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Supersymmetry: Theory

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ABSTRACT

If supersymmetric particles are discovered at high-energy colliders, what can we hope to learn about them? In principle, the properties of supersymmetric particles can give a window into the physics of grand unification, or of other aspects of interactions at very short distances. In this article, I sketch out a systematic program for the experimental study of supersymmetric particles and point out the essential role that e^+e^- linear colliders will play in this investigation.

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SUPERSYMMETRY: THEORY

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If supersymmetric particles are discovered at high-energy colliders, what can we hope to learn about them? In principle, the properties of supersymmetric particles can give a window into the physics of grand unification, or of other aspects of interactions at very short distances. In this article, I sketch out a systematic program for the experimental study of supersymmetric particles and point out the essential role that e^+e^- linear colliders will play in this investigation.

1 Introduction

When we think about the motivation for a future accelerator project, it is easy enough to work out its ability to continue ongoing experimental programs or to test with greater stringency well-understood theoretical models. In these areas, we can anticipate the goals of the next generation of experiments, and we can reasonably compute the effectiveness of a new facility in meeting these goals. And, when the price of a step in accelerator energy is in the billions of dollars, it is reasonable that the physics community and the taxpayers should ask for a program of experimentation that is well-considered and analyzed.

But these programmatic goals are the last thing on our minds when we dream about new facilities for experimental particle physics. What we are really hoping for is the discovery of new physical processes, of new particles and interactions. To the extent that these process are truly unanticipated, we cannot be quantitative about the capabilities of a proposed machine to discover them. Thus, we have a 'Catch-22' situation. In such a situation, there is a danger that simplistic or naive criteria can dominate the discussion of the relative merits of possible future facilities.

To go beyond naive arguments, it is necessary to look into the details of theoretical models. I do not mean by this that we must accept one of these models as the truth. Rather, we must investigate models that are plausible, and consider a large enough class of models to illustrate the range of options for the next level of elementary particle physics. Let me emphasize that arguments should flow from the study of models toward facilities; that is, we must first ask what are the crucial problems that our field faces and then, given the various options for their solution, what facilities will provide the experimental data that we require.

If we ask what are the major problems facing elementary particle physics, people give many answers. However, there is a single best answer: We must understand the mechanism for the spontaneous breaking of the weak-interaction gauge symmetry, $SU(2) \times U(1)$. This answer springs out ahead of the others for three reasons. First, this problem is the single glaring flaw in the standard gauge theory of strong, weak, and electromagnetic interactions. From the precision experiments on the electroweak interactions done over the last six years at SLAC, CERN, and Fermilab, we know that the weak interactions are described by a gauge theory with spontaneous symmetry breaking, and we know that this theory is not powerful enough to break its own symmetry. Thus, some external agent is required. Second, because the mass scale of electroweak symmetry breaking is the scale of the W and Z masses, we know that the agent of this symmetry breaking must be close at hand, accessible to this or the next generation of accelerators. Third, because the spontaneous breaking of $SU(2) \times U(1)$ is the origin of the masses of all quarks, leptons, and gauge bosons, the solution to any of the other pressing problems of elementary particle physics—CP violation, quark mixing, neutrino masses, or whatever—must be based on finding the correct solution to this problem. Thus, to discuss the relative merits of future collider facilities, we must enumerate a variety of possible solutions to the problem of electroweak symmetry breaking and understand their implications.

In a study of this kind, special consideration must be given to the possibility that Nature is supersymmetric at the TeV energy scale. It is by now well appreciated among elementary particle physicists that supersymmetry, the proposed symmetry which connects fermions and bosons, is an interesting option for the next layer of physics beyond the standard model. The motivations for believing that supersymmetry is a property of Nature have been discussed in many places, and I will not repeat them all here. Two useful reviews of phenomenological supersymmetry are those of Nilles[1] and Haber and Kane.[2] A recent list of encouragements for supersymmetry has been given in ref. [3]. In the present context, I would like to emphasize only one of these motivations, the fact that supersymmetry can provide the explanation for the spontaneous breaking of the weak-interaction gauge symmetry. (This aspect of supersymmetry has recently been reviewed in ref. [4].) Supersymmetry models naturally contain Higgs scalar fields; they are on the same epistomological footing as the scalar fields associated with quarks and leptons. These particular scalar fields obtain negative $(\text{mass})^2$ from radiative corrections due to the top quark Yukawa coupling; thus, if the top quark is the heaviest standard particle, the required pattern of electroweak symmetry breaking is naturally explained. Finally, supersymmetry is the unique setting in which the Higgs field and other fundamental scalar fields do not obtain much larger postive contributions to

their masses from radiative corrections. In this sense, supersymmetric models are the only models of electroweak symmetry breaking through fundamental scalar fields which all of the conceptual problems of this idea are naturally remedied.

Many other explanations of $SU(2) \times U(1)$ breaking are possible. These explanations are based on strong-coupling dynamics. For example, they may postulate the pair condensation of new fermions or bosons to create a vacuum asymmetry. However, the fact that supersymmetric models have a weak-coupling mechanism of symmetry breaking puts them in a special position. The array of mathematical methods to study strong-coupling problems in quantum field theory is rather limited. To work out the consequences of strong-coupling models of electroweak symmetry breaking, we have to guess the outcome of proposed strong-coupling dynamics. In models of supersymmetry, we can just compute. That does not mean that supersymmetric models are intrinsically more plausible, but it does mean that we can analyze them in great detail. Indeed, if we assume that supersymmetry is the explanation of weak interaction symmetry breaking, we can trace out the whole program of experimentation that would follow from this assumption and understand exactly what this program asks of future high-energy colliders. This gives a way of evaluating and comparing planned colliders which is scientific and yet corresponds to our hopes for their success. In principle, we should perform such an analysis for every proposed model of electroweak symmetry breaking. It is illuminating, in any event, to carry out the analysis for this one tractable case.

We have now arrived at the starting point for this article. I will assume that Nature is supersymmetric at the TeV energy scale, and that supersymmetry provides the explanation for $SU(2) \times U(1)$ symmetry breaking. Let me assume also that the first signal of supersymmetry has been discovered, perhaps already at LEP 2 or at the Fermilab collider, perhaps at the LHC or at an e^+e^- linear collider. What happens next?

In Section 2, I will trace out the first consequence of this assumption: The crucial problem for experiments at the next generation of colliders will be the exploration of the mechanism of supersymmetry breaking. This problem can be solved systematically by a three-stage experimental program that I will describe in Sections 3–5. In Section 6, I will review the role that hadron colliders have to play in this program. It is interesting, but limited by several well-known aspects of supersymmetry phenomenology. Finally, in Section 7, I will review the analysis and point out the crucial questions that belong in the province of e^+e^- experiments. In the accompanying article, Fujii[5] will discuss the measurements at e^+e^- linear colliders which would provide the answers to these questions and thus give the foundation for the next stage of development of elementary particle physics.

2 The Issue of Supersymmetry Breaking

If supersymmetry is the mechanism of electroweak symmetry breaking, we expect that it will be visible at the weak interaction scale. In fact, this hypothesis implies that the W and Z masses belong to the mass scale of supersymmetry. We can use that idea to to estimate the masses of the supersymmetric partners of quarks, leptons, and gauge bosons. If we assume that the value of the weak scale is a natural consequence of supersymmetry dynamics, that is, that there is no fine adjustment of parameters to make this scale especially small, we obtain limits on the masses of supersymmetry partners. The most characteristic of these are limits on the masses of the W and gluon partners

$$m(\widetilde{w}) < 250\text{GeV} , \qquad m(\widetilde{g}) < 800\text{GeV} , \tag{1}$$

for some reasonable limits on allowable fine adjustment.[6] Under the most commonly used hypotheses for supersymmetry phenomenology, in which there is a conserved 'R-parity' which makes the lightest supersymmetric partner stable, these particles will decay to unobserved neutral states, giving signatures of missing energy and unbalanced momentum which should be visible both at lepton and at hadron colliders.[b] In brief, if this scheme is chosen by Nature, we will know it. I would now like to go on from this point. The discovery of supersymmetry will open up a new field of study in elementary particle physics, and it will be the task of the machines that discover supersymmetry also to explore it.

If we discover that quarks, leptons, or gauge bosons have supersymmetric partners, what is the next question that we would like to answer? A simple reply is that we will want to measure the masses of these supersymmetric partners and understand their properties systematically. I would like to address this issue in more detail.

The equation of motion of a supersymmetric extension of the standard model has three parts. Of these, two are highly constrained by supersymmetry: The gauge interactions of superpartners are fixed by their $SU(3)\times SU(2)\times U(1)$ quantum numbers, and the renomalizable couplings of quark, lepton, and Higgs partners are fixed to be equal to the corresponding couplings of the standard model. However, the third piece of the puzzle is a complete mystery. If we wish to understand why the partners of quarks and leptons are heavy, we must

[b] I will not review here schemes of supersymmetry phenomenology in which R-parity is violated; see refs. [2] and [7].

appeal to some mechanism of spontaneous supersymmetry breaking. This mechanism is unknown and is not constrained by a direct connection to any known physics. This mechanism controls the regularities of the supersymmetric mass spectrum and the possible mixings between superpartner states. It also controls the other important qualitative features of the theory. For example, the various sources of the Higgs boson masses which lead eventually to $SU(2)\times U(1)$ breaking have their origin in supersymmetry breaking.

