Scaling Violation in Power Corrections to Energy Correlators from the Light-Ray Operator Product Expansion

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In recent years, energy correlators have emerged as a powerful tool to explore the field theoretic structure of strong interactions at particle colliders. In this Letter we initiate a novel study of the nonperturbative power corrections to the projected N-point energy correlators in the limit where the angle between the detectors is small. Using the light-ray operator product expansion as a guiding principle, we derive the power corrections in terms of two nonperturbative quantities describing the fragmentation of quarks and gluons. In analogy with their perturbative leading-power counterpart, we show that power corrections obey a classical scaling behavior that is violated at the quantum level. This crucially results in a dependence on the hard scale Q of the problem that is calculable in perturbation theory. Our analytic predictions are successfully tested against Monte Carlo simulations for both lepton and hadron colliders, marking a significant step forward in the understanding of these observables.

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Introduction—Within the physics program of the Large Hadron Collider (LHC), energy correlators [1–3] have emerged as a powerful tool to study the properties of strong interactions, such as the precise extractions of the strong coupling constant [4]. From a theoretical viewpoint, these observables have inspired a thorough investigation of their field-theoretic properties [5–27]. Owing to their simplicity, energy correlators inherit the quantum properties of the correlation functions, which encode fundamental information about the underlying field theory [28]. This paves the way to new explorations of quantum chromodynamics using present and future collider data, as reflected in the wide phenomenological interest they have attracted in particle physics [21,29–49], heavy-ion physics [50–56], and nuclear physics [57–62].

An N-point energy correlator is defined by weighing the cross section with the product of the energies of N particles (e.g., within a jet), as a function of their relative angles. One can define the corresponding projected N-point energy correlator (ENC) by integrating the resulting correlator over these angles except for the largest one θ_L, as a univariate function of the angular variable $x_L \equiv (1 - \cos\theta_L)/2$ [3]. In the collinear limit, considered in this Letter, one is interested in the regime in which such angular variable is parametrically small, i.e., $\sqrt{x_L}Q \ll Q$, with Q being the hard momentum transfer of the scattering process. At leading power, the ENC shows a classical scaling behavior $\mathcal{O}(1/x_L)$ in the collinear limit [5–7,9–13,29], that is determined by Lorentz symmetry. This scaling is then violated by quantum effects, which induce a mild additional dependence on the hard scale Q. The evolution of ENC with Q can be obtained using perturbative collinear resummation techniques [3,21,29,49,63–65].

The full exploitation of high-precision experimental data also demands an understanding of the dynamics of ENC beyond perturbation theory. A first-principle understanding of the deep nonperturbative limit in which the angular distance $\sqrt{x_L}$ becomes of the order of the ratio Λ_{QCD}/Q, with Λ_{QCD} being a typical hadronic scale, is currently out of reach. Nevertheless, in the regime $1 \gg \sqrt{x_L} \gg \Lambda_{\mathrm{QCD}}/Q$, one can approximate nonperturbative corrections in a power expansion in Λ_{QCD}/Q, gaining a better analytic control over their properties. The next-to-leading power term of this expansion, commonly denoted as *power correction*, defines the leading nonperturbative correction in this kinematic regime. These power corrections can be studied using a range of analytic techniques, which have been used to investigate observables belonging to the ENC family at lepton colliders both in the bulk of the phase space [66–68] ($x_L \neq 0, 1$) as well as in the back-to-back limit [69] ($x_L \to 1$).

This Letter initiates a novel study of the ENC in the collinear ($x_L \to 0$) limit. We will show that, similarly to their leading-power counterpart, the coefficient of the linear $\mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)$ power correction to the ENC exhibits a classical scaling behavior fixed by symmetry arguments, which is violated at the quantum level in a way that can be predicted using perturbation theory. This phenomenon shares similarities with the violation of the well-known Bjorken scaling [70–73], where the evolution of the nonperturbative structure functions with the scale is fully perturbative [71–73]. Using the light-ray operator product expansion (OPE) [5,7,9–11,74], we are able to calculate the evolution of the power correction with the energy scale Q, hence predicting how they are related at different scales. This result marks a significant step forward in the theoretical understanding of this class of observables beyond the perturbative level.

