ISBN: 978-3-95450-247-9 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2024-MOPS37

DEVELOPMENT OF NUMERICAL TOOLS FOR INTRA-BEAM SCATTERING MODELLING

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Abstract

Intra-Beam Scattering (IBS) is one of the main mechanisms of emittance blowup and performance deterioration in the Large Hadron Collider (LHC) accelerator chain. It is particularly relevant with the recent upgrades implemented across the injector complex to reach the high brightness beams of the High Luminosity LHC (HL-LHC) era. Several studies have focused on developing an accurate formalism to describe IBS. The implementation of IBS in codes such as e.g. MAD-X is widely used in the accelerator physics community. This study presents the latest developments of a Python package for IBS simulations, recently developed at CERN, meant for integration within the Xsuite ecosystem. The new capabilities of the Python code are detailed and a thorough benchmark against existing codes is presented, for various machines of the CERN accelerator complex in different configurations.

INTRODUCTION

One of the main areas of improvement for existing particle accelerators is the increase of beam intensities and reduction of transverse and longitudinal beam sizes. One of the main mechanisms of performance degradation is Intra-Beam Scattering (IBS) - the small angle binary Coulomb scattering of particles within the beam - which impacts the beam emittances and can eventually lead to beam losses. As IBS plays an important role for ion machines [1, 2], damping rings [3], light sources [4] and proton rings with long storage time [5–7], its accurate modelling and study is necessary. While various codes have been developed for this purpose - e.g. SIRE [8], IBStrack [9] or CMAD [10, 11] - none of them is modular enough to incorporate the additional beam dynamics effects required for state of the art simulation studies and each requires long simulation times. This submission presents *xibs* [12], a prototype package for IBS modelling based on previous work [13] and meant for integration in the Xsuite ecosystem [14, 15]. The focus is on the code capabilities with references for implementation details.

ANALYTICAL GROWTH RATES

The evolution of the rms emittances under the influence of IBS can be described analytically with growth rates. These growth rates depend on the machine optics and the beam parameters such as the bunch intensity, the beam energy and the bunch size. Two different formalism - which assume transverse and longitudinal Gaussian beam profiles - are available in the code for their computation.

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Nagaitsev Formalism

A first approach, based on Nagaitsev's formulae [16], provides a fast computation method of the IBS growth rates through symmetric elliptic integrals of the second kind. This method is particularly computationally efficient: only two evaluations of this integral are needed to obtain various terms from which one can compute the growth rates with simple calculations. These terms are defined in [16]. Furthermore, in this approach no term depends on the number of elements in the lattice, so the computing time is relatively constant regardless of the machine size: it is by far the fastest accurate method to compute IBS growth rates. However, the equations established by Nagaitsev do *not* take into account the vertical dispersion. For machines with vertical dispersion, such as e.g. the LHC in the presence of crossing angles, another approach is needed.

Bjorken-Mtingwa Formalism

Growth rates can also be computed according to the theory by Bjorken and Mtingwa (B&M) [17]. The implementation in *xibs* follows that of the MAD-X code [18], for which modifications to the terms of B&M's theory have been made to account for vertical dispersion [19]. In this formalism, intermediate values are computed at every element of the lattice according to Eq. (8) of [19] and then averaged over the machine to obtain the final IBS growth rates. While this method is accurate in the presence of vertical dispersion, it requires computing an integral at each element of the lattice, for each plane, which is much more computationally expensive than the Nagaitsev formalism.

Benchmarks

Table 1 shows a comparison of the growth rates computed with *xibs* to the ones obtained with the MAD-X code, for several configurations of the CERN accelerator complex, as well as the relevant computing times. One can observe a great agreement is found between all implementations, with the Nagaitsev formalism being the fastest approach. The B&M results are expectedly slower due to the element-by-element estimation of the growth rates before averaging, and see their computing time scale up with the size of the machine. The MAD-X implementation, being fully written in Fortran, remains significantly faster than *xibs*.