Supersymmetry breaking also connects the phenomenology of supersymmetry to the truly deep questions about the structure of elementary particles. If Nature is supersymmetric and weakly-coupled at the TeV scale, it is reasonable that the strong, weak, and electromagnetic interactions have their origin in a grand unified symmetry group. Indeed, there is evidence from the precision measurement of the coupling constants at Z^0 energies that the standard model coupling constants extrapolate to a unification when the extrapolation is done with renormalization group equations which incorporate the supersymmetric particle spectrum.[c] It is important to ask what additional hints we can obtain from experiment about the nature of this underlying or unified theory at very short distances. It is obvious that, to get new information, we must go outside those parameters which are already understood from applying supersymmetry relations to the standard model couplings. What is less obvious, but also true, is that the the supersymmetry breaking parameters measured at the weak scale can provide essential clues to the origin of supersymmetry breaking in the physics of very short distances and, just possibly, a glimpse into the structure of the truly fundamental theory.

What, exactly, do we wish to know about supersymmetry breaking? At the first level of any discussion of the physics of supersymmetry breaking, two questions arise. The answers to these questions would take us a long way toward an understanding of supersymmetry breaking and its relation to the other fundamental interactions.

The first of these questions is the mass scale of supersymmetry breaking and, as a closely connected issue, the scale of the transmission of supersymmetry breaking. The interplay of these scales deserves some explanation. To begin, we should recall why it is that the quarks and leptons are expected to be lighter than their superpartners, rather than the other way around. Quarks and leptons can receive mass only if $SU(2) \times U(1)$ is spontaneously broken. However, their partners—squarks and sleptons—are scalars, and there is no principle of quantum field theory that prohibits scalar fields from obtaining a mass. What keeps the quark and lepton partners light is their supersym-

[c] The success of this prediction, and the size of the residual uncertainties, are reviewed in ref. [8].

metry relation to their fermion partners. If supersymmetry is spontaneously broken in some sector of Nature, and this sector communicates with the quarks and leptons and their partners through some interactions, the supersymmetry breaking will be transmitted to the squarks and sleptons to produce scalar masses and other simple interactions. Call the scale of these masses m_S. The Higgs boson masses will also be of scale m_S. The Higgs vacuum expectation value will also be of size m_S, up to coupling constants, and so m_S will determine the location of the weak interaction scale. Then, finally, masses are fed down to the quarks and leptons according to the strength of their coupling to the Higgs sector.

The value of m_S is determined by the underlying physics responsible for spontaneous supersymmetry breaking. Let Λ be the mass scale of spontaneous supersymmetry breaking, and let M be the mass of the particles that connect the symmetry-breaking sector to the quarks, leptons, and standard model gauge bosons. I will refer to M as the 'messenger scale', and it will play a crucial role in our analysis. Though the relation between M, Λ, and m_S is model-dependent, the general form of this relation is given by the equation

$$m_S \sim \frac{\Lambda^2}{M} , \tag{2}$$

so that different choices for Λ and M are correlated by the fact that they must generate $m_S \sim m_Z$.

By default, gravity (or supergravity) is the messenger. This was made clear in the beautiful foundational papers of Cremmer and collaborators,[9] who showed explicitly how supersymmetry breaking is transferred from the original symmetry-breaking sector to the rest of Nature through supergravity interactions. More generally, the messenger interactions might be associated with the grand unified scale or other flavor physics, with some intermediate scale, or with the standard model gauge interactions. The nature of the messenger plays an important role in determining the form and selection rules for the supersymmetry breaking masses and interactions.

If this were our only information about M and Λ, there would be considerable room for speculation. Fortunately, the range of possible theories that lead to M and Λ is limited by additional constraints. These stem from the second problem that the mechanism of supersymmetry breaking must solve, the 'supersymmetric flavor problem'. To understand this issue, let us write the formula for the mass matrix of the scalar partners of the d, s, b quarks. Since in supersymmetry left- and right-handed fermions have independent complex scalar fields as their superpartners, I will write this matrix as a 2×2 matrix

of 3×3 blocks, acting on a vector

$$\begin{pmatrix} \tilde{d}^i_L \\ \tilde{d}^i_R \end{pmatrix} \tag{3}$$

where i is the generation label, $i = 1, 2, 3$. The mass matrix gets contributions from four sources, two of which are supersymmetric—the quark mass matrix m_d of the standard model, and the combination of this term with the Higgs mass parameter μ—and two of which arise from supersymmetry-breaking—scalar field mass matrices m^2_{dL} and m^2_{dR}, and a mixing term generated by a supersymmetry-breaking 3-scalar term involving the Higgs field. The final result is a matrix

$$\mathcal{M}^2 = \begin{pmatrix} m^2_{dL} + m_d m_d^\dagger & -m_d(A + \mu \tan\beta) \\ -m_d^\dagger(A + \mu\tan\beta) & m^2_{dR} + m_d^\dagger m_d \end{pmatrix} . \tag{4}$$

In this equation, $\tan\beta$ is the ratio of the two Higgs field vacuum expectation values required in the minimal supersymmetric extension of the standard model: $\tan\beta = < h^0_2 > / < h^0_1 > = v_2/v_1$. This parameter infects all of supersymmetry phenomenology. I have simplified the expression by writing the 3-scalar term in terms of a constant parameter A. In principle, this could also be a matrix with flavor indices.

The quark mass matrix m_d is not intrinsically diagonal. In standard weak interaction phenomenology, we diagonalize it with matrices V_L and V_R which eventually become ingredients of the Cabbibo-Kobayashi-Maskawa mixing matrix:

$$m_d = V_L \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} V_R . \tag{5}$$

Then the Z^0 couplings are automatically flavor-diagonal; flavor-changing neutral current effects appear only in loop diagrams, and only proportional to products of quark mass differences. This lead to the observed suppression of flavor-changing neutral current processes. However, the mass matrix (4) contains new sources of flavor violation through the supersymmetry-breaking scalar mass matrices m^2_{dL}, m^2_{dR}. Unless the diagonalization of m_d also diagonalizes these matrices, diagrams with supersymmetric particles in loops can provide new and dangerous sources of flavor violation. For example, applying this logic to the contribution to the K_L–K_S mass difference due to gluino exchange, Gabbiani and Masiero[10] have derived the bound

$$\frac{(V_R m^2_{dR} V_R^\dagger)_{12}}{m^2_{\tilde{d}}} < 10^{-2} \left(\frac{m_{\tilde{d}}}{300 \text{ GeV}}\right)^2 . \tag{6}$$

Similar bounds on the flavor violation of the supersymmetry-breaking mass terms have been discussed by many authors.[d]

Why should the supersymmetry-breaking scalar masses be diagonal in the same basis as the standard model mass terms? There are a large number of explanations for this in the literature. These explanations divide into general classes which express the range of possibilities for the underlying physics of supersymmetry breaking. On the one hand, it is possible that the supersymmetry breaking scalar masses are universal among generations, so that the mass matrices m^2_{dL}, m^2_{dR} are proportional to 1 and thus diagonal in any basis. Or these mass matrices may have structure, but they might also have a reason to be diagonal in the basis set by the mass matrix. On the other hand, the mechanism for the specific form of these matrices might be predetermined by the short-distance physics, or it might arise as the result of dynamical effects on larger scales. Thus, we have four classes of models:

1. **Preset Universality.** This is the original schema for supersymmetry model building which was proposed in the early papers of Dimopoulos and Georgi[12] and Sakai.[13] It is realized elegantly, with M equal to the Planck mass $m_{\rm Pl}$, in models in which supersymmetry is broken at a high scale and the breaking is communicated by supergravity.[14] Other agents which couple universally to quarks and leptons can also give models of this structure.

2. **Dynamical Universality.** This class encompasses a broad range of models in which the supersymmetry-breaking mass matrices are fixed in a manner determined only by the standard model gauge couplings of superpartners. It includes the 'no-scale' models in which m^2_L, m^2_R are zero at the fundamental scale and are generated by radiative corrections,[15] a model of Lanzagorta and Ross[16] in which m^2_L, m^2_R are determined by an infrared fixed point, and models studied by Dine, Nelson, Nir, and Shirman[17] in which supersymmetry is broken at a low scale and communicated through the standard model gauge interactions.

3. **Preset Alignment.** This class of models attempts to build up the supersymmetry-breaking mass matrices using the same principles that one uses to construct the standard model quark mass matrices (for example, the successive breaking of discrete symmetries). Then these symmetry principles can insure that the two sets of matrices are diagonal in the same basis, without flavor-degeneracy of scalar masses.[18] In this class of models, it is natural for the messenger scale to be of the order of the grand unification scale.

4. **Dynamical Alignment.** In this class of models, the relative orientation of the supersymmetry-breaking and standard model mass matrices is a free parameter in the underlying theory and is determined to be aligned by

[d]See, for example, ref. [1]; some recent articles are given in ref. [11].

radiative corrections. The one current example of a model in this class has M near the Planck scale,[19], leading to a phenomenology very similar to that of the class just above. A very low value of M might be more natural in this scheme and may lead to some different options.

Here are four broad classes of possibiilities for the mechanism of supersymmetry breaking. It is interesting to lay out the various possibilities in this way, because it makes clear that every specific solution to the supersymmetric flavor problem entails a choice of M and therefore of Λ. If we can recognize experimentally which possibility Nature chooses, we can also infer the nature of the messenger and perhaps the specific origin of supersymmetry breaking.

How can we decide which mechanism is chosen? At a certain level, it is obvious that the answer can be found by measuring the spectrum of superparticle masses. To probe more deeply, we should ask which of these measurements are easy and which are very challenging, and whether the measurements that are reasonably straightforward can actually give us the information we are looking for.

To understand how we will learn about these fundamental issues from measurements, it is necessary to work out the correspondence between properties of the supersymmetry spectrum and the various hypotheses described above. I will descibe that correspondence in Section 5. To prepare the way, we must first discuss two issues that provide the baseline for that analysis, the masses of the gauge boson superpartners and the value of the Higgs sector parameter $\tan\beta$.