ENC and the light-ray OPE—The projected energy correlators (ENC) are correlation functions of the energy flow operators $\mathcal{E}(n)$ in a physical state $|\Psi_q\rangle$ [1,2,5], defined as $\langle\mathcal{E}(n_1)\cdots\mathcal{E}(n_k)\rangle_{\Psi_q} \equiv \langle\Psi_q|\mathcal{E}(n_1)\cdots\mathcal{E}(n_k)|\Psi_q\rangle$, where q^μ is the total momentum of the state $|\Psi_q\rangle$. The energy flow operator $\mathcal{E}(n)$ is defined as [5,75]

$$\mathcal{E}(n) = \mathbb{L}_{\tau=2}[n^i T_{0i}(t, r\vec{n})], \tag{1}$$

where $T_{\mu\nu}$ is the energy-momentum tensor of QCD and the operation $\mathbb{L}_\tau$ is the light transform [76]

$$\mathbb{L}_\tau = \lim_{r\to\infty} r^\tau \int_0^\infty dt. \tag{2}$$

Examples for the state $|\Psi_q\rangle$ include those excited by the electromagnetic current from the vacuum, the decay products of a Higgs boson, or the scattering state of a high-energy collision. Generic final states consist of the ensemble described by the density $\rho_q = \sum_\Psi |\Psi_q\rangle\langle\Psi_q|$.

In recent years, the light-ray OPE has emerged as an efficient tool to study energy correlators in the small angle limit. Originally developed in the context of conformal collider physics [5,7,9], the light-ray OPE has recently been found useful also in QCD [10,11,74]. For the simplest two-point energy correlator (EEC), the light-ray OPE at leading twist reads

$$\lim_{n_1\to n_2} \mathcal{E}(n_1)\mathcal{E}(n_2) = \frac{1}{x_L}\vec{C}\cdot\vec{\mathbb{O}}^{[J=3]}_{\tau=2}(n_2) + \frac{\Lambda_{\mathrm{QCD}}}{x_L^{3/2}}\vec{D}\cdot\vec{\mathbb{O}}^{[J=2]}_{\tau=2}(n_2) + \cdots, \tag{3}$$

where $x_L = (n_1\cdot n_2)/2$ is related to the angular distance of two lightlike directions n_1 and n_2, and $\vec{C}$, $\vec{D}$ are dimensionless OPE coefficients. The light-ray OPE formula in (3) describes the leading small-angle behavior of the EEC. The operator $\vec{\mathbb{O}}^{[J]}_{\tau=2}$ belongs to the leading trajectory of the light-ray operator [76]. For *even* collinear spin J, it can be obtained by a light transform of the following twist $\tau = 2$ local operators [10,11]:

TABLE I. Collinear spin (boost) and classical scaling dimension of various quantities appearing in the OPE (3).

	$\mathbb{L}_\tau$	$\vec{O}^{[J]}_\tau$	$\vec{\mathbb{O}}^{[J]}_\tau$	x_L	Λ_{QCD}, Q
Collinear spin	$1-\tau$	$-J$	$1-(\tau+J)$	2	0
Dimension	$-\tau-1$	$\tau+J$	$J-1$	0	1

$$\vec{\mathbb{O}}^{[J]}_{\tau=2} = \mathbb{L}_2[\vec{O}^{[J]}_{\tau=2}], \qquad \vec{O}^{[J]}_{\tau=2} = \begin{pmatrix} \frac{1}{2^J}\bar{\psi}\gamma^+(iD^+)^{J-1}\psi \\ \frac{-1}{2^J}G^{a,\mu+}(iD^+)^{J-2}G^{a,+}_\mu \end{pmatrix}, \tag{4}$$

where $\gamma^+ = \bar{n}\cdot\gamma$, and $\tau = \Delta - J$, where Δ is the operator dimension. The energy flow operator corresponds to the combination $\mathcal{E} = (1,1)\cdot\vec{\mathbb{O}}^{[J=2]}_{\tau=2}$, with $J = 2$.

