ISBN: 978-3-95450-247-9 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2024-THPC56

SIMULATION OF BEAM LOADING COMPENSATION WITH RF-TRACK

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Abstract

The beam loading effect results in a gradient reduction of accelerating structures due to the excitation of the fundamental mode when the beam travels through the cavity. A recent implementation of this process in the tracking code RF-Track allows the simulation of realistic scenarios, thus revealing the impact of this phenomenon in start-to-end accelerator designs. In this paper, we present the latest update of the beam loading module which allows the simulation of the compensation of this effect.

INTRODUCTION

When a charged particle travels through a conducting cavity, currents arise upon its surface and electromagnetic (EM) fields are created inside of the structure [1]. Accelerating cavities require the presence of the so-called fundamental mode which exhibits a longitudinal electric field which is responsible of the acceleration of the particles. These same charged particles themselves interact with the cavity, exciting the fundamental mode in a decelerating phase and reducing the accelerating gradient available for the following bunches. This phenomenon is known as beam loading (BL) [2].

Based on a power-diffusive model [3], this effect has been implemented in version 2.2.0 of the tracking code RF-Track [4] for both travelling-wave (TW) structures and standing-wave (SW) structures. The expressions that describe gradient reduction in TW and SW respectively are:

$$-\frac{\partial G}{\partial t} = v_g \frac{\partial G}{\partial z} + \left(\frac{\partial v_g}{\partial z} - \frac{v_g}{\frac{r}{Q}} \frac{\partial \frac{r}{Q}}{\partial z} + \frac{\omega}{Q} \right) \frac{G}{2} + \frac{\omega \frac{r}{Q} \mathcal{T} \tilde{I}}{2}, \quad (1)$$

$$-\frac{\partial G}{\partial t} = \frac{\omega G}{2Q_l} - \frac{\omega G_{\text{end}}}{2Q_l} + \frac{\omega \frac{r}{Q} \mathcal{T} \tilde{I}}{2}, \quad (2)$$

where G is the gradient, ω the angular frequency, v_g the group velocity of the cavity, Q its unloaded quality factor, Q_l the loaded quality factor, $\frac{r}{Q}$ the normalized shunt impedance per unit length, $\mathcal{T}$ the time-transit factor and $\tilde{I}$ defined as:

$$\tilde{I} = \beta_z \frac{q_{\text{bunch}}}{T} F, \quad (3)$$

with F being the form-factor of the bunch, q_{bunch} its total charge, β_z its average longitudinal velocity (normalized by the speed of light c) and T the RF period.

Equations (1) and (2) describe how the gradient builds up and dissipates in a structure depending on cell-to-cell power flow (in TW structures), ohmic dissipation, the port influence (which sets up an initial unloaded gradient of G_{end} in SW structures) and the presence of the beam in the cavity.

The standard techniques for BL compensation during operation rely on an understanding of the temporal dynamics of the EM fields described by Eqs (1) and (2). Indeed, an optimal choice of the initial and boundary conditions as well as the injection time of the bunches counteracts the beam-induced contribution in Eqs. (1) and (2), as will be shown in the following sections.

COMPENSATION IN SW STRUCTURES

The temporal build-up of EM fields in SW cavities depends on the filling time, defined as:

$$t_{\text{fill}} = \frac{2Q_l}{\omega}. \quad (4)$$

The solution of Eq. (2) for the unloaded case ($\tilde{I} = 0$) when $t \gg t_{\text{fill}}$ stabilizes at $G \simeq G_{\text{end}}$. The gradient reaches its maximum value G_{end}, as shown in Fig. 1. If a train of electron bunches is injected at a time t_{inj}, a beam-induced field builds up and the total gradient diminishes. Figure 1 shows these cases for the CLEAR electron gun, a 2.6 cell S-band structure [5] described in Tab 1.

