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Colour-Octet NLO QCD Corrections to Hadronic χ_J Decays

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Abstract

In this paper we present a complete next-to-leading order QCD calculation of the $\chi_J({}^3P_J\,; J = 0, 1, 2)$ hadronic decay width. We include the NLO colour-octet contribution, as defined in the Bodwin, Braaten and Lepage formalism. We extract an estimate of the colour-octet parameter H_8 from the charmonium decay data.

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1. Heavy quarkonium (HQ) systems are among the most interesting objects that nature gave us to explore perturbative Quantum Chromodynamics (QCD). The predictions of their production cross sections and decay rates were among the most important tests of the early-time QCD. Nowadays there is a large renewed interest in the physics of heavy quarkonium, due above all to the recent discovery of the surprisingly big discrepancies between data and theory in the high-p_T charmonium cross-section production at the Tevatron [1]. Bodwin, Braaten and Lepage (BBL) [2] recently developed a new formalism based on non-relativistic QCD (NRQCD) [3] to implement in a systematic way both the relativistic and the QCD corrections to the naive Colour Singlet Model (CSM) (for a recent review, see for example [4]). This framework has been successfully applied to solve the charmonium anomaly at the Tevatron [**?**]. Another crucial test of perturbative QCD in the heavy quarkonium systems is given by the P-wave hadronic decay rates. In the CSM the inclusive hadronic decay rate of P-wave HQ states shows a singular infrared behaviour [8] which is a clear signal that such a process is sensitive to at least another non-perturbative parameter beyond the usual wave function. In particular, the infrared problem of χ_J (^3P_J) decay arises from the $\Gamma(\chi_J \to q\overline{q}g)$ subprocess. The amplitude associated to this process diverges when the final gluon becomes soft. This ambiguity spoils the traditional factorization picture even at leading order in α_s in the decay of the χ_1 state into light hadrons (LH). In the BBL theory there is the solution of the χ_J decay problem. In the CSM, the heavy quark pair that participates in the hard annihilation process is in a colour singlet state and has the same quantum numbers as the physical bound state: the non-perturbative transition changes neither the colour nor the spin-parity of the heavy-quark pair. In this picture the χ_J decay occurs through the non-perturbative transition $\chi_J \to Q\overline{Q}[^3P_J^{(1)}]$ (the upper right label indicates the colour state), which is parametrized by the derivative of the wave function $|R'(0)|^2$, followed by the annihilation of the $Q\overline{Q}[^3P_J^{(1)}]$ heavy quark pair. Bodwin, Braaten and Lepage suggested that the HQ wave function contains a non-negligible component in which the heavy quark pair is in a $Q\overline{Q}[^3S_1^{(8)}]$ state. This component leads to the HQ decay through the process $Q\overline{Q}[^3S_1^{(8)}] \to q\overline{q}$. The final state created by the colour-octet contribution is degenerate with the colour-singlet one in the kinematical region of soft final gluon. The colour-octet long-distance matrix element absorbs the infrared sensitivity of the colour-singlet term yielding an IR-finite result [9]. In the NRQCD framework it is therefore possible to give a theoretical prediction of χ_J hadron decays avoiding infrared inconsistencies.

Hadronic χ_J annihilation then gives a very important phenomenological test of the role of the colour-octet mechanism, and of the BBL theory in general. In ref. [9] Bodwin, Braaten and Lepage performed a phenomenological analysis of χ_{cJ} decays using the LO results for both the colour-singlet and the colour-octet contributions. They justified the neglect of the known NLO colour-singlet corrections with the observation that NLO accuracy would require inclusion of the yet unknown NLO colour-octet coefficients. In a

following paper [10] it was argued that it is justified to include in the analysis the available QCD NLO colour-singlet terms, because the octet contribution does not depend on J even at NLO. Therefore the higher-order colour-octet coefficient can be simply reabsorbed in a redefinition of the LO octet wave function without changing the values of α_s and $|R'(0)|^2$ extracted from the fit to the experimental data.