The form of the light-ray OPE (3) is determined by dimensional analysis and Lorentz symmetry. To understand this, we collect in Table I the collinear spin and dimension of all the ingredients entering the OPE (3). By dimensional analysis, imposing that the dimension of both sides of (3) is the same fixes the dimension of the operators in each term of the light-ray OPE. Imposing that the light-ray operators have twist label $\tau = 2$ fixes their collinear spin, which determines the label $J = 3$ for the first term in (3) and $J = 2$ for the $\mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)$ power correction. This completely determines the classical scaling in x_L, e.g., $x_L^{-3/2}$ for the $\mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)$ term. The reason is that the collinear spin of x_L is completely fixed by its transformation properties under a Lorentz boost, which acts as a dilation of angles on the celestial sphere, as depicted in Fig. 1. Using the

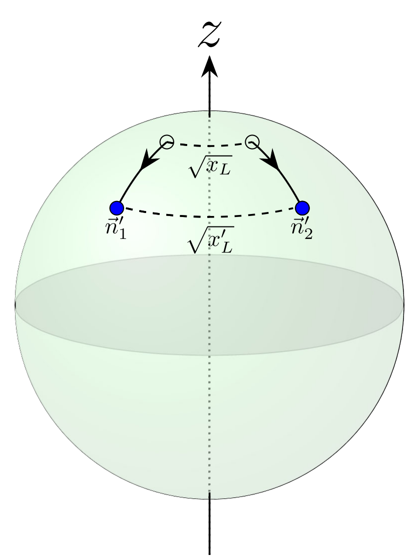

FIG. 1. A boost in the positive z direction increases the distance of two energy flow operators on the celestial sphere.

quantities listed in Table I, it is easy to verify the legitimacy of (3) (cf. [77] for further discussions).

The light-ray OPE in (3) can be generalized to higher point correlators. For N energy flow operators, the observable depends on $N(N-1)/2$ angles for an isotropic state $|\Psi_q\rangle$. We are interested in the projective N-point energy correlator, where the higher dimensional distribution is projected to the axis of the maximal angular distance of the N energy flow operators. In this case the OPE formula reads

$$\int d\bar{\Omega} \lim_{n_i\to n} \mathcal{E}(n_1)\cdots\mathcal{E}(n_N) = \frac{1}{x_L}\vec{C}_N\cdot\vec{\mathbb{O}}^{[J=N+1]}_{\tau=2}(n) + \frac{\Lambda_{\mathrm{QCD}}}{x_L^{3/2}}\vec{D}_N\cdot\vec{\mathbb{O}}^{[J=N]}_{\tau=2}(n) + \cdots, \tag{5}$$

where all n_i directions approach n. The angular integral is over the $N(N-1)/2-1$ angles except the largest separation x_L among the N directions. The general N-point projective energy correlator can be defined as the normalized expectation value of the product of N energy flow operators in a state $|\Psi_q\rangle$:

$$\mathrm{ENC}_{\Psi_q}(x_L, Q) = \frac{4\pi}{\sigma_{\Psi_q} Q^N}\int d\bar{\Omega} \lim_{n_i\to n}\langle\mathcal{E}(n_1)\cdots\mathcal{E}(n_N)\rangle_{\Psi_q}, \tag{6}$$

where $\sigma_{\Psi_q} = \langle\Psi_q|\Psi_q\rangle$ and $Q = q^0$ is the energy of the state, e.g., the center-of-mass energy in $\gamma^*\to q\bar{q}$ or $h\to gg$, or the energy of jets.

For $N=2$ it reduces to the conventional EEC [1,2]. In parton language, the first term in the rhs of Eq. (3) gives rise to the factorization theorem of Refs. [3,29], known at next-to-next-to-leading logarithmic (NNLL) accuracy in QCD [29,49]. The operators $\vec{\mathbb{O}}^{[J=3]}_{\tau=2}$ are mapped onto the hard function while the OPE coefficients are encoded in the jet function. The leading-power EEC exhibits a classical scaling behavior $\mathcal{O}(1/x_L)$, which is mildly violated by quantum corrections that modify its dependence on the energy Q. Analogous considerations hold for the ENC starting from Eq. (5).

We now focus on the power correction to the N-point projective energy correlator by defining

$$\mathrm{ENC}^{\mathrm{N.P.}}_{\Psi_q}(x_L, Q) \equiv \mathrm{ENC}_{\Psi_q}(x_L, Q) - \mathrm{ENC}^{\mathrm{P.T.}}_{\Psi_q}(x_L, Q), \tag{7}$$

where the subscript P.T. denotes the leading power, perturbative part of the energy correlator. Equation (5) predicts a classical scaling behavior $\mathcal{O}(x_L^{-3/2})$ for the leading power correction. The assumption of linearity in Λ_{QCD} is supported by predictions from hadronization [2,78], Wilson loop [66,67], and renormalon [68] models.

