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NONRESTORATION OF SPONTANEOUSLY BROKEN P, CP AND PQ AT HIGH TEMPERATURE

Gia Dvali*
CERN, CH-1211, Geneva 23, Switzerland

Alejandra Melfo
SISSA, 34014 Trieste, Italy, and *Centro de Astrofísica Teórica, Universidad de Los Andes Mérida 5101-A, Venezuela*

Goran Senjanović
International Center for Theoretical Physics 34100 Trieste, Italy
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Abstract

The possibility of P and CP violation at high temperature in models where these symmetries are spontaneously broken is investigated. It is found that in minimal models that include singlet fields, high T nonrestoration is possible for a wide range of parameters of the theory, in particular in models of CP violation with a CP-odd Higgs field. The same holds true for the invisible axion version of the Peccei-Quinn mechanism. This can provide both a way out for the domain wall problem in these theories and the CP violation required for baryogenesis. In the case of spontaneous P violation it turns out that high T nonrestoration requires going beyond the minimal model. The results are shown to hold true when next-to-leading order effects are considered.

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*e-mail adresses: dvali@surya11.cern.ch, melfo@stardust.sissa.it, goran@ictp.trieste.it

I. INTRODUCTION

The phenomenon of spontaneous symmetry breaking has become a cornerstone of modern particle physics. To be able to establish a connection between particle physics and cosmology, it is essential to investigate the behavior of symmetry breaking in the early universe, *i.e.* at high temperature. In spite of common sense prejudice, it is by now known that more heat does not necessarily imply more symmetry [1,2]. Rather, the question of symmetry restoration is quite a complex phenomenon and depends on the dynamics of the theory considered.

Examples have been found with symmetries remaining broken at arbitrarily high temperature, or even exact symmetries becoming broken as the system gets heated up [2–5]. However, some of these examples were artificially created just in order to demonstrate the phenomenon. In our opinion, symmetry nonrestoration becomes relevant only when resulting from minimal and realistic models. This is precisely what we wish to address in this paper. For the sake of focus, we concentrate on the issues of P and CP violation (both weak and strong). The choice of parity and time reversal is in our opinion natural, these being fundamental symmetries of nature. Furthermore, the spontaneous breaking of these symmetries may offer a simple way out of the strong CP problem [6].

There are at least two important reasons to have CP broken at high temperature. Baryogenesis requires CP violation, and if one is to adhere to the appealing idea of CP symmetry being broken spontaneously, its nonrestoration becomes a must. On the other hand, the spontaneous breakdown of a discrete symmetry leads to a domain wall problem, following the phase transition that takes place if the symmetry is restored at high T [7,8]. Avoiding this phase transition may be sufficient to solve the problem, since the thermal production of large domain walls is naturally suppressed for a wide range of the parameters of the theory [9]. In section II, we study CP behavior at high temperature in some $SU(2) \times U(1)$ theories with Higgs doublets and singlets only. It turns out that in minimal such models with doublets only CP is always restored, whereas it can naturally remain broken if there is at least one singlet on top of the usual Higgs doublet.

Section III is devoted to P violation and there we find that nonrestoration of P at high T seems to be in conflict with perturbation theory. Again, the existence of P odd singlets, welcome for the implementation of the minimal see-saw mechanism, works in favor of nonrestoration of P just as in the case of CP.

There is yet another class of theories plagued by the domain wall problem, that is, those based on the Peccei-Quinn solution [10] to the strong CP problem. Once again, symmetry nonrestoration can solve the problem [9]. In section IV we demonstrate in detail how this is achieved.

It has been pointed out that next-to-leading order corrections to the high temperature effective potential may play an important role on the question of nonrestoration, even to the extent of invalidating it in the case of local gauge symmetries [11,12]. However, a recent study [13] involving a Wilson renormalization group approach which simulates nonperturbative effects, seems to encourage the validity of the conventional one loop results. Since the issue is not completely settled, to be on the safe side we show in section V how inclusion of next-to-leading order terms does not affect any of our conclusions.

Focusing on CP forced us to ignore some rather important applications of the idea of

symmetry nonrestoration, in particular a possible solution to the monopole problem in grand unified theories [14,15]. We leave this and related issues for the future.

II. SPONTANEOUS CP VIOLATION AND HIGH T

As with any discrete symmetry, we would like to be able to keep CP broken at high temperature in order to avoid the formation of the dangerous domain walls. In the case of CP, there is yet an additional reason not to restore it in the early universe, at least not until the time of baryogenesis. Simply, CP must be broken in order for matter to be created [16]. This was actually the original motivation of the first application in particle physics of the phenomenon of nonrestoration of symmetries at high temperature [2]. The model presented in [2] however does not satisfy the minimality condition introduced above, since there the Higgs sector is extended to three doublets only in order to have high T symmetry nonrestoration.

A. $C\!\!\!/P$ with two doublets

The simplest and original example of a theory with spontaneous CP violation was presented by T.D. Lee [17]. His model is an extension of the Standard Model with two complex Higgs doublets, with

$$\mathcal{L}_H = \sum_{i=1}^{2} \frac{1}{2}(D_\mu \Phi_i)^\dagger (D^\mu \Phi_i) - V(\Phi_1, \Phi_2) \tag{1}$$

where

$$\begin{aligned} V(\Phi_i, \Phi_2) = & \sum_{i=1}^{2} \left(-\frac{m_i^2}{2}\Phi_i^\dagger \Phi_i + \frac{\lambda_i}{4}(\Phi_i^\dagger \Phi_i)^2 \right) - \frac{\alpha}{4}\Phi_1^\dagger \Phi_1 \, \Phi_2^\dagger \Phi_2 \\ & - \frac{\beta}{4}\Phi_1^\dagger \Phi_2 \, \Phi_2^\dagger \Phi_1 + \frac{1}{8}\left[\Phi_1^\dagger \Phi_2 \left(a\Phi_1^\dagger \Phi_2 + b\Phi_1^\dagger \Phi_1 + c\Phi_2^\dagger \Phi_2 \right) + h.c. \right] \end{aligned} \tag{2}$$

Choosing the parameter $\beta > 0$, one can prove that the minimum of the potential is achieved when the fields acquire vevs

$$\Phi_1 = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad ; \quad \Phi_2 = \begin{pmatrix} 0 \\ v_2 \end{pmatrix} e^{i\theta} \tag{3}$$

The terms in brackets in the potential will force the CP-violating phase θ to be non-zero. This can be readily seen by writing (2) at the minimum (3), and wisely rearranging terms:

$$V(\langle \Phi_1 \rangle, \langle \Phi_2 \rangle) = \sum_{i=1}^{2} \left(-\frac{m_i^2}{2} v_i^2 + \frac{p_i}{4} v_i^4 \right) + \frac{\rho}{4} v_i^2 v_2^2 + \frac{a}{2} v_i^2 v_2^2 [\cos\theta - \delta]^2 \tag{4}$$

where

$$p_1 = \lambda_1 - \frac{b^2}{8a} \quad ; \quad p_2 = \lambda_2 - \frac{c^2}{8a}$$
$$\rho = \alpha + \beta + a + \frac{cb}{4a} \quad ; \quad \delta = \frac{-(bv_1^2 + cv_2^2)}{4av_1v_2} \tag{5}$$

Obviously, for $a > 0$ the minimum will be at $\cos\theta = \delta$, and CP is broken spontaneously.