3 Gaugino Masses

In the discussion of the previous section, we concentrated our attention on the masses of the scalar partners of the quarks and leptons. The masses of the fermionic partners of gauge bosons—gauginos—did not seem to play an important role. But in fact, a precise understanding of gauginos is a prerequisite to any detailed exploration of supersymmetry. This is true for two reasons. First, gaugino masses influence scalar masses through radiative corrections. Second, the nature of the gaugino mass matrix affects the general phenomenology of supersymmetry, as viewed by collider experiments. In this section, I will review both of these issues.

The systematics of gaugino masses forms an essential part of the scalar mass problem due to the diagram shown in Figure 1. The scalar masses are renormalized, as shown, by the transition to a gaugino and a quark or lepton. This process gives a correction to the mass which is described by the

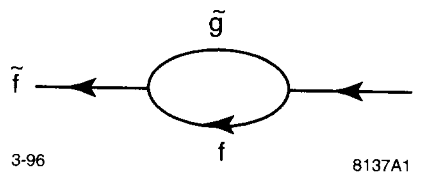

Figure 1: Feed-down of gaugino masses into scalar masses.

renormalization group equation

$$-\frac{dm_{\tilde{f}}^2}{d\log Q} = \sum_i \frac{2}{\pi} C_i \alpha_i m_i^2 \ , \tag{7}$$

where $m_{\tilde{f}}^2$ are the scalar masses and m_i are the gaugino mass parameters generated by supersymmetry breaking. The coupling constants α_i are the standard model couplings, evaluated at the weak interaction scale, normalized as in grand unification: $\alpha_3 = \alpha_s$, $\alpha_2 = \alpha_w$, $\alpha_1 = (5/3)\alpha'$. The C_i are Casimir coefficients:

$$C_1 = \frac{3}{5}Y^2 \ , \qquad C_2 = \begin{cases} \frac{3}{4} & \mathrm{L} \\ 0 & \mathrm{R} \end{cases} \ , \qquad C_3 = \begin{cases} 0 & \ell \\ \frac{4}{3} & \mathrm{q} \end{cases} \ . \tag{8}$$

The renormalization group equation must be integrated from the messenger scale M to the weak scale. One of the ways we can determine the value of the messenger scale is to estimate how far this renormalization group equation has been evolved in order to produce the observed spectrum of scalar masses. To do that, we require the values of the gaugino masses, to set the overall scale of this effect.

At the same time that they renomalize the scalar masses, the gaugino masses evolve by their own renormalization. This means that a simple spectrum of gauginos at one scale will acquire structure as we move to a different scale. The simplest possible picture of gaugino masses is that they are grand-unified, that is, they are all equal at the grand unification scale. From this starting point, the one-loop renormalization group equation gives an interesting pattern at lower scales: The gaugino masses evolve so as to remain proportional to the gauge couplings:

$$\frac{m_1}{\alpha_1} = \frac{m_2}{\alpha_2} = \frac{m_3}{\alpha_3} \ . \tag{9}$$

I will refer to this systematic relation as 'gaugino unification'. The simple relation is corrected by the two-loop terms in the renormalization group equations

and by finite one-loop corrections at the weak scale.[20] The only large correction comes in the finite contributions which relate the short-distance gluino mass to the physical gluino mass,[40] a problem reminiscent of the problems of the quark mass definition in QCD.

It is interesting to ask how broad a class of models obey gaugino unification. Obviously, if there is no grand unification, there is no reason for this relation to be true. However, one of the phenomenological virtues of supersymmetry is that it allows the grand unification of couplings, and so it is reasonable to assume this in model-building. Still, grand unification does not necessarily imply gaugino unification. On one hand, the messenger scale might be well below the grand unification scale, so that the physics of gaugino mass generation is not grand unified. On the other hand, it is possible that the field which breaks supersymmetry is not a singlet of the grand unification group. Thus, a violation of gaugino universality would be a signal of one of these mechanisms and thus would be of great experimental importance. Curiously, though, the simplest models of each type actually respect the relation (9), so that the observation of gaugino universality is not in itself a signature for a particular mechanism of supersymmetry breaking.[22]

The experimental measurement of the gaugino mass parameters m_i brings in some additional issues. The parameter m_3 is the only contribution to the mass of the supersymmetry partner of the gluon, the gluino, up to the usual problems of defining the mass of a colored particle. I will discuss techniques for the measurement of the gluino mass in Section 6. For the supersymmetry partners of W, Z, and γ, however, there are additional effect that contribute to their masses. Even in a supersymmetric situation, the partners of W and Z will obtain mass from the Higgs mechanism. This mass term couples the fermionic parters of the vector bosons to the fermionic partners of the Higgs bosons. These latter particles can obtain mass also from a supersymmetric mass term μ, and we know from the non-observation of light superpartners at the Z^0 that μ is nonzero.[e]

These effects are summarized as a mixing problem involving the vector boson and Higgs boson superpartners. Supersymmetric models necessarily include two Higgs doublets h_1, h_2; therefore, they contain physical charged Higgs fields, which have fermionic partners. Denote the left-handed fermion partners of W^+ and h_2^+ by $\widetilde{w}^+$, $\widetilde{h}_2^+$, and adopt a similar notation for the left-handed fermion partners of W^- and h_1^-. Then the charged fermionic superparticles have a mass matrix, including all three of the effects described

[e] A tiny corner of parameter space is still available; see ref[23].

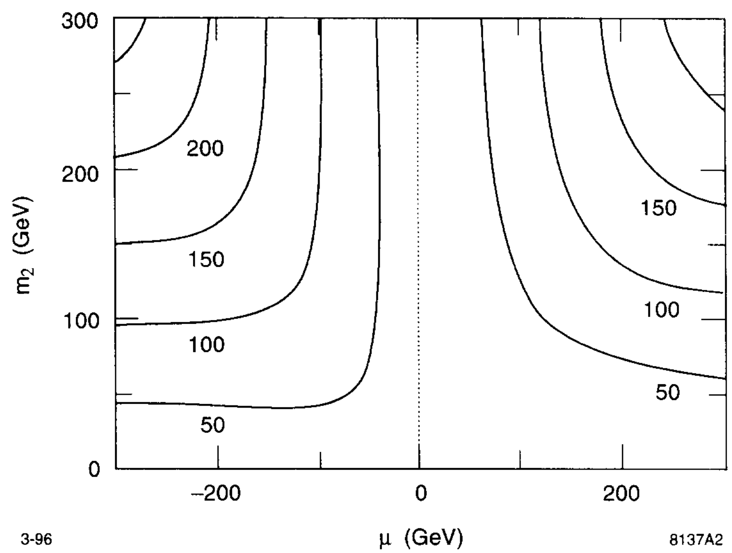

Figure 2: Lines of constant $\widetilde{\chi}_1^+$ mass in the (m_2, μ) plane, for $\tan\beta = 4$.

in the previous paragraph, which takes the form

$$\begin{pmatrix} -i\widetilde{w}^- & \widetilde{h}_1^- \end{pmatrix} \begin{pmatrix} m_2 & \sqrt{2}m_W \sin\beta \\ \sqrt{2}m_W \cos\beta & \mu \end{pmatrix} \begin{pmatrix} -i\widetilde{w}^+ \\ \widetilde{h}_2^+ \end{pmatrix} . \tag{10}$$

This mass matrix is asymmetric, and its diagonalization will generally require a different mixing angle for the positively and negatively charged left-handed fermions. In a similar way, the Z, photon, and neutral Higgs partners have a 4×4 mixing problem:

$$\begin{pmatrix} m_1 & 0 & -m_Z \sin\theta_w \cos\beta & m_Z \sin\theta_w \sin\beta \\ 0 & m_2 & m_Z \cos\theta_w \cos\beta & -m_Z \cos\theta_w \sin\beta \\ -m_Z \sin\theta_w \cos\beta & m_Z \cos\theta_w \cos\beta & 0 & -\mu \\ m_Z \sin\theta_w \sin\beta & -m_Z \cos\theta_w \sin\beta & -\mu & 0 \end{pmatrix} \tag{11}$$

acting on the vector $(-i\widetilde{b}, -i\widetilde{w}^3, \widetilde{h}_1^0, \widetilde{h}_2^0)$. The mass eigenstates of (10) and (11) are called, respectively, 'charginos' and 'neutralinos' and are denoted $\widetilde{\chi}_i^+$, $\widetilde{\chi}_i^0$.

One cannot, then, extract m_1 and m_2 simply by observing the masses of supersymmetric particles. It is also necessary to understand which values of the mixing angles Nature has chosen. Constraints coming from searches for

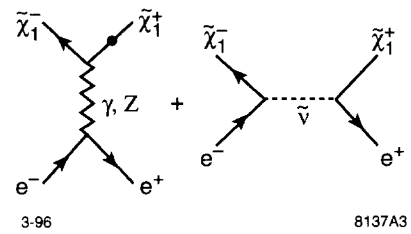

Figure 3: Diagrams contributing to the process $e^-e^+ \to \widetilde{\chi}_1^+ \widetilde{\chi}_1^-$.

charginos and neutralinos are often plotted on the plane of m_2 versus μ, at a constant value of $\tan\beta$. The lines in this plane representing constant mass of the lighter chargino, for $\tan\beta = 4$, are shown in Figure 2. Toward the bottom of this figure, the masses of the lightest charginos and neutralinos are close to m_2 and m_1, and these particles are composed dominantly of the gauge partners. Toward the top of the figure, the lightest chargino and neutralino become degenerate at the value μ and behave like the partners of Higgs bosons. This means that it is essential, both for the extraction of the supersymmetry breaking parameters and for the more general understanding of the signatures of supersymmetry that experiments should determine where we actually sit in the (m_2, μ) plane.