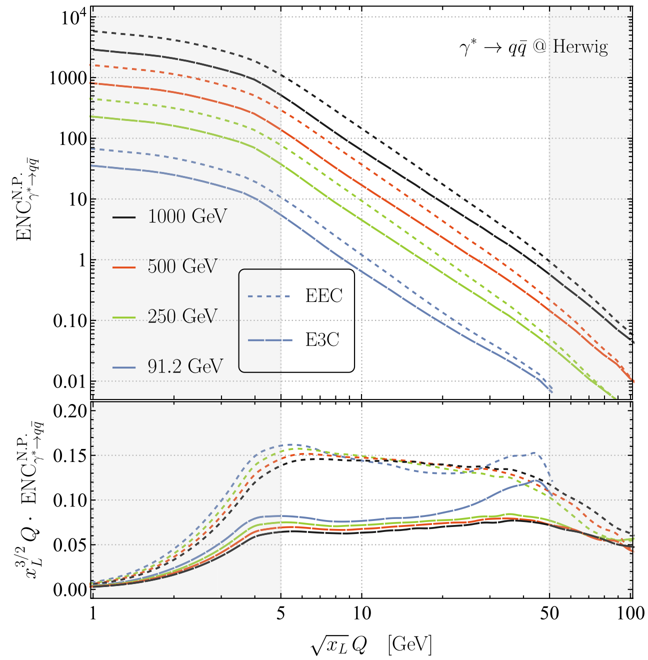

FIG. 2. The upper panel shows the classical scaling of the energy correlators across energy and angular scales. The lower panel highlights the mild quantum scaling violation.

The classical scaling can be verified using Monte Carlo simulations for e^+e^- collision at different Q, as shown in Fig. 2 for the process $\gamma^*\to q\bar{q}$. In the following, we consider the regime $Q\gg Q\sqrt{x_L}\gg\Lambda_{\mathrm{QCD}}$, shown in the unshaded region of the plot, where the scaling is clearly visible.

Scaling violation in power corrections—The above discussion about classical scaling is based only on Lorentz symmetry and classical dimensional analysis. We now show that the OPE also predicts quantum corrections that slightly violate this scaling in the perturbative region $Q\gg Q\sqrt{x_L}\gg\Lambda_{\mathrm{QCD}}$. We start by factoring out the classical scaling of $\mathrm{ENC}^{\mathrm{N.P.}}_{\Psi_q}(x_L, Q)$, and write

$$\mathrm{ENC}^{\mathrm{N.P.}}_{\Psi_q}(x_L, Q) = \frac{\mathrm{ENC}^{\mathrm{N.P.}}_{1,\Psi_q}(K_\perp, Q)}{x_L^{3/2}Q} + \ldots, \tag{8}$$

where $K_\perp = \sqrt{x_L}Q$ characterizes the exchanged transverse momentum scale and we neglected subleading power corrections in the rhs. Classically, no dependence of $\mathrm{ENC}^{\mathrm{N.P.}}_{1,\Psi_q}(K_\perp, Q)$ on Q is expected. At the quantum level, the function $\mathrm{ENC}^{\mathrm{N.P.}}_{1,\Psi_q}(K_\perp, Q)$ mildly depends on Q. This can be appreciated in the lower panel of Fig. 2, where the classical scaling has been removed. We refer to the small residual dependence of $\mathrm{ENC}^{\mathrm{N.P.}}_{1,\Psi_q}(K_\perp, Q)$ on Q as to quantum *scaling violation*.

We now show how the scaling violation is caused by the renormalization group (RG) evolution of $\vec{\mathbb{O}}^{[J]}_{\tau=2}$.

The light-ray OPE (5) provides the following factorized prediction for $\mathrm{ENC}^{\mathrm{N.P.}}_{1,\Psi_q}$:

$$\mathrm{ENC}^{\mathrm{N.P.}}_{1,\Psi_q}(K_\perp, Q) = \Lambda_{\mathrm{QCD}}\vec{D}_N\left(\frac{K_\perp^2}{\mu^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{\mu^2}\right) \cdot \frac{\langle \vec{\mathbb{O}}^{[J=N]}_{\tau=2}(n;\mu)\rangle_{\Psi_q}}{(4\pi)^{-1}\sigma_{\Psi_q} Q^{N-1}}\left(\frac{Q^2}{\mu^2}\right), \quad (9)$$

