# CMS Physics Analysis Summary

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## The strong coupling constant and its running from inclusive jet production at CMS

## The CMS Collaboration

#### **Abstract**

A comprehensive analysis at next-to-next-to-leading order (NNLO) accuracy in quantum chromodynamics (QCD) is presented. It uses double-differential cross section measurements of inclusive jet production from the CMS Collaboration at centreof-mass energies of 2.76, 7, 8, and 13 TeV. The result is a simultaneous determination of the value of the strong coupling constant at the mass of the Z boson *α*s (*M*Z) together with the parton distribution functions of the proton. The value  $\alpha_s(M_Z) = 0.1176^{+0.0014}_{-0.0016}$  is obtained. By using the measurements in different intervals of jet transverse momenta, the running of the strong coupling is probed for energy scales between 100 and 1600 GeV.

#### **1. Introduction 1**

#### **1 Introduction**

Collimated sprays of particles, conventionally called jets, are abundantly produced in high energy proton-proton (pp) collisions at the CERN LHC. At sufficiently high transverse momenta  $(p_{\rm T})$ , jet production is described by quantum chromodynamics (QCD) using perturbative techniques (pQCD), which have now reached next-to-next-to-leading order (NNLO) accuracy.

Description of jet production in hadronic collisions depends on the value of the strong coupling constant *α*<sub>S</sub> and an accurate understanding of the proton structure, encoded in the parton distribution functions (PDFs). Precise measurements of jet production provide a means to determine  $\alpha_S$  at the mass of the Z boson  $\alpha_S(m_Z)$  and the PDFs up to high values of *x*, which is the proton momentum fraction carried by the parton.

Inclusive jet production  $pp \rightarrow jet + X$ , refers to events with at least one jet with  $p_T$  above a certain threshold in the final state and is a key process to test pQCD calculations up to the highest accessible energy scales. In pp collisions at the LHC, inclusive jet production has been extensively studied by the CMS [\[1–](#page-13-0)[6\]](#page-13-1) and ATLAS [\[7](#page-13-2)[–12\]](#page-13-3) Collaborations at several centre-ofmass energies over a wide kinematic range.

This note presents results on PDFs and  $\alpha_S(m_Z)$  obtained simultaneously from a comprehensive QCD analysis at NNLO, where the CMS measurements of inclusive jet production in pp sive QCD analysis at NNLO, where the CMS measurements of inclusive jet production in pp collisions at  $\sqrt{s}$  = 2.76 [\[1\]](#page-13-0), 7 [\[2,](#page-13-4) [3\]](#page-13-5), 8 [\[4\]](#page-13-6) and 13 TeV [\[5\]](#page-13-7) are used together with cross section measurements of deep-inelastic scattering (DIS) at the DESY HERA [\[13\]](#page-14-0). In such a simultaneous extraction, the correlation between the PDFs and  $\alpha_{\rm S}(m_Z)$  is mitigated [\[14\]](#page-14-1) to allow for an unbiased determination. Furthermore, the running of  $\alpha_S$  as a function of the energy scale is demonstrated at NNLO up to 1.6 TeV.

### <span id="page-2-1"></span>**2 Inclusive jet measurements**

In this note, four CMS measurements of the double-differential inclusive jet cross section as a function of the individual jet  $p<sub>T</sub>$  and absolute rapidity |*y*| are considered. These measurements use jets clustered with the anti- $k<sub>T</sub>$  [\[15\]](#page-14-2) algorithm as implemented in the FASTJET [\[16\]](#page-14-3) program with a distance parameter of  $R = 0.7$ . For each of the measurements, the corresponding integrated luminosity  $\mathcal{L}$ , the number of data points  $N_{\text{dv}}$ , and the ranges in  $p_{\text{T}}$  and |*y*| are summarised in Tab. [1.](#page-2-0)

The primary source of experimental systematic uncertainty in jet measurements stems from the calibration of the jet energy scale (JES) and the jet energy resolution (JER). These calibrations and their associated uncertainties are determined using test beam data, Monte Carlo (MC) simulations, and samples of dijet, Z+jet, *γ*+jet and multijet production [\[17,](#page-14-4) [18\]](#page-14-5). The correlation of these uncertainties across the data points within an individual measurements and among

<span id="page-2-0"></span>Table 1: The CMS inclusive jet measurements considered in this study. The columns show the  $\sqrt{s}$ , the integrated luminosity, the number of measured data points, the ranges of individual jet  $p_T$  and  $|y|$ , and the reference.



the measurements at different <sup>√</sup> *s* were examined and are accounted for in the QCD analysis, as described in the following.