In hadron colliders, the dependence on the chargino and neutralino mixing angles is typically folded with other dependences on supersymmetry parameters and must be obtained as part of a grand fit. However, lepton colliders offer certain specific tools which allow one to solve the chargino and neutralino mixing problem experimentally. I will now present two techniques for doing this.

In this discussion, I will present the formulae for e^+e^- cross sections to supersymmetric particle pairs in a rather schematic way. A very useful compilation of the formulae for supersymmetry production in e^+e^- reactions can be found in ref. [24].

We first consider the reaction $e^-e^+ \to \widetilde{\chi}_1^+ \widetilde{\chi}_1^-$, making use of the highly polarized electron beams which are anticipated for linear collider experiments. In ref. [22], some wonderful observations are made about this process. To understand these, imagine first that we study the reaction at very high energy, so high that we can ignore all masses. Now assume that the electron beam can be polarized completely in the right-handed orientation. Since right-handed electrons do not couple to the $SU(2)$ gauge interactions, the second diagram

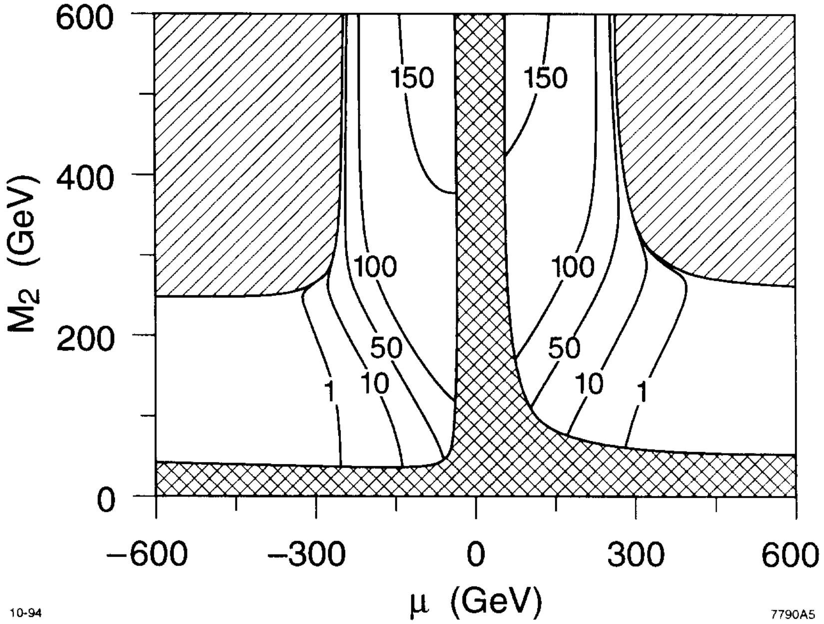

Figure 4: Total cross section for the process $e^-_R e^+ \to \widetilde{\chi}^+_1 \widetilde{\chi}^-_1$, in fb, as a function of m_2 and μ, for $\tan\beta = 4$, from ref. 25. The selected region is that in which the lightest chargino is too heavy to have been discovered at the Z^0 but is accessible to a 500 GeV e^+e^- collider.

in Figure 3 vanishes. In addition, the first diagram in Figure 3 involves only the linear combination of γ and Z^0 which gives the $U(1)$ (hypercharge) gauge boson. But the $U(1)$ gauge boson does not couple to W superpartners. Thus, this diagram only involves the Higgs superpartners. If we project onto the lowest mass eigenstate, the rate of the process $e^-_R e^+ \to \widetilde{\chi}^+_1 \widetilde{\chi}^-_1$, will be proportional to the squares of the mixing angles linking the $\widetilde{h}^-_1$ and $\widetilde{h}^-_2$ to this mass eigenstate.

The promise which is suggested by this high-energy analysis is actually realized under more realistic conditions. In Figure 4, I plot contours of this polarized cross section for an e^+e^- collider at 500 GeV in the relevant region of the (m_2, μ) plane. You can see that the cross section maps out this plane, giving the location chosen by Nature, up to a two-fold $(\mu \leftrightarrow -\mu)$ ambiguity, for any determined value of the chargino mass.

Actually the chargino pair production cross section contains even more in-

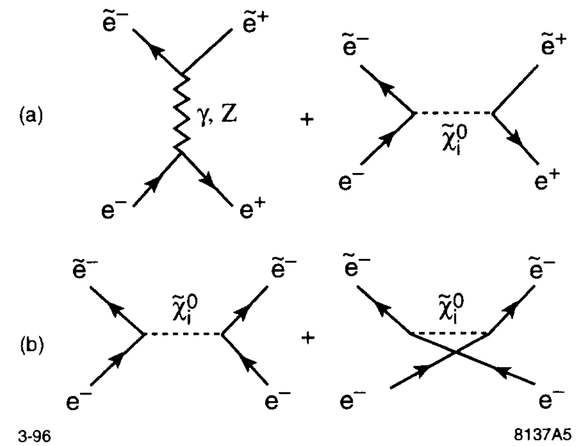

Figure 5: Diagrams contributing to the processes (a) $e^-e^+ \to \widetilde{e}^-\widetilde{e}^+$; (b) $e^-e^- \to \widetilde{e}^-\widetilde{e}^-$.

formation. Going back to the limit of very high energies, the angular distribution for an e^-_R to produce a right-handed fermion is proportional to $(1+\cos\theta)^2$, while the angular distribution to produce a left-handed fermion is $(1-\cos\theta)^2$. Thus, the forward production of $\widetilde{\chi}^-_1$ is given by the mixing angle for $\widetilde{h}^+_2$, while the backward production is controlled by the mixing angle for $\widetilde{h}^-_1$. Thus, measurement of both the total cross section and the forward-backward asymmetry for this process gives the two mixing angles needed to diagonalize the chargino mass matrix (10). In this analysis, one must assume that there are only two of Higgs doublets that the weak scale (as is required for the grand unification of couplings), but there are essentially no other model-dependent assumptions.[25]

A second method for determining the gaugino mixing parameters involves the production of electron partners, selectrons. There are two selectrons, one the partner of e^-_R, the other the partner of e^-_L. (These states can be distinguished most easily by the polarization asymmetry of their production.) I will discuss the expectations for the selectron masses in Section 5; let me note for now that these particles are expected to be among the lightest superpartners.

The Feynman diagrams which contribute to selectron production in e^+e^- annihilation are shown in Figure 5(a). The second diagram involves neutralino exchange. Although this diagram is exotic, it typically dominates, since the lightest neutralino is usually lighter than the Z^0 and the diagram is a t-channel rather than an s-channel exchange. A related process is that of selectron production in e^-e^- collisions. Here the reaction is mediated only by neutralino exchange diagrams, in the t- and u-channel.

To discuss these processes, it is convenient to define 'neutralino functions', in the following way: Let V_{ij} be the orthogonal matrix that diagonalizes (11), with the first index denoting a weak eigenstate and the second denoting a mass eigenstate. Define

$$\begin{aligned} V_{Ri} &= -\frac{1}{\cos\theta_w} V_{1i} \\ V_{Li} &= -\frac{1}{2\cos\theta_w} V_{1i} - \frac{1}{2\sin\theta_w} V_{2i} \, . \end{aligned} \tag{12}$$

Then define, for $a, b = L, R$,

$$\begin{aligned} \mathbf{N}_{ab}(t) &= \sum_i V_{ai} \frac{m_1^2}{m_i^2 - t} V_{bi} \\ \mathbf{M}_{ab}(t) &= \sum_i V_{ai} \frac{m_1 m_i}{m_i^2 - t} V_{bi} \end{aligned} \tag{13}$$

where the sum runs over the four neutralino mass eigenstates, m_i is the mass of the ith neutralino, and m_1 has been introduced to make the functions dimensionless. The neutralino functions are simply related to the production cross sections, for example,

$$\begin{aligned} \frac{d\sigma}{d\cos\theta}(e_R^- e^+ &\to \widetilde{e}_R^- \widetilde{e}_R^+) \\ &= \frac{\pi\alpha^2}{4s} \left[1 + \frac{\sin^2\theta_w}{\cos^2\theta_w} \frac{s}{s - m_Z^2} - \frac{s}{m_1^2} \mathbf{N}_{RR}(t)\right]^2 \beta^3 \sin^2\theta \, . \end{aligned} \tag{14}$$

The functions $\mathbf{N}_{RR}$, $\mathbf{M}_{RL}$, $\mathbf{N}_{LL}$ enter the formulae for the production of $\widetilde{e}_R^- \widetilde{e}_R^+$, $\widetilde{e}_R^- \widetilde{e}_L^+$, and $\widetilde{e}_L^- \widetilde{e}_L^+$, respectively, in e^+e^- annhiliation; the opposite three combinations enter into the production cross sections for e^-e^-.

The neutralino functions are full of information about the neutralino mixing problem. As an example, I plot in Figure 6 the values of the six neutralino functions, extrapolated to $t = 0$, along a contour of constant chargino mass in the (m_2, μ) plane. These variables also map the position in this plane. Though it is not shown here, the relative heights of the curves are sensitive to the value of m_1/m_2 and thus provide a test of gaugino unification. A detailed simulation of selectron pair production which uses these ideas to extract m_1, m_2 and μ has been presented in ref. [22].