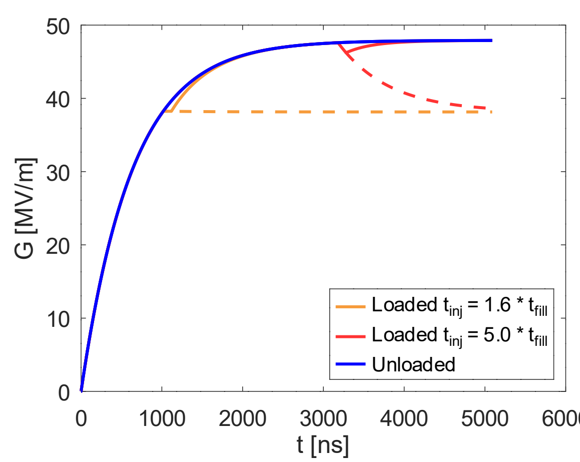

Figure 1: Gradient build-up in a SW photoinjector with $G_{\text{end}} = 48.0$ MV/m. The loaded case corresponds to a train of 150 electron bunches with $q_{\text{bunch}} = -300$ pC with a bunch spacing of 0.67 ns. The dashed lines represent the gradient reduction caused by an infinte train.

As can be observed, one can find an optimal injection time t^*_{inj} where the beam-induced gradient reduction compensates the gradient rise in the early stages of the filling of the cavity.

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To account for this effect, the resolution of Eq. (2) has been extended in RF-Track for $t < t_{\text{inj}}$ so that the previous behavior is captured. Therefore, the new implementation requires the user to specify the injection time of the beam t_{inj}.

Figure 2 shows the comparison between the long-range longitudinal phase-space of a train of 50 electron bunches injected with $q_{\text{bunch}} = -300$ pC in the SW structure at a time $t^*_{\text{inj}} = 1.6t_{\text{fill}}$ and a train injected at $t_{\text{inj}} = 5t_{\text{fill}}$. It shows that BL can be compensated by choosing an early injection time t^*_{inj} at the expense of not benefiting from the maximum energy gain.

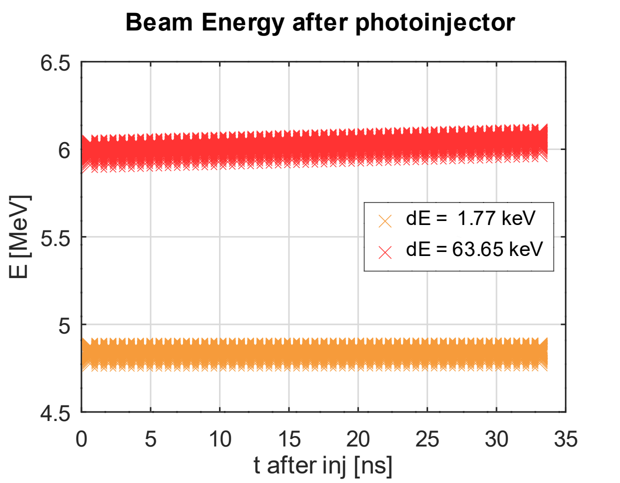

Figure 2: Longitudinal phase-space of two 50-bunch electron beams ($q_{\text{bunch}} = -300$ pC) injected at $t_{\text{inj}} = 5.0t_{\text{fill}}$ (red) and $t_{\text{inj}} = 1.6t_{\text{fill}}$ (orange) simulated with RF-Track. (The x-axis has been shifted for the comparison).

COMPENSATION IN TW STRUCTURES

The idea behind BL compensation in TW structures is similar to the compensation in SW structures: to inject the beam into the structure in a way that the beam-induced gradient compensates the unloaded field build-up. For TW structures, the filling time is defined as:

$$t_{\text{fill}} = \int_0^L \frac{\mathrm{d}z}{v_g(z)}. \tag{5}$$

The creation and propagation of the fundamental accelerating mode in the structure, Eq. (1) for the unloaded case ($\tilde{I} = 0$), should be revised for the initial condition $G(z,t) = 0,\ z \in (0, L)$. This requires a spatial initial condition for the gradient of the first cell, which depends on the input power P_{input} provided by the coupling cell [6] as:

$$G(z=0,t) = \sqrt{\frac{\omega \frac{r}{Q}(0) P_{\text{input}}(t)}{v_g(0)}},\quad t \in [0,\infty). \tag{6}$$