In this work we perform the NLO calculation of the colour-octet coefficient completing the picture of the χ_J hadron decay at order $O({\alpha_s}^3 v^5)$ in the sense of the BBL double expansion. The knowledge of the NLO octet correction allows us to extract the value of the parameter H_8 using the results of ref [10].
An analogous calculation relative to the newly discovered $^1P_1$ state [11] has recently appeared [12].

2. The χ_J state is represented as a Fock-space vector superposition of heavy quark pair states of different spin, angular momentum and colour, possibly accompanied by gluons [2]:

$$|\chi_J\rangle = O(1)|Q\overline{Q}[^3P_J^{(1)}]\rangle + O(v)|Q\overline{Q}[^3S_1^{(8)}]g\rangle + \cdots, \tag{1}$$

where v is the relative velocity between the bound quarks. The first term of eq. (1) represents the conventional colour-singlet configuration and the second one corresponds to a colour-octet heavy quark pair in the $Q\overline{Q}[^3S_1^{(8)}]$ state accompanied by a gluon. The small relative bound quark velocity v splits the physics of heavy quarkonium into two well separated energy scales, allowing a formal factorization of the physical observables into perturbative short-distance kernels describing annihilation of the heavy quark pair and soft non-perturbative coefficients. In the BBL factorization framework, the low energy heavy quarkonium physics is described by the NRQCD Lagrangian which has a physical ultraviolet cutoff Λ . The short-distance annihilation effects are implemented including 4-fermion interactions in the Lagrangian:

$$\delta\mathcal{L}_{4-fermions} = \Sigma_n \frac{f_n(\Lambda)}{m^{\delta_n - 4}}\mathcal{O}_n(\Lambda), \tag{2}$$

where m is the mass of the heavy quark. Both the NRQCD operators $\mathcal{O}_n$ and the short-distance coefficients f_n depend on Λ, but their product does not. The operators $\mathcal{O}_n$ have well-defined scaling rules with velocity v and the coefficients f_n have a QCD perturbative definition. Equation (2) can be actually read as a double α_s and v expansion. For our study, the relevant operators $\mathcal{O}_n$ are:

$$\mathcal{O}_1(^3P_0) = \frac{1}{3}\psi^\dagger \left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\sigma\right)\phi\ \phi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\sigma\right)\psi \tag{3}$$

$$\mathcal{O}_1(^3P_1) = \frac{1}{2}\psi^\dagger \left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\times\sigma\right)\phi\cdot\phi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\times\sigma\right)\psi \tag{4}$$

$$\mathcal{O}_1(^3P_2) = \psi^\dagger \left(-\frac{i}{2}\overleftrightarrow{D}^{(i}\sigma^{j)}\right)\phi\ \phi^\dagger\left(-\frac{i}{2}\overleftrightarrow{D}^{(i}\sigma^{j)}\right)\psi \tag{5}$$

$$\mathcal{O}_8(^3S_1) = \psi^\dagger\sigma T^a\phi\cdot\phi^\dagger\sigma T^a\psi \tag{6}$$

The χ_J hadronic decay width can be written as:

$$\Gamma(\chi_J \to LH) = 2\ \mathrm{Im}f_1(^3P_J)\ \frac{\langle\chi_J|\mathcal{O}_1(^3P_J)|\chi_J\rangle}{m^4} + 2\ \mathrm{Im}f_8(^3S_1)\ \frac{\langle\chi_J|\mathcal{O}_8(^3S_1)|\chi_J\rangle}{m^2} \tag{7}$$

The short-distance coefficients can be extracted by matching NRQCD and full QCD amplitudes [2]. The NRQCD matrix elements can be determined phenomenologically or calculated on the lattice.

