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University of Trondheim UNIT

Theoretical Physics Seminar in Trondheim

No 11 1995

Massive quark-antiquark and gluon production in high energy spin polarized electron positron collisions

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ISSN 0365–2459
October 1995

Abstract

The quark-antiquark production cross section, including gluon radiative corrections to order α_s, for high energy spin-polarized electron-positron collisions is obtained. The gluon radiative corrections are functions of the maximum recorded scaled gluon momentum $X_g = E_g/E$ and are expressed by gluon form factors $F_i(\bar{m}_f, X_g)$. Formulae for form factors are given for all values of X_g, $0 < X_g \leq 1 - \bar{m}_f^2$, where $\bar{m}_f$ is the scaled quark mass for flavour f, $\bar{m}_f = m_f/E$. Exact formulae in terms of the form factors are given for the radiative corrections to the cross section, the forward-backward asymmetry, the left-right asymmetry and the forward-backward, left-right asymmetry. Explicit analytic formulae for $\bar{m}_f \leq 0.2$ are given for the radiative corrections to the cross section and the asymmetries for elastic quark-antiquark production, small values of X_g, and for the case that almost all gluons are recorded, $1 - X_g \ll 1$.

1 Introduction

The recent advance in technologies for obtaining high-energy spin-polarized electron beams [1] has given new possibilities for obtaining new and more accurate knowledge of the electroweak interaction [2].

The theoretical study of polarization effects in gluon bremsstrahlung and related radiative gluon effects from collisions of high energy spin-polarized electrons and positrons was initiated a long time ago [3]. In these bremsstrahlung calculations, where also the linear and circular polarization of gluons was obtained, the quark and antiquark were assumed to be massless. Mass effects are taken into account in radiative gluon effects, i.e. radiative corrections to quark-antiquark cross sections and various asymmetries [4, 5, 6] involving spin-polarized electrons and positrons.

In the present paper we obtain the quark-antiquark cross section with radiative gluon corrections to order α_s for initially spin-polarized electrons and positrons. The effects of the gluon is taken into account in gluon form factors which are functions of $X_g = E_g/E$, the maximum value recorded of the scaled gluon energy. The form factors $F_i(\bar{m}_f, X_g)$ are given by exact formulae, except $F_3(\bar{m}_f, X_g)$, related to the radiative corrections to the forward-backward asymmetry of the cross section, which cannot be expressed in terms of known functions for general values of X_g, $0 < X_g \leq 1 - \bar{m}_f^2$, where $\bar{m}_f$ is the scaled quark mass m_f/E. Since the form factor formulae are very complicated we show that it is possible to find approximate formulae representing well the exact results for regions of moderate values of the scaled mass.

We discuss forward-backward, left-right and forward-backward,left-right asymmetries [7]. With the approximate form factors we obtain explicit formulae for radiative corrections to cross sections and asymmetries. In particular we obtain the radiative corrections to elastic quark-antiquark production, $X_g \ll 1$. At the other end of the gluon spectrun $1 - X_g \ll 1$, the radiative corrections to quark-antiquark production, when almost all gluons are recorded, are obtained. Our present results valid for all values of X_g are compared to our results in ref. [6] at the Z_0 resonance and for $X_g = 1 - \bar{m}_f^2$, and to recent calculations for unpolarized electrons and positrons [8, 9].

2 The cross section

The $e_+e_- \to q\bar{q}g$-cross section for production of quark-antiquark of flavour f is given in reference [5]

$$
\begin{aligned}
\frac{d^4\sigma_f^{q\bar{q}g}}{d\Omega dx d\bar{x}} &= \frac{\alpha^2}{2\pi}\frac{\alpha_s}{s}\frac{1}{(1-x)(1-\bar{x})} \\
&\times \Big\{ \mathcal{F}_1(x,\bar{x}) \Big(h_f^{(1)}(s, P_-^{||}P_+^{||})(1+\cos^2\theta) \\
&\quad + P_-^{\perp}P_+^{\perp}\sin^2\theta \Big[h_f^{(3)+}(s)\cos(2\phi-\phi_+-\phi_-) - h_f^{(4)}(s)\sin(2\phi-\phi_+-\phi_-) \Big] \Big) \\
&\quad + \frac{\bar{m}_f^2}{2}\mathcal{F}_2(x,\bar{x}) \Big(\Big[h_f^{(1)}(s, P_-^{||}P_+^{||}) - h_f^{(5)}(s, P_-^{||}P_+^{||}) \Big]\cos^2\theta \\
&\qquad + P_-^{\perp}P_+^{\perp}\sin^2\theta \Big[h_f^{(3)-}(s)\cos(2\phi-\phi_+-\phi_-) \\
&\qquad\qquad - h_f^{(4)}(s)\sin(2\phi-\phi_+-\phi_-) \Big] \Big) \\
&\quad + 2h_f^{(2)}(s, P_-^{||}P_+^{||})\mathcal{F}_3(x,\bar{x})\cos\theta + h_f^{(1)}(s, P_-^{||}P_+^{||})\mathcal{F}_4(x,\bar{x}) \\
&\quad + \frac{\bar{m}_f^2}{4}\Big[h_f^{(1)}(s, P_-^{||}P_+^{||}) - h_f^{(5)}(s, P_-^{||}P_+^{||}) \Big]\mathcal{F}_5(x,\bar{x}) \Big\},
\end{aligned}
\tag{2. 1}
$$

with the coupling constants

$$
\begin{aligned}
h_f^{(1)}(s, P_-^{||}P_+^{||}) &= Q_f^2\Xi - 2Q_f Ref(s)(v\Xi - a\xi)v_f \\
&\quad + |f(s)|^2 \Big[(v^2+a^2)\Xi - 2va\xi \Big] \Big(v_f^2 + a_f^2 \Big), \\
h_f^{(2)}(s, P_-^{||}P_+^{||}) &= -2Q_f Ref(s)(a\Xi - v\xi)a_f \\
&\quad -2|f(s)|^2 \Big[(v^2+a^2)\xi - 2va\Xi \Big] v_f a_f, \\
h_f^{(3)\pm}(s) &= Q_f^2 - 2Q_f Ref(s)vv_f + |f(s)|^2(v^2-a^2)(v_f^2 \pm a_f^2), \\
h_f^{(4)}(s) &= 2Q_f Imf(s)av_f, \\
h_f^{(5)}(s, P_-^{||}P_+^{||}) &= 2|f(s)|^2 \Big[(v^2+a^2)\Xi - 2va\xi \Big] a_f^2,
\end{aligned}
\tag{2. 2}
$$

where $\mathbf{P}_-$ and $\mathbf{P}_+$ are the electron and positron polarizations respectively and where the longitudinal polarizations are given by

$$\Xi = 1 - P_+^{||} P_-^{||}, \qquad \xi = P_-^{||} - P_+^{||}, \tag{2. 3}$$

and

$$f(s) = \frac{1}{4\sin^2 2\theta_w} \frac{s}{s - M_z^2 + iM_z\Gamma_z^{tot}}. \tag{2. 4}$$

It is convenient to introduce the functions $\Psi_1 - \Psi_5$ from reference [6], where the integration over parts of the phase space is specified by $\bar{m}_f < x < 1$ and $\bar{\mu} < x_g < X_g$ where $\bar{\mu}$, the scaled gluon mass parameter, describes the infrared gluon singularity to be removed by the radiative correction to the $e_+e_- \to q\bar{q}$ cross section, and X_g is the maximum scaled gluon energy recorded. The functions $\Psi_1 - \Psi_5$ are given by

$$\begin{aligned}
\frac{1}{2}\int \frac{dx d\bar{x}}{(1-x)(1-\bar{x})}\left(\mathcal{F}_1(x,\bar{x}) + \frac{\bar{m}_f^2}{2}\mathcal{F}_2(x,\bar{x}) \right) &= \Psi_1(X_g,\bar{\mu}) - 3\Psi_2(X_g,\bar{\mu}), \\
\frac{\bar{m}_f^2}{4}\int \frac{dx d\bar{x}}{(1-x)(1-\bar{x})}\mathcal{F}_2(x,\bar{x}) &= \Psi_4(X_g,\bar{\mu}) - 3\Psi_5(X_g,\bar{\mu}), \\
\frac{1}{2}\int \frac{dx d\bar{x}}{(1-x)(1-\bar{x})}\mathcal{F}_3(x,\bar{x}) &= \Psi_3(X_g,\bar{\mu}), \qquad (2.\ 5) \\
\frac{1}{2}\int \frac{dx d\bar{x}}{(1-x)(1-\bar{x})}\left(\mathcal{F}_1(x,\bar{x}) + \mathcal{F}_4(x,\bar{x}) + \frac{\bar{m}_f^2}{4}\mathcal{F}_5(x,\bar{x}) \right) &= \Psi_1(X_g,\bar{\mu}) + \Psi_2(X_g,\bar{\mu}), \\
\frac{\bar{m}_f^2}{8}\int \frac{dx d\bar{x}}{(1-x)(1-\bar{x})}\mathcal{F}_5(x,\bar{x}) &= \Psi_4(X_g,\bar{\mu}) + \Psi_5(X_g,\bar{\mu}).
\end{aligned}$$

This gives the cross section

$$\frac{d^2\sigma_f^{q\bar{q}g}}{d\Omega} = \frac{\alpha^2}{\pi}\frac{\alpha_s}{s}$$

$$\times \Big[\ h_f^{(1)}(s, P_-^{\|}P_+^{\|})\Big\{\ \Psi_1(X_g,\bar{\mu})\Big(\ 1+\cos^2\theta\ \Big) + \Psi_2(X_g,\bar{\mu})\Big(\ 1-3\cos^2\theta\ \Big)\ \Big\}$$

$$-h_f^{(5)}(s, P_-^{\|}P_+^{\|})\Big\{\ \Psi_4(X_g,\bar{\mu})\Big(\ 1+\cos^2\theta\ \Big) + \Psi_5(X_g,\bar{\mu})\Big(\ 1-3\cos^2\theta\ \Big)\ \Big\}$$

$$+2h_f^{(2)}(s, P_-^{\|}P_+^{\|})\Psi_3(X_g,\bar{\mu})\cos\theta$$

$$+P_-^{\perp}P_+^{\perp}\sin^2\theta\Big\{\ \Big[\ h_f^{(3)+}(s)\cos(2\phi-\phi_+-\phi_-) - h_f^{(4)}(s)\sin(2\phi-\phi_+-\phi_-)\ \Big]$$

$$\times[\ \Psi_1(X_g,\bar{\mu}) - 3\Psi_2(X_g,\bar{\mu})\ ]$$

$$+\Big[\ h_f^{(3)-}(s)\cos(2\phi-\phi_+-\phi_-) - h_f^{(4)}(s)\sin(2\phi-\phi_+-\phi_-)\ \Big]$$

$$\times[\ \Psi_4(X_g,\bar{\mu}) - 3\Psi_5(X_g,\bar{\mu})\ ]\ \}\ ], \qquad (2.\ 6)$$

which shows the structure of the angular dependence of the cross section.