The case of a CLIC TW accelerating structure (CLIC AS) [6,7], whose characteristics are specified in Table 1, is shown in Fig. 3. This structure is ramped-up from 0 to a desired value $P_{\text{input}}(t^*) = P_{\text{end},0}$ at time t^*, which gives an accelerating gradient in the first cell of $G(0,t^*) = G_{\text{end},0}$. Then, for $t \in [t^*, t^* + t_{\text{fill}})$, this final power configuration propagates through all the structure until a steady unloaded gradient is reached. Usually, single-bunch beams are injected at $t > t^* + t_{\text{fill}}$ so that they can benefit from the maximum energy gain.

Table 1: Accelerating Structures Specifications [5–7]

	Unit	SW Photoinj	CLIC AS	Empty cav.
r/Q	[kΩ/m]	3.40	12.1	2.29
Q_l	[-]	6000	12000 (unl)	7200 (unl)
f	[GHz]	3.00	12.00	12.00
v_g	[% c]	-	1.75	-

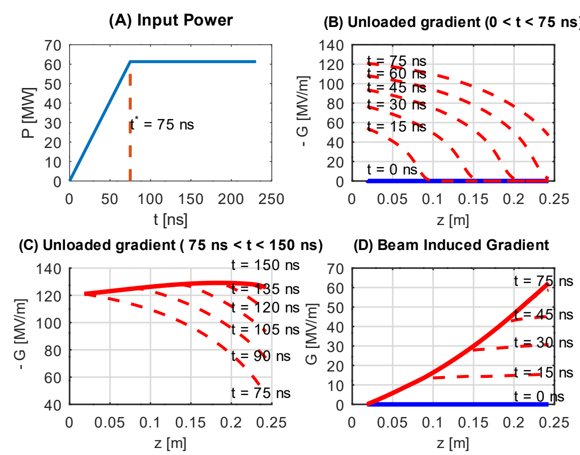

Figure 3: Transient build-up of the gradient in a CLIC AS with $t_{\text{fill}} = 67$ ns. (A) Ramp-up input power until $P_{\text{end},0} = 61$ MW. (B) Unloaded gradient rise until $G_{\text{end},0} = 121$ MV/m is achieved at $t^* = 75$ ns. (C) Unloaded gradient rise until the steady state. (D) Beam-induced field for $q_{\text{bunch}} = -600$ pC, $N_{\text{bunches}} = 312$ and bunch-spacing 0.5 ns.

To compensate BL in TW structures, RF-Track's BL implementation has been modified to reproduce the unloaded gradient build-up for arbitrary P_{input} profiles by expanding the resolution of Eq. (1) to times prior to injection. Therefore, the user now has the possibility to provide a one-dimensional mesh for P_{input} that RF-Track can cubic-interpolate, as well as the injection time of the beam with respect to the origin of P_{input}.

Figure 4 shows the comparison between the compensated and uncompensated scenario for the case where P_{input} follows the profile described in Fig. 3A. In both cases, a beam of 312 electron bunches with $q_{\text{bunch}} = -600$ nC and bunch-spacing 0.5 ns is injected with an average initial energy per bunch of 90 MeV .

In one case, injection occurs at $t_{\text{inj}} = 150$ ns where the unloaded gradient had reached its maximum value (typical single-bunch operation choice), which leads to an energy spread of 5.97 MeV from the first-to-last bunch (see Fig. 4C).

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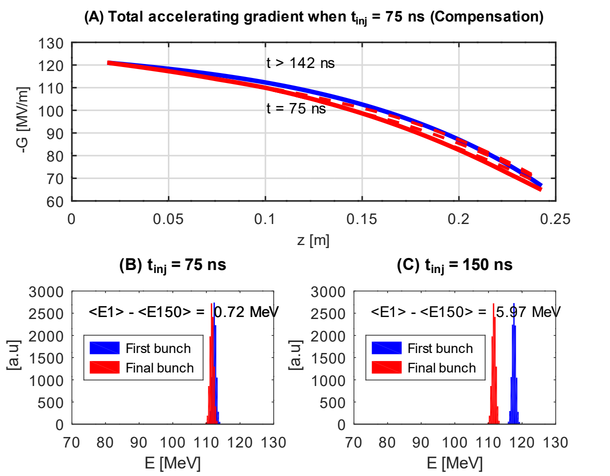

Figure 4: Case of 312 electron bunches with $\langle I \rangle = -1.2$ A and $\langle E_0 \rangle = 90$ MeV/bunch.