We are interested in the possibility that CP remains broken at arbitrarily high temperature. For this to happen in T.D. Lee's model, we need not only to have the vev's of *both* Φ_1 and Φ_2 nonzero at high T, but also to keep the CP-violating phase from vanishing.

To get an idea of how both vev's may be kept different from zero, consider a simple model with two real scalar fields (ϕ_1, ϕ_2), and a potential with a Z_2 symmetry $\phi_1 \to -\phi_1$, $\phi_2 \to -\phi_2$

$$V(\phi_1, \phi_2) = \sum_{i=1}^{2} \left(-\frac{m_i^2}{2}\phi_i^2 + \frac{\lambda_i}{4}\phi_i^4 \right) - \frac{\alpha}{2}\phi_1^2\phi_2^2 + \beta_1\,\phi_1^3\phi_2 + \beta_2\,\phi_2^3\phi_1 \tag{6}$$

One can always choose $\alpha > 0$, $\beta_1, \beta_2 > 0$, and require

$$\lambda_1\lambda_2 > \alpha^2 \tag{7}$$

so that the potential is bounded from below. The potential has extrema at $\langle\phi_1\rangle = v_1$, $\langle\phi_2\rangle = v_2$ satisfying

$$[-m_1^2 + \lambda_1 v_1^2 - \alpha v_2^2 + 3\beta_1 v_1 v_2]v_1 + \beta_2 v_2^3 = 0 \tag{8a}$$

$$[-m_2^2 + \lambda_2 v_2^2 - \alpha v_1^2 + 3\beta_2 v_2 v_1]v_2 + \beta_1 v_1^3 = 0 \tag{8b}$$

With negative mass terms, both vevs are nonzero. Admittedly, this model does not belong to the class of minimal models as defined in this paper, since one can break the Z_2 symmetry with just one vev; however, we include it in order to illustrate the role of the linear terms in symmetry nonrestoration.

At high temperature, the effective potential acquires the additional terms [1,18,19]

$$\Delta V = \frac{T^2}{24}\left[(3\lambda_1 - \alpha)\phi_1^2 + (3\lambda_2 - \alpha)\phi_2^2 + 6(\beta_1 + \beta_2)\phi_1\phi_2)\right] \tag{9}$$

By asking, e.g. $\alpha > 3\lambda_1$, one can keep one of the mass terms negative at any temperature, while (7) forces the other to be positive. However, the cubic terms in (8) guarantee that only one negative mass term suffices to have *both* vevs non zero at high T. In other words, the field with the negative mass term acquires a vev and "forces" the other to get one also, via the linear terms in the potential. The reader must have noticed that we can redefine the fields at high T so that just one of them has a nonvanishing vev. However, she should keep in mind that the same holds true at $T = 0$; the point is that the symmetry breaking patterns at high and low T are equal.

One can hope that the potential in T.D. Lee's model, being of a similar form as (6), will exhibit a similar behavior, allowing both vev's to remain non zero at high temperature.

Unfortunately, it is readily found out that it does so at the expense of having the phase θ going to zero, thus restoring CP, as we now show.

The high temperature corrections to the effective potential for a model with N Higgs doublets can be found by generalizing Weinberg's formula [1] for complex doublets. Write the most general potential for N complex doublets as [1]

$$V = -\sum_{i=1}^{N} m_i^2 \Phi_i^\dagger \Phi_i + \sum_{i,j,k,l=1}^{N} \lambda_{ijkl} \Phi_i^\dagger \Phi_j \Phi_k^\dagger \Phi_l \tag{10}$$

Then the high T correction is

$$\Delta V(T) = \sum_{i,j,k=1}^{N} \frac{T^2}{6} \left(2\lambda_{ijkk} + \lambda_{kijk}\right) \Phi_i^\dagger \Phi_j \tag{11}$$

For the two doublet model (2), this gives

$$\Delta V(T) = \frac{T^2}{6}\left[(6\lambda_1 - 2\alpha - \beta)\Phi_1^\dagger\Phi_1 + (6\lambda_2 - 2\alpha - \beta)\Phi_2^\dagger\Phi_2 + \frac{3}{2}(b+c)(\Phi_1^\dagger\Phi_2 + h.c)\right] \tag{12}$$

The potential at high T can then be cast in the same form (4), where now the masses m_i^2 are replaced by $m_i^2(T)$

$$\begin{aligned} m_1^2(T) &= -m_1^2 + 2T^2\left(\lambda_1 - \frac{\alpha}{3} - \frac{\beta}{6} - \frac{b(b+c)}{16a}\right) \simeq 2T^2\nu_1^2 \\ m_2^2(T) &= -m_2^2 + 2T^2\left(\lambda_2 - \frac{\alpha}{3} - \frac{\beta}{6} - \frac{c(b+c)}{16a}\right) \simeq 2T^2\nu_2^2 \end{aligned} \tag{13}$$

for $T >> m$; and δ becomes $\delta(T)$:

$$\delta(T) = -\left[\frac{bv_1^2 + cv_2^2 + T^2(b+c)}{4av_1v_2}\right] \tag{14}$$

Again, as in the simpler model, one can have one and only one mass negative at high T, due to the condition analogous to (7), i.e.

$$p_1 p_2 > \frac{\rho^2}{4} \tag{15}$$

since now

$$\nu_1^2 = p_1 - \sigma\ ; \quad \nu_2^2 = p_2 - \sigma\ ; \quad \text{with} \quad \sigma = \frac{\rho}{2} - \frac{\alpha}{6} - \frac{\beta}{3} - \frac{a}{2} < \frac{\rho}{2} \tag{16}$$

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[1] Obviously we do not worry about the potential being hermitian. Needless to say, the reader should take care of this in choosing her potential, and then safely proceed to use our formula for $\Delta V(T)$

Requiring $\nu_1^2, \nu_2^2 < 0$ will give $p_1 p_2 < \sigma^2 < \rho^2/4$, which contradicts (15).

Considering only the θ-dependent part, we see as before that there is a minimum for $\theta = \delta(T)$. However, it is not difficult to see that with only one mass term negative, both vevs cannot be nonzero at high T, due to the fact that the mass terms now depend on the coupling constants. Taking $\nu_2^2 < 0$, the requirement that v_1 be real gives

$$|\nu_2^2|\frac{\rho}{2} > \nu_1^2 p_2 \tag{17}$$

together with (15), this is also enough to ensure that v_2 is real. Substituting for ν_1^2 and ν_2^2 one gets

$$\frac{\rho}{2}\left(\frac{\rho}{2} - p_2\right) > \frac{\rho}{2}(\sigma - p_2) > (p_1 - \sigma)\, p_2 > \left(p_1 - \frac{\rho}{2}\right) p_2 \tag{18}$$

Which again implies $p_1 p_2 < \rho^2/4$, contradicting (15).