The Ψ-functions are obtained from reference [6]

$$\begin{aligned}
\Psi_1(X_g,\bar{\mu}) &= \beta\left(1+\frac{\bar{m}_f^2}{2}\right)\left[F_1(\bar{m}_f,X_g) - F_V(\beta,\bar{\mu}) - \frac{6}{2+\bar{m}_f^2}ReF_V'(\beta)\right], \\
\Psi_2(X_g,\bar{\mu}) &= \beta\left[F_2(\bar{m}_f,X_g) - \frac{\bar{m}_f^2}{2}F_V(\beta,\bar{\mu}) - ReF_V'(\beta)\right], \\
\Psi_3(X_g,\bar{\mu}) &= \left(1-\bar{m}_f^2\right)\left[F_3(\bar{m}_f,X_g) - F_V(\beta,\bar{\mu})\right], \qquad (2.\ 7) \\
\Psi_4(X_g,\bar{\mu}) &= \beta\frac{3\bar{m}_f^2}{4}\left[F_4(\bar{m}_f,X_g) - F_V(\beta,\bar{\mu}) - \frac{2}{3}\frac{5-2\bar{m}_f^2}{\bar{m}_f^2}ReF_V'(\beta)\right], \\
\Psi_5(X_g,\bar{\mu}) &= \beta\frac{\bar{m}_f^2}{4}\left[F_5(\bar{m}_f,X_g) - F_V(\beta,\bar{\mu}) - \frac{2}{\bar{m}_f^2}ReF_V'(\beta)\right],
\end{aligned}$$

where the form factors $F_i(\bar{m}_f, X_g)$ here are functions of X_g, and $\beta^2 = 1-\bar{m}_f^2$. $F_V(\beta,\bar{\mu})$ and $F_V'(\beta)$ are radiative correction functions occuring in the $e_+e_- \to q\bar{q}$ cross section, reference [6],

$$\frac{d^2\sigma_f^{q\bar{q}}}{d\Omega} = \frac{3\alpha^2}{4s}\beta$$

$$\times\Bigg\{ \Big[h_f^{(1)}(s,P_-^{\|}P_+^{\|})\left(1+\frac{\bar{m}_f^2}{2}\right)\left(1+\frac{4\alpha_s}{3\pi}\left\{F_V(\beta,\bar{\mu})+\frac{6}{2+\bar{m}_f^2}ReF_V'(\beta)\right\}\right)$$

$$-\frac{3}{2}h_f^{(5)}(s,P_-^{\|}P_+^{\|})\frac{\bar{m}_f^2}{4}\left(1+\frac{4\alpha_s}{3\pi}\left\{F_V(\beta,\bar{\mu})+\frac{2(5-2\bar{m}_f^2)}{3\bar{m}_f^2}ReF_V'(\beta)\right\}\right)\Big](1+\cos^2\theta)$$

$$+\frac{\bar{m}_f^2}{4}\left[\,2h_f^{(1)}(s,P_-^{\|}P_+^{\|})-h_f^{(5)}(s,P_-^{\|}P_+^{\|})\,\right]\left(1+\frac{4\alpha_s}{3\pi}\left\{F_V(\beta,\bar{\mu})+\frac{2}{\bar{m}_f^2}ReF_V'(\beta)\right\}\right)(1-3\cos^2\theta)$$

$$+2\beta\left[\,h_f^{(2)}(s,P_-^{\|}P_+^{\|})\left(\,1+\frac{4\alpha_s}{3\pi}F_V(\beta,\mu)\,\right)+\frac{a_f}{v_f}h_f^{(4)}(s)\frac{4\alpha_s}{3\pi}\,2ImF_V'(\beta)\,\right]$$

$$\times\left(1+\frac{4\alpha_s}{3\pi}F_V(\beta,\bar{\mu})\right)\cos\theta$$

$$+\left(\,1-\bar{m}_f^2\right)P_-^{\perp}P_+^{\perp}\sin^2\theta \tag{2. 8}$$

$$\times\Big(\,\left[\,h_f^{(3)+}(s)\cos(2\phi-\phi_+-\phi_-)-h_f^{(4)}(s)\sin(2\phi-\phi_+-\phi_-)\,\right]\left(\,1+\frac{4\alpha_s}{3\pi}F_V(\beta,\bar{\mu})\,\right)$$

$$+\left[\,h_f^{(3)-}(s)\cos(2\phi-\phi_+-\phi_-)-h_f^{(4)}(s)\sin(2\phi-\phi_+-\phi_-)\,\right]\frac{4\alpha_s}{3\pi}\,ReF_V'(\beta)\,\Big)\,\Bigg\},$$

where the formulas for $F_V(\beta,\mu)$ and $F_V'(\beta)$ are given by

$$F_V(\beta,\bar{\mu}) = \left(\,1+\frac{1+\beta^2}{2\beta}\ln\frac{1-\beta}{1+\beta}\,\right)\ln\frac{1-\beta^2}{\mu^2}-\frac{1}{\beta}(\,1+\beta^2-\frac{1}{2}\,)\ln\frac{1-\beta}{1+\beta}-2$$

$$+\frac{1+\beta^2}{2\beta}\left[\,-\frac{1}{2}\ln^2\frac{1-\beta}{1+\beta}+2\ln\frac{1-\beta}{1+\beta}\ln\frac{2\beta}{1+\beta}+2L_2(\frac{1-\beta}{1+\beta})+\frac{2\pi^2}{3}\,\right],$$

$$F_V'(\beta) = \frac{\bar{m}_f^2}{4\beta}\left(\,\ln\frac{1-\beta}{1+\beta}+i\pi\,\right).$$

Addition of Eq. (2.6) and (2.8) gives the cross section for quark-antiquark production with emission of a gluon with scaled energy less than X_g

$$\frac{d^2\sigma_f}{d\Omega} = 3\beta\frac{\alpha^2}{4s}$$

$$\times\Bigg[\ h_f^{(1)}(s, P_-^{||}P_+^{||})\Bigg\{\ \left(1+\frac{\bar{m}_f^2}{2}\right)\left(1+\frac{4\alpha_s}{3\pi}F_1(\bar{m}_f, X_g)\right)(1+\cos^2\theta)$$

$$+\left(\ \frac{\bar{m}_f^2}{2}+\frac{4\alpha_s}{3\pi}F_2(\bar{m}_f, X_g)\ \right)(1-3\cos^2\theta)\ \Bigg\}$$

$$-h_f^{(5)}(s, P_-^{||}P_+^{||})\frac{\bar{m}_f^2}{4}\Bigg\{\ 3\left(1+\frac{4\alpha_s}{3\pi}F_4(\bar{m}_f, X_g)\right)(1+\cos^2\theta)$$

$$+\left(1+\frac{4\alpha_s}{3\pi}F_5(\bar{m}_f, X_g)\right)(1-3\cos^2\theta)\ \Bigg\}$$

$$+2\beta\Bigg\{\ h_f^{(2)}(s, P_-^{||}P_+^{||})\left(1+\frac{4\alpha_s}{3\pi}F_3(\bar{m}_f, X_g)\right)+\frac{a_f}{v_f}h_f^{(4)}(s)\frac{4\alpha_s}{3\pi}\ 2ImF_V'(\beta)\ \Bigg\}\cos\theta$$

$$+P_-^{\perp}P_+^{\perp}\sin^2\theta\Bigg\{\ \Big[\ h_f^{(3)+}(s)\cos(2\phi-\phi_+-\phi_-)-h_f^{(4)}(s)\sin(2\phi-\phi_+-\phi_-)\ \Big]$$

$$\times\Bigg[\ 1-m_f^2+\frac{4\alpha_s}{3\pi}\Bigg[\left(1+\frac{\bar{m}_f^2}{2}\right)F_1(\bar{m}_f, X_g)-3F_2(\bar{m}_f, X_g)\ \Bigg]\Bigg]$$

$$+\Big[\ h_f^{(3)-}(s)\cos(2\phi-\phi_+-\phi_-)-h_f^{(4)}(s)\sin(2\phi-\phi_+-\phi_-)\ \Big]$$

$$\times\frac{\alpha_s}{\pi}\bar{m}_f^2\left[\ F_4(\bar{m}_f, X_g)-F_5(\bar{m}_f, X_g)\ \right]\Bigg\}\Bigg]. \qquad (2.\ 9)$$

The infrared function $F_V(\beta, \mu)$ as well as the real part of

$$F_V'(\beta) \quad = \quad \frac{\bar{m}_f^2}{4\beta}\left(\ln\frac{1-\beta}{1+\beta}+i\pi\right), \qquad (2.\ 10)$$

have disappeared. Only the imaginary part $ImF_V'(\beta) = \pi\bar{m}_f^2/(4\beta)$, survives, usually a small term.

3 The form factors

The analytical formulae for the form factors as functions of the maximum scaled gluon energy X_g is given in Appendix 1. The formulae for $F_1(\bar{m}_f, X_g)$, $F_2(\bar{m}_f, X_g)$, $F_4(\bar{m}_f, X_g)$ and $F_5(\bar{m}_f, X_g)$ are exact. The formula for $F_3(\bar{m}_f, X_g)$ which cannot be obtained in terms of known functions is given as a formula which is exact for $\bar{m}_f = 0$ and is a good approximation for all values of X_g for scaled quark masses $\bar{m}_f \leq 0.3$.

The form factors are shown in Fig. 1 for $F_1(\bar{m}_f, X_g)$ and $F_3(\bar{m}_f, X_g)$ and in Fig. 2 for $F_2(\bar{m}_f, X_g)$, $F_4(\bar{m}_f, X_g)$ and $F_5(\bar{m}_f, X_g)$. In Fig. 3 we demonstrate the accuracy of the approximate formula for $F_3(\bar{m}_f, X_g)$ Eq. (A.11).

The exact formulae for the form factors are complicated. It is therefore useful to have simple approximate formulae representing $F_i(\bar{m}_f, X_g)$. Formulae which are exact for massless quarks and in the infrared limit $X_g \to 0$ for massive quarks, and correct to order $\bar{m}_f^2$ and $\bar{m}_f^2 \ln^2(4/\bar{m}_f^2)$ in the high energy gluon limit $X_g = 1 - \bar{m}_f^2$ are given by

$$\begin{aligned} F_1(\bar{m}_f, X_g) &= \left[2\ln\left(\frac{\sqrt{1-X_g} + \sqrt{1-\bar{m}_f^2 - X_g}}{\bar{m}_f}\right) - 1 \right] \\ &\times \left[2\ln X_g + \frac{1}{2}(3-X_g)(1-X_g) \right] \\ &+\frac{3}{4}\left[1-(1-X_g)^2 \right] - \frac{1}{2}(1-X_g^2) + 2L_2(1-X_g) + \frac{\bar{m}_f^2}{4}, \end{aligned} \tag{3. 1}$$

$$\begin{aligned} F_2(\bar{m}_f, X_g) &= \frac{\bar{m}_f^2}{2} F_1(\bar{m}_f, X_g) + X_g\left(\frac{1}{2} - \frac{\pi^2}{8}\bar{m}_f\right) \\ &-\bar{m}_f^2\left(\frac{1}{8} + \frac{\pi^2}{12} - \frac{3}{8}\ln\frac{4}{\bar{m}_f^2} + \frac{1}{16}\ln^2\frac{4}{\bar{m}_f^2} \right), \end{aligned} \tag{3. 2}$$

$$\begin{aligned} F_3(\bar{m}_f, X_g) &= F_1(\bar{m}_f, X_g) - \frac{3}{4}\left[1-(1-X_g)^2 \right] - \frac{1}{2}(3-X_g)(1-X_g)\ln(1-X_g) \\ &+\left(\bar{m}_f^2 \ln \bar{m}_f^2 + 2\bar{m}_f \right) X_g \left[1 - 2X_g\left(1-X_g^2\right) \right] \\ &+\frac{3\bar{m}_f^2}{4}\left(-\frac{2}{3} + \frac{\pi^2}{18} + \frac{1}{3}\ln\frac{4}{\bar{m}_f^2} + \frac{1}{12}\ln^2\frac{4}{\bar{m}_f^2} \right), \end{aligned} \tag{3. 3}$$

$$F_4(\bar{m}_f, X_g) = F_1(\bar{m}_f, X_g) + \left(1 - \ln\frac{4}{\bar{m}_f^2} \right)\left(1 + \frac{X_g}{2} \right), \tag{3. 4}$$

$$F_5(\bar{m}_f, X_g) = F_1(\bar{m}_f, X_g) + \left(2 - \ln\frac{4}{\bar{m}_f^2} \right)\left(1 + \frac{X_g}{2} \right). \tag{3. 5}$$

These formulae represent well the form factors for all X_g for $\bar{m}_f \leq 0.2$. By checking with Eqs. (3.1)-(3.11) in reference [4] it is seen that Eqs. (3.1)-(3.3) for $\bar{m}_f \to 0$ are identical to the form factors for massless quarks. Likewise, by checking with Eqs. (6.1)-(6.5) in reference [6] the formulae given here are seen to give the correct form factors in the limit $X_g = 1 - \bar{m}_f^2$.