The JES uncertainties are subsumed into eight main categories.

- Uncertainties related to the absolute scale are associated with the JES correction within the barrel region, corresponding to pseudorapidities |*η*| < 1.3. This correction is obtained from a global fit using Z+jet, *γ*+jet, and multijet data, where extrapolations to higher  $p<sub>T</sub>$  are performed by using MC simulations (*AbsoluteScale*). It includes a correction for initial-state and final-state radiation (ISR and FSR) (*AbsoluteMPFBias*). Additional uncertainties are introduced to address response differences from different MC event generators (*Fragmentation*) and the single-particle response in the electromagnetic calorimeter (ECAL) and in the hadronic calorimeter (HCAL) (*SinglePionECAL, SinglePionHCAL*).
- Relative JER uncertainties represent the *η*-dependent uncertainty from the JER corrections (*RelativeJER*).
- The relative *η* correction of the JES calibration results in a uniform detector response across different detector regions. It includes a log-linear  $p_T$  dependence. In addition, an uncertainty is included, associated with the procedure to derive this correction in different regions of the detector (*RelativePt*).
- Other relative contributions, such as *RelativeFSR*, correspond to uncertainties in the *η*-dependent corrections for ISR and FSR, and are summarised in an additional uncertainty.
- The statistical uncertainties in the determination of the relative corrections in different regions of *η* for different uncertainty sources are merged into one uncertainty (*RelativeStat*).
- Jet-flavour dependent uncertainties (*FlavourQCD*) encompass differences between data and MC simulations when applied to various jet flavour mixtures. The uncertainty is based on response differences to jets from uds/c/b-quarks and gluons between PYTHIA and HERWIG.
- Two time-dependent uncertainties address the JES variation over time during the data taking periods (*TimePt* and *TimeEta*).
- Uncertainties associated to the corrections for additional pp interactions within the same or nearby bunch crossings (pileup) are considered as well.

A detailed description of the JES calibration and the corresponding uncertainties is given in Refs. [\[17,](#page-14-4) [18\]](#page-14-5). Further uncertainties specific to each individual measurement are discussed in the original publications.

The following correlation scheme for JES-related uncertainties across the different CMS mea-The following correlation scheme for jeb-related uncertainties across the different Civib measurements is used in this analysis. For the measurement at  $\sqrt{s}$  = 7 TeV [\[2\]](#page-13-4), the correlation scheme for JES-related uncertainties is used as described in Ref. [\[3\]](#page-13-5). In particular, the sources *SinglePionECAL* and *SinglePionHCAL* are decorrelated as a function of *η*. The uncertainties in pileup, the relative statistical contribution, and the time-dependent sources are considered uncorrelated between individual measurements. The *SinglePion* uncertainties in the 7 TeV data set are treated following the correlation scheme introduced in Ref. [\[3\]](#page-13-5). The JER uncertainties are treated as uncorrelated between the individual measurements. The correlations in the remaining JES uncertainties between different data sets are implemented as follows.

• The uncertainty *AbsoluteMPFBias* is correlated between the 2.76, 8, and 13 TeV mea-

#### **3. Theoretical predictions 3**

surements. This uncertainty has not been considered for the 7 TeV measurement.

