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DISCRETE AMBIGUITIES IN EXTRACTING WEAK PHASES FROM B DECAYS [1]

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ABSTRACT

Phases of elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, as obtained using decays of B mesons to $\pi^+\pi^-$, $\pi^\pm K^\mp$, and $\pi^+ K^0$ or $\pi^-\overline{K}^0$, are shown to have a class of discrete ambiguities. In most cases these can be eliminated using other information on CKM phases.

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A promising source of information about the mechanism of CP violation is the study of rate asymmetries in the comparison of B and $\overline{B}$ decays to specific final states. These asymmetries often involve unknown strong-interaction phase shifts. A method was recently proposed [1] to circumvent this difficulty using time-dependent B^0 and $\overline{B}^0$ decays and time-integrated rates for $B^0 \to \pi^- K^+$, $\overline{B}^0 \to \pi^+ K^-$, and $B^+ \to \pi^+ K^0$ or $B^- \to \pi^- \overline{K}^0$. (The last two rates are predicted to be equal.) Within an assumption of flavor SU(3) for strong phase shifts and for diagrams dominated by tree (but not penguin) graphs, it was possible to exhibit six equations in six unknowns and thus to demonstrate the existence of solutions for all parameters of interest. However, the Monte Carlo method employed in Ref. [1] indicated the presence of discrete ambiguities. Using numerical methods in the present note, we clarify these ambiguities, and show that they may be eliminated for the most part using other information already known about phases of the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

The amplitudes of the processes $B^0 \to \pi^+\pi^-$ and $B^0 \to \pi^- K^+$ are defined as $A_{\pi\pi}$ and $A_{\pi K}$, while that for $B^+ \to \pi^+ K^0$ is defined as A_+. The amplitudes for the corresponding charge-conjugate decay processes are denoted by $\overline{A}_{\pi\pi}$, $\overline{A}_{\pi K}$, A_-, respectively. It was shown in Ref. [1] that one can measure six independent combinations of the following six parameters: the strangeness-preserving tree amplitude $\mathcal{T}$, the strangeness-preserving and -violating penguin amplitudes $\mathcal{P}$ and $\tilde{\mathcal{P}}'$, the weak phases α and γ, and the strong phase δ. These combinations may be expressed as

$$A \equiv \frac{1}{2}(|A_{\pi\pi}|^2 + |\overline{A}_{\pi\pi}|^2) = \mathcal{T}^2 + \mathcal{P}^2 - 2\mathcal{T}\mathcal{P}\cos\delta\cos\alpha \quad , \tag{1}$$

$$B \equiv \frac{1}{2}(|A_{\pi\pi}|^2 - |\overline{A}_{\pi\pi}|^2) = -2\mathcal{T}\mathcal{P}\sin\delta\sin\alpha \quad , \tag{2}$$

$$C \equiv \mathrm{Im}\,(e^{2i\beta} A_{\pi\pi}\overline{A}^*_{\pi\pi}) = -\mathcal{T}^2\sin 2\alpha + 2\mathcal{T}\mathcal{P}\cos\delta\sin\alpha \quad , \tag{3}$$

$$D \equiv \frac{1}{2}(|A_{\pi K}|^2 + |\overline{A}_{\pi K}|^2) = (\tilde{r}_u\mathcal{T})^2 + \tilde{\mathcal{P}}'^2 - 2\tilde{r}_u\mathcal{T}\tilde{\mathcal{P}}'\cos\delta\cos\gamma \quad , \tag{4}$$

$$E \equiv \frac{1}{2}(|A_{\pi K}|^2 - |\overline{A}_{\pi K}|^2) = 2\tilde{r}_u\mathcal{T}\tilde{\mathcal{P}}'\sin\delta\sin\gamma \quad , \tag{5}$$

$$F \equiv |A_+|^2 = |A_-|^2 = \tilde{\mathcal{P}}'^2 \quad . \tag{6}$$

Here $\tilde{r}_u \equiv r_u f_K/f_\pi$, where $r_u \equiv |V_{us}/V_{ud}| = 0.23$. The quantities $A - C$ are measured in time-dependent rates for B^0 or $\overline{B}^0 \to \pi\pi$, D and E by comparing rates for $B^+ \to \pi^- K^+$ and $B^- \to \pi^+ K^-$, and F via the rate for the process $B^+ \to \pi^+ K^0$, which is predicted to be dominated by a single penguin amplitude and hence to have the same rate as $B^- \to \pi^- \overline{K}^0$.