It is useful to study in particular the two cases, elastic quark-antiquark production $X_g \ll 1$ with gluon radiative correction and on the other hand the close to the total quark-antiquark-gluon production $1 - X_g \ll 1$.

In the case of elastic q, $\bar{q}$ production the radiative correction is dominated by the infrared singularity effect which is contained in the function $G(z)$, Eq. (A.6). This logarithmic dominance is clearly demonstrated in Fig. 3 for $F_3(\bar{m}_f, X_g)$. In fact the form factors can in this elastic region for $X_g \leq 0.2$, $\bar{m}_f \leq 0.2$ be described by

$$F_1(\bar{m}_f, X_g) = 2\left(\ln\frac{4}{\bar{m}_f^2} - 1 \right)\left(\ln X_g + \frac{3}{4} - X_g \right) - \frac{1}{2}(1 + 4X_g) + \frac{\pi^2}{3}, \quad (3.\ 6)$$

$$F_2(\bar{m}_f, X_g) = \frac{\bar{m}_f^2}{2} F_1(\bar{m}_f, X_g) + X_g \left(\frac{1}{2} - \frac{\pi^2}{8}\bar{m}_f \right), \quad (3.\ 7)$$

$$F_3(\bar{m}_f, X_g) = F_1(\bar{m}_f, X_g) - \frac{3}{2}[\, X_g + \ln(1 - X_g)\,], \quad (3.\ 8)$$

$$F_4(\bar{m}_f, X_g) = F_1(\bar{m}_f, X_g) + \left(1 - \ln\frac{4}{\bar{m}_f^2} \right)\left(1 + \frac{X_g}{2} \right), \quad (3.\ 9)$$

$$F_5(\bar{m}_f, X_g) = F_1(\bar{m}_f, X_g) + \left(2 - \ln\frac{4}{\bar{m}_f^2} \right)\left(1 + \frac{X_g}{2} \right). \quad (3.\ 10)$$

For the case that close to all quark-antiquark events, $1 - X_g = \xi \ll 1$ are recorded we define

$$\Delta F_i(\bar{m}_f, 1 - \xi) = F_i(\bar{m}_f, 1 - \xi) - F_i(\bar{m}_f, 1 - \bar{m}_f^2), \quad (3.\ 11)$$

where the form factors for total recording of q-$\bar{q}$-g events to first order in $\bar{m}_f^2$, are given in reference [6] and can be obtained from reference [9]

$$
\begin{aligned}
F_1(\bar{m}_f, 1-\bar{m}_f^2) &= \frac{3}{4}\left(1+3\bar{m}_f^2\right), \\
F_2(\bar{m}_f, 1-\bar{m}_f^2) &= \frac{3}{4}\left[\frac{2}{3}-\frac{\pi^2}{6}\bar{m}_f-\bar{m}_f^2\left(\frac{1}{3}+\frac{\pi^2}{9}+\ln\frac{\bar{m}_f}{2}+\frac{1}{3}\ln^2\frac{\bar{m}_f}{2}\right)\right], \\
F_3(\bar{m}_f, 1-\bar{m}_f^2) &= \frac{3}{4}\left[\frac{8}{3}\bar{m}_f+\bar{m}_f^2\left(\frac{7}{3}+\frac{\pi^2}{18}-\frac{2}{3}\ln\frac{\bar{m}_f}{2}+\frac{1}{3}\ln^2\frac{\bar{m}_f}{2}\right)\right], \qquad (3.\ 12) \\
F_4(\bar{m}_f, 1-m_f^2) &= \frac{3}{4}\left(3+4\ln\frac{\bar{m}_f}{2}\right), \\
F_5(\bar{m}_f, 1-\bar{m}_f^2) &= \frac{3}{4}\left(5+4\ln\frac{\bar{m}_f}{2}\right).
\end{aligned}
$$

The changes in the form factors Eqs. (3.11) are obtained for $\bar{m}_f^2 \le \xi \ll 1$ as

$$
\begin{aligned}
\Delta F_1(\bar{m}_f, 1-\xi) &= \left[\,2\ln\left(\sqrt{\frac{\xi}{\bar{m}_f^2}}+\sqrt{\frac{\xi}{\bar{m}_f^2}-1}\right)-1\,\right][\,2\ln(1-\xi)+\xi\,] \\
&\quad +\xi-2m_f^2, \qquad (3.\ 13) \\
\Delta F_2(\bar{m}_f, 1-\xi) &= \frac{\bar{m}_f^2}{2}\Delta F_1(\bar{m}_f, 1-\xi)-\frac{1}{2}\left(\xi-\bar{m}_f^2\right), \qquad (3.\ 14) \\
\Delta F_3(\bar{m}_f, 1-\xi) &= \Delta F_1(\bar{m}_f, 1-\xi)-\frac{\xi}{2}(2+\xi)\ln\frac{\xi}{\bar{m}_f^2}-10\bar{m}_f\left(\xi-\bar{m}_f^2\right), \qquad (3.\ 15) \\
\Delta F_4(\bar{m}_f, 1-\xi) &= \Delta F_1(\bar{m}_f, 1-\xi)-\frac{\xi-\bar{m}_f^2}{2}\left(1-\ln\frac{4}{\bar{m}_f^2}\right), \qquad (3.\ 16) \\
\Delta F_5(\bar{m}_f, 1-\xi) &= \Delta F_1(\bar{m}_f, 1-\xi)-\frac{\xi-\bar{m}_f^2}{2}\left(2-\ln\frac{4}{\bar{m}_f^2}\right). \qquad (3.\ 17)
\end{aligned}
$$

With the approximate formulae for the form factors Eqs. (3.6)-(3.10) for $X_g \ll 1$ and Eqs. (3.13)-(3.16) for $\xi \ll 1$ we shall be able to give analytical formulae for radiative corrections to cross sections and asymmetries.

4 Radiative corrections to cross sections and asymmetries

We give in the following formulae for cross sections and quark forward-backward, left-right and forward-backward, left-right asymmetries including radiative corrections, as functions of the maximum recorded scaled gluon energy X_g. Exact formulae in terms of the form factors are then specialized to two cases: Radiative corrections to elastic quark-antiquark production, $e_+e_- \to q\bar{q}$ where $X_g \ll 1$, and on the other hand the case when almost all gluons are recorded, $\xi = 1 - X_g \ll 1$. For these two cases analytic formulae are obtained with the use of approximations developed in Sec. 3. We shall here not discuss transversely polarized electrons and positrons.

The differential cross sections is then given by

$$\frac{d^2\sigma_f}{d\Omega} = 3\beta\frac{\alpha^2}{4s}$$

$$\times\Bigg[h_f^{(1)}(s, P_-^{||}P_+^{||})\Bigg\{ \left(1+\frac{\bar{m}_f^2}{2}\right)\left(1+\frac{4\alpha_s}{3\pi}F_1(\bar{m}_f, X_g)\right)(1+\cos^2\theta)$$

$$+\left(\frac{\bar{m}_f^2}{2}+\frac{4\alpha_s}{3\pi}F_2(\bar{m}_f, X_g)\right)(1-3\cos^2\theta)\Bigg\}$$

$$-h_f^{(5)}(s, P_-^{||}P_+^{||})\frac{\bar{m}_f^2}{4}\Bigg\{ 3\left(1+\frac{4\alpha_s}{3\pi}F_4(\bar{m}_f, X_g)\right)(1+\cos^2\theta)$$

$$+\left(1+\frac{4\alpha_s}{3\pi}F_5(\bar{m}_f, X_g)\right)(1-3\cos^2\theta)\Bigg\}$$

$$+2\beta\Bigg\{ h_f^{(2)}(s, P_-^{||}P_+^{||})\left(1+\frac{4\alpha_s}{3\pi}F_3(\bar{m}_f, X_g)\right)+\frac{a_f}{v_f}h_f^{(4)}(s)\frac{8\alpha_s}{3\pi}ImF_V'(\beta)\Bigg\}\cos\theta\Bigg]. \quad (4.\ 1)$$

The total cross section

$$\sigma_f(X_g) = 4\pi\beta\frac{\alpha^2}{s}\ h_f^{(1)}(s, P_-^{||}P_+^{||})$$

$$\times\Bigg[\left(1+\frac{\bar{m}_f^2}{2}\right)\left(1+\frac{4\alpha_s}{3\pi}F_1(\bar{m}_f, X_g)\right)-\frac{3\bar{m}_f^2}{4}\frac{h_f^{(5)}(s, P_-^{||}P_+^{||})}{h_f^{(1)}(s, P_-^{||}P_+^{||})}\left(1+\frac{4\alpha_s}{3\pi}F_4(\bar{m}_f, X_g)\right)\Bigg], \quad (4.\ 2)$$

written in terms of the Born approximation $e_+e_- \to q\bar{q}$ cross section σ_f^0 and the radiative correction $\Delta_{tot}^f(X_g)$, becomes

$$\sigma_f(X_g) = \sigma_f^0\left(1-\Delta_{tot}^f(X_g)\right), \quad (4.\ 3)$$

with

$$\sigma_f^0 = 4\pi\beta\frac{\alpha^2}{s}h_f^{(1)}(s,P_-^{||}P_+^{||})\left[1+\frac{\bar{m}_f^2}{2}-\frac{3\bar{m}_f^2}{4}\frac{h_f^{(5)}(s,P_-^{||}P_+^{||})}{h_f^{(1)}(s,P_-^{||}P_+^{||})}\right], \quad (4.\ 4)$$

and

$$\Delta_{tot}^f(X_g) = -\frac{4\alpha_s}{3\pi}\left[F_1(\bar{m}_f,X_g)+\frac{3\bar{m}_f^2}{4}\frac{h_f^{(5)}(s,P_-^{||}P_+^{||})}{h_f^{(1)}(s,P_-^{||}P_+^{||})}\frac{F_1(\bar{m}_f,X_g)-F_4(\bar{m}_f,X_g)}{1+\frac{\bar{m}_f^2}{2}-\frac{3\bar{m}_f^2}{4}\frac{h_f^{(5)}(s,P_-^{||}P_+^{||})}{h_f^{(1)}(s,P_-^{||}P_+^{||})}}\right], \quad (4.\ 5)$$

which to order $\bar{m}_f^2$ is

$$\Delta_{tot}^f(X_g) = -\frac{4\alpha_s}{3\pi}\left[F_1(\bar{m}_f,X_g)+\frac{3\bar{m}_f^2}{4}\frac{h_f^{(5)}(s,P_-^{||}P_+^{||})}{h_f^{(1)}(s,P_-^{||}P_+^{||})}\Big(F_1(\bar{m}_f,X_g)-F_4(\bar{m}_f,X_g)\Big)\right], \quad (4.\ 6)$$

where the effect of the electron and positron polarizations is contained in the ratio $h_f^{(5)}(s,P_-^{||}P_+^{||})/h_f^{(1)}(s,P_-^{||}P_+^{||})$. It then follows from Eq. (2.2) that the radiative correction to the total cross section at the Z_0 resonance is independent of the polarization of the electron or positron for all values of X_g. This can be understood since the radiative correction is a final state effect and the decay of the almost real particle Z_0 at the resonance is independent of its spinn state. Note however that the total cross section Eq. (4.2) is changed at the Z_0 resonance by the factor

$$1-P_-^{||}P_+^{||}-\frac{2va}{v^2+a^2}(P_-^{||}-P_+^{||}), \quad (4.\ 7)$$

which is a Z_0 production effect. In particular note that if $P_-^{||}=P_+^{||}=\pm 1$, no spin-1 Z_0 particle can be produced and the cross section is zero.