From the growth rates, one can compute the evolution of the rms beam emittances due to IBS from step N to step $N+1$, with dt the time step, according to [20, 21]:

$$\begin{aligned} \varepsilon_{x,y}^{N+1} &= \varepsilon_{x,y}^{N} * e^{dt/\tau_{x,y}} , \\ \sigma_{\delta,z}^{N+1} &= \sigma_{\delta,z}^{N} * e^{dt/2\tau_{z}} . \end{aligned} \tag{1}$$

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Table 1: Comparison of Growth Rates and Computing Times Obtained in MAD-X and *xibs* for Several Setups

Simulation Setup	Code	τ_x [s^{-1}]	τ_y [s^{-1}]	τ_z [s^{-1}]	Computing Time [ms]
PS Protons (Injection Energy)	MAD-X	$-1.07 \cdot 10^{-4}$	$5.03 \cdot 10^{-5}$	$3.37 \cdot 10^{-4}$	29.2
	xibs (B&M)	$-1.07 \cdot 10^{-4}$	$4.97 \cdot 10^{-5}$	$3.37 \cdot 10^{-4}$	181
	xibs (Nagaitsev)	$-1.04 \cdot 10^{-4}$	$4.67 \cdot 10^{-5}$	$3.41 \cdot 10^{-4}$	32.5
SPS Ions (Top Energy)	MAD-X	$1.47 \cdot 10^{-3}$	$-8.88 \cdot 10^{-6}$	$2.07 \cdot 10^{-3}$	75.4
	xibs (B&M)	$1.47 \cdot 10^{-3}$	$-8.85 \cdot 10^{-6}$	$2.07 \cdot 10^{-3}$	261
	xibs (Nagaitsev)	$1.47 \cdot 10^{-3}$	$-8.77 \cdot 10^{-6}$	$2.07 \cdot 10^{-3}$	33.8
LHC Ions (Top Energy, no xing)	MAD-X	$3.98 \cdot 10^{-5}$	$-8.38 \cdot 10^{-9}$	$6.14 \cdot 10^{-5}$	346
	xibs (B&M)	$3.98 \cdot 10^{-5}$	$-8.35 \cdot 10^{-9}$	$6.15 \cdot 10^{-5}$	802
	xibs (Nagaitsev)	$3.98 \cdot 10^{-5}$	$-8.32 \cdot 10^{-9}$	$6.16 \cdot 10^{-5}$	36.8

Figure 1 shows the computed evolutions for LHC protons at 6.8 TeV, $\beta^* = 30$ cm and with crossing angles (160 µrad at IP1 / IP5), which introduce vertical dispersion.

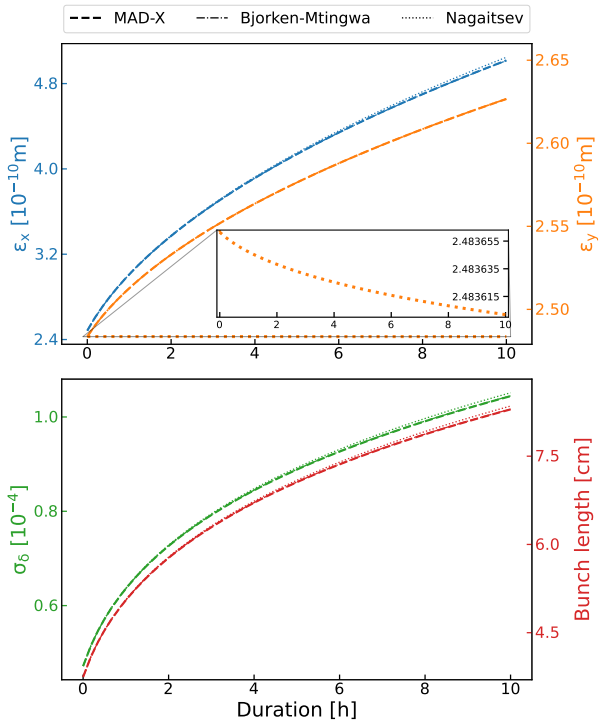

Figure 1: Analytical evolution of transverse (top) and longitudinal (bottom) emittances for LHC protons at 6.8 TeV and $\beta^* = 30$ cm, with crossing angles.