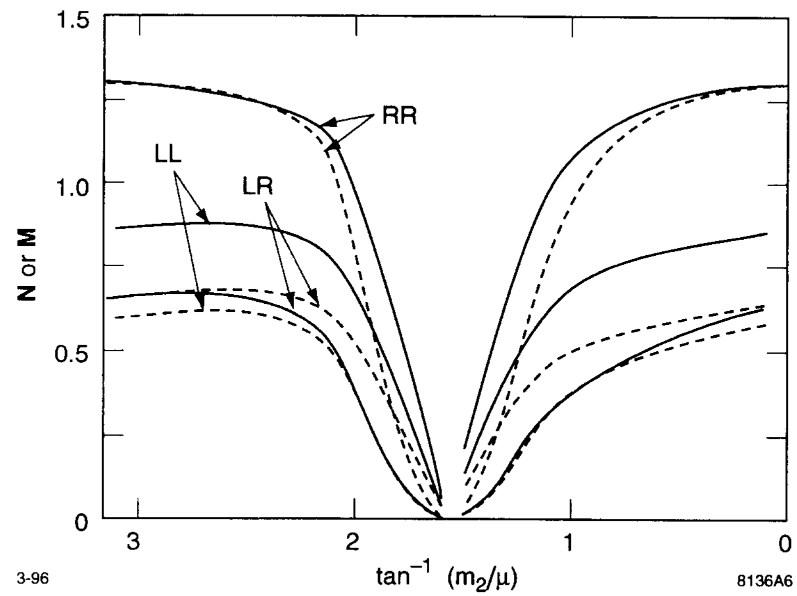

Figure 6: Values of the 'neutralino functions' $\mathbf{N}_{ij}$, $\mathbf{M}_{ij}$, at $t = 0$, as a function of the angle in the (m_2, μ) plane: $\alpha = \tan^{-1}(\mu/m_2)$. The solid curves denote the predictions for selectron production in e^+e^- collisions, the dotted curves for selectron production in e^-e^- collisions.

4 Determination of $\tan\beta$

The set of parameters needed for a precise understanding of the spectrum of superpartners also includes $\tan\beta = v_2/v_1$, the ratio of the Higgs field vacumm expectation values. We have already seen that $\tan\beta$ appears as a parameter in the gaugino mixing problem. This parameter also plays a role in the formula for the scalar masses. Through the supersymmetrization of the gauge interactions, all quark and lepton partners receive a 'D-term' contribution to their masses of the form

$$\Delta m_D^2 = -m_Z^2 \left(\frac{\tan^2\beta - 1}{\tan^2\beta + 1}\right)(I^3 - Q\sin^2\theta_w) \ , \tag{15}$$

where I^3 and Q are the electroweak quantum numbers.[f] More generally, any discussion of the experimental signatures of supersymmetry brings in many sources of dependence on $\tan\beta$, through the production and decay amplitudes for gauginos and Higgs bosons. Thus, it is important to find a model-independent method for determining this paramter.

Unfortunately, there is no method known which systematically determines $\tan\beta$ throughout its whole range of possible values. I will discuss here four methods, of which the first two apply mainly for small or intermediate values of $\tan\beta$ and the last gives a bound rather than a value.

The first method for determining $\tan\beta$ goes back to the chargino production cross section discussed in Section 3. I argued there that it is possible to determine the mixing angles needed to diagonalize the chargino mass matrix; from these, one can deduce the off-diagonal elements of this mass matrix. But note from (10) that the ratio of these elements is just equal to $\tan\beta$. Since these off-diagonal elements are related by supersymmetry to the vertices which give mass to the W boson, this ratio is model-independent. In ref. [25], it was remarked that the determination of the chargino mass matrix discussed there could be interpreted as a $\tan\beta$ measurement. Then this parameter could be determined with an accuracy of was 3% at $\tan\beta = 4$, for a parameter set in which the lightest chargino was a roughly equal mixture of gaugino and Higgsino.

A second method for determining $\tan\beta$ has been proposed by Nojiri.[26] This involves a beautiful supersymmetry observable for linear colliders, the polarization of the τ leptons produced in $\widetilde{\tau}$ decay. The τ polarization is now known to be straightforwardly measurable in e^+e^- experiments. The polarization of τ's from $\widetilde{\tau}$ decay contains information on the mixing of the two $\widetilde{\tau}$ eigenstates and on the decay pattern. For a full discussion of the extraction of this information, see ref. [26]. For the purpose of this discussion, I will simply point out that the mixing of $\widetilde{\tau}_L$ and $\widetilde{\tau}_R$ can be determined from the $\widetilde{\tau}$ cross sections and polarization asymmetry. In the following discussion, I will assume for simplicity that the lightest τ partner is an unmixed $\widetilde{\tau}_R$.

[f] If the theory contains additional gauge bosons, there are additional D terms. I include these in the model-dependent part of the scalar masses.

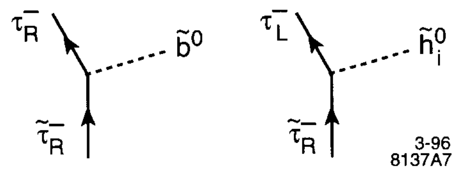

Figure 7: Components of the decay $\widetilde{\tau}_R \to \tau\widetilde{\chi}_1^0$.

The dominant decay of this scalar should be to $\tau\widetilde{\chi}_1^0$. In terms of weak-interaction eigenstates, there are two amplitudes that contribute to this decay; these are shown in Figure 7. On one hand, the $\widetilde{\tau}_R^-$ can decay to a τ_R^- with the emission of a $\widetilde{b}^0$. On the other hand, the $\widetilde{\tau}_R^-$ can decay to a τ_L^- with the emission of a $\widetilde{h}_1^0$. These two processes give rise to a nontrivial τ polarization, given to first order by

$$P(\tau^-) = 1 - \frac{\cos^2\theta_w}{\sin^2\theta_w}\frac{m_\tau^2}{m_W^2}\frac{1}{\cos^2\beta}\frac{p(\widetilde{h}_1^0)}{p(\widetilde{b}^0)} \ , \tag{16}$$

where $p(\widetilde{h}_1^0)$ and $p(\widetilde{b}^0)$ are the probabilities that the lightest neutralino appears as one of these states. If we know the content of the lightest neutralino mass eigenstate in terms of weak eigenstates—and I have given methods for determining this in the previous section—this formula can be solved for $\cos\beta$. This technique should give $\tan\beta$ measurements below the 10% level even when the Higgsino component of the lightest neutralino is rather small.

Ideally, $\tan\beta$ can be determined from the branching ratios of the heavy Higgs bosons of supersymmetry. If the A^0 boson of the Higgs sector has a mass well above the Z^0 mass, the lightest Higgs boson h^0 has branching ratios close to those of the Higgs boson of the minimal standard model. However, the heavy Higgs bosons H^0 and A^0 have couplings which reflect the ratio of the two Higgs vacuum expectation values. For example,

$$\frac{\Gamma(H^0 \to t\bar{t})}{\Gamma(H^0 \to b\bar{b})} = \left(\frac{m_t}{m_b}\cot^2\beta\right)^2 \left(1 - \frac{4m_t^2}{m_H^2}\right)^{1/2} . \tag{17}$$

Unfortunately, these heavy Higgs bosons have masses of order 500 GeV in typical models, and they must be pair-produced (except in $\gamma\gamma$ collisions); thus, they may be difficult to find in the early stages of linear collider experimentation.

As a last resort, there is one interesting determination of $\tan\beta$ that will be available from the LHC. The processes $H^0, A^0 \to \tau^+\tau^-$ is a possible mode for observing the heavy Higgs bosons at the LHC, but only if the branching ratios to $\tau^+\tau^-$ are enhanced by a large value of $\tan\beta$.[27,28] If this signature can be observed, then $\tan\beta > 10$, which is already sufficient information to evaluate the scalar mass contribution (15) to a reasonable accuracy.

5 The Pattern of Scalar Masses

In the previous two sections, I have discussed the experimental determination of the parameters which provide the baseline for a discussion of the spectrum of scalar masses. With these parameters known, we can examine the pattern of scalar masses systematically. Let me now discuss how this can be done, and what variety of patterns the various models of Section 2 produce.

In general, the formula for a scalar partner mass has three components. First, there is the underlying supersymmetry-breaking mass term. At least for the light generations, for which we can ignore the Yukawa couplings to the Higgs sector, this term is not renormalized at the level of one-loop renormalization group equations. Second, there is the contribution fed down from the gaugino masses, obtained by integrating the renormalization group equation (7). Finally, there is the D-term contribution (15). Once $\tan\beta$ is known, this last contribution can be computed in a model-independent way and subtracted; I will define the reduced scalar partner masses

$$\overline{m}_f^2 = m_f^2 - \Delta m_D^2(I^3, Q) \; . \tag{18}$$

Next, we must deal with the mass contribution due to gauginos. The result of integrating (7) can be conveniently written

$$\overline{m}_f^2 = \overline{m}_{f0}^2 + \Big(\sum_i 2C_i \frac{\alpha_i^2 - \alpha_{iM}^2}{b_i \alpha_2^2}\Big) \cdot m_2^2 \; . \tag{19}$$

In this equation, $i = 1, 2, 3$ runs over the standard model gauge groups. The C_i are the Casimirs from (8). The b_i are the renormalization group coefficients; these are given by $b_i = (-33/5, -1, 3)$ for $i = 1, 2, 3$ in minimal supersymmetry. Finally, the α_{iM} are the values of the coupling constants at the messenger scale M. In writing this equation, I have assumed gaugino universality to convert the gaugino masses to the single value m_2, which should be precisely known. I emphasize again that gaugino universality is an assumption, but one which can be confirmed or refuted experimentally as part of the broader exploration of supersymmetry.