- The uncertainties in *AbsoluteScale*, *SinglePionECAL*, *SinglePionHCAL*, and *RelativeJER* are correlated between the 2.76 and 8 TeV measurements, because the corresponding corrections were obtained using 8 TeV data and applied to both measurements. These uncertainties are uncorrelated between the other data sets for which the corrections have been derived specifically.
- The *RelativeFSR* and *Fragmentation* uncertainties are fully correlated between the 2.76, 7, and 8 TeV measurements, but uncorrelated with the 13 TeV measurement, because of the use of different MC tunes and event generators for Run 1 and Run 2 data.
- The *RelativePt* uncertainties are uncorrelated between the individual measurements, since these reflect differences between linear and logarithmic fits in the  $p_T$  extrapolation of the residual corrections, different for the different years.
- The *FlavorQCD* uncertainty is treated correlated between the 7 and 8 TeV measurements, but uncorrelated with the 2.76 and 13 TeV measurements.

Different alternative correlation scenarios for the JES uncertainties were investigated and no significant impact on the results was observed, as quantified in Section [4.](#page-5-0)

While the JER uncertainties are uncorrelated across the measurements at different <sup>√</sup> *s*, the correlations of these uncertainties within each individual measurement have been investigated. In particular, the JER uncertainty is treated uncorrelated in |*y*| for the measurements at 2.76, 7, and 8 TeV, because the residual |*y*| dependence of these data sets was not fully accounted for. An additional uncorrelated uncertainty of 1.5% is assigned to the JER uncertainty in the 2.76 TeV measurement to take into account a statistical contribution to the JER uncertainty. Alternative JER uncertainty correlation scenarios are tested and lead to insignificant changes in the results.

### **3 Theoretical predictions**

The NNLO pQCD calculations used in this study are performed assuming five active massless quark flavours in the leading-colour (LC) and leading-flavour-number approximation using the NNLOJET program [\[19\]](#page-14-6). The subleading colour contributions for the NNLO corrections, neglected in Ref. [\[19\]](#page-14-6), have been recently calculated [\[20\]](#page-14-7) and have a very small impact for inclusive jet production with  $R = 0.7$ . The renormalisation  $\mu_{\rm r}$  and factorisation  $\mu_{\rm f}$  scales in the calculation are set to the individual jet  $p<sub>T</sub>$ , following the studies in Ref. [\[21\]](#page-14-8). The cross section predictions for bins in  $p_T$  and |*y*| of the respective measurements are stored in the form of interpolation tables in the APPLFAST [\[22\]](#page-14-9) format, allowing for a fast evaluation of the predictions under variations of *α*<sub>S</sub>, PDFs, or the QCD scales. The numerical uncertainty in the grids is below 1% in most bins and increases to about 5% in the forward region at high  $p_{\rm T}$ , remaining significantly smaller than the statistical uncertainties in the measurements. However, in the forward region at very high  $p_{\text{T}}$ , a few bins have grid uncertainties exceeding  $10\%$ , which are excluded from the QCD interpretation. As recommended by the authors of NNLOJET, the statistical uncertainties of the theory predictions are multiplied by a factor of two.

The QCD predictions are modified with a multiplicative correction for contributions of the dominant electroweak (EW) effects. These include Sudakov-type and other high-energy logarithms resulting from the virtual exchange of soft or collinear massive weak gauge bosons, calculated at next-to-leading order [\[23\]](#page-14-10). The contribution of EW effects becomes important at large jet  $p_{\rm T}$ , reaching 11% at the highest  $p_{\rm T}$  probed in the 13 TeV measurement. Subleading EW

effects from the emissions of a real weak gauge bosons have been estimated in the 13 TeV measurement to be smaller than 1% at high  $p<sub>T</sub>$  and are neglected. The EW corrections for the 13 TeV measurement were taken from the original publication. For the 7 and 8 TeV measurements, these corrections have been updated by using more recent PDF sets. The EW corrections are omitted for the 2.76 TeV measurement because of their negligible impact for  $p_{\text{T}}$  smaller than 600 GeV.

