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IMPACT OF OCTUPOLES ON THE SCHOTTKY SPECTRA OF BUNCHED BEAMS

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Abstract

Schottky monitors serve as non-invasive tools for beam diagnostics, providing insights into crucial bunch characteristics such as tune, chromaticity, bunch profile, or synchrotron frequency distribution. However, octupole magnets commonly used in circular storage rings to mitigate instabilities through the Landau damping mechanism, can significantly affect the Schottky spectrum. Due to the amplitude-dependent incoherent tune shift of individual particles, the satellites of the Schottky spectrum are smeared out as the octupolar field increases. This study investigates the impact of octupoles and their incorporation into theory, with the goal of improving beam and machine parameter evaluation from measured spectra. Theoretical findings are validated through macro-particle simulations conducted across a range of octupole strengths, encompassing typical operational conditions at the Large Hadron Collider.

INTRODUCTION

Octupole magnets are commonly used in circular accelerators to counteract instabilities via the Landau damping mechanism [1]. Experimental observations indicate that octupoles can exert a significant influence on the Schottky spectra. This prompts a need to investigate their impact on existing formulas used to extract parameters from Schottky spectra, and assess their applicability in the presence of octupolar fields. Moreover, previous experiences also indicate that octupoles significantly reduce the coherent components in the spectrum. This reduction is highly beneficial, given that current theories [2, 3] for parameter extraction only focus on non-coherent spectra. The following section explores theoretically how octupoles can modify the transverse spectra while the third section compares numerical simulations of Schottky spectra against the theoretical predictions.

THEORETICAL SPECTRUM

The transverse Schottky spectrum is defined as the Power Spectral Density (PSD) of the dipole moment of the beam. For a single particle i defined by a given synchrotron amplitude $\widehat{\tau}_i$, transverse actions J_{x_i} and J_{y_i}, and a transverse horizontal tune $Q = Q_I + Q_F$ (with Q_I and Q_F, respectively, the integer and fractional part of the betatron tune), the transverse horizontal[1] Schottky spectrum around the harmonic n of the revolution frequency is given by [2, 4, 5]:

$$P_i^{\pm}(\omega, \widehat{\tau}_i, J_{x_i}, J_{y_i}) = \frac{q^2 f_0 \beta_x J_{x_i}}{2} \sum_{p=-\infty}^{\infty} J_p^2\left(\chi^{\pm}_{\widehat{\tau}_i, n\mp Q_I}\right) \times \delta\left(\pm Q_F + pQ_{s_i} - \tilde{\omega}\right), \quad (1)$$

where $\pm$ denotes the upper and lower sidebands, $\tilde{\omega} = (\omega - n\omega_0)/\omega_0$ is the normalised frequency centred around harmonic n, and

$$\chi^{\pm}_{\widehat{\tau},n} = \left(n \mp \frac{Q'}{\eta}\right)\omega_0 \widehat{\tau},$$

where q is the charge of the particle, $f_0 = \omega_0/2\pi$ the revolution frequency, β_x the beta function at the location of the Schottky monitor, $J_p(\cdot)$ the Bessel function of order p, $\delta(\cdot)$ the Dirac delta, Q' the horizontal chromaticity, η the slippage factor, and Ω_{s_i} the angular synchrotron frequency.

Octupoles are responsible for a transverse, action-dependent, tune shift given by [1]:

$$\Delta Q_x = \alpha_x J_x + \alpha_{xy} J_y,$$

and the fractional part of the horizontal tune writes

$$Q_F(J_x, J_y) = Q_{F_0} + \alpha_x J_x + \alpha_{xy} J_y. \quad (2)$$

As shown in Refs. [5–7], assuming no coherent motion, the expected value of the multiparticle PSD for a bunch of N particles is equal to the sum of the single-particle PSDs. Considering a given normalised probability density function $f(\widehat{\tau}, J_x, J_y)$, the multiparticle Schottky spectrum writes:

$$P^{\pm}(\omega) = N \iiint_0^{\infty} f(\widehat{\tau}, J_x, J_y) P_i^{\pm}(\omega, \widehat{\tau}, J_x, J_y)\, d\widehat{\tau}\, dJ_x\, dJ_y. \quad (3)$$