In the other case, injection occurs at $t^*_{\text{inj}} = 75$ ns, so that the beam-induced gradient reduction compensates the transient build-up of the unloaded gradient (see Fig. 4A), resulting in an energy spread of only 0.72 MeV from the first-to-last bunch (see Fig. 4B).

MULTISPECIES BL COMPENSATION

In some specific scenarios, like positron sources, one can find mixed-species bunches where particles of different type propagate through the structure. According to the fundamental theorem of BL [8], each particle excites the fundamental mode in such a way that it gets decelerated and deposits energy in the cavity it has passed through. Therefore, two bunches with opposite charges will excite the fundamental mode with opposite phase.

To allow the simulation of BL effects in multi-species bunches, RF-Track's BL module has recently been modified to take into account this particle-dependent phase of the BL field.

Figure 5 shows the longitudinal phase-space of a 20-bunch-beam of interleaved electrons and positron bunches, with $|q_{\text{bunch}}| = 8.42$ nC and different bunch-spacing when it goes through an empty cavity (specified in Table 1) with an initial energy of 2.4 GeV.

Figure 5 shows that bunches of the same charge but opposite sign can cancel each other's BL effect if a time separation of nT ($n \in \mathbb{N}$) is achieved, while a separation of $(2n+1)\frac{T}{2}$ results in a cumulative transient effect bunch-to-bunch. For the BL compensated case, the same-species bunch-to-bunch energy spread is cancelled but because of the fundamental theorem of BL there is a charge-dependent energy loss/gain.

CONCLUSIONS

Studying the initial and boundary conditions of the power-diffusive partial differential equations accounting for the beam loading effect, compensation techniques for both standing and travelling wave structures have been discussed. The

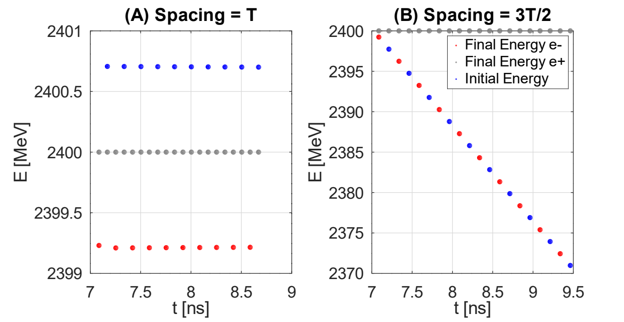

Figure 5: BL compensation for electron (blue) and positron (red) bunches with $|q_{\text{bunch}}| = 8.42$ nC after 10 Empty Cavities as in Table 1. (A) Case where the bunches are spaced an RF-period apart. (B) Case where the bunches are spaced 1.5 RF periods apart.

already-exisiting BL module in RF-Track (v. 2.2.0) has been modified to solve these equations prior to injection and reproduce the unloaded build-up of the gradient in this scenario. In addition, the code's capability has been extended to accept an arbitrary bunch profile as user input, enabling the implementation and test of non-trivial BL compensation schemes in TW structures.

This allows RF-Track to include RF considerations while performing beam dynamics simulations, therefore enabling the simulation of realistic scenarios such as those for the CLIC accelerating structure.

In addition, the BL module has been expanded to consider this effect for multi-species beams. These three modifications open the door to the simulation of complex scenarios such as start-to-end industrial compact linacs, where the BL effect has a considerable impact, or simulations of the CLIC positron source where both electrons and positrons could be transported together.

ACKNOWLEDGEMENTS

The authors would like to thank Dr. John Patrick Farmer and Dr. Walter Wuensch for the useful discussions on the BL effect.

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