To compare the fixed-order predictions to the measured particle-level cross sections, the predictions are corrected for the non-perturbative (NP) effects from hadronisation and the underlying event. These NP corrections have been derived as ratios from event generators with parton showers, in which NP effects are switched on and off. The NP corrections are applied through  $p_T$ - and |y|-dependent correction factors as provided in the original publications of the individual measurements. These use the generator tunes matching best the conditions of the corresponding measurements. The values of NP corrections vary between  $5-20\%$  at low  $p_T$ and decrease with increasing  $p_{\text{T}}$ . The associated uncertainties range from 2 to 4% at low  $p_{\text{T}}$ , depending on  $|y|$ , and are negligible at high  $p<sub>T</sub>$ . Despite slight differences in the way the NP corrections were obtained for each measurement, the impact of the NP corrections in the QCD analysis is found to be insignificant considering their uncertainties. In particular, the impact of the NP corrections in the QCD analysis was studied by performing individual fits with a subset of measurements with  $p_T > 150$ , 200, and 300 GeV. This way, the kinematic ranges with decreasing importance of NP corrections are consequently probed. The difference in the results is found to be negligible. The main result is based on the full  $p_{\rm T}$  spectra probed by the CMS jet data.

## <span id="page-5-0"></span>**4 QCD analysis**

In the QCD analysis, the four CMS measurements of the double-differential cross sections of inclusive jet production are used together with the combined neutral current (NC) and charged current (CC) ep DIS cross sections measured at the HERA collider at DESY [\[13\]](#page-14-0).

Theoretical predictions for the DIS cross sections are calculated at fixed order in QCD at NNLO accuracy using the QCDNUM code [\[24\]](#page-14-11), with  $\mu_{\rm r}$  and  $\mu_{\rm f}$  set to the squared four-momentum transfer  $Q^2$ . The contribution of massive c and b quarks in the DIS cross sections is treated in the Thorne-Roberts general-mass variable-flavour number scheme [\[25](#page-14-12)[–27\]](#page-14-13). The masses of c and b quarks are set as  $m_c = 1.47 \pm 0.06$  GeV and  $m_b = 4.5 \pm 0.25$  GeV, respectively. The fraction of the strange quark in the sea  $f_s = x\overline{s}/(x d + x\overline{s})$  is assumed to be  $0.4 \pm 0.08$ . The low-*Q*<sup>2</sup> region in DIS, where resummation effects from small-*x* logarithms become important, is removed from the fit by using only the DIS measurements with  $Q^2 > 10 \,\text{GeV}^2$ . This cut is varied by  $2.5 \text{ GeV}^2$  up and down. The published DIS cross sections are already corrected by the dominant EW effects.

The PDFs for the gluon, valence u- and d-quark, and  $\overline{u}$  and  $\overline{d}$  quark densities are parametrized at a starting (evolution) scale of  $Q_0^2 = 1.9 \,\text{GeV}^2$ , varied by  $0.2 \,\text{GeV}^2$ . The  $Q^2$  dependence of the PDFs and of α<sub>s</sub> is obtained by solving the DGLAP evolution equations at NNLO in pQCD as implemented in QCDNUM. The functional form of the PDFs for each parton *i* at the starting scale is taken to be

$$
xq_i(x) = A_i x^{B_i} (1 - x)^{C_i} P_i(x)
$$
\n(1)

where *A* is a normalization parameter and  $P_i(x) = (1 + D_i x + E_i x^2)$  is a polynomial that interpolates between the small- and large-*x* behaviour given by the *B* and *C* parameters. The *D* and *E* terms in the polynomial expansion are only included following the results of an op<span id="page-6-1"></span>Table 2: The values of  $\chi^2$  per number of degrees of freedom,  $N_{\rm dof}$ , as obtained in the fit to HERA and CMS jet data. The partial  $\chi^2$  is reported for each of the individual datasets considered, together with the contribution to the  $\chi^2$  from correlated uncertainty sources.



timization procedure. The parameters  $A_{\mathbf{u}_v}$  and  $A_{\mathbf{d}_v}$  are determined during the fit from the valence-quark number sum rule, while the value of *A*<sup>g</sup> is obtained from the momentum sum rule. In the present work, the parametrization of the most recent NNLO QCD analysis [\[5\]](#page-13-7) of the CMS measurement of inclusive jet production at 13 TeV is used. Further parameters are added, alternatively, to estimate the parametrization uncertainty.