We conclude then that the only way to have both fields with a nonvanishing vev at high temperature is to set the phase θ to zero. In other words, the field with a negative mass term can "force" the other to acquire a vev, but it drags it in the same direction in $U(1)$ space.

Notice that in [2] the fact that both vevs can be nonzero was overlooked, but it was still concluded correctly that with two doublets only, CP would become a good symmetry at high T.

B. $\not{C}\not{P}$ and natural flavor conservation

A common feature of models with two Higgs doublets as the one in the previous section is that they allow for flavor-violating interactions in neutral current phenomena. As shown in [20–22], the minimal model for spontaneous CP violation involving doublets only that conserves flavor, requires three of them.

To see why, consider a Lagrangian with two complex Higgs as in(1), (2), and an extra symmetry D_1

$$\Phi_1 \longrightarrow -\Phi_1 \qquad u_{iR} \longrightarrow -u_{iR} \tag{19}$$

(where $u_{a\,R}$ are up quarks and hereafter $a, b, ..$ are flavor indices). The Yukawa interactions are written now

$$\mathcal{L}_Y = (\bar{u}\bar{d})^a_L h^1_{ab} \Phi_1 d^b_R + (\bar{u}\bar{d})^a_L h^2_{ab} (i\tau_2) \Phi^*_2 u^b_R \tag{20}$$

so that flavor violation through neutral Higgs exchange is avoided. However, now the symmetry prohibits the terms of the type $\Phi_1^\dagger \Phi_1 \Phi_1^\dagger \Phi_2$ in the Higgs potential, and therefore at the minimum we have the phase $\theta = 0$ or $\pi/2$, both leading to CP conservation.

The way out is to have three doublets, and an additional symmetry D_2 that prevents it from coupling to the quarks: $\Phi_3 \to -\Phi_3$, with other fields unchanged. The most general potential invariant under $SU(2) \times U(1) \times D_1 \times D_2$ is

$$V = \sum_{i=1}^{3} \left[-m_i^2 \Phi_i^\dagger \Phi_i + \lambda_i (\Phi_i^\dagger \Phi_i)^2 \right]$$
$$+ \sum_{i<j} \left[-\alpha_{ij} (\Phi_i^\dagger \Phi_i)(\Phi_j^\dagger \Phi_j) - \beta_{ij} (\Phi_i^\dagger \Phi_j)(\Phi_j^\dagger \Phi_i) + \gamma_{ij} (\Phi_i^\dagger \Phi_j \Phi_i^\dagger \Phi_j + h.c.) \right] \tag{21}$$

It can be shown [20–22] that choosing $\beta_{ij}, \gamma_{ij} > 0$, the above potential has a minimum at

$$\Phi_i = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_i e^{i\theta_i} \end{pmatrix} \tag{22}$$

where only two of the θ_i (say, θ_1 and θ_3) are relevant. Extremization with respect to θ yields [21]

$$\gamma_{12} v_2^2 \sin 2\theta_1 + \gamma_{13} v_3^2 \sin 2(\theta_1 - \theta_3) = 0 \tag{23a}$$

$$\gamma_{13} v_1^2 \sin 2(\theta_1 - \theta_3) + \gamma_{23} v_2^2 \sin 2\theta_3 = 0 \tag{23b}$$

Notice that to have CP violation, we need all three v_i and both θ_1, θ_3 to be nonzero.

It can be shown [22] that the CP violating solution of (23) is indeed a minimum. When the phases take this value, the remaining potential is

$$V(v_i) = \sum_{i=1}^{3} \left(-\frac{m_i}{2} v_i^2 + \frac{p_i}{4} v_i^4 \right) - \sum_{i<j} \frac{(\alpha_{ij} + \beta_{ij})}{4} v_i^2 v_j^2 \tag{24}$$

where

$$p_1 = \lambda_1 - \frac{\gamma_{12}\gamma_{13}}{\gamma_{23}} \tag{25}$$

and analogous expressions for p_2, p_3.

Once again, we are interested in whether the CP symmetry can remain broken at high temperatures. It is straightforward using (11) to calculate the masses at high temperature

$$m_i^2(T) = -m_i^2 + \frac{T^2}{6} \left[6p_i - \sum_{j \neq i} (2\alpha_{ij} + \beta_{ij}) \right] \simeq \frac{T^2}{3} \nu_i^2 \tag{26}$$

Due to the high degree of symmetry of the potential, temperature contributions are independent of the phases, so equations (23) are the same.

For the potential to be bounded from below, a set of constraints analogous to (7) has to be imposed on the couplings, namely

$$p_i > 0 \qquad p_i p_j > a_{ij} \qquad \text{for each } i < j \tag{27a}$$

$$p_1 p_2 p_3 - p_1 a_{23}^2 - p_2 a_{13}^2 - p_3 a_{12}^2 - 2a_{12}a_{13}a_{23} > 0 \tag{27b}$$

with $a_{ij} \equiv \alpha_{ij} + \beta_{ij}$, and we choose $\alpha_{ij} > 0$, so $a_{ij} > 0$.

It is easy to prove that (27a) prevents us from taking all three of the mass terms negative at high T, as we could have expected. Necessary conditions would be

$$\sum_{j \neq i} a_{ij} > 3p_i \tag{28}$$

Multiplying these equations by pairs and adding them results in a contradiction with eq. (27a). But it turns out that with only two negative mass terms, all three vevs cannot be nonzero at arbitrarily high temperature. Take for example $\nu_1^2 > 0$, $\nu_2^2, \nu_3^2 < 0$. We need v_1 to be real, that is, minimizing (24)

$$v_1^2 = \left(\frac{T^2}{3}\right) \frac{-\nu_1^2(p_2p_3 - a_{23}^2) + \nu_2^2(p_3a_{12} + a_{23}a_{13}) + \nu_3^2(p_2a_{13} + a_{23}a_{12})}{p_1p_2p_3 - p_1a_{23}^2 - p_2a_{13}^2 - p_3a_{12}^2 - 2a_{12}a_{13}a_{23}} > 0 \tag{29}$$

We have already required the denominator to be positive. For the numerator to be positive also, necessary (though not sufficient) conditions are

$$\bar{\nu_2}^2(p_3a_{12} + a_{23}a_{13}) + \bar{\nu_3}^2(p_2a_{13} + a_{23}a_{12}) > -\bar{\nu_1}^2(p_2p_3 - a_{23}^2) \tag{30}$$

where

$$\begin{aligned} \bar{\nu_1}^2 &= 3p_1 - a_{12} - a_{13} < \nu_1^2 \\ \bar{\nu_2}^2 &= a_{12} + a_{23} - 3p_2 > \nu_2^2 \\ \bar{\nu_3}^2 &= a_{13} + a_{23} - 3p_3 > \nu_3^2 \end{aligned} \tag{31}$$

Inserting (30) in (31), one gets

$$\begin{aligned} &-2p_2p_3(a_{12} + a_{13}) - a_{23}(p_2a_{13} + p_3a_{12}) > \\ &p_1p_2p_3 - p_1a_{23}^2 - p_2a_{13}^2 - p_3a_{12}^2 - 2a_{12}a_{13}a_{23} + 2p_1(p_2p_3 - a_{23}^2) \end{aligned} \tag{32}$$

which in view of (27) cannot be satisfied.