For the following, we consider that the particle distribution along the x, y and z directions are independent, i.e. the distribution function can be factorised. For a Gaussian transverse bunch profile we have:

$$f(\widehat{\tau}, J_x, J_y) = f_z(\widehat{\tau}) f_x(J_x) f_y(J_y) \quad (4)$$

$$= f_z(\widehat{\tau}) \times \frac{e^{-J_x/\epsilon_x}}{\epsilon_x} \times \frac{e^{-J_y/\epsilon_y}}{\epsilon_y} \quad (5)$$

Combining Eqs. (1), (2), and (3) yields:

$$P^{\pm}(\omega) \propto \sum_p \iint_0^{\infty} dJ_x d\widehat{\tau}\, f_z(\widehat{\tau}) f_x(J_x) J_p^2\left(\chi^{\pm}_{\widehat{\tau}, n\mp Q_I}\right) J_x \int_0^{\infty} dJ_y f_y(J_y)\, \delta\left(\pm Q_{F_0} \pm \alpha_x J_x \pm \alpha_{xy} J_y + pQ_s(\widehat{\tau}) - \tilde{\omega}\right), \quad (6)$$

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[1] The derivation focuses on the horizontal plane for clarity but the calculations can be extended to the vertical plane in a similar manner.

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where the dependency on $\widehat{\tau}$ of $Q_s(\widehat{\tau})$ has been introduced. The rightmost integral is given by

$$\frac{1}{|\alpha_{xy}|}\int_0^\infty dJ_y f_y(J_y)\,\delta\left(J_y - \frac{\pm Q_{F_0} \pm \alpha_x J_x + pQ_s - \tilde{\omega}}{\mp\alpha_{xy}}\right)$$
$$= \begin{cases} \frac{1}{|\alpha_{xy}|\epsilon_y} e^{\frac{\pm Q_{F_0} \pm \alpha_x J_x + pQ_s - \tilde{\omega}}{\pm\alpha_{xy}\epsilon_y}} & \text{if } \frac{\pm Q_{F_0} \pm \alpha_x J_x + pQ_s - \tilde{\omega}}{\mp\alpha_{xy}} > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

Under the assumption that α_x and α_{xy} are of opposite sign — which is the case for the LHC octupoles — the first condition of Eq. (7) can be written $J_x > \pm(\tilde{\omega} \mp Q_{F_0} - pQ_s)/\alpha_x$, which allows to replace the lower bound of the integral over J_x by

$$A^\pm(\tilde{\omega},\widehat{\tau}) = \max\left(0,\ \pm\left(\tilde{\omega} \mp Q_{F_0} - pQ_s(\widehat{\tau})\right)/\alpha_x\right), \tag{8}$$

and Eq. (6) becomes:

$$\begin{aligned} P^\pm(\omega) &= \frac{Nq^2 f_0 \beta_x}{2|\alpha_{xy}|\epsilon_y} e^{\frac{\pm Q_{F_0} - \tilde{\omega}}{\pm\alpha_{xy}\epsilon_y}} \sum_p \int_0^\infty d\widehat{\tau} f_z(\widehat{\tau}) J_p^2\left(\chi^\pm_{\widehat{\tau},n\mp Q_I}\right) e^{\frac{pQ_s(\widehat{\tau})}{\pm\alpha_{xy}\epsilon_y}} \int_{A^\pm(\tilde{\omega},\widehat{\tau})}^\infty dJ_x f_x(J_x) J_x e^{\frac{\alpha_x J_x}{\alpha_{xy}\epsilon_y}} \\ &= \frac{Nq^2 f_0 \beta_x}{2|\alpha_{xy}|\epsilon_x\epsilon_y a^2} e^{\frac{\pm Q_{F_0} - \tilde{\omega}}{\pm\alpha_{xy}\epsilon_y}} \sum_p \int_0^\infty d\widehat{\tau} f_z(\widehat{\tau}) J_p^2\left(\chi^\pm_{\widehat{\tau},n\mp Q_I}\right) e^{\frac{pQ_s(\widehat{\tau})}{\pm\alpha_{xy}\epsilon_y} - aA^\pm(\tilde{\omega},\widehat{\tau})} \left(aA^\pm(\tilde{\omega},\widehat{\tau}) + 1\right), \end{aligned} \tag{9}$$