The fit is performed using the XFITTER [\[28,](#page-14-14) [29\]](#page-15-0) program. The values of the PDF parameters and of  $\alpha_S$  are obtained in the minimization procedure using MINUIT [\[30\]](#page-15-1) with a  $\chi^2$  goodness-of-fit (GoF) function that includes experimental and theoretical (PDF) uncertainties,

$$
\chi^{2}(\mathbf{b}_{exp}, \mathbf{b}_{th}) = \sum_{i=1}^{N_{data}} \frac{\left[D_{i} - T_{i}(1 - \sum_{j} \gamma_{ij}^{exp} b_{j,exp})\right]^{2}}{\delta_{i,uncor}^{2} T_{i}^{2} + \delta_{i,stat}^{2} D_{i} T_{i}} + \sum_{i}^{N_{data}} \log \frac{\delta_{i,uncor}^{2} T_{i}^{2} + \delta_{i,stat}^{2} D_{i} T_{i}}{\delta_{i,uncor}^{2} D_{i}^{2} + \delta_{i,stat}^{2} D_{i}^{2}} + \sum_{j=1}^{N_{exp,sys}} b_{j,exp}^{2}.
$$
\n(2)

<span id="page-6-0"></span>Here,  $D_i$ ,  $T_i$  are the measurements and theory predictions, respectively. Further,  $\delta_{i, \text{stat}}$ ,  $\delta_{i, \text{uncor}}$ denote relative statistical and uncorrelated systematic uncertainties. The correlated uncertainty sources are accounted for by the  $b_{exp}$  set of parameters, with the impact of each source on the theory point  $\sigma_i^{\text{exp}}$ <sup>exp</sup> described by the matrices  $\gamma_{ij}^{\text{exp}}$ . Asymmetric uncertainties in the PDF parameters are obtained from a  $\Delta \chi^2 = 1$  criterion using an iterative Hessian approach, following the prescription of Ref. [\[31\]](#page-15-2). The uncertainties quoted in the following, correspond to 68% confidence level (CL).

#### **5 Results**

In Fig. [1,](#page-7-0) the measurements are shown divided by the NNLO LC QCD predictions, corrected for the NP and EW effects. In general, a very good agreement between the theory predictions and the data is observed. The consistency of all the experimental data used is quantified by the goodness of the fit  $\chi^2$  from Eq. [\(2\)](#page-6-0), presented in Tab. [2.](#page-6-1) The total  $\chi^2$  per number of data points *N*<sub>dp</sub> is 1680/1453. A somewhat higher  $\chi^2/N_{\text{dp}}$  for the HERA DIS data is investigated in detail in the original work [\[13\]](#page-14-0). The partial  $\chi^2/N_{\rm dp}$  of 427/453 is obtained for the CMS jet measurements.

In Fig. [2,](#page-8-0) the impact of the CMS jet data on the PDF extraction (*HERA+CMS* fit) is illustrated, considering the Hessian fit uncertainties. The fractional uncertainties in the PDFs resulting

<span id="page-7-0"></span>

Figure 1: Cross sections of inclusive jet production in pp collisions as measured by CMS (black Figure 1: Cross sections of inclusive jet production in pp collisions as measured by CM5 (black markers) at  $\sqrt{s}$  of 2.76, 7, 8 and 13 TeV, using the anti- $k_T$  clustering algorithm with *R* = 0.7, as a function of individual jet  $p_{\text{T}}$  in bins of absolute rapidity  $|y|$ . Shown is the total uncertainty of every data point (vertical bar). The data are divided by the NNLO LC QCD prediction corrected by the NP and EW effects. The PDFs obtained in this note are used. The total theory uncertainty (shaded band) includes the PDF and scale variation uncertainty.