Defining:

$$H_1 = \frac{\langle\chi_J|\mathcal{O}_1(^3P_J)|\chi_J\rangle}{m^4} \qquad H_8(\Lambda) = \frac{\langle\chi_J|\mathcal{O}_8(^3S_1;\Lambda)|\chi_J\rangle}{m^2}. \tag{8}$$

we can rewrite the χ_J width as follows

$$\Gamma(\chi_J \to LH) = \hat{\Gamma}_1(^3P_J^{(1)} \to LH)H_1 + \hat{\Gamma}_8(^3S_1^{(8)} \to LH)H_8 = \hat{\Gamma}_1(J)H_1 + \hat{\Gamma}_8 H_8 \tag{9}$$

Velocity and mass scaling of the matrix elements of the relevant NRQCD operators are $H_1 \sim mv^5$, $H_8 \sim mv^5$. The QCD leading-order colour-singlet short-distance coefficients are of $O(\alpha_s^2)$ for χ_0 and χ_2 and of $O(\alpha_s^3)$ for χ_1 states. On the other hand, the QCD lowest order colour-octet short-distance process $Q\overline{Q}[^3S_1^{(8)}] \to q\overline{q}$ is of $O(\alpha_s^2)$ while the

process $Q\overline{Q}[^3S_1^{(8)}] \to gg$ with both gluons on the mass shell is forbidden. Therefore a consistent perturbative picture of the χ_1 decay at order ${\alpha_s}^3 v^5$ requires the calculation of the QCD NLO colour-octet contribution. For ease of reference, we collect here the expression for the χ_J decay widths including the NLO colour-singlet terms [8] and the LO colour-octet terms [9]

$$\Gamma(\chi_0 \to LH) = \tfrac{4}{3}\pi {\alpha_s}^2 H_1 [1 + \tfrac{\alpha_s}{\pi} C_0] \quad + \; n_{\rm Lf}\, \tfrac{\pi}{3} {\alpha_s}^2 \left[\tfrac{16}{27}\tfrac{\alpha_s}{\pi} H_1 \log\tfrac{m}{\mathcal{E}} + H_8\right]$$

$$\Gamma(\chi_2 \to LH) = \tfrac{4}{3}\pi {\alpha_s}^2 H_1 [1 + \tfrac{\alpha_s}{\pi} C_2] \tfrac{4}{15} \; + \; n_{\rm Lf}\, \tfrac{\pi}{3} {\alpha_s}^2 \left[\tfrac{16}{27}\tfrac{\alpha_s}{\pi} H_1 \log\tfrac{m}{\mathcal{E}} + H_8\right] \qquad (10)$$

$$\Gamma(\chi_1 \to LH) = \tfrac{4}{3}\pi {\alpha_s}^2 H_1 [\quad\; \tfrac{\alpha_s}{\pi} C_1] \quad + \; n_{\rm Lf}\, \tfrac{\pi}{3} {\alpha_s}^2 \left[\tfrac{16}{27}\tfrac{\alpha_s}{\pi} H_1 \log\tfrac{m}{\mathcal{E}} + H_8\right]$$

where

$$C_0 = \left(\tfrac{454}{81} - \tfrac{1}{144}\pi^2 - \tfrac{11}{3}\log 2\right) C_A + \left(-\tfrac{7}{3} + \tfrac{\pi^2}{4}\right) C_F + n_{\rm Lf}\, \left(-\tfrac{16}{27} + \tfrac{2}{3}\log 2\right)$$

$$C_2 = \left(\tfrac{2239}{216} - \tfrac{337}{384}\pi^2 - 2\log 2\right) C_A - 4 C_F + n_{\rm Lf}\, \left(-\tfrac{11}{18} + \tfrac{2}{3}\log 2\right) \qquad (11)$$

$$C_1 = \left(\tfrac{587}{54} - \tfrac{317}{288}\pi^2\right) + n_{\rm Lf}\, \tfrac{28}{81}$$

$n_{\rm Lf}$ is the number of light quarks: $n_{\rm Lf} = 3$ for charmonium and $n_{\rm Lf} = 4$ for bottomonium states and $\alpha_s = \alpha_s(m)$. H_1 is related to the derivative of the wave function through the relation:

$$H_1 = \frac{9}{2\pi} \frac{|R'(0)|^2}{m^4} \left[1 + O(v^2)\right] \qquad (12)$$