Thus, once again, the CP violating phase disappears at high temperature. As in the two-doublet case, here too the problem is that CP violation is achieved through the relative phase of the vevs of the doublets.

C. $\not{C}P$ with a singlet field

It should be clear from the previous examples that when the CP phase is related to the relative phases of doublet fields, high temperature effects will make it vanish. We therefore look for models in which CP violation is broken spontaneously by the vev of just one field, which may be easier to keep at high temperature.

The simplest such model is a minimal extension of the Standard Model with

a) a real singlet field S which transforms under CP as $S \to -S$.

b) an additional down quark, with both left and right components D_L^a and D_R^a singlets under $SU(2)$.

The interaction Lagrangian for the down quarks, symmetric under CP, contains the terms

$$\begin{aligned}\mathcal{L}_Y = &(\bar{u}\bar{d})_L^a h_a \Phi D_R + (\bar{u}\bar{d})_L^a h_{ab} \Phi d_R^b \\ &+ M_D \bar{D}_L D_R + M_a (\bar{D}_L d_R^a + h.c.) \\ &+ i f_D S(\bar{D}_L D_R - \bar{D}_R D_L) + i f_a S(\bar{D}_L d_R^a - \bar{d}_R^a D_L)\end{aligned} \tag{33}$$

Clearly, when S gets a vev (at a scale σ much bigger than the weak scale M_W) CP is spontaneously broken by the terms in the last line. A model of this kind was developed by Bento and Branco [23], in the version where the singlet is a complex field and gets a complex vev, and with an additional symmetry under which S and D_R are odd, all other fields even.

We will for simplicity keep S real (and impose no further symmetries), noting that the analysis goes over the same lines as in [23], and referring the reader there for details. Suffice it to say that CP violation is achieved by complex phases appearing in the CKM matrix through the mixings of d and D quarks, which are of the order σ/M_D. These phases remain in the limit $M_D, \sigma \to \infty$ when the heavy quarks decouple. This should not come as a surprise, since in the decoupling limit the theory reduces to the minimal standard model, which in general has complex Yukawa couplings and a complex CKM matrix. Also, flavor-violating currents are suppressed by powers of M_W/σ, disappearing in the decoupling limit. Thus the measure of the departure from the standard model is the dimensionless parameter M_W/M_D, and for the theory to be experimentally testable M_D should not be much bigger than 1 TeV.

To leading order, the high-temperature behavior of the $\Phi - \sigma$ system is very simple. The most general potential can be written as

$$\begin{aligned}V(\Phi, S) = &-m_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger \Phi)^2 \\ &- \frac{m_S^2}{2} S^2 + \frac{\lambda_S}{4} S^4 - \frac{\alpha}{2} \Phi^\dagger \Phi S^2\end{aligned} \tag{34}$$

and it has a minimum at

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad ; \quad \langle S \rangle = +\sigma \tag{35}$$

At high T, the masses are replaced by

$$\begin{aligned}m_\Phi^2(T) &= -m_\Phi^2 + \frac{T^2}{24}(12\lambda_\Phi - \alpha) \\ \frac{m_S^2(T)}{2} &= -\frac{m_S^2}{2} + \frac{T^2}{24}(3\lambda_S - 2\alpha)\end{aligned} \tag{36}$$

We can have $m_S^2 < 0$ always by requiring $2\alpha > 3\lambda_S$, and thus $\sigma \neq 0$ at any temperature. The only further restriction is the usual $\lambda_\Phi > \alpha^2/\lambda_S$.

It seems then that in this model, one can have CP broken at any temperature. Remember however that up to now we have only considered the leading order contributions to the effective potential in calculating the masses (36). A complete analysis should include the next-to leading order corrections, as we already mentioned in the Introduction. We can anticipate that for a singlet field these effects will not change the picture much, but we leave a detailed analysis for a separate section.

III. SPONTANEOUS P VIOLATION AND HIGH T

Spontaneous P violation has been already discussed in the second paper of ref. [2], mostly in connection with strong CP violation. It was concluded there that in the minimal models of spontaneous P violation, left-right asymmetry may persist to high temperatures. The analysis however was carried out without considering carefully the role of the gauge couplings, which is now known to be fundamental [15], and which as we will show may invalidate that conclusion.

Let us recall the salient features of the minimal left-right symmetric theories [24] based on a $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry. The fermions are in doublet representations

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad ; \quad \begin{pmatrix} u \\ d \end{pmatrix}_R$$
$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad ; \quad \begin{pmatrix} \nu \\ e \end{pmatrix}_R \tag{37}$$

The minimal Higgs sector of the theory consists of

- the bi-doublets (one or more) Φ needed to provide Yukawa couplings and fermion masses
- two multiplets Δ_L and Δ_R which may be either doublets or triplets under $SU(2)_L$ and $SU(2)_R$, and which are in charge of breaking P spontaneously.

For the sake of completeness, we remind the reader of the essence of spontaneous P violation and we do it in a simplified toy example which has all the relevant features of the theory. More precisely, we take Δ_L and Δ_R as real scalar fields and assume a left-right symmetric potential

$$\begin{aligned} V &= -\frac{m^2}{2}(\Delta_L^2 + \Delta_R^2) + \frac{\lambda}{4}(\Delta_L^4 + \Delta_R^4) + \frac{\lambda'}{2}\Delta_L^2\Delta_R^2 \\ &= -\frac{m^2}{2}(\Delta_L^2 + \Delta_R^2) + \frac{\lambda}{4}(\Delta_L^2 + \Delta_R^2)^2 + \frac{\lambda' - \lambda}{2}\Delta_L^2\Delta_R^2 \end{aligned} \tag{38}$$

A simple inspection of V is enough to convince oneself that for $m^2 > 0$ and $\lambda' - \lambda > 0$, the global minimum of the theory is obtained for

$$\langle \Delta_L \rangle^2 = 0 \quad ; \quad \langle \Delta_R \rangle^2 = \frac{m^2}{\lambda} \tag{39}$$

or viceversa. Thus the left-right symmetry is broken spontaneously. Of course in realistic models, besides Δ's being non-trivial representations under the gauge group, we do need a field Φ. One can then try to take one or more of the coupling constants between Φ and the Δ's negative, thus achieving a negative mass term for the Δ's at all temperatures.