<span id="page-8-0"></span>

Figure 2: The fractional Hessian uncertainties in the u-valence  $u_v$  (upper left), d-valence  $d_v$ (upper right), gluon g (lower left), and sea quark  $\Sigma$  (lower right) distributions, shown as a function of *x* at the factorisation scale  $Q^2 = m_t^2$ . The results of the HERA+CMS fit (blue hatched area) are compared to the results of the HERA-only fit using  $\alpha_{\rm S}(m_Z) = 0.118$  (orange hatched area). The uncertainties are given at 68% CL.

from the present QCD analysis are shown in comparison to the results of an alternative fit which includes only HERA DIS data (*HERA-only* fit). In the HERA-only fit, due to poor sensitivity of the DIS data to  $\alpha_{S}(m_Z)$ , its value is fixed to the one of the HERA+CMS fit. The PDFs are shown at the factorization scale  $Q^2 = m_t^2$ , where  $m_t$  is the top quark mass. Significant improvement of the uncertainties in all PDFs is observed, once CMS jet measurements are considered.

In Fig. [3,](#page-9-0) the PDFs obtained in the HERA+CMS fit are compared to those obtained by the global fitting groups HERAPDF20 [\[13\]](#page-14-0), NNPDF40 [\[33\]](#page-15-3), CT18NNLO [\[32\]](#page-15-4) and MSHT20 [\[34\]](#page-15-5). Note that these global PDFs are based on inclusive HERA DIS data and partially include the CMS jet measurements considered in the present note, except for the HERAPDF20 PDF set. By adding the CMS jet data, the valence PDF, in particular the d valence distribution, is observed to shift with respect to HERAPDF20 towards the results of the global PDFs, in particular NNPDF40 and MSHT20.

In the HERA+CMS fit, together with the PDFs, the value of the strong coupling at the  $m_Z$  is obtained as  $\alpha_{\rm S}(m_Z) = 0.1176 \pm 0.0009$ , where the uncertainty denotes the Hessian fit uncertainties.

<span id="page-9-0"></span>

Figure 3: Ratios of different global PDF sets at NNLO, in particular CT18 [\[32\]](#page-15-4), NNPDF4 [\[33\]](#page-15-3), MSHT20 [\[34\]](#page-15-5), and HERAPDF20 [\[13\]](#page-14-0) to the result of the present study. The u-valence  ${\tt u}_v$  (upper left), d-valence d<sub>*v*</sub> (upper right), gluon g (lower left), and sea quark Σ (lower right) distributions are shown as functions of *x* at the factorisation scale  $Q^2 = m_t^2$ . The Hessian PDF uncertainties at 68% CL are shown.

Following the strategy of Ref. [\[5\]](#page-13-7), further uncertainties in PDFs and  $\alpha_{\rm S}(m_Z)$  are considered, corresponding to the variations of the fit input parameters (*model uncertainty*), the uncertainty originating from missing higher-order corrections, and the one accounting for alternative parametrization shape (*parametrization uncertainty*). For every variation, an alternative fit is performed. The contributions to the model uncertainty are added in quadrature, resulting in  $\delta \alpha_{\rm S}(m_Z)$  =  $^{+0.0006}_{-0.0004}$  (model). The uncertainty of missing higher orders in the calculation of the jet cross sections is evaluated by varying the scales  $\mu_\text{r}$  and  $\mu_\text{f}$  (*scale uncertainty*) independently by a factor of two up and down, avoiding cases with  $\mu_f/\mu_r = 4$  or 1/4. For each scale choice, an individual fit is performed, and the maximum difference to the central result is considered as an uncertainty of  $\delta \alpha_{\rm S}(m_Z) = ^{+0.0009}_{-0.0012}$  (scale). The parametrization uncertainty is estimated by extending the functional form of the PDFs by additional parameters *D* and *E*, added one at a time. An uncertainty  $\delta \alpha_{S}(m_Z) = +0.00004$  (param.) is obtained from an envelope of the results of the corresponding fits and is added linearly to the aforementioned uncertainties.

