quark mass. The fact that the total charmonium cross section is just $1/(1+8)$ of the subthreshold cross section must therefore be considered as fortuitous. This ratio was recently advocated as universal for colour-singlet production [19] giving the fraction of both diffractive events in deep-inelastic ep scatterings and bound states in heavy-quark production. Our analysis shows that bound state production does not obey this rule. Moreover there is no reason to expect this ratio to hold. In fact, considering the complete system rather than restricting to the $Q\overline{Q}$ pair suggests that the colour-singlet fraction is $1:1$ rather than $1:9$ [20].

Finally we discuss photoproduction of charmonium. We first note that the inelastic J/ψ cross section is defined only within cuts. These cuts are necessary since the quasielastic process, $\gamma \mathrm{p} \rightarrow \mathrm{J}/\psi + \mathrm{p}$, and the forward-elastic reaction, $\gamma \mathrm{p} \rightarrow \mathrm{J}/\psi + \mathrm{X}$, where the J/ψ is isolated in rapidity, cannot be described in perturbative QCD using a (single) gluon distribution in the nucleon. In order to stay away from diffractive production one typically restricts z (E_ψ/E_γ in the nucleon rest frame) to values less than 0.9 and applies additional cuts as, e.g., on p_T or on the number of tracks. Since the cross section rises quickly towards $z = 1$, $F[J/\psi]$ is not well defined here[4]. Comparing the cross sections for open charm and (inelastic) J/ψ photoproduction we find

$$\begin{aligned} F[J/\psi] &\approx (1-2.5)\% \quad \text{at low } \sqrt{\mathrm{s}} \\ &\approx (0.5-1.4)\% \quad \text{at high } \sqrt{\mathrm{s}}\ , \end{aligned} \tag{5}$$

numbers in reasonable agreement with the value obtained from hadroproduction. While the J/ψ long-distance factor thus appears to be universal, photoproduction of χ_{cJ} seems to be problematic for the CEM: NA14 [21] puts an upper limit of about 8% for the fraction of J/ψ's from χ_{cJ} decays while hadroproduction experiments suggest a value four times greater. So far these measurements are not very precise, but, if confirmed, χ_{cJ} photoproduction might indicate limitations on the CEM, requiring refinements of its simplest variant.

In summary, we find an impressive agreement between data on quarkonium production in hadronic collisions and the CEM: Total cross sections, x_F distributions, and p_T spectra are all well described by the assumption that the non-perturbative bound-state formation is governed by an average, universal, long-distance factor. Since only a single non-perturbative ingredient is required for any given bound state, the CEM has great predictive power. Two possible deviations from this simple scenario have been pointed out: a difference in the $\psi(2S)/J/\psi$ ratio in hadronic collisions and a different χ_{cJ} production fraction in hadro- and photoproduction. More data, also for bottomonium, are eagerly awaited.

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