where μ is a factorization scale that separates the perturbative matrix element of light-ray operators and the nonperturbative OPE coefficients $\vec{D}_N$, which also depend on Λ_{QCD}. The light-ray operator $\vec{\mathbb{O}}^{[J]}_{\tau=2}(n;\mu)$ satisfies the DGLAP equation [10,11,74,79]

$$\mu\frac{d}{d\mu}\vec{\mathbb{O}}^{[J]}_{\tau=2}(n;\mu) = \gamma^{[J]}_{\tau=2}(\mu)\cdot\vec{\mathbb{O}}^{[J]}_{\tau=2}(n;\mu), \quad (10)$$

where $\gamma^{[J=N]}_{\tau=2}$ is the anomalous dimension matrix of the twist-2 light-ray operators, which admits the perturbative expansion $\gamma^{[J]}_{\tau=2}(\mu) = \sum_{k=0}^{\infty}(\alpha_s(\mu)/4\pi)^{k+1}\gamma^{[J],(k)}_{\tau=2}$. The leading order expression $\gamma^{[J],(0)}_{\tau=2}$ can be found in [80,81], while higher-order calculations of $\gamma^{[J]}_{\tau=2}$ and their inverse Mellin transform can be found in [82–90]. Renormalization group invariance implies that

$$\mu\frac{d\vec{D}_N}{d\mu} = -\vec{D}_N\cdot\gamma^{[J=N]}_{\tau=2}. \quad (11)$$

By observing that $\vec{D}_N$ does not depend on Q explicitly, Eq. (9) suggests that one can predict the Q dependence of $\mathrm{ENC}^{\mathrm{N.P.}}_{1,\Psi_q}$ at *fixed* $K_\perp$ solely from the Q dependence of the matrix element of light-ray operators. Specifically, we let $\mu = K_\perp$ in the OPE coefficient, and evolve the matrix element of the light-ray operators from Q to $K_\perp$ using the renormalization group equation (10). Since the physical state $|\Psi_q\rangle$ is μ independent, at fixed $K_\perp$ the Q dependence of $\mathrm{EEC}^{\mathrm{N.P.}}_{1,\Psi_q}$ is given by

$$\mathrm{ENC}^{\mathrm{N.P.}}_{1,\Psi_q}(K_\perp, Q) = \Lambda_{\mathrm{QCD}}\vec{D}_N\left(1, \frac{\Lambda^2_{\mathrm{QCD}}}{K_\perp^2}\right)\cdot U_N(K_\perp, Q)\cdot\frac{\langle\vec{\mathbb{O}}^{[J=N]}_{\tau=2}(n;Q)\rangle_{\Psi_q}}{(4\pi)^{-1}\sigma_{\Psi_q}Q^{N-1}}, \quad (12)$$

where $U_N(K_\perp, Q)$ is the evolution operator

$$U_N(K_\perp, Q) \equiv \mathbb{P}\exp\left(-\int_{K_\perp}^{Q}\frac{d\mu}{\mu}\gamma^{[J=N]}_{\tau=2}(\mu)\right). \quad (13)$$

This is one of the main results of this Letter.

We finally discuss the implications of Eq. (12) for the factorization formula of the projected correlator [3,29]:

$$\mathrm{ENC}_{\Psi_q}(x_L, Q) = \int_0^1 dx\frac{x^N}{x_L}\vec{J}_N\left(\frac{x_L x^2 Q^2}{\mu^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{\mu^2}\right)\cdot\vec{H}\left(x, \frac{Q}{\mu}\right). \quad (14)$$

Based on the equivalence [74] of the light-ray OPE and Eq. (14), we can extend the above factorization formula to include the power corrections derived in this Letter. The hard function $\vec{H}$ encodes the probability of producing a parton with energy fraction x and it corresponds to the normalized matrix element of the twist $\tau = 2$ operators in Eq. (5). The jet function $\vec{J}_N$ is sensitive to the fragmentation at small angular scales and, as such, it encodes also the nonperturbative dynamics. It is mapped [74] onto the OPE coefficients of Eq. (5), from which we can deduce the following expansion:

$$\vec{J}_N\left(\frac{x_L x^2 Q^2}{\mu^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{\mu^2}\right) = \vec{J}^{\mathrm{P.T.}}_N\left(\frac{x_L x^2 Q^2}{\mu^2}, \alpha_s(\mu)\right) + \frac{\Lambda_{\mathrm{QCD}}}{x\sqrt{x_L}Q}\vec{J}^{(1)}_N\left(\frac{x_L x^2 Q^2}{\mu^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{\mu^2}\right) + \cdots, \quad (15)$$

where $\vec{J}^{\mathrm{P.T.}}_N$ is the perturbative jet function [3,21,29,49] and $\vec{J}^{(1)}_N = \vec{D}_N|_{K_\perp\to xK_\perp}$ is the corresponding power correction.