The forward-backward asymmetry, $A_{FB}^f(X_g)$, for all values of X_g is obtained from Eq. (4.1) as

$$A_{FB}^f(X_g) = A_{FB}^{f\ 0}\Big(1-\Delta_{FB}^f(X_g)\Big), \quad (4.\ 8)$$

with

$$A_{FB}^{f\ 0} = \frac{3}{4}\beta\frac{h_f^{(2)}(s,P_-^{||}P_+^{||})}{h_f^{(1)}(s,P_-^{||}P_+^{||})}\left[1+\frac{\bar{m}_f^2}{2}-\frac{3\bar{m}_f^2}{4}\frac{h_f^{(5)}(s,P_-^{||}P_+^{||})}{h_f^{(1)}(s,P_-^{||}P_+^{||})}\right]^{-1}, \quad (4.\ 9)$$

and the radiative correction may be written in the form

$$\Delta^f_{FB}(X_g) = -\left[\Delta^f_{tot}(X_g) + \frac{4\alpha_s}{3\pi}\left\{ F_3(\bar{m}_f, X_g) + \frac{a_f}{v_f}\frac{h^{(4)}_f(s)}{h^{(2)}_f(s, P^{||}_- P^{||}_+)} 2ImF'_V(\beta) \right\}\right] \times \left[1 - \Delta^f_{tot}(X_g) \right]^{-1}, \qquad (4.\ 10)$$

where $\Delta^f_{tot}(X_g)$ is given in Eq. (4.5). To order α_s and $\bar{m}^2_f$ the radiative correction to the asymmetry is given by

$$\Delta^f_{FB}(X_g) = -\frac{4\alpha_s}{3\pi}\left[F_3(\bar{m}_f, X_g) - F_1(\bar{m}_f, X_g) + \frac{a_f}{v_f}\frac{h^{(4)}_f(s)}{h^{(2)}_f(s, P^{||}_- P^{||}_+)}\frac{\pi}{2}\bar{m}^2_f - \frac{3\bar{m}^2_f}{4}\frac{h^{(5)}_f(s, P^{||}_- P^{||}_+)}{h^{(1)}_f(s, P^{||}_- P^{||}_+)}\left(F_1(\bar{m}_f, X_g) - F_4(\bar{m}_f, X_g) \right)\right]. \qquad (4.\ 11)$$

The left-right asymmetry, $A^f_{LR}(X_g)$, for polarized electrons with polarization $P^{||}_-$ is given by

$$A^f_{LR}(X_g) = \frac{\sigma_f(X_g, -P^{||}_-) - \sigma_f(X_g, P^{||}_-)}{\sigma_f(X_g, -P^{||}_-) + \sigma_f(X_g, P^{||}_-)}.$$

With the definitions

$$\Delta h^{(i)}_f = h^{(i)}_f(s, -P^{||}_-) - h^{(i)}_f(s, P^{||}_-),$$

$$Sh^{(i)}_f = h^{(i)}_f(s, -P^{||}_-) + h^{(i)}_f(s, P^{||}_-),$$

the asymmetry can be written

$$A^f_{LR}(X_g) = \frac{\Delta h^{(1)}_f}{Sh^{(1)}_f}\left\{ 1 - \frac{3\bar{m}^2_f}{4}\left(\frac{\Delta h^{(5)}_f}{\Delta h^{(1)}_f} - \frac{Sh^{(5)}_f}{Sh^{(1)}_f} \right)\left(1 + \frac{4\alpha_s}{3\pi}F_4(\bar{m}_f, X_g) \right) \times \left[\left(1 + \frac{\bar{m}^2_f}{2}\right)\left(1 + \frac{4\alpha_s}{3\pi}F_1(\bar{m}_f, X_g)\right) - \frac{3\bar{m}^2_f}{4}\frac{Sh^{(5)}_f}{Sh^{(1)}_f}\left(1 + \frac{4\alpha_s}{3\pi}F_4(\bar{m}_f, X_g)\right) \right]^{-1} \right\}. \qquad (4.\ 12)$$

To order α_s and $\bar{m}_f^2$ the asymmetry simplifies to

$$A_{LR}^f(X_g) = A_{LR}^{f\,0}\left(1 - \Delta_{LR}^f(X_g)\right),$$

with

$$A_{LR}^{f\,0} = \frac{\Delta h_f^{(1)}}{Sh_f^{(1)}}\left[1 - \frac{3\bar{m}_f^2}{4}\left(\frac{\Delta h_f^{(5)}}{\Delta h_f^{(1)}} - \frac{Sh_f^{(5)}}{Sh_f^{(1)}}\right)\right], \tag{4. 13}$$

and the radiative correction

$$\Delta_{LR}^f(X_g) = \frac{\alpha_s}{\pi}\bar{m}_f^2\left(\frac{\Delta h_f^{(5)}}{\Delta h_f^{(1)}} - \frac{Sh_f^{(5)}}{Sh_f^{(1)}}\right)\left(F_4(\bar{m}_f, X_g) - F_1(\bar{m}_f, X_g)\right). \tag{4. 14}$$

The left-right asymmetry being at the Z_0 resonance purely an initial-state effect for production of the almost real Z_0 particle, $A_{LR}^f = 2avP_-/(v^2+a^2)$, $\Delta_{LR}^f = 0$, has for energies outside the Z_0-mass where $\Delta h_f^{(5)}(s, P_-^{||}P_+^{||})/\Delta h_f^{(1)}(s, P_-^{||}P_+^{||})$ is different from $Sh_f^{(5)}(s, P_-^{||}P_+^{||})/Sh_f^{(1)}(s, P_-^{||}P_+^{||})$, radiative as well as mass corrections.

<u>The forward-backward, left-right asymmetry,</u> $A_{FB,LR}^f(X_g)$, is to order α_s and $\bar{m}_f^2$ given by

$$A_{FB,LR}^f(X_g) = A_{FB,LR}^{f\,0}\left(1 - \Delta_{FB,LR}^f(X_g)\right),$$

with

$$A_{FB,LR}^{f\,0} = \frac{3\beta}{4}\frac{\Delta h_f^{(2)}}{Sh_f^{(1)}}\left(1 - \frac{\bar{m}_f^2}{2} + \frac{3\bar{m}_f^2}{4}\frac{Sh_f^{(5)}}{Sh_f^{(1)}}\right), \tag{4. 15}$$

and

$$\Delta_{FB,LR}^f(X_g)$$

$$= -\frac{4\alpha_s}{3\pi}\left[F_3(\bar{m}_f, X_g) - F_1(m_f, X_g) - \frac{3\bar{m}_f^2}{4}\frac{Sh_f^{(5)}}{Sh_f^{(1)}}\left(F_1(\bar{m}_f, X_g) - F_4(\bar{m}_f, X_g)\right)\right]. \tag{4. 16}$$

From the formulae given in this Section it is clear that the simple relations at the Z_0 resonance, reference [6],

$$A_{FB}(P_-) : A_{FB}(0) : A_{FB,LR}(P_-) = P_{Z_0}(P_-) : P_{Z_0}(0) : P_-,$$

do not hold outside the resonance, in particular the radiative correction differ considerably for the different asymmetries.

We show in Figs. 4-6 the radiative corrections $\Delta^b_{tot}(X_g)$, $\Delta^b_{FB}(X_g)$ and $\Delta^b_{FB,LR}(X_g)$ for b-quarks for two values of energy, 91.2 GeV and 200 GeV. The accuracy of the approximate formulae for the radiative correction is demonstrated. The effect of electron polarization on the radiative correction is at the Z_0 resonance, 91.2 GeV, essentially zero, and very small for 200 GeV. For the asymmetries, however, the polarization effects are large. We demonstrate the large effects of electron polarization on $A_{FB}(X_g)$ and $A_{FB,LR}(X_g)$ in Figs. 7 and 8 respectively. We use $\alpha_s(91.2\ GeV) = 0.118$ and $\alpha_s(200\ GeV) = 0.104$.

5 Radiative corrections to elastic quark-antiquark production

The radiative corrections to the $e_+e_- \rightarrow q\bar{q}$ process is obtained by the use of equation (3.6)-(3.10) for $X_g \ll 1$. One obtains the analytic formulae for the radiative correction to the total elastic cross section to order $\bar{m}_f^2$ from Eq. (4.6)

$$\Delta^f_{tot}(X_g) = -\frac{4\alpha_s}{3\pi}\left[2\left(\ln\frac{4}{\bar{m}_f^2}-1\right)\left\{\ln X_g + \frac{3}{4} - X_g + \frac{3\bar{m}_f^2}{8}\frac{h_f^{(5)}(s,P_-^{||}P_+^{||})}{h_f^{(1)}(s,P_-^{||}P_+^{||})}\right\} -\frac{1}{2}(1-3X_g) + \frac{\pi^2}{3}\right]. \quad (5.\ 1)$$

For the asymmetries to order $\bar{m}_f^2$, Eqs. (4.11), (4.14) and (4.16) one finds

$$\Delta^f_{FB}(X_g) = \frac{\alpha_s}{\pi}\left[2(\ X_g + \ln(1-X_g)\ ) - \frac{a_f}{v_f}\frac{h_f^{(4)}(s)}{h_f^{(2)}(s,P_-^{||}P_+^{||})}\frac{2\pi}{3}\bar{m}_f^2 + \bar{m}_f^2\left(\ln\frac{4}{\bar{m}_f^2}-1\right)\frac{h_f^{(5)}(s,P_-^{||}P_+^{||})}{h_f^{(1)}(s,P_-^{||}P_+^{||})}\right], \quad (5.\ 2)$$

$$\Delta^f_{LR}(X_g) = -\frac{\alpha_s}{\pi}\bar{m}_f^2\left(\ln\frac{4}{\bar{m}_f^2}-1\right)\left(\frac{\Delta h_f^{(5)}}{\Delta h_f^{(1)}} - \frac{Sh_f^{(5)}}{Sh_f^{(1)}}\right), \quad (5.\ 3)$$

$$\Delta^f_{FB,LR}(X_g) = \frac{\alpha_s}{\pi}\left[2(\ X_g + \ln(1-X_g)\ ) + \bar{m}_f^2\left(\ln\frac{4}{\bar{m}_f^2}-1\right)\frac{Sh_f^{(5)}}{Sh_f^{(1)}}\right]. \quad (5.\ 4)$$

The effects of electron-positron polarization are always contained in the $h_f^{(i)}(s, P_-^{||} P_+^{||})$ functions. Note that these factors depend strongly on energy, for instance at the Z_0 resonance $\Delta h^{(5)}/\Delta h^{(1)}$ equals $Sh^{(5)}/Sh^{(1)}$ and the left-right asymmetry being then a purely initial state effect, has no radiative correction. It should be noticed that $\Delta_{tot}^f(X_g)$ containing the large factor $\ln(4/\bar{m}_f^2) \ln X_g$ is in general much larger than the radiative correction to the asymmetries which are ratios of cross sections.

6 Radiative corrections when almost all gluons are recorded

When all gluons are recorded, X_g has the maximum value $X_g = 1 - \bar{m}_f^2$. The variable $\xi = 1 - X_g$ introduced in Sec. 3 describes for $\xi \ll 1$ the effects on radiative corrections when a small part of gluons have escaped recording; ξ is sometimes called "cut".