We denote by $\mathcal{E}$ a momentum scale that regularizes the soft divergence associated to the $\chi_J \to q\overline{q}g$ process. $\mathcal{E}$ was usually related to the binding energy of quarkonium. Notice that neither the colour-octet term nor the coefficient of the divergent logarithm depends on the quarkonium spin J; this fact makes a universal renormalization of the parameter H_8 possible. In fact, considering only the universal piece (U_Γ) of χ_J widths we get:

$$U_\Gamma = n_{\rm Lf}\,\frac{\pi}{3}{\alpha_s}^2\left[\frac{16}{27}\frac{\alpha_s}{\pi}H_1\left(\log\frac{m}{\Lambda}+\log\frac{\Lambda}{\mathcal{E}}\right)+{H_8}^{(b)}\right] \quad (13)$$

$$= n_{\rm Lf}\,\frac{\pi}{3}{\alpha_s}^2\left[H_8(\Lambda)+\frac{16}{27}\frac{\alpha_s}{\pi}H_1\log\frac{m}{\Lambda}\right] = n_{\rm Lf}\,\frac{\pi}{3}{\alpha_s}^2 H_8(m). \quad (14)$$

The Λ-dependence of the colour-singlet coefficient is consistent with that specified by the RGE for H_8 [2].

3. We perform the calculation of the full NLO QCD colour-octet contribution to the χ_J decay widths term using the dimensional regularization scheme to regularize UV, IR and collinear divergences. We work in D$= 4-2\epsilon$ dimensions. If we define:

$$\hat{\Gamma}_8^{(0)} = \pi{\alpha_s}^2\frac{1-\epsilon}{3-2\epsilon}\left(\frac{4\pi\mu^2}{M^2}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)} \quad (15)$$

where $M \equiv 2\,m$, then the D-dim Born colour-octet short-distance coefficient assumes the form:

$$\hat{\Gamma}_8^{(Born)} = n_{\rm Lf}\,\hat{\Gamma}_8^{(0)} \quad (16)$$

The NLO correction to $\hat{\Gamma}_8$ consists of real and virtual emission of gluons.

Real emission

The real correction to the short-distance colour-octet χ_J annihilation term is represented by the two processes $Q\overline{Q}[^3S_1^{(8)}] \to ggg$ and $Q\overline{Q}[^3S_1^{(8)}] \to q\overline{q}g$. The calculation of the D=4 $Q\overline{Q}[^3S_1^{(8)}] \to ggg$ amplitude can be obtained via crossing from the results of ref [7]. This amplitude is completely IR and collinear finite because the two-gluon leading order amplitude vanishes. The calculation of the three-gluon real contribution is therefore straightforward. We obtain [1]:

$$\hat{\Gamma}_8^{(ggg)} = \hat{\Gamma}_8^{(0)}\frac{\alpha_s}{\pi}\,5\,\left(-\frac{73}{4}+\frac{67}{36}\pi^2\right) \quad (17)$$

[1]The expression reported here assumes implicitly $N_c = 3$, since the explicit N_c dependence of the matrix elements is not reported in the result of ref. [7]

On the contrary the process $Q\overline{Q}[^3S_1^{(8)}] \to q\bar{q}g$ shows IR and collinear poles that we expect will cancel when adding the virtual correction. The D-dimension $Q\overline{Q}[^3S_1^{(8)}] \to q\bar{q}g$ amplitude that we obtain is in agreement with ref [7] in the D=4 limit. It leads to the following width contribution

$$d\hat{\Gamma}_8^{(q\bar{q}g)} = n_{\rm Lf}\,\frac{64\pi^3{\alpha_s}^3}{M^2(3-2\epsilon)}\left[t^2+u^2+2M^2s-\epsilon(t+u)^2\right]\left[\frac{C_F}{tu}-\frac{C_A}{(s-M^2)^2}\right]d(PS)[q\bar{q}g] \quad (18)$$