We considered [1] a set of representative CKM elements parametrized [2] as shown in Table I, where ρ and η are the real and imaginary parts of $V^*_{ub}/|V_{cd}V_{cb}|$. For each of these points, the phase shifts $\delta = 5.7°$, $36.9°$, $84.3°$, $95.7°$, $143.1°$, and $174.3°$ were chosen. (We shall not be concerned here with $\sin\delta = 0$, a singular case in which the above equations no longer provide sufficient information.) Monte Carlo results indicated that the equations sometimes had more than one solution.

We have used an exact numerical method to obtain all solutions of Eqs. (1-6) for the points p_1, p_2, p_3 and the six phases δ. We express the five observables A, B, C, D, E

Table I: Points in the (ρ, η) plane and angles of the unitarity triangle.

Point	ρ	η	α (deg.)	β (deg.)	γ (deg.)
p_1	-0.30	0.15	20.0	6.6	153.3
p_2	0	0.35	70.7	19.3	90.0
p_3	0.36	0.27	120.3	22.9	36.9

in terms of five unknowns $\mathcal{T}, \mathcal{P}, \alpha, \gamma, \delta$ by substituting the measured value of $\tilde{\mathcal{P}}' = \sqrt{F}$ and noting that $\tilde{r}_u$ also is well-measured. The solution then proceeds as follows:

- We eliminate γ from Eq. (4) and Eq. (5) to get

$$D = \tilde{r}_u^2 \mathcal{T}^2 + F - 2\tilde{r}_u \mathcal{T}\sqrt{F}\cos\delta \left(1 - \frac{E^2}{4\tilde{r}_u^2 \mathcal{T}^2 F \sin^2\delta}\right)^{1/2} . \quad (7)$$

  When both the sides are squared, this equation becomes a quadratic in $x \equiv \sin^2\delta$ whose coefficients depend only on $\mathcal{T}$. For each of the solutions which is real and lies between 0.0 and 1.0 (since $x = \sin^2\delta$),

- we eliminate α from Eq. (1) and Eq. (2) to get

$$A = \mathcal{T}^2 + \mathcal{P}^2 - 2\mathcal{T}\mathcal{P}\cos\delta \left(1 - \frac{B^2}{4\mathcal{T}^2\mathcal{P}^2 \sin^2\delta}\right)^{1/2} . \quad (8)$$

  When both the sides of this equation are squared, we obtain a quadratic in $y \equiv \mathcal{P}^2$ whose coefficients involve only $\mathcal{T}$ and x, which is a known function of $\mathcal{T}$. We proceed with those values of y that are real and positive.

- Now we know all the other unknowns $\mathcal{P}, \alpha, \gamma, \delta$ as explicit functions of a single unknown $\mathcal{T}$. We can now check for those values of $\mathcal{T}$ which satisfy Eq.(3). We can increase the accuracy of our solutions as much as we want by decreasing the step size in $\mathcal{T}$ and using the *zero crossing algorithm*, where a solution corresponds to that value of $\mathcal{T}$ where increasing $\mathcal{T}$ by a small amount changes the sign of [L.H.S. - R.H.S.] in Eq. (3).