Let us concentrate in the version of the theory which incorporates the see-saw mechanism with Δ_L and Δ_R being triplets [25]. Since we wish to keep $\langle \Delta_R \rangle$ nonzero at high temperature, it is enough to look at the $\Delta_R - \Phi$ system and, as in [2], consider a simplified model in which the potential is written

$$V = -m_\Delta^2 \Delta_R^\dagger \Delta_R + \lambda_\Delta (\Delta_R^\dagger \Delta_R)^2 + -m_\Phi^2 Tr\Phi^\dagger\Phi + \lambda_\Phi (Tr\Phi^\dagger\Phi)^2 - 2\alpha Tr\Phi^\dagger\Phi\Delta_R^\dagger\Delta_R \qquad (40)$$

where Δ_R is a triplet under $SU(2)_R$, has $B-L$ number 2, and other couplings are taken to be small. The high temperature masses are [2]

$$m_\Phi^2(T) = -m_\Phi^2 + T^2\left\{\frac{5}{6}\lambda_\Phi - \frac{1}{3}\alpha + \frac{3}{16}g^2\right\} \qquad (41a)$$

$$m_\Delta^2(T) = -m_\Delta^2 + T^2\left\{\frac{1}{2}\lambda_\Delta - \frac{2}{3}\alpha + \frac{3}{8}(g'^2 + 2g^2)\right\} \qquad (41b)$$

where g'^2 is the $U(1)$ gauge coupling, g^2 the $SU(2)_R$ one. We have to keep $m_\Delta^2(T)$ negative at high T while preserving the boundedness condition $\lambda_\Phi\lambda_\Delta > \alpha^2$, thus we arrive at

$$\begin{aligned}\lambda_\Phi &> \frac{\alpha^2}{\lambda_\Delta} \\ &> \frac{9}{4}\left[\frac{1}{2}\lambda_\Delta + \frac{3}{8}(g'^2 + 2g^2)\right]\end{aligned} \qquad (42)$$

λ_Φ as a function of λ_Δ has a minimum at $\lambda_\Delta = (3/4)(g'^2 + 2g^2)$, so we must have

$$\lambda_\Phi > \frac{27}{16}(g'^2 + 2g^2) \qquad (43)$$

If we now use $g'^2 = g^2/2$ and take $g^2 = 1/4$, we see that nonrestoration of P requires $\lambda_\Phi > 1$ in conflict with perturbation theory. Including other couplings does not help, since new conditions on the couplings coming from the mass matrices have to be imposed (since it is not illustrative, we omit here the numerical analysis required to prove this).

Although physically less attractive, one can in principle use doublets to break P spontaneously. This is actually the case studied in [2]. It is easily found that with doublets the condition equivalent to (43) is down by a factor of half. Thus this case may be considered borderline.

Now, for the implementation of the see-saw mechanism in its minimal form, it turns out that a parity odd singlet field is needed [26]. The singlet field S will couple to the Δ fields with a left-right symmetric term

$$MS(\Delta_L^\dagger\Delta_L - \Delta_R^\dagger\Delta_R) \qquad (44)$$

Without the lower bound imposed by the gauge couplings, the situation in this case goes along the same lines as that of section II C: the vev of the singlet can be kept nonzero at high temperatures with the aid of the bi-doublet field Φ, or even of the Δ's. Exactly as it worked with CP, now P may remain broken at high temperature, and the presence of more fields coupled to S than in the CP case only makes it easier.

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[2] We use the normalization $Tr\Phi^\dagger\Phi = \Phi_a\Phi_a/2$; $\Delta_R^\dagger\Delta_R = \Delta_R^a\Delta_R^a$, where a sums over six real fields.

IV. STRONG CP PROBLEM AND HIGH T

The strong CP problem arises in QCD when nonperturbative effects, resulting from the existence of instanton solutions, induce effective terms in the Lagrangian that violate CP. The resulting CP violating phase is

$$\bar{\Theta} = \Theta + \arg\det(M) \tag{45}$$

where Θ is the coefficient of the $\epsilon_{\alpha\beta\mu\nu}F_a^{\alpha\beta}F_a^{\mu\nu}$ term, and M is the quark's mass matrix. $\bar{\Theta}$ is constrained experimentally to be zero to a very high precision ($\bar{\Theta} < 10^{-9}$), giving rise to a "naturalness" problem [27].

A. The invisible axion solution

The most popular solution to the strong CP problem is the Peccei-Quinn mechanism [10], in which the phase $\bar{\Theta}$ is identified with the pseudo-Goldstone boson resulting from the spontaneous breakdown of a global symmetry $U(1)_{PQ}$. Observational constraints require this breakdown to occur at a scale M_{PQ} much bigger than the electroweak scale, making the axion "invisible" [28,29]. Besides the axion field a, the breaking of $U(1)_{PQ}$ produces a network of global strings [30]. As we go around each minimal string, the phase $\bar{\Theta} = a/M_{PQ}$ winds by 2π. Instanton effects appear later, when the temperature has reached the QCD scale Λ_{QCD}. Their effects in the Higgs sector can be mimicked by an effective term

$$\Delta V = \Lambda_{QCD}^4(1 - cosN\bar{\Theta}) \tag{46}$$

where N is the number of quark flavors. It becomes energetically favorable for $\bar{\Theta}$ to choose one out of the discrete set of values $2\pi k/N$ ($k = 1, 2, ..N$). But since we must have $\Delta\bar{\Theta} = 2\pi$ around a string, this results in the formation of N domain walls attached to each string [31]. For $N > 1$, these domain walls are stable and therefore in conflict with standard cosmology.

Clearly, without the global strings no walls will be formed: above $T \simeq \Lambda_{QCD}$, $\bar{\Theta}$ would be aligned having some typical value $\bar{\Theta}_0$ which after the QCD phase transition would relax to the nearest minimum. We wish then to study in detail the high temperature behavior of the invisible axion mechanism, well above the scale M_{PQ}.

For concreteness we concentrate on the minimal extension of the original Peccei-Quinn model [29]. The potential for the PQ model with the doublets ϕ_i (i=1,2) both having $Y = 1$ and a $SU(2) \times U(1)$ singlet S may be written as

$$\begin{aligned} V_{PQ} = &\sum_i \left[-\frac{m_i^2}{2}\phi_i^\dagger\phi_i + \frac{\lambda_i}{4}(\phi_i^\dagger\phi_i)^2\right] - \frac{\alpha}{2}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) - \frac{\beta}{2}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ &- \frac{m_s^2}{2}S^*S + \frac{\lambda_s}{4}(S^*S)^2 - \sum_i(\frac{\gamma_i}{2}\phi_i^\dagger\phi_i)S^*S - M(\phi_1^\dagger\phi_2 S + \phi_2^\dagger\phi_1 S^*) \end{aligned} \tag{47}$$

Besides the $SU(2)_L \times U(1)_Y$ local gauge symmetry, V_{PQ} has a chiral $U(1)_{PQ}$ symmetry (ϕ_1 couples to say down quarks, and ϕ_2 to up quarks)

$$\phi_1 \to e^{i\alpha}\phi_1 \;\;;\;\; \phi_2 \to e^{-i\alpha}\phi_2 \;\;;\;\; S \to e^{2i\alpha}S \tag{48}$$