where $s=(q+\overline{q})^2$, $t=(q+g)^2$, $u=(\overline{q}+g)^2$. Performing the Mandelstam variable substitution $s=M^2(1-x)$, $t=M^2xy$, $u=M^2x(1-y)$, the phase space assumes the following form:

$$d(PS)[q\bar{q}g] = \frac{M^2}{128\pi^3}\left(\frac{4\pi\mu^2}{M^2}\right)^{2\epsilon}\frac{1}{\Gamma(1-\epsilon)}x\left[x^2(1-x)y(1-y)\right]^{-\epsilon}dxdy. \quad (19)$$

Integrating over the phase space we get

$$\hat{\Gamma}_8^{(q\bar{q}g)} = n_{\rm Lf}\,\hat{\Gamma}_8^{(0)}\frac{\alpha_s}{\pi}f(\epsilon)\left[C_F\left(\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}\right)+C_A\frac{1}{2\epsilon}+C_F\left(\frac{19}{4}-\frac{2}{3}\pi^2\right)+\frac{11}{6}C_A\right] \quad (20)$$

where

$$f(\epsilon) = \left(\frac{4\pi\mu^2}{M^2}\right)^{\epsilon}\Gamma(1+\epsilon)\ . \quad (21)$$

Virtual emission

The diagrams contributing to NLO virtual emission are shown in fig. (1). In table 1 we list the contribution $\mathcal{D}_k$ of each diagram with the relative colour factors. The virtual colour-octet width can be written as

$$\hat{\Gamma}_{8,QCD}^{(Virtual)} = n_{\rm Lf}\,\hat{\Gamma}_8^{(0)}f(\epsilon)\frac{\alpha_s}{\pi}\sum\nolimits_k(\mathcal{D}_k f_k) \quad (22)$$

Summing all the virtual diagrams, we find

$$\hat{\Gamma}_{8,QCD}^{(Virtual)} = n_{\rm Lf}\,\hat{\Gamma}_8^{(0)}\left\{1+\frac{\alpha_s}{\pi}f(\epsilon)\left[2b_0\frac{1}{\epsilon_{\rm UV}}-C_F\left(\frac{1}{{\epsilon_{\rm IR}}^2}+\frac{3}{2\epsilon_{\rm IR}}\right)-C_A\frac{1}{2\epsilon_{\rm IR}}\right.\right.$$
$$\left.\left.+\quad\frac{\pi^2}{v}\left(C_F-\frac{1}{2}C_A\right)+A\right]\right\} \tag{23}$$

where

$$b_0=\frac{1}{2}\left(\frac{11}{6}C_A-\frac{2}{3}n_{\rm Lf}\,T_F\right) \tag{24}$$

$$A=C_F\left(-8+\frac{2}{3}\pi^2\right)+C_A\left(\frac{50}{9}+\frac{2}{3}\log 2-\frac{\pi^2}{4}\right)-\frac{10}{9}n_{\rm Lf}\,T_F \tag{25}$$

$$v\equiv\left[1-\frac{4m^2}{(p_Q+p_{\overline{Q}})^2}\right]^{-\frac{1}{2}}. \tag{26}$$

The Coulomb singularity disappears by performing the matching between the NLO full QCD and NRQCD amplitudes, yielding a finite result in the $v\to 0$ limit. Summing the real and the virtual emission corrections, we obtain the α_s NLO colour-octet decay width for χ_J states; we give the result for C_F =4/3, C_A = 3 and T_F =1/2:

$$\hat{\Gamma}_8^{(NLO)} = \hat{\Gamma}_8^{(Born)}\left[1+\frac{{\alpha_s}^{\overline{MS}}(\mu)}{\pi}\left(\frac{107}{6}-\frac{3}{4}\pi^2+2\log 2+\right.\right.$$
$$\left.\left.-\quad\frac{5}{9}n_{\rm Lf}+4b_0\log\frac{\mu}{2\,m}\right)\right]+\hat{\Gamma}_8^{(0)}\frac{{\alpha_s}^{\overline{MS}}(\mu)}{\pi}\,5\,\left(-\frac{73}{4}+\frac{67}{36}\pi^2\right). \tag{27}$$