Summarising the findings above, the value of  $\alpha_{\mathsf{S}}(m_{\mathsf{Z}})$  is obtained as

<span id="page-9-1"></span>
$$
\alpha_{\rm S}(m_Z) = 0.11759^{+0.0009}_{-0.0009} \, (\text{fit})^{+0.0006}_{-0.0004} \, (\text{model})^{+0.0009}_{-0.0012} \, (\text{scale})^{+0.}_{-0.00004} \, (\text{param.}), \tag{3}
$$

corresponding to the total uncertainty of  $^{+0.0014}_{-0.0016}$  (tot). This value is in good agreement with

<span id="page-10-0"></span>

Figure 4: The value of  $\alpha_{S}(m_Z)$  obtained in this analysis (red marker), compared to a selection of recent results obtained by using different methods (black markers) with their uncertainties (horizontal error bars). Also, the PDG world average is shown (shaded band).

previous determinations from hadron colliders, particularly with the CMS result based on a subset of the same data [\[5\]](#page-13-7) and a dijet measurement at 13 TeV [\[35\]](#page-15-6). It also aligns well with the ATLAS measurement from the Sudakov peak of the Z-boson  $p_{\text{T}}$  [\[36\]](#page-15-7) and transverse energy-energy correlations in multijet events [\[37\]](#page-15-8), as well as the CDF result from the Z-boson  $p_T$ distribution [\[38\]](#page-15-9). Finally, the result agrees well with PDG category average of the global PDF fits [\[39\]](#page-15-10), with the estimate from lattice QCD [\[40\]](#page-15-11), and with the PDG world average of  $\alpha_{\rm S}(m_Z) = 0.1180 \pm 0.0009$  [\[39\]](#page-15-10), as shown in Fig. [4.](#page-10-0) Applying alternative possible scenarios for the correlations of JES uncertainties between the data sets, mentioned in Section [2,](#page-2-1) resulted in a maximum shift in  $\alpha_{\rm S}(m_Z)$  of 0.41%, considered negligible compared to the fit uncertainty of 0.8% and the total uncertainty of 1.3%.

Following the same approach, the value of  $\alpha_S$  in five different ranges of  $\mu_r = p_T$  is extracted, illustrating the running of the strong coupling,  $\alpha_{\rm S}(\mu_{\rm r})$  at NNLO. For this purpose, the CMS measurements of inclusive jet production are split into exclusive ranges of individual jet  $p_{\text{T}}$ . In each independent  $p_{\text{T}}$  range, the PDFs and  $\alpha_{\text{S}}(m_{\text{Z}})$  are extracted simultaneously. The values of  $\alpha_S(m_Z)$  obtained in each individual fit are evolved as  $\alpha_S(\mu_r)$  using the five-loop five-flavour renormalization group equation (RGE) encoded in CRUNDEC [\[41\]](#page-15-12) version 0.5.2. The same RGE is used to obtain the corresponding uncertainties. The fit, model, scale, and parametrization uncertainties are computed as in case of Eq. [\(3\)](#page-9-1). In the estimate of the scale uncertainty, only variations in  $\mu_f$  are considered, because the  $\mu_r$  dependence is probed in this study. In each  $p_T$ range,  $\mu_{\rm r}$  is calculated at NNLO QCD as a cross section-weighted average  $\langle p_{\rm T} \rangle$ . Results are summarised in Table [3](#page-11-0) and shown in Fig. [5,](#page-11-1) where the values of  $\alpha_{\rm S}(\mu_{\rm r})$  are compared to the evolution of the world average of  $α_S(m_Z)$  performed at five-loop order in QCD using CRUN-DEC [\[41\]](#page-15-12).

<span id="page-11-1"></span>

Figure 5: Values of *α*<sub>S</sub> as a function of the renormalization scale, obtained in the present QCD analysis at NNLO. The results (black markers) are shown with their total uncertainties (vertical error bars). For comparison, the RGE at five loops is shown using the current world-average value  $\alpha_{\rm S}(m_Z) = 0.1180 \pm 0.0009$  [\[39\]](#page-15-10) (red line) together with its associated total uncertainty (shaded band).