The radiative correction to the cross section is in this region of X_g, to order $\bar{m}_f^2$,

$$\Delta_{tot}^f(X_g) = -\frac{\alpha_s}{\pi} \left[1 + 3\bar{m}_f^2 + \frac{3\bar{m}_f^2}{2} \left(\ln \frac{4}{\bar{m}_f^2} - 1 \right) \frac{h_f^{(5)}(s, P_-^{||} P_+^{||})}{h_f^{(1)}(s, P_-^{||} P_+^{||})} \right.$$

$$\left. + \frac{4}{3} \left\{ \left[2 \ln \left(\sqrt{\frac{\xi}{\bar{m}_f^2}} + \sqrt{\frac{\xi}{\bar{m}_f^2} - 1} \right) - 1 \right] \left(2 \ln(1 - \xi) + \xi \right) + \xi - 2\bar{m}_f^2 \right\} \right] . \quad (6.\ 1)$$

The corresponding radiative correction to the forward-backward asymmetry is obtained from Eq. (4.11) as

$$\Delta_{FB}^f(X_g) = \frac{\alpha_s}{\pi} \left[1 - \frac{8}{3}\bar{m}_f + \frac{\bar{m}_f^2}{3} \left(2 - \frac{\pi^2}{6} - \ln \frac{4}{\bar{m}_f^2} - \frac{1}{4} \ln^2 \frac{4}{\bar{m}_f^2} \right) \right.$$

$$+ \frac{3\bar{m}_f^2}{2} \left(\ln \frac{4}{\bar{m}_f^2} - 1 \right) \frac{h_f^{(5)}(s, P_-^{||} P_+^{||})}{h_f^{(1)}(s, P_-^{||} P_+^{||})} - \frac{a_f}{v_f} \frac{h_f^{(4)}(s)}{h_f^{(2)}(s, P_-^{||} P_+^{||})} \frac{2\pi}{3} \bar{m}_f^2$$

$$\left. + \frac{2}{3} \left(\xi(2 + \xi) \ln \frac{\xi}{\bar{m}_f^2} + 20\bar{m}_f(\xi - \bar{m}_f^2) \right) \right] . \quad (6.\ 2)$$

The left-right asymmetry Eq. (4.14) becomes in the present region of X_g

$$\Delta_{LR}^f(X_g) = -\frac{3\alpha_s}{2\pi} \bar{m}_f^2 \left(\ln \frac{4}{\bar{m}_f^2} - 1 \right) \left(\frac{\Delta h_f^{(5)}}{\Delta h_f^{(1)}} - \frac{Sh_f^{(5)}}{Sh_f^{(1)}} \right) , \quad (6.\ 3)$$

which in the present approximation (to order $\bar{m}_f^2$) is independent of ξ.

The radiative correction to the forward-backward, left-right asymmetry Eq. (4.16) is given by Eq. (4.11) with $h_f^{(5)}(s, P_-^{||} P_+^{||})/h_f^{(1)}(s, P_-^{||} P_+^{||})$ replaced by $Sh_f^{(5)}/Sh_f^{(1)}$ and $h_f^{(4)}(s)/h_f^{(2)}(s, P_-^{||} P_+^{||})$ left out, as in Eqs. (5.2) and (5.4).

It is seen from the Eqs. (6.1)-(6.3) that even a very small value of ξ, of the order $\bar{m}_f^2$, meaning that a very small part of the gluons have been left undetected, whould give effects in $\Delta_{tot}^f(X_g)$, $\Delta_{FB}^f(X_g)$ and $\Delta_{LR}^f(X_g)$ which are as big as and bigger than the mass effects.

A Appendix

With the definition $z^2 = 1 - \bar{m}_f^2/(1-X_g)$, the form factors $F_1(\bar{m}_f, X_g)$, $F_2(\bar{m}_f, X_g)$ and $F_4(\bar{m}_f, X_g)$, $F_5(\bar{m}_f, X_g)$ are given by the analytical formulae:

$$\begin{aligned}
F_1(\bar{m}_f, X_g) &= G(z) - \frac{10-\bar{m}_f^2}{4(2+\bar{m}_f^2)} + \frac{1}{\beta}\left[\frac{3}{8}(2-\bar{m}_f^2) + \frac{(1-\bar{m}_f^2)^2}{2(2+\bar{m}_f^2)}\right]\ln\frac{1+\beta}{1-\beta} \\
&\quad - \frac{2}{\beta(2+\bar{m}_f^2)}\left[\left(-2+\frac{7-z^2}{8}\frac{\bar{m}_f^2}{1-z^2}\right)z\frac{\bar{m}_f^2}{1-z^2}\right. \\
&\quad \left. +\frac{1}{2}\left(1+\frac{m_f^4}{8}-\left(\frac{\beta^2-z^2}{1-z^2}\right)^2\right)\ln\frac{1+z}{1-z}\right], \qquad \text{(A.1)}
\end{aligned}$$

$$F_2(\bar{m}_f, X_g) = \frac{\bar{m}_f^2}{2}G(z) + J(\bar{m}_f, z) + J(-\bar{m}_f, z), \qquad \text{(A.2)}$$

$$\begin{aligned}
F_4(\bar{m}_f, X_g) &= G(z) + \frac{2-\bar{m}_f^2}{8} - \frac{1}{2\beta}\left(1-\bar{m}_f^2+\frac{\bar{m}_f^4}{8}\right)\ln\frac{1+\beta}{1-\beta} \\
&\quad - \frac{1}{6\beta}\left[\left(2-\frac{5-3z^2}{4}\frac{\bar{m}_f^2}{1-z^2}\right)z\frac{\bar{m}_f^2}{1-z^2}\right. \\
&\quad \left. -\left(1-\bar{m}_f^2+\frac{3\bar{m}_f^4}{8}-\left(\frac{\beta^2-z^2}{1-z^2}\right)^2\right)\ln\frac{1+z}{1-z}\right], \qquad \text{(A.3)}
\end{aligned}$$

$$\begin{aligned}
F_5(\bar{m}_f, X_g) &= -2G(z) + 3F_4(\bar{m}_f, X_g) + \beta\ln\frac{1+\beta}{1-\beta} \\
&\quad + M(\bar{m}_f, z) + M(-\bar{m}_f, z), \qquad \text{(A.4)}
\end{aligned}$$

where the $G(z)$, $J(\bar{m}_f, z)$ and $M(\bar{m}_f, z)$ functions are defined by

$$G(z) = 2 - 4\ln\beta + 6\ln\frac{1+\beta}{2} - \left(3 - \frac{2-\bar{m}_f^2}{\beta}\right)\ln\frac{1+\beta}{1-\beta}$$

$$
+\frac{2-\bar{m}_f^2}{\beta}\left[\frac{\pi^2}{6}+\ln\frac{1+\beta}{2}\ln\frac{1+\beta}{1-\beta}+2L_2\left(\frac{1-\beta}{1+\beta}\right)+L_2(\beta^2)\right.
$$

$$
-4L_2(\beta)+2L_2\left(\frac{1+\beta}{2}\right)-2L_2\left(\frac{1-\beta}{2}\right)+\ln\left(\frac{2}{1+\beta}\frac{\beta^2-z^2}{1-z^2}\right)\ln\frac{1+z}{1-z}
$$

$$
\left.+\Phi_z\left(\frac{1-z}{2}\right)+\Phi_z\left(\frac{1-\beta}{1-z}\right)+\Phi_z\left(\frac{1+z}{1+\beta}\right)\right]
$$

$$
+2\ln\frac{\beta+z}{\beta-z}-\frac{2}{\beta}\left(2z\frac{\bar{m}_f^2}{1-z^2}+\frac{\beta^2-z^2}{1-z^2}\ln\frac{1+z}{1-z}\right), \tag{A.5}
$$

where

$$
\Phi_z(f(z)) = L_2(f(z))-L_2(f(-z)), \tag{A.6}
$$

with $L_2(z)$ the Euler dilogarithm

$$
L_2(z) = -\int_0^z\frac{dy}{y}\ln(1-y), \tag{A.7}
$$

$$
J(\bar{m}_f,z)=\frac{1}{4\beta}\left[\left(1-3\bar{m}_f^2\right)\left(\beta-z\frac{\bar{m}_f^2}{1-z^2}\right)\right.
$$

$$
+\bar{m}_f^2\left\{\frac{3-\bar{m}_f^2}{2}\ln\frac{1+\beta}{1-\beta}\right.
$$

$$
-\frac{1}{4}\left(2(2-\bar{m}_f^2)+\frac{1}{\beta^2}\frac{\beta^2-z^2}{1-z^2}\left[\frac{\beta^2-z^2}{1-z^2}-4\beta^2\right]\right)\ln\frac{1+z}{1-z}
$$

$$
+\left[\left(\frac{2+\bar{m}_f}{4\beta}\right)^2(1-\bar{m}_f)\left\{\frac{1}{1-z}\left(1+\frac{1}{1-z}-\frac{1}{\bar{m}_f}\right)\right.\right.
$$

$$
\left.+\frac{1}{1+z}\left(\frac{\bar{m}_f}{2+\bar{m}_f}\right)\left(1+\frac{1}{1+z}+\frac{1}{2+\bar{m}_f}\right)\right\}
$$

$$
\left.+\frac{\bar{m}_f(3+\bar{m}_f)(2+\bar{m}_f)}{8(1+\bar{m}_f)}\left\{\frac{1}{1-z}+\frac{1}{1+z}\left(\frac{\bar{m}_f}{2+\bar{m}_f}\right)\right\}\right]
$$

$$
\times(1+\bar{m}_f-z)\ln\left[\left(\frac{1-z}{1+z}\right)\left(\frac{1+\bar{m}_f+z}{1+\bar{m}_f-z}\right)^2\right]
$$

$$+\frac{1+\bar{m}_f}{2}\left(\frac{(1+\bar{m}_f)^2-z^2}{1+\bar{m}_f-z^2}\right)\ln\left[\left(\frac{1-z}{1+z}\right)\left|\frac{1+\bar{m}_f+z}{1+\bar{m}_f-z}\right|\right]$$

$$+\frac{1}{\bar{m}_f}(1-\bar{m}_f^2-2\bar{m}_f)(1+\bar{m}_f)\Big(H(\bar{m}_f,\beta)-H(\bar{m}_f,z)\Big)\Big\}\Big], \qquad \text{(A.8)}$$

where

$$\begin{aligned}H(\bar{m}_f,z) &= \ln\frac{2}{2+\bar{m}_f}\ln\frac{1+z}{1-z}+\Phi_z\left(\frac{1-z}{2}\right)-\Phi_z\left(\frac{1-z}{2+\bar{m}_f}\right)-\Phi_z\left(\frac{-\bar{m}_f}{1-z}\right)\\ &+\Phi_z\left(\frac{\sqrt{1+\bar{m}_f}-z}{\sqrt{1+\bar{m}_f}(1+\sqrt{1+\bar{m}_f})}\right)-\Phi_z\left(\frac{\sqrt{1+\bar{m}_f}-z}{\sqrt{1+\bar{m}_f}(1-\sqrt{1+\bar{m}_f})}\right)\\ &+\Phi_z\left(\frac{\sqrt{1+\bar{m}_f}-z}{\sqrt{1+\bar{m}_f}-1}\right)-\Phi_z\left(\frac{\sqrt{1+\bar{m}_f}-z}{\sqrt{1+\bar{m}_f}+1}\right),\end{aligned} \qquad \text{(A.9)}$$

and

$$\begin{aligned}M(\bar{m}_f,z) = \frac{1}{8\beta}\Bigg[\ & 4\beta+2\bar{m}_f^2\ln\frac{1-\beta}{1+\beta}\\ &+\bar{m}_f^2\left\{\frac{1}{1-z}\left(\frac{1}{1-z}-\frac{1}{\bar{m}_f}\right)+\frac{1}{1+z}\left(\frac{\bar{m}_f}{2+\bar{m}_f}\right)\left(\frac{1}{1+z}+\frac{1}{2+\bar{m}_f}\right)\right\}\\ &\times(1+\bar{m}_f-z)\ln\left[\left(\frac{1-z}{1+z}\right)\left(\frac{1+\bar{m}_f+z}{1+\bar{m}_f-z}\right)^2\right]\\ &-\frac{\bar{m}_f^2}{2}\left(1-\frac{z}{1+\bar{m}_f}\right)\ln\left|\frac{1+\bar{m}_f+z}{1+\bar{m}_f-z}\right|\\ &\times\left\{(4-\bar{m}_f^2)\left(\frac{1}{1-z}+\frac{1}{1+z}\frac{\bar{m}_f}{2+\bar{m}_f}\right)\right.\\ &\left.-\bar{m}_f^2\left[\frac{1}{1-z}\left(\frac{1}{1-z}-\frac{1}{\bar{m}_f}\right)+\frac{1}{1+z}\left(\frac{\bar{m}_f}{2+\bar{m}_f}\right)\left(\frac{1}{1+z}+\frac{1}{2+\bar{m}_f}\right)\right]\right\}\\ &-4z\frac{\bar{m}_f^2}{1-z^2}-2\bar{m}_f^2\left(1+2\frac{\bar{m}_f^2}{(4-\bar{m}_f^2)^2}\right)\ln\frac{1-z}{1+z}+4\bar{m}_f(1+\bar{m}_f)[\,H(\bar{m}_f,\beta)-H(\bar{m}_f,z)\,]\Bigg].\end{aligned} \qquad \text{(A.10)}$$

The form factor $F_3(\bar{m}_f, X_g)$ cannot be obtained in closed form in terms of known functions. For values of $\bar{m}_f \leq 0.3$ a useful formula is

$$
\begin{aligned}
F_3(\bar{m}_f, X_g) &= F_1(\bar{m}_f, X_g) - \frac{3}{4}\left[\, 1 - (1 - X_g)^2 \,\right] - \frac{1}{2}(3 - X_g)(1 - X_g)\ln(1 - X_g) \\
&\quad + \left(\, \bar{m}_f^2 \ln \bar{m}_f^2 + 2\bar{m}_f \,\right) X_g \left[\, 1 - 2X_g\left(1 - X_g^2\right) \,\right] \\
&\quad + \frac{3\bar{m}_f^2}{4}\left(-\frac{2}{3} + \frac{\pi^2}{18} + \frac{1}{3}\ln\frac{4}{\bar{m}_f^2} + \frac{1}{12}\ln^2\frac{4}{\bar{m}_f^2} \right), \qquad \text{(A.11)}
\end{aligned}
$$

where $F_1(\bar{m}_f, X_g)$ is given in Eq. (A.1). The accuracy of this formula is demonstrated in Fig. 3.