For this simulation the growth rates were re-computed every 10 min of beam time, and in between the simulated beam parameters are updated every second of beam time using Eq. (1). The complete process took 40s for MAD-X, 46s for *xibs* B&M and 3s for *xibs* Nagaitsev. The old code this work started from [13] took 13 minutes for the same task, as its computation of the rates is an order of magnitude slower.

These results showcase both the accuracy of the implemented methods as well as the shortcomings of the Nagaitsev formalism in the presence of vertical dispersion, where it inaccurately predicts a shrinkage of the vertical emittance.

Auto Re-Computing

It is possible in *xibs* to automatically re-compute the growth rates based on the evolution of rms emittances, based on a user-given threshold and reference values taken at the last growth rates update. Figure 2 shows the advantage of this method, for SPS protons at top energy and with pushed beam parameters to exacerbate the IBS effects.

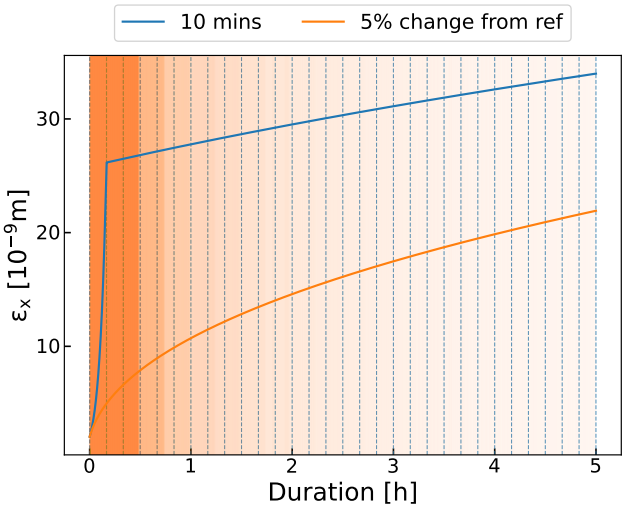

Figure 2: Evolution of the horizontal emittance for SPS protons at top energy due to IBS, with (blue) and without (orange) auto-recomputing of the growth rates. Vertical lines indicate when the growth rates were re-computed.

Thanks to the initially high frequency of IBS growth rate re-computing where the beam parameters evolve quickly and a gradually less frequent re-computing, this feature provides the most accurate results without unnecessary computing of the IBS growth rates.

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KICK-BASED IBS

While analytical models are useful to calculate the rms emittances evolution in time, a different approach is needed to study the interplay of IBS with other effects such as Space Charge or beam-beam, or simply to include IBS in tracking simulations. The *xibs* package offers classes to compute and apply IBS momenta kicks to particles used in Xsuite tracking simulations, with two formalism currently available.

Simple Kicks

Above transition energy, transverse emittances and momentum spread can grow indefinitely when considering IBS alone [22]. A first simplified IBS kick was implemented as introduced in [23], directly based on analytical growth rates. Each particle receives a kick at each turn according to:

$$\Delta p_u = \sigma_{p_u} \sqrt{2 T_{\mathrm{IBS},u}^{-1} T_{\mathrm{rev}} \sigma_z \sqrt{\pi} \rho(z)} R \,, \tag{2}$$

where u is the considered plane, R a number taken from the standard normal distribution, $T_{\mathrm{IBS},u}$ the IBS growth rate, T_{rev} the revolution period, σ_z the bunch length and σ_{p_u} the standard deviation of the bunch's momentum. Figure 3 shows the effect of such a kick on the longitudinal plane of a positron bunch in the CLIC Damping Ring, with typical beam parameters and using an arbitrary growth rate $1/\tau_z$ of $3 \cdot 10^{-4}$ s^{-1} for showcasing (around 100 times the actual rate). This formalism is restricted to cases above transition as it always leads to an increase of the emittances.