Choosing the renormalization scale $\mu=m$ (the same of the colour-singlet terms in eqs. (10)) we obtain the following expression of the α_s NLO imaginary part of $f_8(^3S_1)$:

$$\text{Im } f_8(^3S_1) = \frac{\pi}{6}\left({\alpha_s}^{\overline{MS}}(m)\right)^2\left[n_{\rm Lf}+n_{\rm Lf}\frac{{\alpha_s}^{\overline{MS}}(m)}{\pi}\left(\frac{107}{6}-\frac{3}{4}\pi^2-9\log 2+\right.\right.$$
$$\left.\left.-\quad\frac{5}{9}n_{\rm Lf}+\frac{2}{3}n_{\rm Lf}\log 2\right)+\frac{{\alpha_s}^{\overline{MS}}(m)}{\pi}\,5\left(-\frac{73}{4}+\frac{67}{36}\pi^2\right)\right] \tag{28}$$

Always keeping $\mu = m$, we report below the numerical colour-octet corrections for charmonium and bottomonium:

$$\frac{\hat{\Gamma}_8^{(NLO)}}{\hat{\Gamma}_8^{(Born)}} = 1 + 3.9\ \frac{{\alpha_s}^{\overline{MS}}(m_c)}{\pi} \qquad \text{[charm]} \tag{29}$$

$$\frac{\hat{\Gamma}_8^{(NLO)}}{\hat{\Gamma}_8^{(Born)}} = 1 + 3.8\ \frac{{\alpha_s}^{\overline{MS}}(m_b)}{\pi} \qquad \text{[bottom]} \tag{30}$$

Using the results of ref. [10] it is straightforward to obtain the best fit of the parameter H_8 for charmonium χ_c including the NLO QCD effects, and we get the value ${H_8}^{(c)}(m_c) = 3.1 \pm 0.5$ MeV. Taking $m_c = 1.5$ GeV we obtain $\langle \chi_{cJ}|\mathcal{O}_8(^3S_1; m_c)|\chi_{cJ}\rangle = (6.8 \pm 1.1) \times 10^{-3}$ GeV^3. For completeness we recall that the fits of the other parameters obtained in [10] are $H_1 = 13.7 \pm 2.3$ MeV and ${\alpha_s}^{\overline{MS}}(m_c) = 0.286 \pm 0.031$. As discussed in the introduction these results are not affected by the inclusion of the NLO colour-octet corrections. Using the NRQCD scaling rules, we can obtain an estimate of the bottom octet matrix element ${H_8}^{(b)}(m_b) \simeq 0.66$ MeV.

We now want to analyse the renormalization scale dependence of the NLO colour-octet decay widths compared with the leading-order ones. The results are shown in figs. 2 and 3 for charmonium and bottomonium states, respectively. The normalization of the bottomonium width is achieved by using the estimate of the colour-octet parameter ${H_8}^{(b)}(m_b)$ obtained above through the NRQCD scaling rules. For the running of two-loop α_s we use the input $\Lambda_{n_f=5}^{\overline{MS}} = 160$ MeV extracted from the fitted value of ${\alpha_s}^{\overline{MS}}(m_c)$. The pictures show that the inclusion of NLO corrections significantly reduces the scale dependence of the processes.

To conclude we notice that the calculation presented here can be used to compute the strong NLO $q\overline{q} \to Q\overline{Q}[^3S_1^{(8)}]$ contribution to the total ψ hadronic production cross section and the NLO colour-octet fragmentation function of the gluon into ψ. Work on these issues is in progress.