Figure captions

Fig. 1
The exact form factors $F_1(\bar{m}_f, X_g)$ and $F_3(\bar{m}_f, X_g)$. The value of the parameter $\bar{m}_f = m_f/E$ is attached to the curves.

Fig. 2
The exact form factors $F_2(\bar{m}_f, X_g)$, $F_4(\bar{m}_f, X_g)$ and $F_5(\bar{m}_f, X_g)$. The value of the parameter $\bar{m}_f = m_f/E$ is attached to the curves.

Fig. 3
The approximate formula for $F_3(\bar{m}_f, X_g)$, Eq. (A.11), (dashed curves) compared to the exact numerical values (solid curves). The value of the parameter $\bar{m}_f = m_f/E$ is attached to the curves.

Fig. 4
The radiative correction to the total cross section $\Delta^b_{tot}(X_g)$ for b-quarks for two energies 91.2 GeV and 200 GeV. For 91.2 GeV the exact (solid) curve is compared to the approximate $\Delta^b_{tot}(X_g)$ (dotted curve) obtained by the use of the approximate values for $F_1(\bar{m}_f, X_g)$ and $F_3(\bar{m}_f, X_g)$ Eqs. (3.1) and (3.3) respectively. The dashed curve for 91.2 GeV describes the low X_g approximation Eq. (5.1) and the dot-dashed curve the small $\xi = 1 - X_g$ approximation Eq. (6.1).

Fig. 5
The radiative correction to the forward-backward asymmetry for b-quarks $\Delta^b_{FB}(X_g)$ at the resonance, 91.2 GeV, (solid curve) which is essentially independent of electron polarization, and at 200 GeV for no polarization (dashed curve) and $P^{||}_{-} = 0.63$ (dotted curve).

Fig. 6
The radiative correction to the forward-backward asymmetry for b-quarks $\Delta^b_{FB,LR}(X_g)$ at the resonance, 91.2 GeV, (solid curve) which is essentially independent of electron polarization, and at 200 GeV for no polarization (dashed curve) and $P^{||}_{-} = 0.63$ (dotted curve).

Fig. 7
The dependence of $A^b_{FB}(X_g)$ for b-quarks on the electron polarization for $X_g = 0.99$ (solid curve), $X_g = 0.9$ (dotted curve) and $X_g = 0.1$ (dashed curve).

Fig. 8

The dependence of $A^b_{FB,LR}(X_g)$ for b-quarks on the electron polarization for $X_g = 0.99$ (solid curve), $X_g = 0.9$ (dotted curve) and $X_g = 0.1$ (dashed curve).

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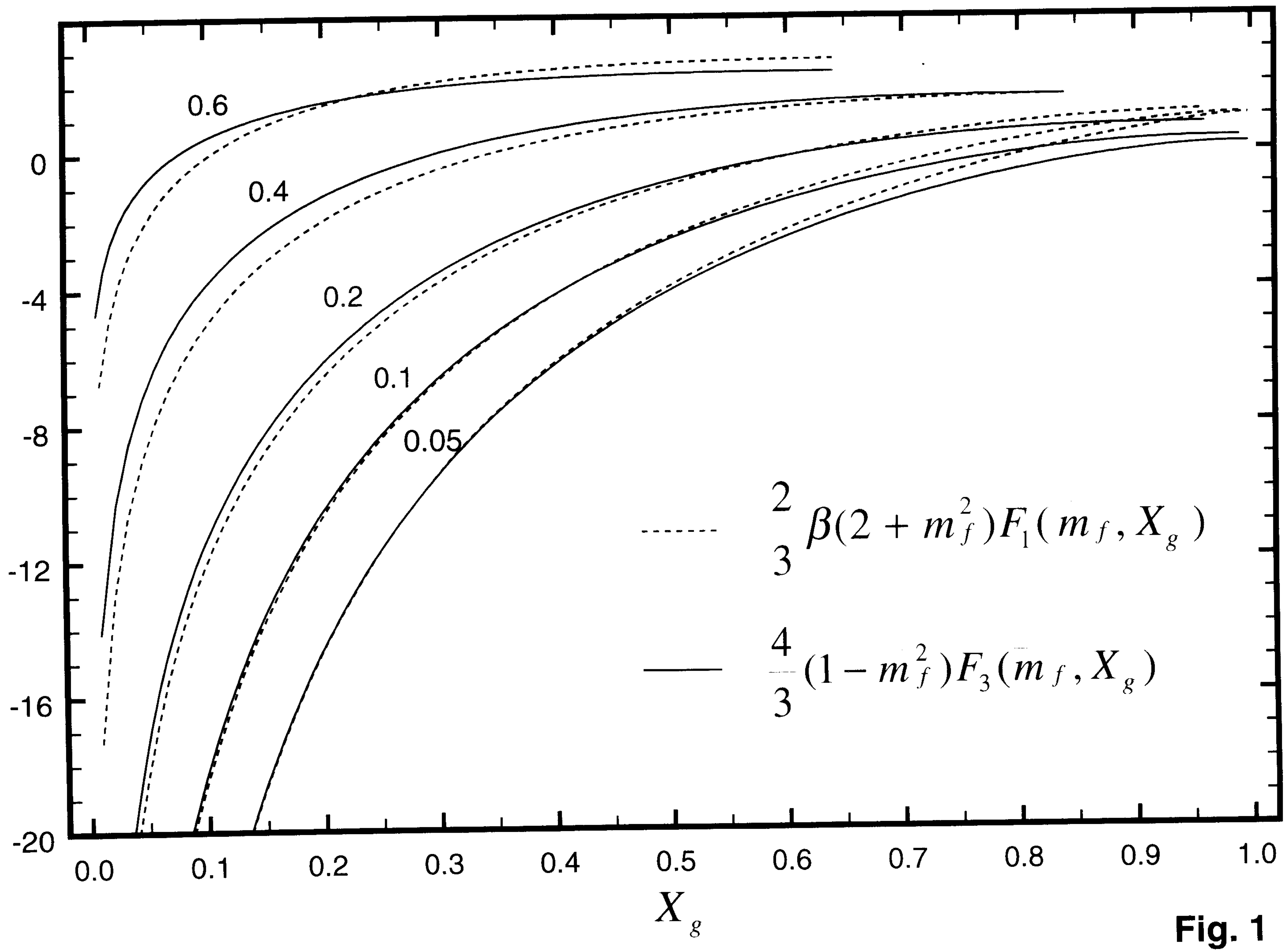
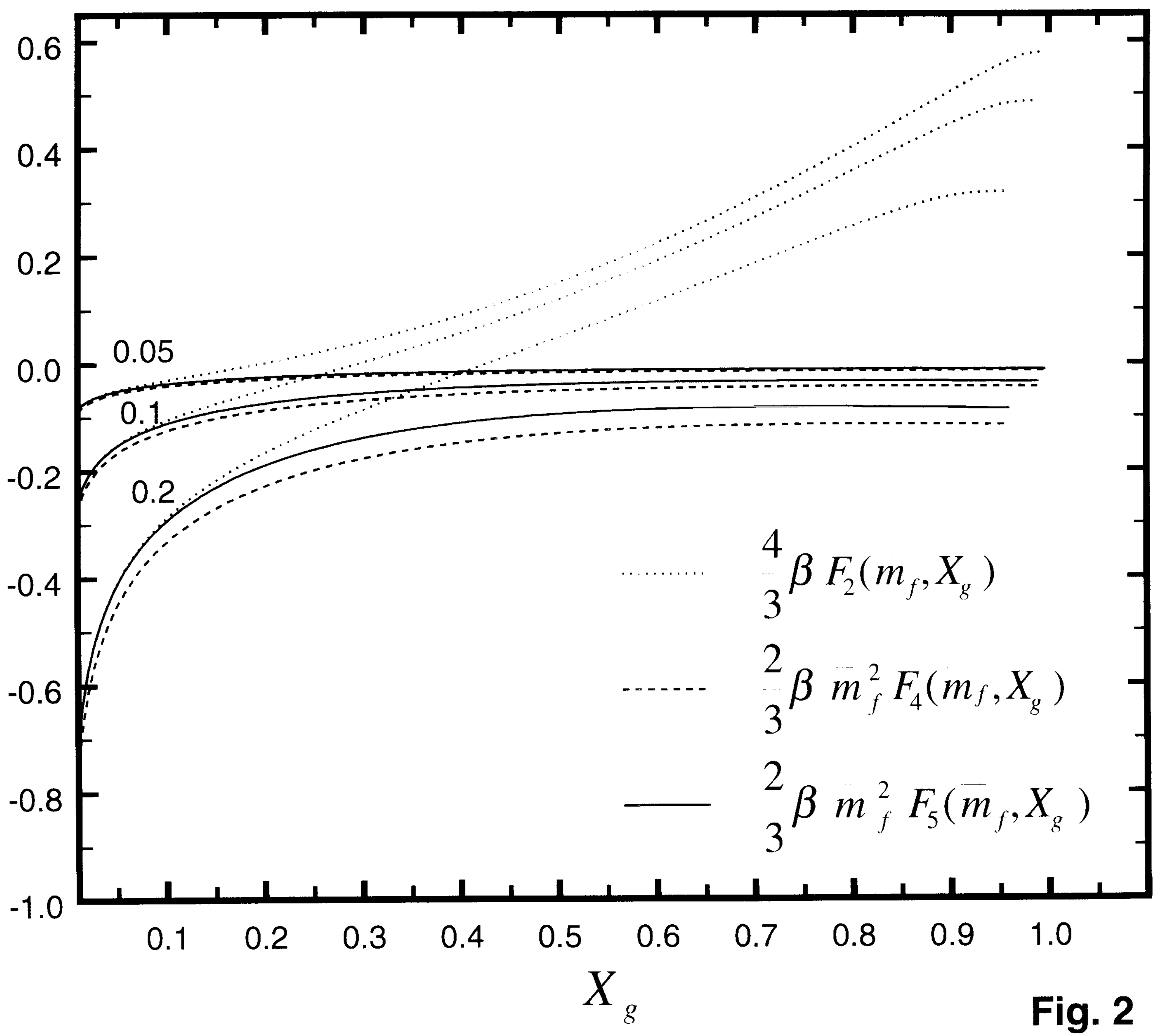