As many as 8 solutions were found for some sets of input parameters. The results are summarized in Tables II – IV for points $p_1 - p_3$. We calculate $A - E$ for the input values $\mathcal{T} = 1$, $\tilde{\mathcal{P}}' = 1$, $\mathcal{P} = \tilde{\mathcal{P}}' r_u \sin\gamma / \sin\alpha$ [assuming flavor SU(3) for the input], and the input strong phases shown in the Tables. The equations are then inverted using the method described above to obtain the output phases. In some cases the numerical algorithm gives two closely related or identical sets of output phases; we have indicated these with equal numbers. These are probably identical solutions arrived at through two different branches of the step-by-step method described above, with small differences associated with rounding errors. Nonetheless, we feel this point could benefit from further study.

Table II: Output values of weak and strong phases, for given values of input strong phases, in degrees, for the point p_1 with $\alpha_{\text{in}} = 20.0°$ and $\gamma_{\text{in}} = 153.3°$.

δ_{in}	α_{out}	γ_{out}	δ_{out}	Notes
5.7	20.0	153.4	5.7	(a)
	10.4	106.1	1.4	(b)
36.9	20.0	153.4	36.9	(a)
84.3	20.0	153.4	84.3	(a)
	70.6	153.8	84.6	(c)
	21.5	28.7	97.5	(b)
	71.8	26.4	95.5	(b)
	59.3	93.1	23.6	(d)
	29.9	70.8	136.5	(b)
	82.8	83.4	152.3	(d)
95.7	18.8	25.0	83.6	(b)
	71.5	24.8	83.5	(b)
	20.0	153.4	95.7	(a)
	72.8	155.0	96.4	(c)
	60.8	82.6	22.7	(d)
	29.3	91.9	140.3	(b)
	83.2	96.0	154.0	(e)
143.1	72.2	16.6	48.6	(b)
	14.4	14.6	50.8	(b)
	20.0	153.2	143.3	(a,1)
	20.0	153.4	143.1	(a,1)
	76.3	162.9	132.0	(c)
	66.4	49.6	15.7	(b)
	81.7	132.6	162.4	(c)
174.3	74.1	3.1	40.2	(b)
	15.0	3.0	41.1	(b)
	15.9	176.7	140.9	(c)
	75.0	176.9	139.9	(b)
	20.0	153.4	174.3	(a)
	81.1	140.7	176.8	(c)

(a) Correct solution; (b) $\beta > \pi/4$; (c) $\alpha + \gamma > \pi$;
(d) potential ambiguity; (e) β or γ too small.
Numbers denote solutions probably identical to one another.

Table III: Output values of weak and strong phases, for given values of input strong phases, in degrees, for the point p_2 with $\alpha_{\rm in} = 70.7°$ and $\gamma_{\rm in} = 90.0°$. Notes are as for Table II.

$\delta_{\rm in}$	$\alpha_{\rm out}$	$\gamma_{\rm out}$	$\delta_{\rm out}$	Notes
5.7	5.7	173.8	88.7	(e)
	84.6	173.9	89.1	(c)
	5.8	6.3	91.6	(b)
	70.7	90.0	5.7	(a)
	171.5	82.0	170.7	(c)
	17.8	62.9	158.1	(b)
	98.6	89.2	174.0	(c)
36.9	15.2	127.2	82.6	(d)
	82.0	39.0	91.0	(b)
	70.7	90.0	36.9	(a)
	92.5	88.8	140.6	(c)
84.3	71.1	90.7	86.3	(a,1)
	70.7	90.0	84.3	(a,1)
95.7	69.2	80.2	89.1	(d)
	69.5	98.9	90.8	(d)
	24.2	87.5	71.3	(b)
	65.8	88.0	74.2	(d)
	19.8	83.7	52.1	(b)
	70.7	90.0	95.7	(a)
143.1	30.8	34.3	88.8	(b)
	61.3	33.7	88.5	(b)
	31.1	145.4	91.1	(d)
	61.0	146.2	91.5	(c)
	50.2	86.2	29.3	(d)
	19.5	81.1	21.8	(b)
	39.8	86.4	135.2	(b)
	70.7	90.0	143.1	(a)
174.3	31.4	174.8	91.6	(c)
	58.9	174.8	91.6	(c)
	46.8	85.8	4.4	(b)
	19.5	80.8	3.4	(b)
	43.3	87.1	173.1	(b)
	70.7	90.0	174.3	(a)

Table IV: Output values of weak and strong phases, for given values of input strong phases, in degrees, for the point p_1 with $\alpha_{\rm in} = 120.3°$ and $\gamma_{\rm in} = 36.9°$. Notes are as for Table II.