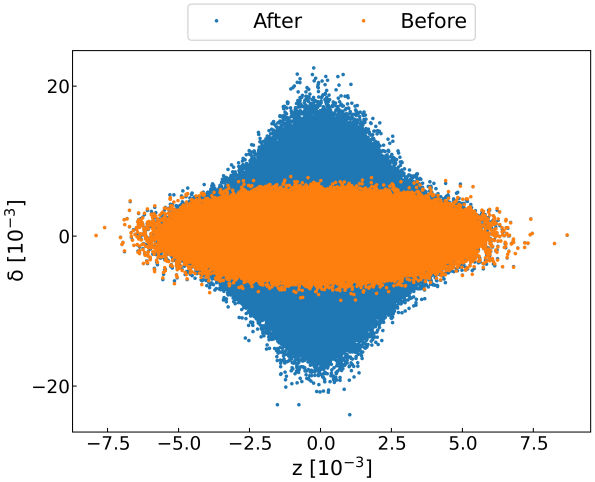

Figure 3: Effect of the simple IBS kick on the longitudinal plane of a positron bunch in the CLIC Damping Ring.

Kinetic Kicks

A general IBS kick, valid above and below transition, was implemented based on the kinetic theory of gases approach introduced by Zenkevich [24, 25], providing a momentum kick with a similar form to the Langevin equation:

$$\Delta p_u = -K_u p_u \sigma_z \sqrt{\pi} \rho(z) \Delta t + R \sigma_{p_u} \sqrt{2 C_u \sigma_z \sqrt{\pi} \rho(z) \Delta t} \,, \tag{3}$$

where K_u and C_u are functions of the friction and diffusion terms, respectively, evaluated using Nagaitsev's formalism as detailed in [26].

Benchmarks

Figure 4 shows the analytical evolution of rms emittances together with the emittance evolution of a macro-particle distribution tracked with Xsuite in the SPS Ring. The ion beam is initialized with 10^4 macro-particles as a Gaussian distribution in all three planes, with $\varepsilon_x^n = 10^{-6}$ m, $\varepsilon_y^n = 2.5 \cdot 10^{-7}$ m and $\sigma_z = 10 \cdot 10^{-2}$ m and IBS kicks are applied at every turn. Two identical distributions are initialized and tracked for 10^3 turns, each being applied a different kick formalism.

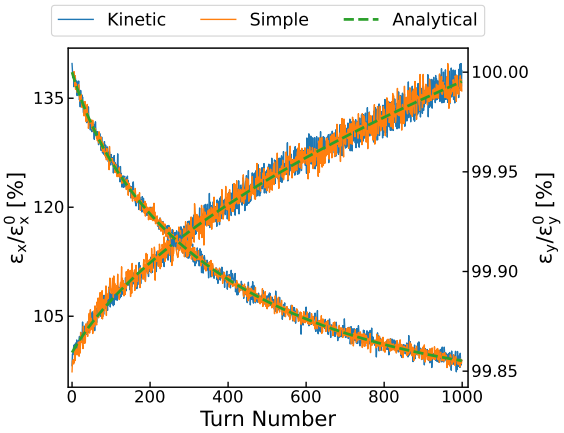

Figure 4: Comparison of averaged horizontal (increasing) and vertical (decreasing) emittances from tracking with simple and kinetic kicks as well as analytical values using Eq. (1).

The tracking results show emittance values averaged over 3 turns. Good agreement is observed between both kick formalism and the analytical results.

Auto-recomputing of kicks coefficient is also available, and a recent release has added GPU computing compatibility for a speedup of up to an order of magnitude.

CONCLUSIONS AND FUTURE PLANS

A new Python package implementing modular analytical and kick-based approaches to IBS modelling was presented. These have been benchmarked for various accelerators against existing codes as well as between themselves with very good agreement, while being over an order of magnitude faster than the previous work it is based on. Additional features such as automatic growth rates re-computation and GPU compatibility are also offered, and current effort is being put into the native integration of the current functionality into Xsuite, with further optimisation of critical code parts as compiled kernels. In the long term, a full GPU-accelerated Particle-in-Cell implementation of 6D particle-to-particle Coulomb interactions is planned.

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