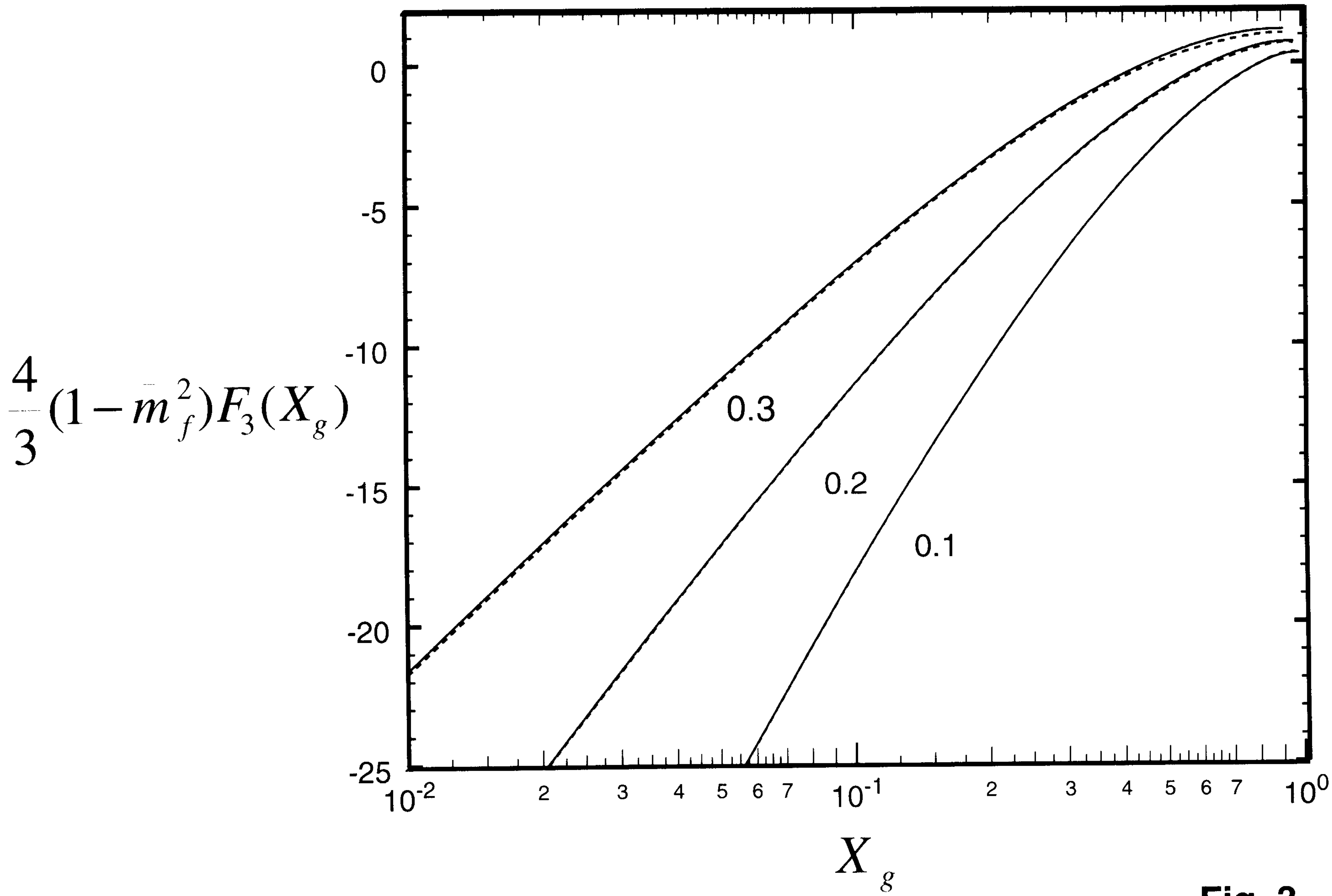
0
-4
-8
-12
-16
-20
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
X_g
0.6
0.4
0.2
0.1
0.05
$\frac{2}{3}\beta(2+m_f^2)F_1(m_f, X_g)$
$\frac{4}{3}(1-m_f^2)F_3(m_f, X_g)$

Fig. 1



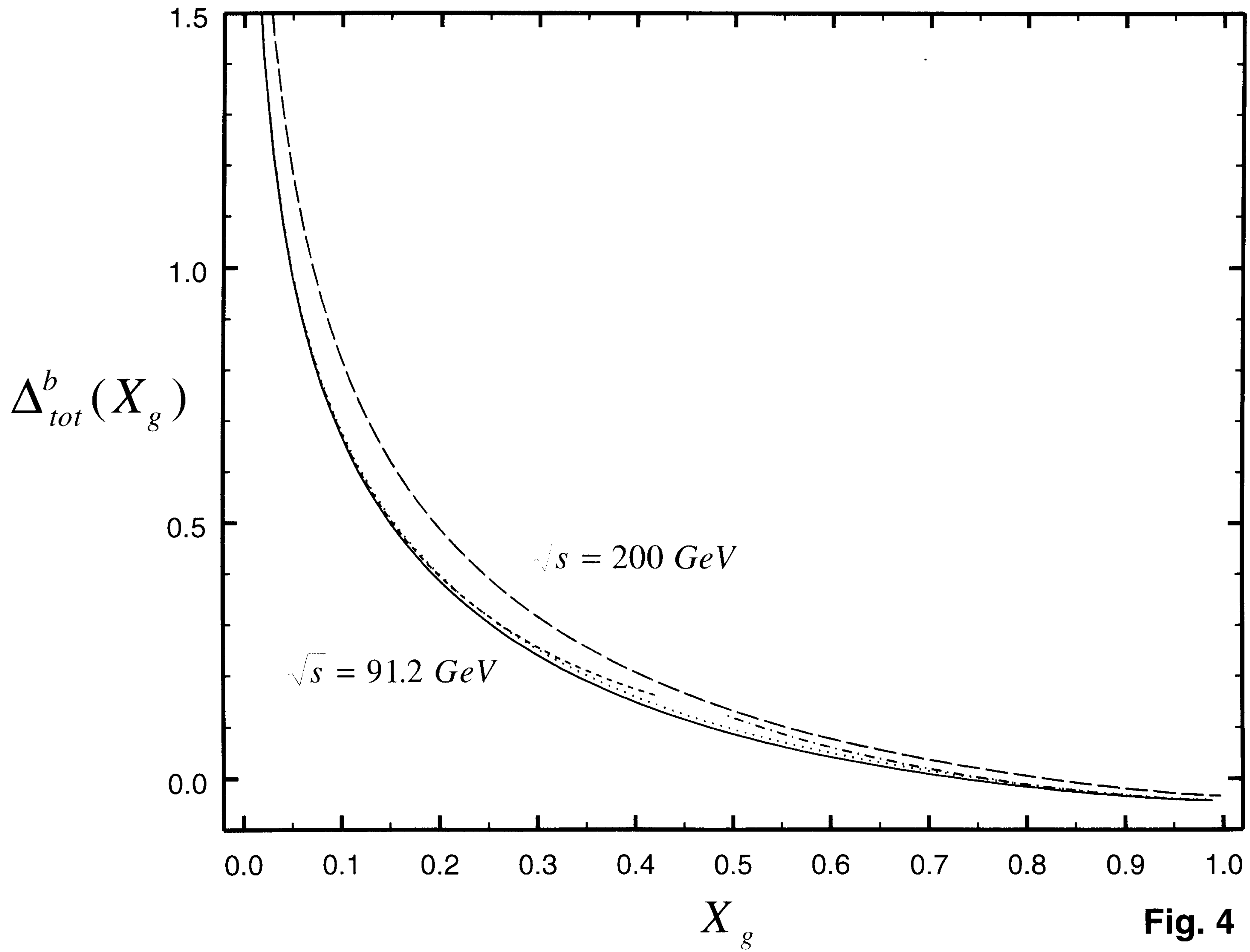
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0.05
0.1
0.2
$\frac{4}{3}\beta\, F_2(m_f, X_g)$
$\frac{2}{3}\beta\, \bar{m}_f^2 F_4(m_f, X_g)$
$\frac{2}{3}\beta\, m_f^2\, F_5(\bar{m}_f, X_g)$
X_g

Fig. 2



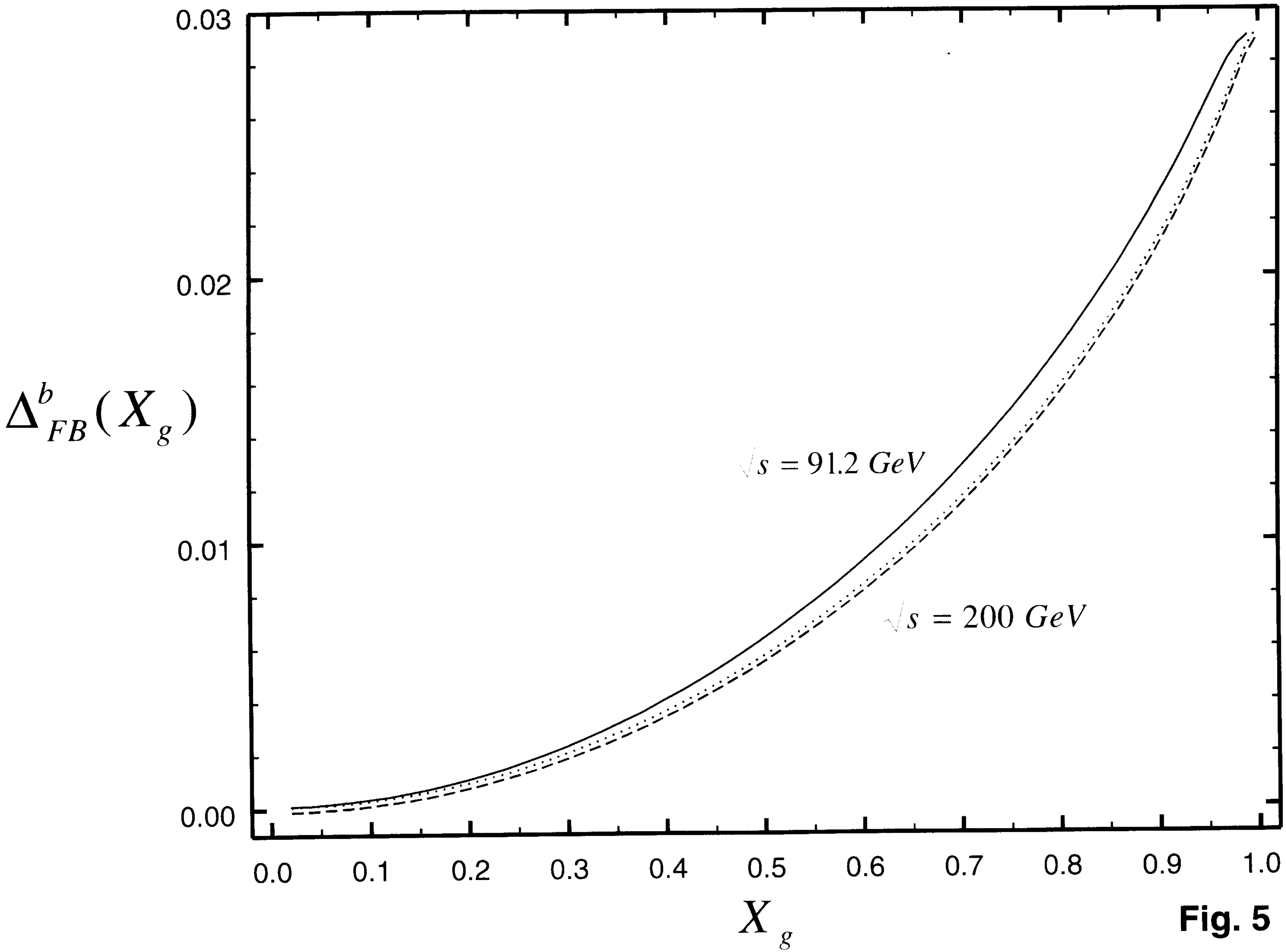
0
-5
-10
-15
-20
-25
10-2
2
3
4
5
6
7
10-1
2
3
4
5
6
7
100
0.3
0.2
0.1
$\frac{4}{3}(1-\bar{m}_f^2)F_3(X_g)$
X_g

Fig. 3



1.5
1.0
0.5
0.0
$\Delta^b_{tot}(X_g)$
$\sqrt{s} = 200\ GeV$
$\sqrt{s} = 91.2\ GeV$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
X_g

Fig. 4



0.03
0.02
0.01
0.00
$\Delta^{b}_{FB}(X_g)$
$\sqrt{s} = 91.2\ GeV$
$\sqrt{s} = 200\ GeV$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
X_g