In Section 2, the class of models exhibiting dynamical universality included models in which the messengers of supersymmetry breaking were the standard model gauge interactions. In models of this type, gaugino masses are generated directly by one-loop diagrams involving the supersymmetry breaking sector, and scalar masses are generated at the two-loop level. I have already noted that these models can naturally lead to gaugino unification. A particular model of Dine, Nelson, Nir, and Shirman[17] gives also gives a simple spectrum of scalar masses:

$$\overline{m}^2 = \Big(\sum_i 2C_i \frac{\alpha_i^2}{\alpha_2^2}\Big) \cdot m_2 \; ; \tag{20}$$

in this formula, the coefficient 2 depends on the model assumptions, while the general structure is characteristic of this mechanism for the communication of supersymmetry breaking.

The simplicity of the formula (20) and its curious resemblance to (19) motivates us to consider the following device for exhibiting the spectrum of quark and lepton superpartners. We plot the ratio $\overline{m}/m_2$ against a weighted combination of Casimirs,

$$C = \big(\sum_i C_i \frac{\alpha_i^2}{\alpha_2^2}\big)^{1/2} \; . \tag{21}$$

The prediction of (20) is that the superpartner spectrum is a straight line on this plot. Thus it is reasonable to call this device the 'Dine-Nelson plot'.

Models in which the scalar masses come dominantly from the renormalization group effect (19) also have a relatively simple form on the Dine-Nelson plot. In Figure 8, I have plotted the contributions from renormalization-group running for three values of the messenger scale—a low scale $M = 100$ TeV, the grand unification scale 2×10^{16} GeV, and the fundamental scale of superstring theory, 10^{18} GeV. As a comparison, I have also plotted the result (20). It is important to note that the Casimir C is not continuously variable but rather takes only fives distinct values, those for the $SU(2) \times U(1)$ multiplets of the standard model, ℓ_R, L_L, d_R, u_R, and Q_L. Of these, the values of C for d_R and u_R (and also the gaugino contributions to their scalar masses) are highly degenerate. So the Dine-Nelson plot is really defined by the value of the masses at these specific points. The curves in Figure 8 are intended only to guide the eye.

The device of the Dine-Nelson plot gives us a concrete way to view the distinctions between the various classes of models reviewed in Section 2. In

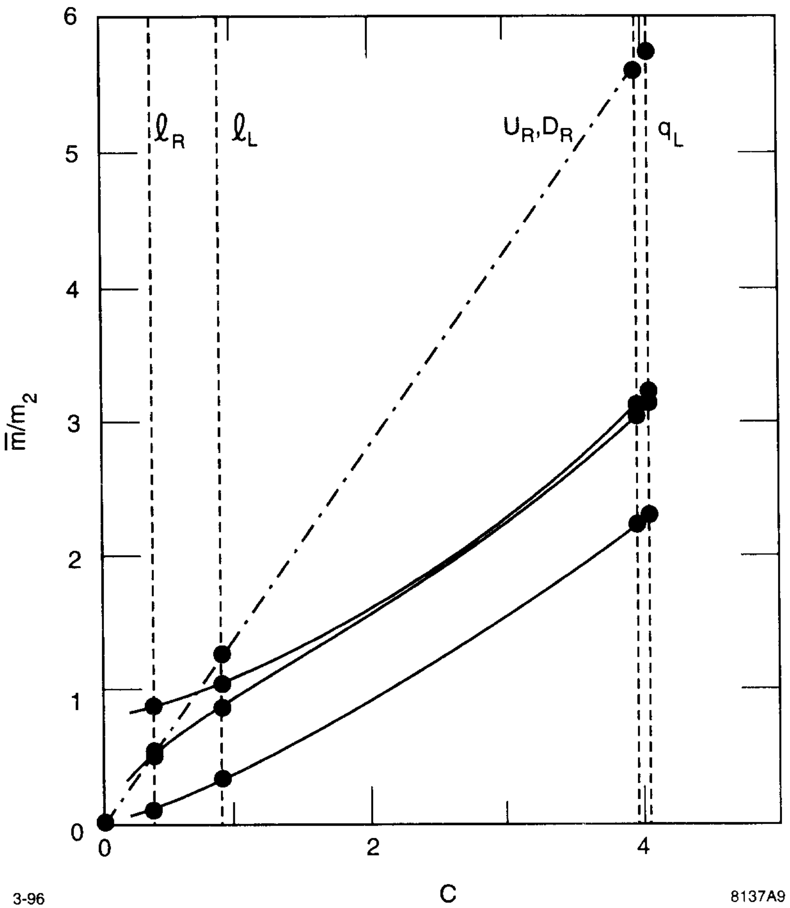

Figure 8: Some reference models displayed on the Dine-Nelson plot. The solid lines show the integration of the renormalization group equation for two values of the messenger scale. The dotted line shows the linear relation predicted in the model of Dine, Nelson, Nir, and Shirman.

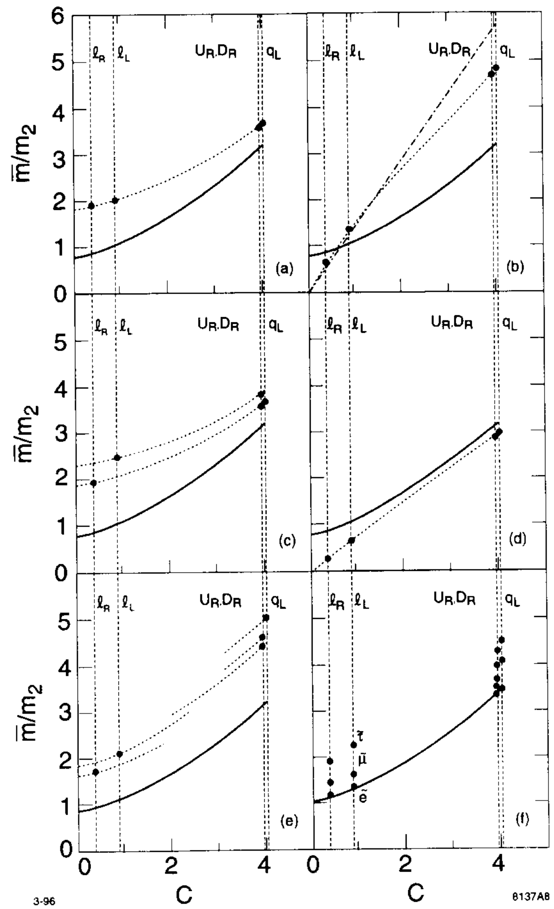

Figure 9: Six classes of models of supersymmetry breaking, displayed as patterns on the Dine-Nelson plot. The solid reference line is the result of integrating the renormalization group equation from the Planck scale. The models (a)-(f) are described in the text.

Figure 9, I have plotted the spectrum of quark and lepton partners for each of six representative models. The values of the masses are compared to the position of the top solid curve from Figure 8, representing the gaugino loop contribution of Figure 1 integrated from the superstring scale.

Model (a) is a typical model with preset universal scalar masses communicated by supergravity.[14] The values of the masses sit a constant distance in $(\text{mass})^2$ above the solid curve. This increment is positive, so that the theory does not develop an instability at the fundamental scale.[g] Notice that the sleptons are typically lighter than the squarks, but the ratio of these masses depends on the size of the original supersymmetry-breaking mass term relative to that generated by renormalization group corrections.

Model (b) is the Dine-Nelson-Nir-Shirman model.[17] I have made some small improvements of the formula (20), evaluating the coupling constants at a more realistic scale of about 100 TeV, and then adding the renormalization group enhancements as the masses come down to the weak interaction scale. The dashed line is copied from Figure 8. Notice that in this class of models the slepton masses are rather small, and also different by a factor 2 between the partners of left- and right-handed leptons.

Model (c) is a variant of the supergravity models which has been considered in ref. [30]. Here the original supersymmetry-breaking scalar masses are universal among generations for a given set of gauge quantum numbers. However, the values of these masses depend on the quantum numbers, for example, differing for the particles that belong to 10 and $\overline{5}$ representations of $SU(5)$. Models in which there are large contributions to the scalar masses from new D terms due to extended gauge interactions, as in refs. [31,32], and the superstring-based models of ref. [33], generate patterns similar to these.

Model (d) is a model with dynamical universality presented by Choi.[34] In this model, the original supersymmetry breaking masses are zero, so that the final masses are determined only by the renormalization group effect, as in 'no-scale' models. However, for Choi, the messenger scale is F_a, the axion decay constant, and the messenger interactions are those associated with Peccei-Quinn symmetry breaking.

Model (e) illustrates an idea for dynamical universality due to Lanzagorta and Ross.[16] In this model, the supersymmetry-breaking masses are driven to the fixed points of the renormalization group equations for a more complex underlying theory at a high scale. The locations of the fixed points depend on the standard model quantum numbers of the quark and lepton partners, but not on the generation. In principle, the pattern of soft masses is predicted by the underlying model.

Model (f) is an example of a model with preset alignment. In such a scheme, the three supersymmetry-breaking mass parameters for each set of standard-model quantum numbers are distinctly different. Though this is not required in these models, I have drawn the figure to suggest that the masses, for each set of quantum numbers, have an asymptote which is the solid line; this would suggest that the messenger scale is the Planck or string scale, and that the discrete symmetries which regulate the alignment of the mass matrices are characteristic of superstring or other deep-level physics.