For $\beta > 0$, the minimum is found at

$$\langle \Phi_i \rangle = \begin{pmatrix} 0 \\ v_i \end{pmatrix} \quad ; \quad \langle S \rangle = v_S \tag{49}$$

To have $U(1)_{PQ}$ broken at any temperature, it is enough to keep the vev of the singlet nonzero for all T. From our analysis of the previous section for a potential with three doublets, one can already expect that keeping the vev of only one field nonzero will not be difficult. In this model then the conditions on the potential parameters cannot be an obstacle for nonrestoration, but we present them here for the sake of completeness. Taking $v_S \gg v_i$, the conditions over the couplings are, to leading order

$$\lambda_i > 0 \quad , \quad \lambda_S > 0 \quad ; \quad \lambda_i \lambda_S > \gamma_i^2 \quad ; \quad \lambda_1 \lambda_2 > (\alpha + \beta)^2 \tag{50a}$$

$$\begin{aligned} & M v_s^3 \left[\frac{v_1^3}{v_2}(\lambda_1 \lambda_S - \gamma_1^2) + \frac{v_2^3}{v_1}(\lambda_2 \lambda_S - \gamma_2^2) - 2 v_1 v_2 (\lambda_S(\alpha + \beta) + \gamma_1 \gamma_2 \right] \\ & + v_S^2 v_1^2 v_2^2 \left[\lambda_1 \lambda_2 \lambda_S - \lambda_1 \gamma_2^2 - \lambda_2 \gamma_1^2 - \lambda_S (\alpha + \beta)^2 - 2 \gamma_1 \gamma_2 (\alpha + \beta) \right] > 0 \end{aligned} \tag{50b}$$

It is easily proven that (50a) imply that the first line of eq. (50b) is positive. A sufficient condition for boundedness will then require (50a) and the second line of (50b) to be positive, the same conditions that were required in the three-doublet model of section II B (eq.(27)).

The mass term of the singlet at high temperature will be

$$m_S^2(T) = -m_S^2 + \frac{T^2}{3}(\lambda_S - \gamma_1 - \gamma_2) \tag{51}$$

so that imposing $\gamma_1 + \gamma_2 > \lambda_S$, we get the $U(1)_{PQ}$ symmetry broken at all temperatures. We already know that at high T one cannot have all three vevs nonzero, and notice that because of the linear terms in (47), having $v_S \neq 0$ forces v_1, v_2 to vanish.

Up to this order then, it seems quite natural to keep the vev of S nonzero at high T; again we leave the next-to-leading order considerations for the next session. The learned reader will notice that the same holds true for Kim's version [28] of the invisible axion idea.

B. Spontaneous P or CP violation

Another well-known solution to the strong CP problem is based on the idea of spontaneous CP or P violation [6]. Here, the symmetries can be used to set $\bar{\Theta}_{\text{tree}} = 0$ and the effective $\bar{\Theta}$ is then finite and calculable in perturbation theory, and in many models small enough. The high T behavior of these theories is completely analogous to the one discussed in section II and III, and thus we can conclude that the solution of the domain wall problem favors models with singlets. However, before *the* model is found we find it fruitless to study this question in detail.

V. NEXT-TO-LEADING ORDER CONTRIBUTIONS

In a series of recent papers, Bimonte and Lozano [11,12] have addressed the issue of next-to-leading order contributions to the effective potential. As was already pointed out in [19], in a theory with a $\lambda\phi^4$ potential, the next-to-leading order contributions to the mass^2 are of order

$$m^2(T) \propto \lambda^{3/2} T^2 \tag{52}$$

while higher loop corrections do not contribute significantly. The point is that in a theory with two fields where one of the self-coupling constants is required to be larger than the other (as we did to avoid symmetry restoration), the larger constant will enter in corrections to the other field's mass. Thus one has to make sure that the results to leading order are maintained when including such terms.

In fact, in the case of gauge symmetries, it was concluded [12] that the inclusion of these effects can alter significantly the phase diagram of the theory. This is mainly due to the fact that in the gauge case the coupling constants cannot be as small as one wishes, but are bounded from below by the value of the gauge coupling. In the case of singlets [11], although the effects are not so dramatic, they do alter the parameter space for symmetry nonrestoration. Since in this investigation the models that allow for nonrestoration at high T were based on singlet fields, we will only consider here the next-to-leading order corrections in the case of global symmetry.

We begin by reviewing briefly the contributions of next-to-leading corrections in the effective potential of a $O(N_1) \times O(N_2)$-symmetric model, although we refer the reader to [11] for details. Take two real fields ϕ_1, ϕ_2, transforming as vectors under $O(N_1), O(N_2)$ respectively, and write the potential

$$V(\phi_1, \phi_2) = \sum_i \left(-\frac{m_i^2}{2}|\phi_i|^2 + \frac{\lambda_i}{4}|\phi_i|^4 \right) - \frac{\alpha}{2}|\phi_1|^2|\phi_2|^2 \tag{53}$$

The temperature contributions to the effective masses are calculated to leading order to be

$$\Delta m_1^2(T) = T^2 \nu_1^2 = T^2 \left[\lambda_i \left(\frac{2+N_1}{12} \right) - \frac{N_2}{12}\alpha \right] \tag{54}$$

(and a similar expression for Δm_2) while to next-to-leading, $\Delta m_i \equiv T x_i$ is found by solving the coupled pair of equations

$$\begin{aligned} x_1^2 &= \nu_1^2 - \left(\frac{2+N_1}{4\pi} \right) \lambda_1 x_1 + \frac{N_2}{4\pi}\alpha x_2 \\ x_2^2 &= \nu_2^2 - \left(\frac{2+N_2}{4\pi} \right) \lambda_2 x_2 + \frac{N_1}{4\pi}\alpha x_1 \end{aligned} \tag{55}$$

Symmetry is restored when such solutions are real and positive. The conditions under which those solutions do not exist, and therefore the $O(N_2)$ symmetry is *not* restored can be found to be

$$\alpha \left(\frac{N_1}{2+N_2} \right) [1 - f(\lambda_1, \alpha)] > \lambda_2 \tag{56a}$$

$$\lambda_1 \lambda_2 > \alpha^2 \tag{56b}$$

where

$$f(\lambda_1, \alpha) = \frac{3(2+N_1)}{8\pi^2} \left(\sqrt{\lambda_1^2 + \left(\frac{16\pi^2}{3(2+N_1)} \right) \left(\lambda_1 - \frac{N_2}{2+N_1} \alpha \right)} - \lambda_1 \right) \tag{57}$$

is a function that can take values from 0 to 1. The leading order conditions are (56) with $f = 0$. One can see then why the parameter space is reduced: it gets more difficult to fulfill (56a). The behavior with the number of fields also becomes nontrivial, since $(1 - f)$ is a decreasing function of N_1, and the two factors of α in (56a) compete (up to leading order, it is always preferable to keep nonzero the vev of the field in the smallest representation).