Fig. 5

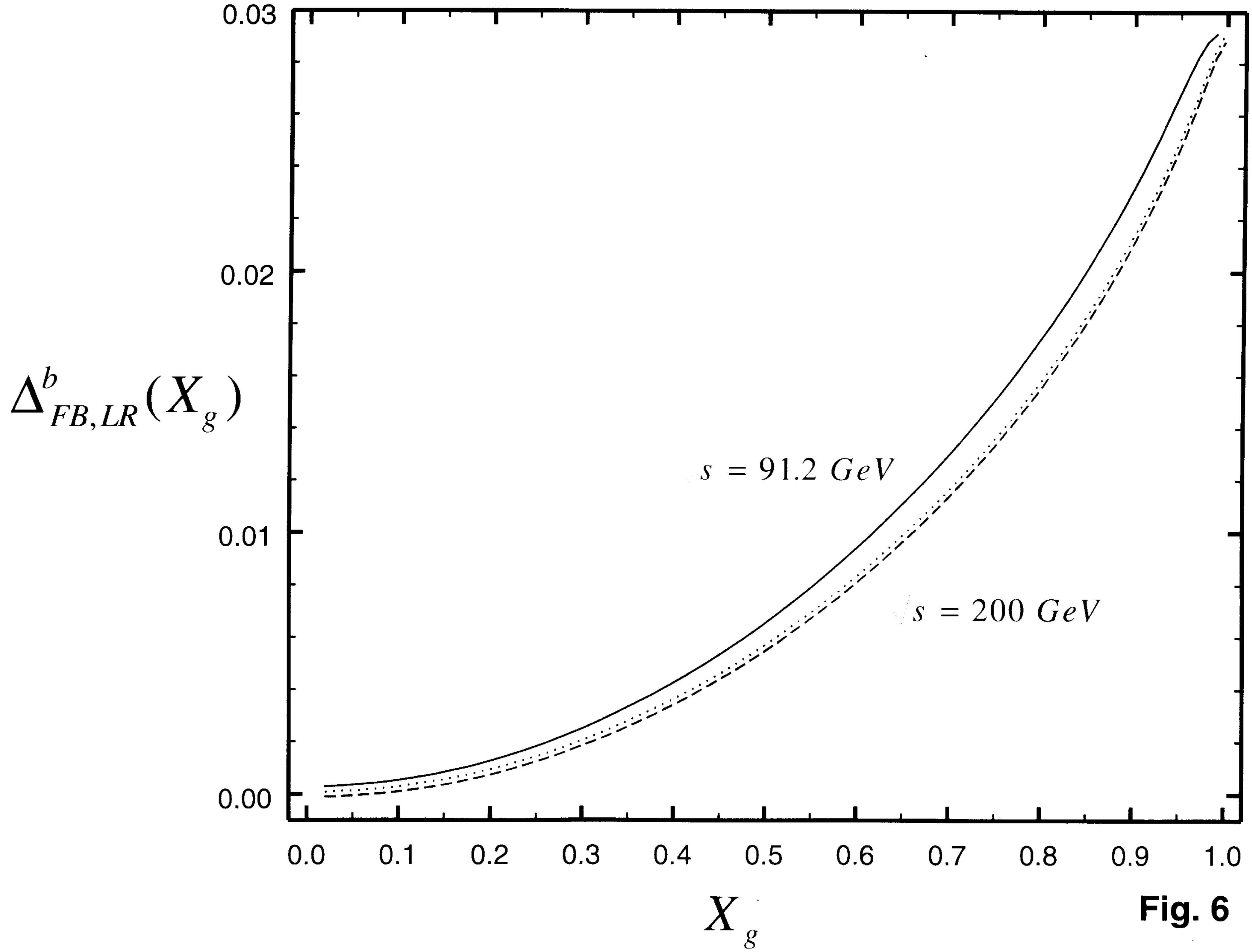
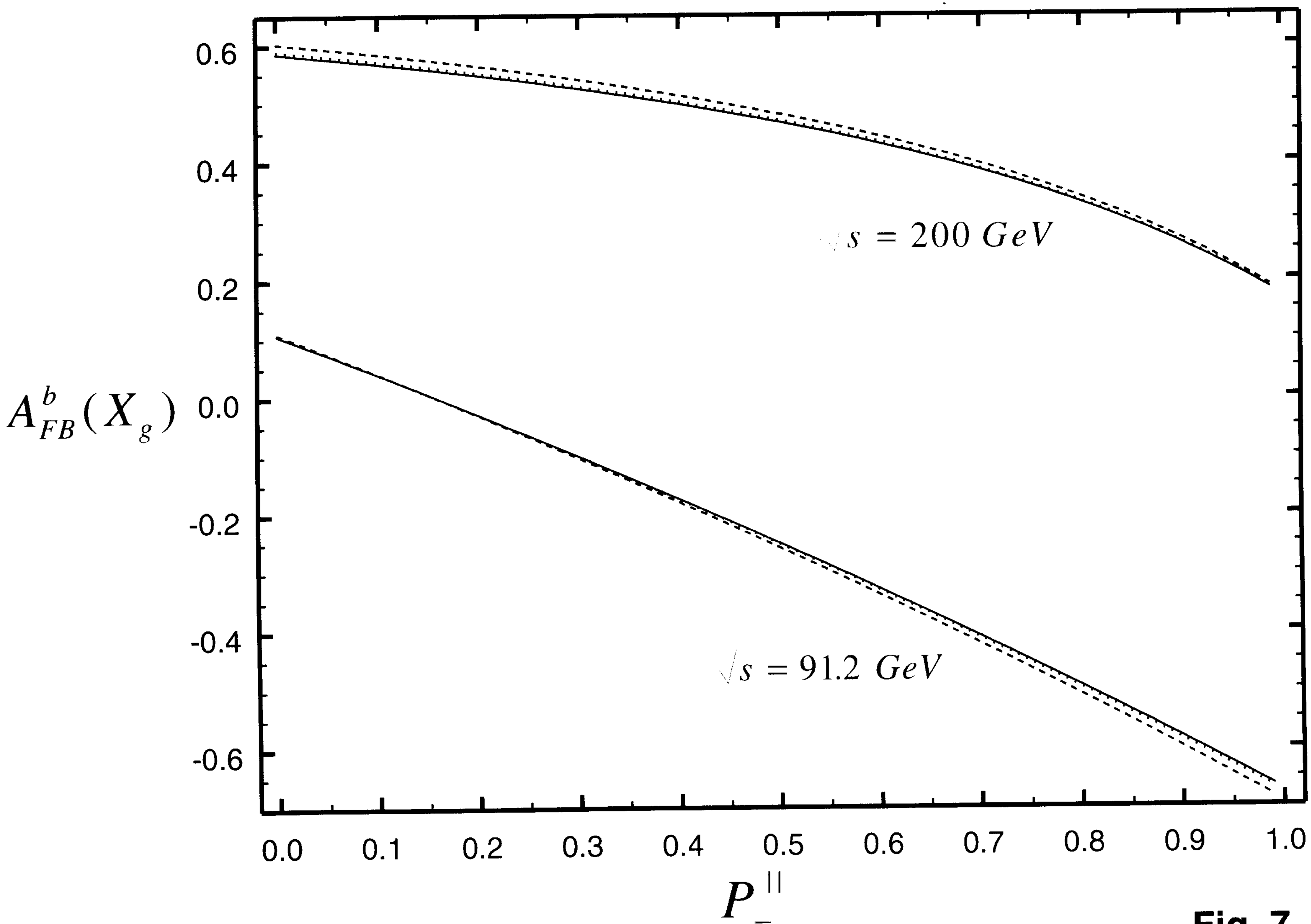
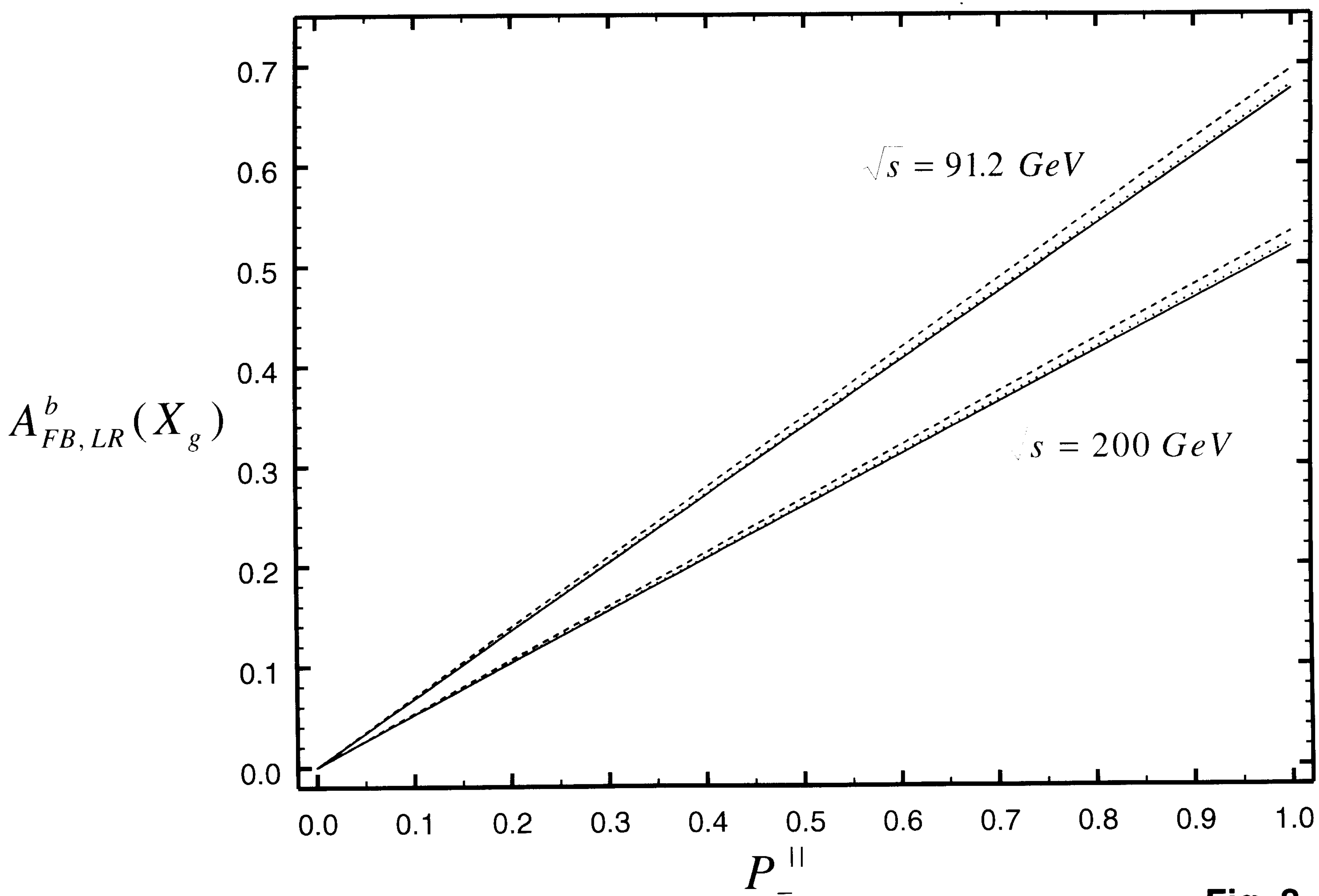

0.03
0.02
0.01
0.00
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
$\Delta^b_{FB,LR}(X_g)$
$\sqrt{s} = 91.2\ GeV$
$\sqrt{s} = 200\ GeV$
X_g

Fig. 6



$A^b_{FB}(X_g)$
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
$\sqrt{s} = 200\ GeV$
$\sqrt{s} = 91.2\ GeV$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
$P_-^{||}$

Fig. 7



$A^{b}_{FB,LR}(X_g)$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
$P_{-}^{||}$
$\sqrt{s} = 91.2\ GeV$
$\sqrt{s} = 200\ GeV$

Fig. 8