Each one of the possibilities presented here is interesting as a plausible model of the origin of supersymmetry breaking. The range of possibilities is fun to think about, and is certainly not exhausted by these cases. That there should be such a wide range is no surprise. In physics, every time we open another door to speculation, manifold possibilities are revealed, and the one chosen by experiment is often one that seemed least likely at the beginning. The real surprise in this figure is how different models of physics coming from a very deep level of Nature present distinctly different patterns. These patterns will be visible in data that can be collected at the weak interaction scale, data that we will gather with the coming generation of high-energy colliders.

6 Superspectrum Experiments at Hadron Colliders

Now we have set out the essential problems of supersymmetry experimentation. We must first set the scale of supersymmetry partner masses by measuring the gaugino masses and testing gaugino universality. Then we must identify the scalar states associated with each flavor and helicity of quarks and leptons, and we must measure their masses with sufficient precision to recognize their pattern on the Dine-Nelson plot. I have already explained the role that linear collider experiments will play in the background issues of determining the gaugino spectrum. It is also straightforward to measure slepton masses at a linear collider, and also squark masses if the squarks are kinematically accessible. The experimental issues connected with all of these measurements are explained in the accompanying paper by Fujii.[5]

On the other hand, supersymmetric particles can also be found at hadron colliders. I have already noted that, if Nature has chosen supersymmetry as the explanation of the weak interaction scale, supersymmetric particles must be visible at the LHC. Thus, any proper understanding of the role of linear e^+e^- colliders must take into account the anticipated results of hadron collider experiments. Will the e^+e^- results cover much the same ground as these investigations, or do they bring some distinctly different ingredient to the study?

[g] There are realistic models which avoid this constraint in which our vacuum is not the global minimum of the potential but is stable over cosmological time; see ref. [29].

In this section, I would like to briefly review studies of supersymmetry experiments for hadron colliders, mainly for the LHC. These studies have, for the most part, been directed to shorter-term goals than the ones I have emphasized here, to the first discovery of supersymmetry, rather than to the systematic experimental pursuit of the new physics. It should be easy for the LHC (or, if we are lucky about Nature's choice of supersymmetry parameters, for the Tevatron) to discover supersymmetry. The cross section for gluino pair production in hadronic collisions is an order of magnitude larger than that for production of a quark of comparable mass, and the expected signature of multijet events with large missing energy is striking and characteristic.

To go deeper than the observation of anomalies, however, will be difficult at hadron colliders. The reasons for this do not come from considerations of relative cleanliness and such experimental matters which are debated between the hadron and lepton physics communities. Rather, they come from the specific predictions of supersymmetry phenomenology. The difficulties and the promise of hadron collider experiments can be made clearer by reviewing some of the techniques which have been developed to date for obtaining information on the supersymmetry spectrum in this environment.

Before beginning this review, I would like to recall the expectation, both in this generation of accelerators and the next, that hadron and lepton collider experiments should probe roughly the same regions of the parameter space of supersymmetry. The reason for this is that colored superpartners receive large positive mass enhancements from their coupling to gluons and gluinos. This is most clear in the gaugino sector. I argued in Section 3 that gaugino unification should at least be a useful guide to the general properties of the supersymmetry spectrum. According to (9), the short-distance gluino mass m_3 should be roughly three times the mass parameter m_2. To convert to physical mass values, we must note that m_2 is essentially an upper bound to the lightest chargino mass, while m_3 receives a 15% upward radiative correction when converted to the 'pole' mass which determines the kinematics of gluino production. Another similarly large correction, which may be of either sign, may appear if the gluino and squark masses differ by a large ratio.[40] Thus,

$$m(\widetilde{g}) > 3.5(m(\widetilde{\chi}_1^+)^2 - m_W^2)^{1/2} \ . \tag{22}$$

Thus, a chargino discovery at 80 GeV which might be made at LEP 2 would correspond to a gluino at 300 GeV which might be discovered at the Tevatron. A linear collider at 1 TeV would be able to search for charginos up to 500 GeV; the corresponding gluino mass is 1700 GeV, which is roughly the search limit of the LHC if $m_{\tilde{g}} \ll m_{\tilde{q}}$.[28] Both of these values are a factor of two beyond the naturalness limits given in (1). In a similar way, the sleptons are

expected to be lighter than the squarks, though the precise relation is more model-dependent. Figure 8 contains spectra in which the ratio of squark to right-handed slepton masses varies from 2 to 6. Of course, this correspondence does not mean that the hadron and lepton colliders are competing to discover the same information. In fact, as we will see, quite the reverse is true.

Hadron colliders provide many striking signatures of supersymmetry. The most basic signature is that of missing energy in multijet events. But the production of supersymmetric particles can also lead to interesting multilepton and Z^0 plus lepton topologies. A summary of event rates at the LHC for a variety of increasingly exotic reactions is shown in Figure 10.[35] These exotic final states arise from decays in which the gluino or squark which is the primary product of the hadronic reaction decays to a neutralino or chargino, which then decays by a cascade to reach the lightest superparticle.[36] An example of such a cascade decay is

$$\begin{aligned} pp \to \quad & \widetilde{g} \to q\overline{q} + \widetilde{\chi}_2^+ \to \ell^+\nu + \widetilde{\chi}_2^0 \to q\overline{q} + \widetilde{\chi}_1^0 \\ + \quad & \widetilde{g} \to q\overline{q} + \widetilde{\chi}_3^0 \to Z^0 + \widetilde{\chi}_1^0 \end{aligned} \tag{23}$$

The appearance of these many topologies is a strength of the hadronic window into supersymmetry, but it is also its weakness. First, because superpartners are pair-produced, and each partner decays with missing energy, it is not possible to reconstruct a superpartner as a mass peak. The reaction shown in (23) illustrates that supersymmetry reactions can contain sources of missing energy from ν or Z^0 emission in addition to that from the final neutralinos. Of course, in hadronic collisions, the initial parton energies and polarizations are also unknown. Thus, analyses of supersymmetry parameters must be based on overall hadronic reaction rates, or on other observables which integrate over the underlying kinematic parameters. To interpret such variables, one needs a trustworthy model of the reaction being studied. But now we come to the second problem. The pattern of squark and gluino decays is influenced by the spectrum and mixings of charginos and neutralinos and changes as the parameters of these states move in the $(m_2, \mu, \tan\beta)$ space. If one relies only on data from hadronic supersymmetry processes, the dependence on these parameters enters as an essential modelling ambiguity.

To clarify these issues, I would like to describe a number of methods proposed in the literature for the detailed measurement of supersymmetry parameters. Before turning to strongly interacting particles, I will comment on color neutral states. Hadronic collisions can also access the chargino and neutralino states directly, through the reactions

$$q\overline{q} \to \widetilde{\chi}^0\widetilde{\chi}^0, \qquad q\overline{q} \to \widetilde{\chi}^0\widetilde{\chi}^+ \ . \tag{24}$$

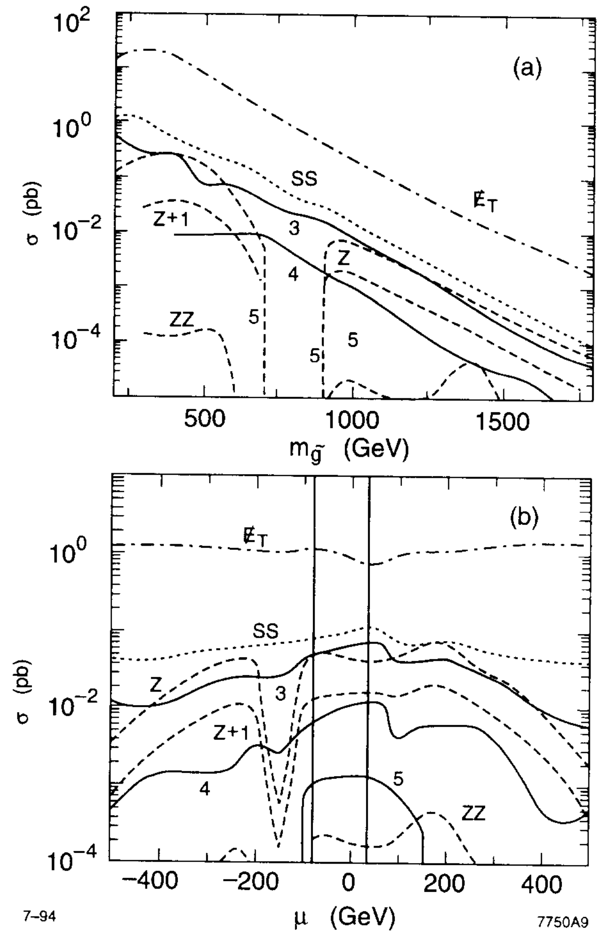

Figure 10: Cross sections for various signatures of supersymmetry observable at the LHC, from ref. 35. The various curves give the cross sections for missing transverse energy, same-sign dilepton production, multilepton events, and multilepton + Z events. The cross sections are shown (a) as a function of the mass of the gluino, for $m(\widetilde{g}) = m(\widetilde{q})/2$ and $\mu = -150$ GeV, (b) as a function of the parameter μ for a fixed gluino mass equal to 750 GeV.

The second of these reactions is a potential source of trilepton events, and therefore has been discussed as an interesting mode for the discovery of supersymmetry at the Tevatron collider.[38] Baer and collaborators have noticed that this reaction can also give some spectral information: The dilepton spectrum in trilepton events falls off sharply at dilepton masses equal to the mass difference $m(\widetilde{\chi}_2^0) - m(\widetilde{\chi}_1^0)$, allowing a measurement of this parameter of the neutralino mass matrix.[39] Sleptons can also be discovered at hadron colliders. An analysis of the slepton signal at LHC, using as the signature acoplanar isolated leptons, is given in ref. [37]. This signal unfortunately has a very low rate, and also sums the contributions of the partners of left- and right-handed sleptons, so it is not promising for an accurate mass determination.