Diag.	$\mathcal{D}_k$	f_k
a	$\left[-\frac{1}{2\epsilon_{\rm UV}}+\frac{1}{2\epsilon_{\rm IR}}\right]$	C_F
b	$\left[\frac{1}{2\epsilon_{\rm UV}}-\frac{1}{{\epsilon_{\rm IR}}^2}-\frac{2}{\epsilon_{\rm IR}}-4+\frac{2}{3}\pi^2\right]$	$C_F-\frac{1}{2}C_A$
c	$\left[\frac{3}{2\epsilon_{\rm UV}}-\frac{2}{\epsilon_{\rm IR}}-1\right]$	$\frac{1}{2}C_A$
d	$\left[-\frac{1}{2\epsilon_{\rm UV}}-\frac{1}{\epsilon_{\rm IR}}-2-3\log 2\right]$	C_F
e	$\left[\frac{\pi^2}{v}+\frac{1}{2\epsilon_{\rm UV}}+\frac{1}{\epsilon_{\rm IR}}-2+3\log 2\right]$	$C_F-\frac{1}{2}C_A$
f	$\left[\frac{3}{2\epsilon_{\rm UV}}+\frac{8}{3}+\frac{13}{3}\log 2\right]$	$\frac{1}{2}C_A$
g	$\left[\frac{5}{6}\frac{1}{\epsilon_{\rm UV}}+\frac{31}{18}\right]$	C_A
h	$\left[-\frac{2}{3\epsilon_{\rm UV}}-\frac{10}{9}\right]n_{\rm Lf}$	T_F
i	$\left[\frac{1}{{\epsilon_{\rm IR}}^2}-\frac{\pi^2}{6}\right]$	$2C_F-C_A$
j	$\left[-\frac{1}{{\epsilon_{\rm IR}}^2}+\frac{\pi^2}{6}\right]$	$2C_F-\frac{1}{2}C_A$

Table 1: Partial virtual QCD corrections to the process $Q\overline{Q}[^3S_1^{(8)}] \to q\overline{q}$

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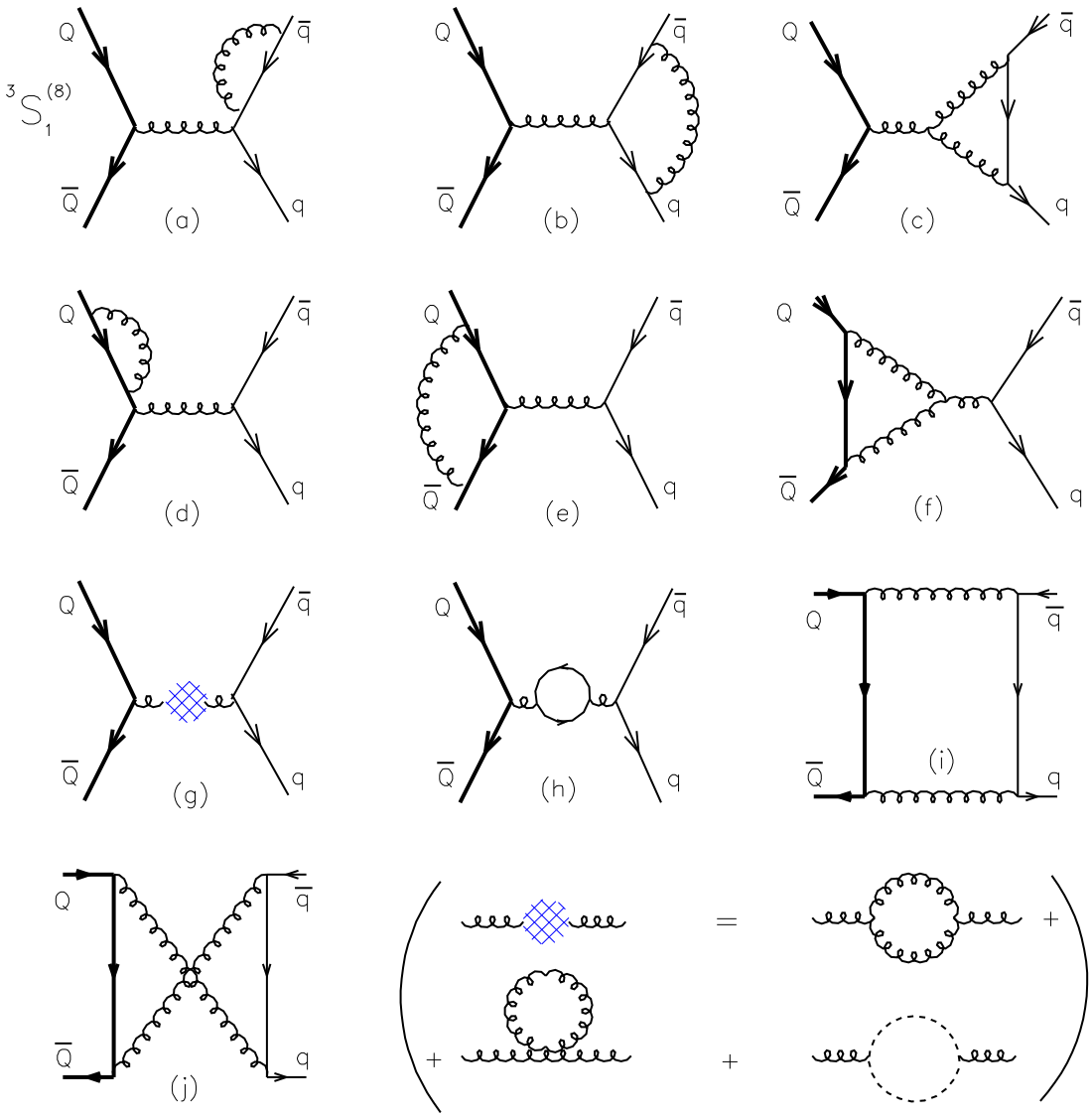