where

$$a = \frac{1}{\epsilon_x} - \frac{\alpha_x}{\alpha_{xy}\epsilon_y},$$

which is always positive as α_x and α_{xy} are assumed to be of opposite sign. The second line of Eq. (9) has been obtained by introducing the Gaussian transverse bunch profile defined in Eq. (4) to solve the second integral over J_x, using the fact that $a > 0$. The summation over the Bessel satellites is relatively limited as, for LHC spectra, satellites with order $|p|$ greater than ~ 30 have a negligible power [7] and do not need to be computed. The integral in Eq. (9) can be evaluated numerically in order to plot the theoretical Schottky spectrum and assess the impact of octupoles on the latter.

Parameter Extraction

Beam and machine parameters such as emittance, tune or chromaticity can be extracted from the standard Schottky spectra by computing respectively, the zeroth, first, and third moment of the upper and lower sidebands of the spectrum. In this section, we assess theoretically how the detuning from octupoles affects these estimates. The following result for the emittance holds in general, while for tune and chromaticity we have assumed a Gaussian transverse bunch profile.

Emittance The cumulative power within each sideband is given by

$$\begin{aligned} P_\pm &= \frac{1}{2\pi}\int_{-\infty}^\infty P^\pm(\omega)d\omega \\ &= \frac{N}{2\pi}\iiint_0^\infty d\widehat{\tau} dJ_x dJ_y f(\widehat{\tau},J_x,J_y) \int_{-\infty}^\infty d\omega P_i^\pm(\omega,\widehat{\tau},J_x,J_y) \\ &= \frac{Nq^2 f_0^2 \beta_x}{2}\iiint_0^\infty f(\widehat{\tau},J_x,J_y) J_x \overbrace{\sum_{p=-\infty}^\infty J_p^2\left(\chi^\pm_{\widehat{\tau}_i,n\mp Q_I}\right)}^{=1} \\ &= \frac{Nq^2 f_0^2 \beta_x \epsilon_x}{2}, \end{aligned}$$

where we used Eqs. (1) and (3) and the fact that the geometric emittance is defined as the mean action. The lower and upper sidebands have the same cumulative power, that is proportional to the emittance and not affected by the octupoles.

Tune If we denote as $\tilde{P}_\pm(\omega)$ the sideband's PSD normalised to unity, following a similar approach as for the cumulative power, the mean frequency of the transverse sidebands (i.e. its "centre of mass") is equal to

$$\begin{aligned} \mu_\pm &= \int_{-\infty}^\infty \omega\tilde{P}^\pm(\omega)d\omega \\ &= \omega_0(n \pm Q_{F_0} \pm 2\alpha_x\epsilon_x \pm \alpha_{xy}\epsilon_y), \end{aligned}$$

where we used the equality $\sum_p pJ_p^2(x) = 0$. The centre of mass of each sideband will be shifted in opposite directions by $\omega_0(2\alpha_x\epsilon_x + \alpha_{xy}\epsilon_y)$ because of the octupolar field.

Chromaticity The variance of the sidebands can be obtained in a similar fashion, using the property of the Bessel function $\sum_p p^2 J_p^2(x) = \frac{x}{2}$. One has:

$$\begin{aligned} \sigma_\pm^2 &= \int_{-\infty}^\infty d\omega\ (\omega - \mu_\pm)^2 \tilde{P}^\pm(\omega) \\ &= \sigma_{\pm,0}^2 + \omega_0^2(2\alpha_x^2\epsilon_x^2 + \alpha_{xy}^2\epsilon_y^2), \end{aligned}$$

where $\sigma_{\pm,0}^2$ is the variance of the spectrum without octupoles. Both sidebands have an increased variance which, in principle, invalidates the chromaticity formula [3]. However, for typical LHC conditions, the increase in width is negligible and does not impact the chromaticity estimate.

SIMULATIONS

To support the theoretical results of the previous section, macroparticle simulations have been conducted with PyHEADTAIL [8, 9] for typical LHC conditions of an ion fill. The Schottky spectra have been calculated using the method presented in [10]. The relevant simulation parameters are summarised in Table 1 and different octupole currents I_{oct} have been included in the simulations, leading to amplitude detuning according to the formulas of Ref. [11].