<span id="page-11-0"></span>Table 3: The extracted  $\alpha_S(m_Z)$  and the corresponding  $\alpha_S(Q)$  values for each  $p_T$  range with their total uncertainties. For  $\alpha_{S}(Q)$ , the individual uncertainty contributions (fit, scale, model, parametrization) are listed.

$p_T$ (GeV)	$ Q\rangle$	$\alpha_{\rm S}(m_Z)$ (tot)	$\alpha_{\rm S}(Q)$	(fit)	(scale)	(model)	(param.)	(tot)
$74 - 220$	103.06	$+0.0013$ $-0.0012$ 0.1182	0.1160	$+0.0011$ $-0.0011$	$+0.000018$ $-0.00005$	$+0.0006$ $-0.0004$	$+0.0$ $-0.0$	$+0.0012$ $-0.0011$
220-395	266.63	$0.1184 + 0.0011$ <sub>-0.0012</sub>	0.1019	$+0.0007$ $-0.0007$	$+0.0$ $-0.0004$	$+0.0004$ $-0.00029$	$+0.0$ $-0.000033$	$+0.0008$ $-0.0009$
395-638	464.31	$+0.0012$ 0.1179 $-0.0012$	0.0947	$+0.0007$ $-0.0007$	$+0.0$ $-0.000032$	$+0.0004$ $-0.00027$	$+0.$ $-0.000008$	$+0.0008$ $-0.0008$
638-1410	753.66	$+0.0013$ 0.1184	0.0898	$+0.0006$ $-0.0006$	$+0.0$ $-0.00004$	$+0.0004$ $-0.00026$	$+0.000005$ $-0.$	$+0.0007$ $-0.0007$
1410-3103	1600.5	$0.1170 + 0.0020$ $-0.0016$	0.0821	$+0.0007$ $-0.0007$	$+0.0004$ $-0.0$	$+0.0005$ $-0.00034$	$+0.00004$ $-0.0$	$+0.0010$ $-0.0008$

#### **6. Conclusions 11**

#### **6 Conclusions**

The value of the strong coupling constant in the minimal subtraction scheme at the scale of the mass of the Z boson,  $\alpha_{\rm S}(m_Z)$ , is extracted together with the parton distribution functions (PDFs) of the proton in an analysis at next-to-next-to-leading order (NNLO) in quantum chromodynamics (QCD). The analysis includes electroweak effects at next-to-leading order and  $\alpha$  modynamics (QCD). The analysis includes electroweak effects at hext-to-leading order and  $\gamma s = 2.76$ , non-perturbative corrections. The CMS measurements of inclusive jet production at  $\sqrt{s} = 2.76$ , 7, 8 and 13 TeV, using the anti- $k<sub>T</sub>$  clustering algorithm with a distance parameter of  $R = 0.7$ , are used together with the combined deep inelastic scattering (DIS) cross sections from HERA. The simultaneous extraction of  $\alpha_{\rm S}(m_Z)$  and the PDFs mitigates a possible bias from their correlations. By using the CMS jet data in the fit, the precision of PDFs is significantly improved and the value of  $\alpha_S(m_Z) = 0.1176^{+0.0014}_{-0.0016}$  is obtained, in good agreement with the world average. It is the most precise measurement from jet cross sections, to date. Furthermore, by extracting the value of  $\alpha_{\rm S}$  in different bins of jet  $p_{\rm T}$ , the running of  $\alpha_{\rm S}$  is tested at NNLO in QCD up to scales of 1.6 TeV. The observed running of  $\alpha_{\mathsf{S}}$  is in agreement with the prediction of the QCD renormalization group equation.

### **A JEC correlation of inclusive jet measurements**

<span id="page-12-0"></span>The JEC correlation scheme as illustrated in Fig. [6](#page-12-0) has been applied in the QCD analysis of the CMS inclusive jet measurements at <sup>√</sup> *s* = 2.76, 7, 8, and 13 TeV. A filled black box indicates that the corresponding two uncertainty sources are treated as correlated.



Figure 6: Sketch of the treatment of JES sources of uncertainty among the measurements at different <sup>√</sup> *s*. A filled black box indicates that the corresponding two uncertainty sources are treated as correlated.

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