The $O(N_1) \times O(N_2)$ toy model can mimick models with more complicated symmetries involving two fields with N_1 and N_2 real components, in the approximation where their interaction is just of the type $\alpha |\phi_1|^2 |\phi_2|^2$. In particular, no approximation needs to be done in the doublet+singlet case.

In Figure 1 we show how symmetry nonrestoration depends in the number of fields when the next-to-leading order effects are included, i.e., we find the values of N_1 and N_2 for which the conditions (56) are satisfied when the parameters of the potential are fixed. The plot shows the situation for two sets of ratios of the couplings: $\lambda_1 : \alpha : \lambda_2 = 1 : 1/3 : 1/9$ and $1 : 1/10 : 1/100$. Notice that $N_2 < N_1$ is still preferred. As the ratio N_2/N_1 increases, it becomes necessary for nonrestoration to take smaller ratio λ_2/λ_1.

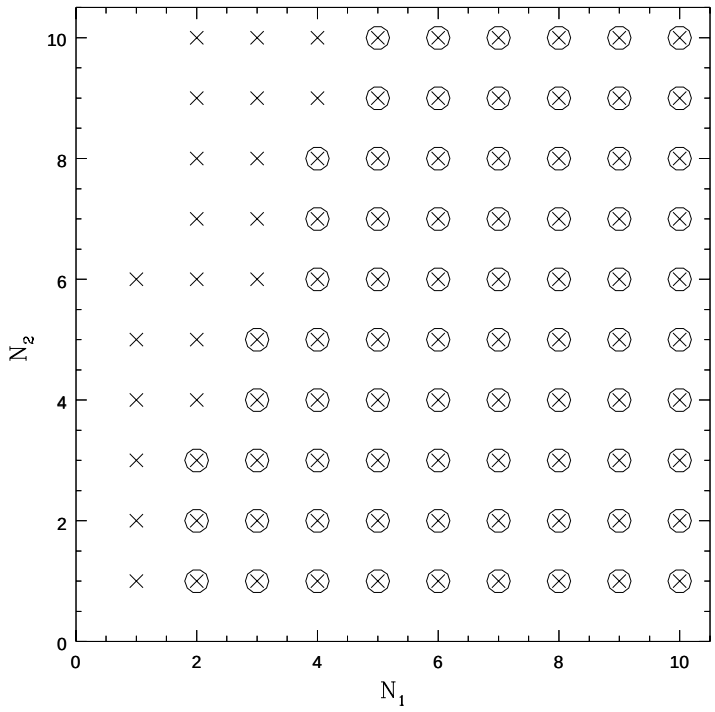

FIG. 1. Symmetry nonrestoration in a model with $O(N_1) \times O(N_2)$ symmetry. Points indicate the values of N_1, N_2 for which the vev of the $O(N_2)$ vector can be kept nonzero at high temperature, for fixed values of the potential's parameters: circles correspond to $\lambda_1 = 0.1$, $\alpha = 0.03$, $\lambda_2 = 0.01$, crosses to $\lambda_1 = 0.1$, $\alpha = 0.01$, $\lambda_2 = 0.001$

The cases of $N_1 = 4, N_2 = 1$ (a complex doublet plus a real singlet, as required for CP violation in section II C), that of $N_1 = 8, N_2 = 2$ (two doublets and one complex singlet, as in the invisible axion model of section IV) and that of $N_1 = 8, N_2 = 1$ (two doublets and a singlet, as in the parity-violating model of section III) lie in the non-restoration region.

The relevant question is how big is the region in parameter space where nonrestoration occurs. In Figure 2 we show that region for the case of the CP violation with a real singlet, in λ_Φ, α space, when λ_S is kept at a fixed value. Varying λ_S basically 'rescales' the whole picture in the α axis. The corresponding region with only leading-order effects is also shown. Although the parameter space is reduced by higher order corrections, the difference with the leading order case is not dramatic.

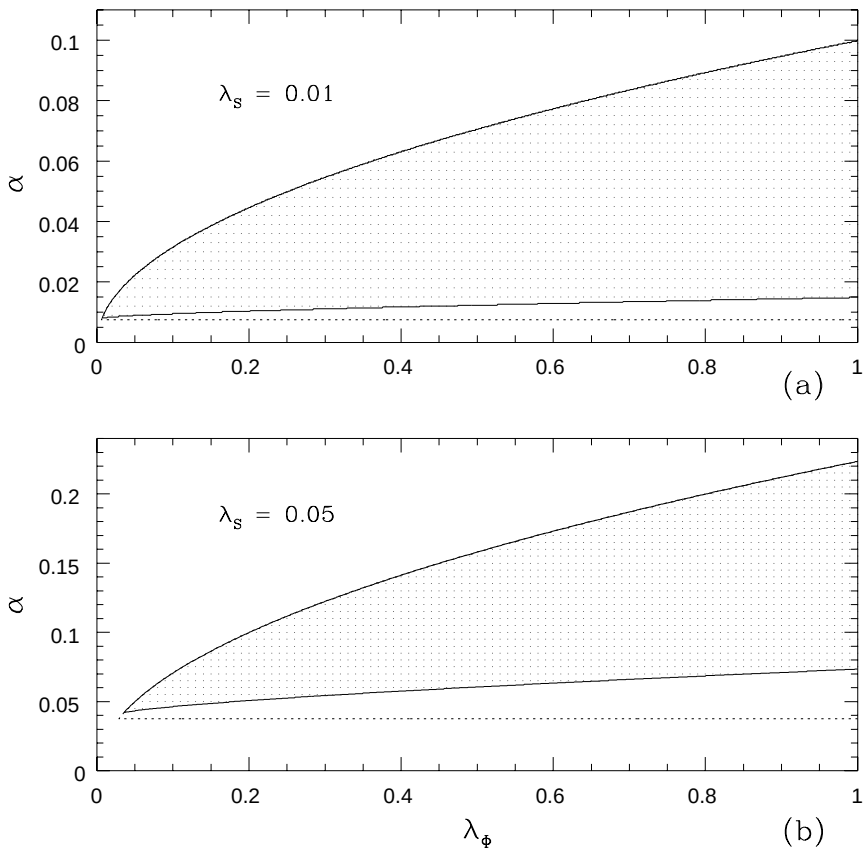

FIG. 2. The region of symmetry nonrestoration for the model of$\not{C}P$ with a real, CP odd singlet, for two values of the singlet's self coupling constant λ_s as indicated. When only leading order effects are taken into account, the region extends up to the dotted line

For the Peccei-Quinn model, the next-to-leading order calculations are only approximated by an $O(8) \times O(2)$ model, in the limit where in (47), $\lambda_1 = \lambda_2 = 2\alpha \equiv \lambda_\Phi$, $\beta = 0$, and $\gamma_1 = \gamma_2 \equiv \gamma$.