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Figure 1 presents the simulated and theoretical spectrum for a Gaussian longitudinal profile, with a frequency range corresponding to the surrounding of the $n = 427725^{\text{th}}$ harmonic of the revolution frequency — as in the LHC Schottky Monitor. We observe an excellent agreement between the theory obtained from Eq. (9) and the simulations. The centres of gravity of the sidebands are shifted and the Bessel satellites are smeared out as the octupolar field increases.

CONCLUSION

This study highlighted the significant influence of octupole magnets on the Schottky spectra of bunched beams within circular accelerators. Through theoretical analyses and simulations, we showed how octupoles modify the spectrum via amplitude-dependent tune shifts. While their presence can modify the classical parameter extraction methods, it also diminishes coherent components, offering potential advantages. We presented how to adapt the current parameter extraction methods to account for octupole effects, ensuring accurate beam diagnostics from Schottky spectra.

Table 1: PyHEADTAIL Simulation Parameters

LHC circumference	26.659 km
Intensity	1.76×10^8 ions per bunch
Energy per ion	36.9 TeV
Ion charge	82 e
Ion mass	193.687 GeV/c^2
Normalised emittance	$\epsilon_x = \epsilon_y = 1.5\,\mu$m
Tune	$Q_x = 64.28,\ Q_y = 59.30$
Chromaticity	$Q'_x = Q'_y = 15$
Slippage factor	3.44×10^{-4}
RF harmonic	35640
RF voltage	8.22 MV
Bunch length (RMS)	$\sigma = 0.31$ ns
Number of macroparticles	100 000
Number of turns	10 000
Number of averaged spectra	200