For the strongly interacting superpartners, we should hope that the hadron colliders can give us accurate mass measurements. Let us consider first the gluino mass measurement. This is simplest if the supersymmetry parameters are such that $m_{\widetilde{g}} < m_{\widetilde{q}}$, and I will restrict my attention to that case for a moment. There is one proposed estimator for the gluino mass that does peak sharply, proposed some time ago by Barnett, Gunion, and Haber.[40] These authors suggested that one should select events with like-sign dileptons and combine a lepton momentum with the momentum vectors of the nearest appropriately hard jets. The resulting mass distribution roughly tracks the gluino mass and has a width of about 15%. Baer, Chen, Tata, and Paige have criticized this analysis for omitting some backgrounds, but have introduced their own observable applicable simply to missing energy events.[41] In events with missing transverse energy greater than some criterion E_c, and with two jets in one hemisphere with transverse energy greater than E_c, they examine the mass distribution of these two jets. Mass distributions generated by Monte Carlo are shown in Figure 11 for sets of three values of the gluino mass differing by 15%. This analysis makes plausible that such integral variables can produce a gluino mass estimate of reasonable accuracy.

In order to understand whether the gluino is in fact lighter than the squarks, and to measure the mass ratio, a number of techniques can be employed. For example, the $\widetilde{q}_R$ typically decays dominantly into the lightest neutralino, so if these particles are light the missing energy signature is stronger and the jet multiplicity is smaller. The use of jet multiplicity to probe the ratio of the squark and gluino masses in discussed in ref. [41]. An additional amusing probe of the squark-gluino mass ratio has been studied by Basa[42] and by the ATLAS collaboration.[28] If squarks and sleptons are comparable in mass, one of the major processes for supersymmetry production at the LHC is

$$q + q \rightarrow \widetilde{q}\widetilde{q} \qquad (25)$$

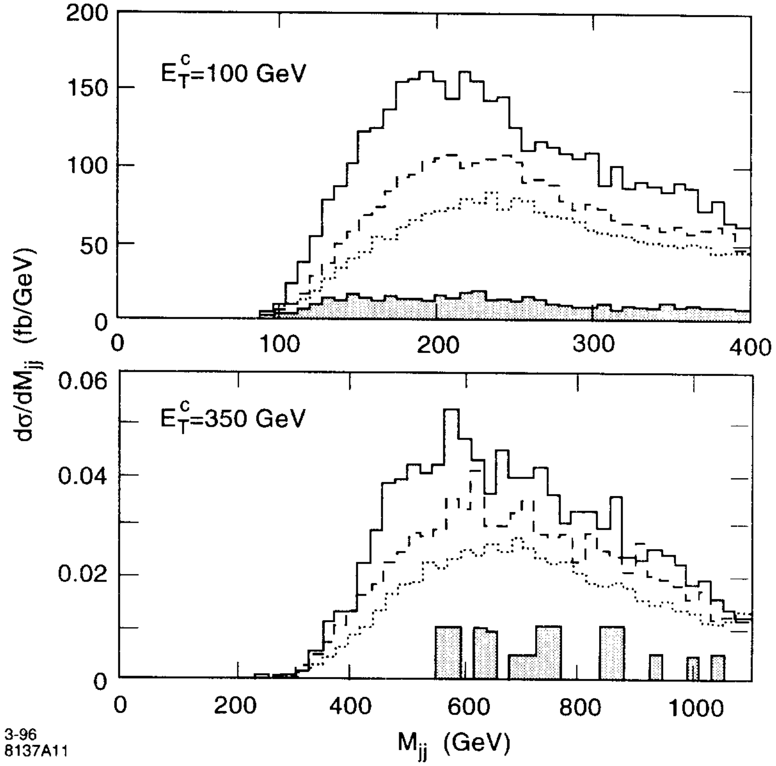

Figure 11: Mass distribution of the estimator of Baer *et al.*, ref. 41, for two different ranges of gluino mass: (a) using $E_c = 100$ GeV, the distributions are shown for $m(\widetilde{g}) = 296$, 340, 369 GeV; (b) using $E_c = 350$ GeV, the distributions are shown for $m(\widetilde{g}) = 773$, 885, 966 GeV. The simulation assumes that the squarks are much heavier than the gluinos.

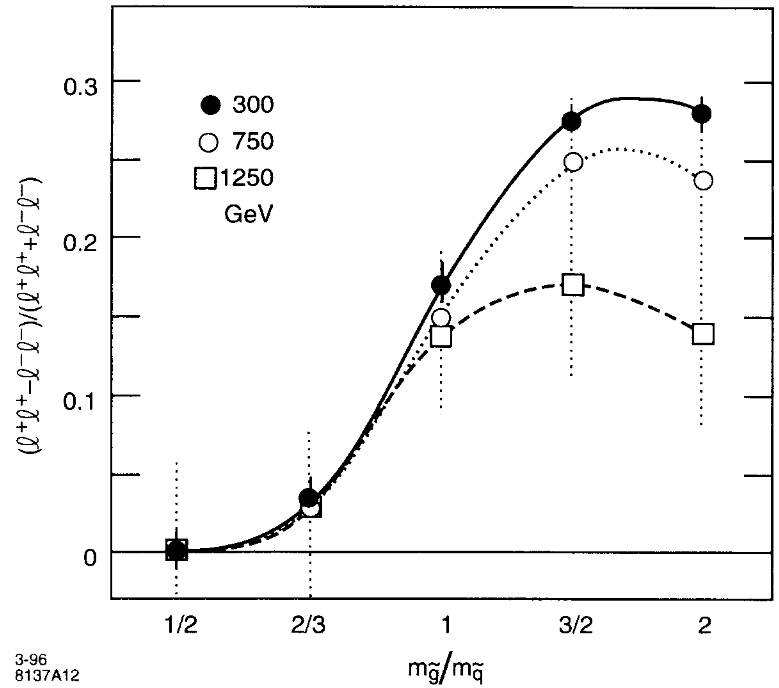

Figure 12: Dependence of the asymmetry between dilepton events with $\ell^+\ell^+$ to those with $\ell^-\ell^-$, as a function of the mass ratio of the squark and gluino, from ref. 28. The three curves represent three different values of the gluino mass.

by t-channel gluino exchange. Since there are more up quarks than down quarks in the proton, this reaction produces an excess of $\ell^+\ell^+$ over $\ell^-\ell^-$ like-sign dilepton events. The asymmetry peaks when the squark and gluino masses are roughly comparable, as shown in Figure 12. On the other hand, the total rate of like-sign dilepton events falls as the gluinos become heavier than the squarks. Thus, it is possible at least in principle to determine the mass ratio from these two pieces of information.

These observables give the flavor of supersymmetry mass determinations in hadronic collisions. There will be considerable information available, if one can learn how to use it. This information resides in integrated reaction rates for various supersymmetry production processes, and in the rates of exotic multilepton reactions. Unfortunately, the spectral pattern is coupled in these observables to the detailed model of squark and gluino decay, which contains the full complexity of the chargino and neutralino mixing problem.

The task of separating these components and extracting the supersymmetry mass parameters purely from hadronic cross sections seems like a nightmare. In fact, none of the analyses I have just described have yet been carried out as systematic surveys over parameter space. It is not so easy to choose a parameter space of sufficiently low dimension that it can be surveyed systematically.

On the other hand, I have emphasized in Section 3 that the experimental environment of e^+e^- colliders provides tools which allow a model-independent determination of the chargino and neutralino mixing parameters. Armed with the results of e^+e^- experiments, the hadron experimenters will be able accurately model the decays of strongly interacting superpartners, and thus to convert their supersymmetry signals to squark and gluino masses. I have argued that the e^+e^- results will play an essential role in turning the wealth of cross sections that the hadron machines will observe into information with fundamental value.

7 Conclusions

In this article, I have tried to sketch out the experimental program that would follow from the discovery of supersymmetry at the weak interaction scale. It is an important question whether supersymmetry is present at the TeV scale, and whether it is the mechanism of electroweak symmetry breaking. But, if indeed Nature chooses this mechanism, what we have to learn at the next generation of colliders goes far beyond this single piece of information. The spectrum of supersymmetric particles contains information which bears directly on the physics of very short distances, perhaps even down to the unification or gravitational scale. The challenge will be to extract this information and study its lessons.

Pursuing this goal, I have set out a three-step program to clarify the physics of the supersymmetry mass spectrum. To set the scale of superpartner masses, we first need to measure the gaugino masses and the Higgs sector parameter $\tan\beta$. In the process, we must test the hypothesis of gaugino unversality. Then, incorporating all of this information, we can measure the slepton and squark masses and try to recognize their pattern as characteristic of a specific messenger of supersymmetry breaking.

Electron-positron colliders have a major role to play in this program. Using their access to the simplest supersymmetry reactions and the handle of polarization, they can make model-independent measurements of the uncolored gaugino masses. They can also provide accurate, helicity-specific measurements of the slepton masses. If the squarks are sufficiently light, they can also provide specific spectral information about the squarks. The details of these experiments will be discussed in the paper of Fujii.[5]

Hadron collider experiments can be expected to pin down the masses of the heavier states of supersymmetry, the squarks and gluinos. However, the observables which are useful for hadron experiments require information on the decay pattern of strongly-interacting superpartners, and thus the interpretation of experimental results from hadron collider will also rely on the precision information available from e^+e^- colldiers.

If supersymmetry is a part of Nature at the weak scale, we can look forward to an exciting future, with experimental information from many sources coming to bear on the deepest questions about the fundamental interactions. Linear colliders have an essential role to play in the grand synthesis that these models promise.

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