$\delta_{\rm in}$	$\alpha_{\rm out}$	$\gamma_{\rm out}$	$\delta_{\rm out}$	Notes
5.7	141.6	5.0	40.0	(e)
	126.8	4.9	40.3	(b)
	128.4	175.2	138.8	(c)
	143.2	175.3	138.6	(c)
	135.7	34.0	6.5	(d)
	120.3	36.9	5.7	(a)
	136.2	134.6	176.1	(c)
	151.4	131.9	176.7	(c)
36.9	120.4	36.8	37.0	(a,1)
	120.9	34.2	39.5	(a,1)
	129.8	154.5	131.9	(c)
	146.4	155.8	130.6	(c)
	120.2	37.2	36.8	(a,1)
	135.2	128.8	157.9	(c)
	152.7	125.7	161.5	(c)
84.3	145.8	38.2	84.6	(c)
	120.3	36.9	84.3	(a)
	147.4	143.6	95.9	(c)
	121.9	143.8	95.9	(c)
	137.6	86.7	47.8	(c)
	111.1	85.1	40.4	(c)
	132.3	97.2	148.9	(c)
	158.6	100.7	156.0	(c)
95.7	146.8	140.0	82.3	(c)
	118.6	142.4	83.8	(c)
	148.8	37.2	96.0	(c)
	120.3	36.9	95.7	(a)
	139.7	105.3	51.2	(c)
	109.6	98.6	41.2	(c)
	130.9	87.8	148.5	(c)
	160.5	94.6	157.2	(c)
143.1	120.3	36.9	143.1	(a,1)
	158.9	25.4	129.1	(c)
	120.3	36.9	143.1	(a,1)
	120.2	37.0	143.3	(a,1)
	121.7	42.6	148.4	(d)
174.3	109.9	171.3	25.7	(c)
	113.4	7.4	150.5	(b)
	159.4	6.1	145.1	(e)
	106.6	158.1	10.3	(c)
	120.3	36.9	174.3	(a)

The correct solutions in Tables II – IV are labeled (a). Solutions with $\beta > \pi/4$ [labeled (b)] imply $\rho > 1/2$, which is disfavored by the constraint [1] $(\rho^2 + \eta^2)^{1/2} = |V_{ub}/V_{cd}V_{cb}| = 0.27 \pm 0.09$. Solutions (c) with $\alpha + \gamma > \pi$ similarly imply CKM parameters outside the currently allowed range of (ρ, η), as do those (e) with β or γ too small. A few of the unphysical solutions show up on the plots of Ref. [1], but many were not found because attention was restricted to values of α and γ in rough accord with known CKM constraints.

One source of discrete ambiguity is the approximate symmetry $\alpha \leftrightarrow \pi/2 - \alpha$ or $\alpha \leftrightarrow 3\pi/2 - \alpha$. This substitution leaves B, D, E, F, and the first term in C unchanged for fixed values of γ, δ, $\mathcal{T}$, $\mathcal{P}$, and $\tilde{\mathcal{P}}'$. The substitution does affect the interference terms between $\mathcal{T}$ and $\mathcal{P}$ in A and C, but small changes in the parameters seem to be able to compensate for this effect.

Another frequently encountered discrete ambiguity involves the interchange $\gamma \leftrightarrow \delta$, which leaves D and E invariant. Of course, α changes under this replacement.

Many solutions thus can be rejected as unphysical. Those "wrong" solutions which remain [labeled (d)] are sources of potential ambiguity. While the existence of discrete ambiguities undercuts the ability of the method to point toward new physics, the procedure serves as a consistency check of the standard CKM picture and as a potential source of further constraints on parameters *within that context*.

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