Under such approximation, the region where nonrestoration is allowed is presented in Figure 3, for the same range of parameters as in Figure 2. It is evident comparing both figures that nonrestoration does not depend only on the ratio N_2/N_1.

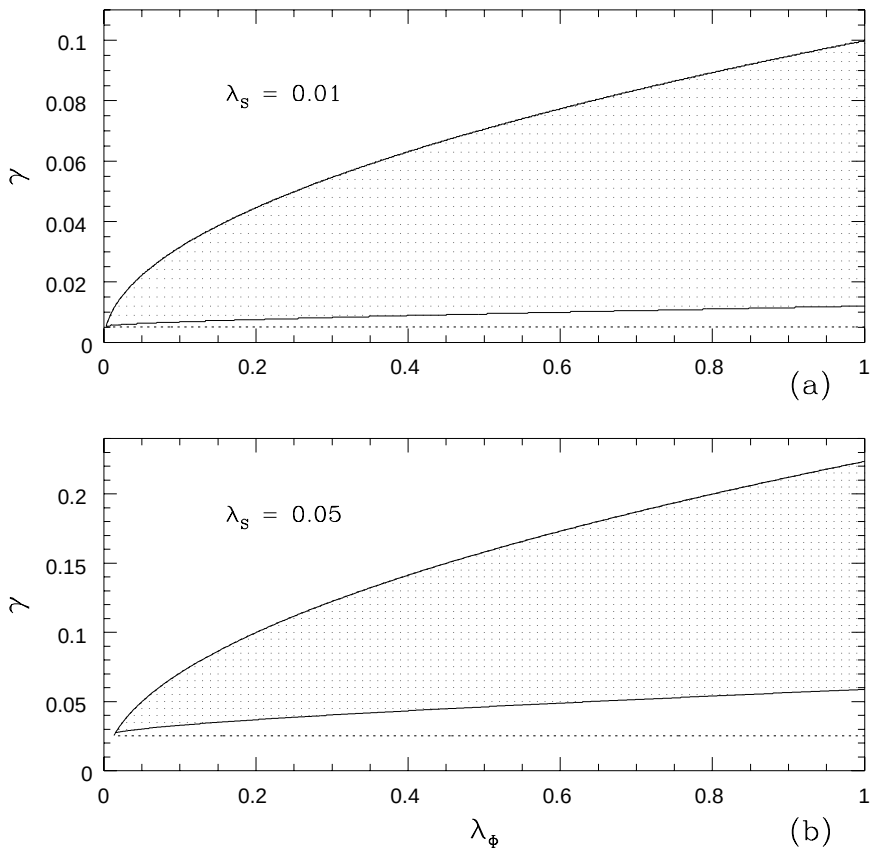

FIG. 3. The region of symmetry nonrestoration for a $O(8) \times O(2)$ model, an approximation of the Peccei-Quinn model

As for the model of P violation with a singlet of section III, it can be imitated by a $O(8) \times O(1)$ model if the quartic coupling with the two doublet fields is taken negative. One can also choose the couplings with the bi-doublet negative, and then consider an approximated model with some of the self and mixed couplings small. The nonrestoration region is clearly bigger than in the weak or strong CP cases.

VI. OUTLOOK AND CONCLUSIONS

In this paper we have studied the phenomenon of symmetry nonrestoration at high temperature, focusing on some minimal models of spontaneous T and P violation. We were motivated by the fundamental role that these symmetries play in nature and by the possibility of using them in solving the strong CP problem. We find that symmetry nonrestoration seems to require singlet fields and that it seems to work in accord with perturbation theory. This provides the hope for solving the domain wall problem and having baryogenesis operate at very high temperature as we now discuss briefly.

Domain Wall Problem: Avoiding the phase transition is not enough to solve the domain wall problem, since thermal fluctuations are in principle able to produce topological

defects at any time. As was shown in [9], thermal production of domain walls and strings can be naturally suppressed. We briefly sketch how this suppression occurs for the two models admitting nonrestoration presented here, and refer to [9] for details.

Consider the nucleation of a large spherically symmetric domain wall or a closed loop of string. The production rate per unit time per unit volume at a temperature T will be given by [32]

$$\Gamma = T^4 \left(\frac{S_3}{2\pi T}\right)^{3/2} e^{-S_3/T} \tag{58}$$

where S_3 is the energy of the closed defect. The suppression factor $e^{-S_3/T}$ is readily calculated in the limit where the defect's radius is much bigger than its width. For the domain walls produced in the model of CP violation with a singlet, we get

$$\frac{S_3}{T} \gg \frac{16\pi}{3\sqrt{6}} \frac{\sqrt{2\alpha - 3\lambda_S}}{\lambda_S} \tag{59}$$

Analogously, for the Peccei-Quinn model the thermal production of large loops of strings is suppressed by [3]

$$\frac{S_3}{T} \gg 4\pi^2 \frac{\sqrt{\gamma_1 + \gamma_2 - \lambda_S}}{\lambda_S} \tag{60}$$

We see that in both cases, it suffices to take the singlet's self-coupling λ_S small to avoid significant thermal production of defects.

The considered models with singlets involve a high scale M_H much bigger than the weak scale M_W, and it is noteworthy that the smallness of λ_S is intimately related to this hierarchy. Strictly speaking one could just fine tune the combination of m_S^2 and $\lambda_S v_S^2$ to be small, but this is not stable under radiative corrections. It is maybe more natural to take all the mass parameters of the model m_Φ and m_S to be small, i.e. of order M_W, and the singlet's self and mixed couplings of order $(M_W/v_S)^2$. In such case it is obvious that both (59) and (60) become enormous, suppressing completely the production of defects. Of course, the nature of the fine-tuning is finally a matter of taste. However, the second possibility has the clear prediction of keeping both Higgs doublets light in the invisible axion model, as is commonly assumed and experimentally verifiable.

Of course, all the above still does not guarantee the absence of domain walls. One needs to assume initial conditions in which the singlet field has a uniform value over a region of roughly the comoving size of the present horizon. This is equivalent to assume that the so-called horizon problem has been solved, for example by means of a period of primordial inflation.

Baryogenesis

The issue of baryogenesis in the context of broken symmetries at high T has been discused in [33] with emphasis on the theories where the $SU(2)\times U(1)$ gauge symmetry of the standard

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[3] We note that the normalization of the kinetic term we use here differs from that of [9].

model never gets restored. This implies massive fermions at high T, but it can still be shown that baryogenesis may take place along the usual lines of the out-of-equilibrium decays of superheavy lepto-quark gauge and Higgs bosons.

Now, in the examples we have discussed both with P and CP violation at high T, and including the Peccei-Quinn mechanism, the $SU(2) \times U(1)$ symmetry gets restored as in the more conventional scenarios. Thus fermions become massless and the creation of baryon asymmetry proceeds as usual. Of course, this implies embedding of the models discussed into GUTs, a task beyond the scope of our paper.

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