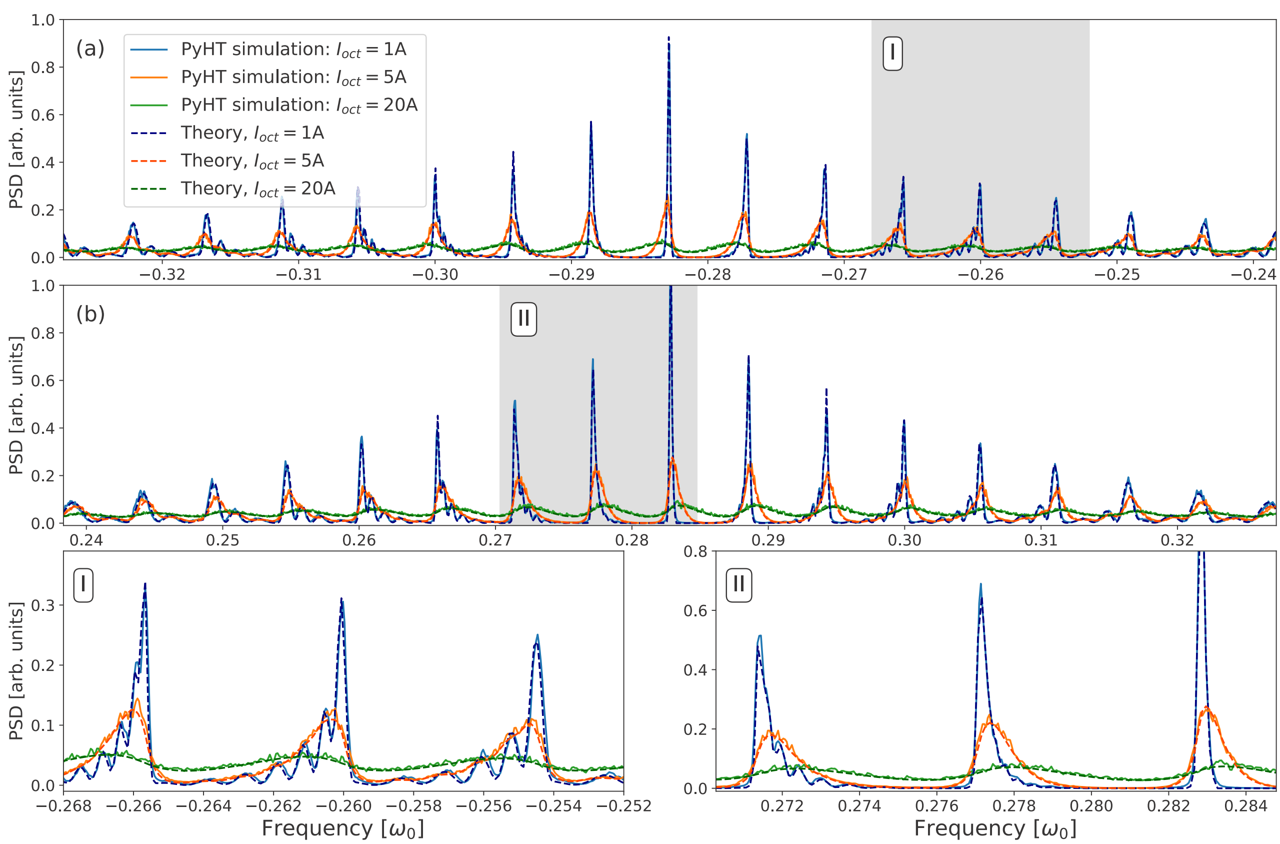

Figure 1: Comparison of simulated and theoretical Schottky spectrum obtained trough Eq. (9) of the lower (a) and upper (b) sidebands of the transverse horizontal Schottky spectrum for various octupole currents.

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REFERENCES

[1] J. Gareyte, J.-P. Koutchouk, and F. Ruggiero, "Landau damping dynamic aperture and octupole in LHC," CERN, Tech. Rep. LHC Project Report 91 (revised), 1997, revised version number 1 submitted on 2003-08-21 14:12:02. `https://cds.cern.ch/record/321824`

[2] D. Boussard, "Schottky noise and beam transfer function diagnostics," Tech. Rep. CERN-SPS-86-11-ARF, 1986, pp. 416–452. `doi:10.5170/CERN-1987-003-V-2.416`

[3] K. Lasocha, "Non-Invasive Beam Diagnostics with Schottky Signals and Cherenkov Diffraction Radiation," Presented 12 Dec 2022, Ph.D. dissertation, Jagiellonian U., 2022. `https://cds.cern.ch/record/2846538`

[4] M. Betz, O. R. Jones, T. Lefevre, and M. Wendt, "Bunched-beam Schottky monitoring in the LHC," *Nucl. Instrum. Methods Phys. Res., A*, vol. 874, pp. 113–126, 2017. `doi:10.1016/j.nima.2017.08.045`

[5] K. Lasocha and D. Alves, "Estimation of transverse bunch characteristics in the LHC using Schottky-based diagnostics," *Phys. Rev. Accel. Beams*, vol. 25, p. 062 801, 2022. `doi:10.1103/PhysRevAccelBeams.25.062801`

[6] C. Lannoy, *Statistical properties of Schottky spectra*, CERN note, in preparation.

[7] C. Lannoy, D. Alves, N. Mounet, T. Pieloni, and K. Łasocha, "Statistical Properties of Schottky Spectra," in *Proc. IBIC'23, Saskatoon, Canada*, Saskatoon, Canada, 2023, pp. 438–442. `doi:10.18429/JACoW-IBIC2023-WEP035`

[8] K. Li, A. Oeftiger, *et al.*, *PyHEADTAIL code repository*. `https://github.com/PyCOMPLETE`

[9] K. S. B. Li *et al.*, "Code development for collective effects," in *Proceedings of ICFA Advanced Beam Dynamics Workshop on High-Intensity and High-Brightness Hadron Beams (HB'16), Malmö, Sweden*, 2016, pp. 362–367. `doi:10.18429/JACoW-HB2016-WEAM3X01`

[10] C. Lannoy, K. Lasocha, T. Pieloni, D. Alves, and N. Mounet, "Impact of beam-coupling impedance on the Schottky spectrum of bunched beam," *J. Instrum.*, vol. 19, no. 03, P03017, 2024. `doi:10.1088/1748-0221/19/03/P03017`

[11] N. Mounet, "The LHC Transverse Coupled-Bunch Instability," Presented Jan 2012, Ph.D. dissertation, 2012. `doi:10.5075/epfl-thesis-5305`