Figure 1: Virtual Feynman diagrams contributing to the process $Q\overline{Q}[^3S_1^{(8)}] \to q\overline{q}$

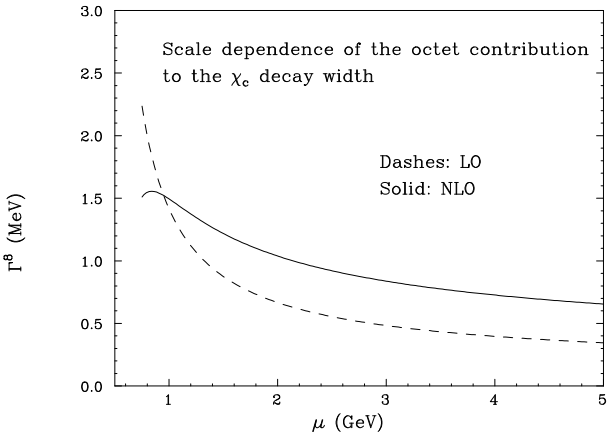

Figure 2: Renormalization scale dependence of the colour-octet contribution to χ_c hadronic decay width $\Gamma_8 = \hat{\Gamma}_8 H_8$

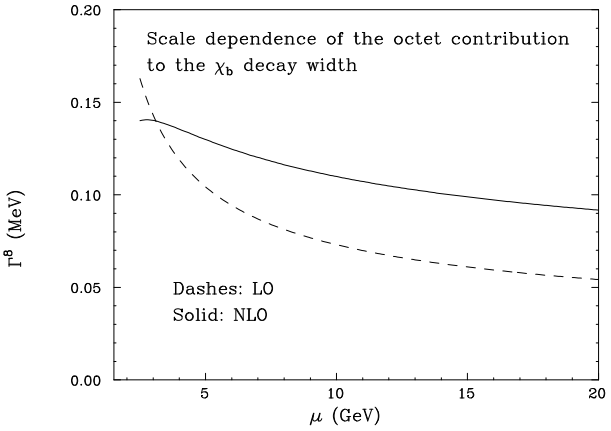

Figure 3: Same as fig. 2 but for χ_b