Monte Carlo validation—We can now explicitly use Eq. (12) to relate the power correction at a reference scale Q_0 to that at a scale Q. We can express the solution in terms of two nonperturbative functions of $K_\perp$ defining the two components of $\vec{D}$ at a reference scale Q_0, which can be extracted from the fragmentation of quarks and gluons. At the leading-logarithmic order we find (cf. [77] for details)

$$\begin{pmatrix}\mathrm{ENC}^{\mathrm{N.P.}}_{1,\gamma^*\to q\bar{q}}(Q)\\ \mathrm{ENC}^{\mathrm{N.P.}}_{1,h\to gg}(Q)\end{pmatrix}^T = \begin{pmatrix}\mathrm{ENC}^{\mathrm{N.P.}}_{1,\gamma^*\to q\bar{q}}(Q_0)\\ \mathrm{ENC}^{\mathrm{N.P.}}_{1,h\to gg}(Q_0)\end{pmatrix}^T\cdot U^{\mathrm{LL}}_N(Q_0, Q), \quad (16)$$

where $K_\perp$ is fixed and kept implicit and $U^{\mathrm{LL}}_N(Q_0, Q) = [\alpha_s(Q)/\alpha_s(Q_0)]^{\gamma^{[N],(0)}_{\tau=2}/(2\beta_0)}$.

We extract the functions $\mathrm{ENC}^{\mathrm{N.P.}}_{1,\gamma^*\to q\bar{q}}(Q_0)$ and $\mathrm{ENC}^{\mathrm{N.P.}}_{1,h\to gg}(Q_0)$ from $\gamma^*\to q\bar{q}$ and $h\to gg$ at $Q_0 = 250$ GeV for 2-, 3-, and 4-point correlators and predict their distribution at a different c.m. energy $Q\in$ 91.2–500 GeV. Specifically, we use events generated with MADGRAPH5 [91], showered with HERWIG7.2 [92] (Specifically, we use the dot-product preserving shower and corresponding tune from Ref. [93].) and analyzed with Rivet [94]. (We have repeated the analysis also with HERWIG7.3 [95] and PYTHIA8 [96], finding consistent

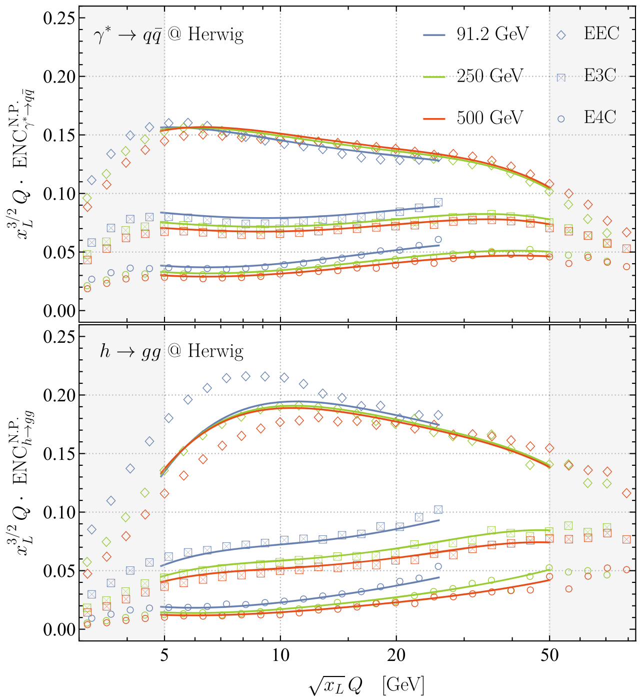

FIG. 3. Comparison of analytic and Monte Carlo predictions for the quantum scaling violation for N-point projected correlators in $\gamma^* \to q\bar{q}$ and $h \to gg$.

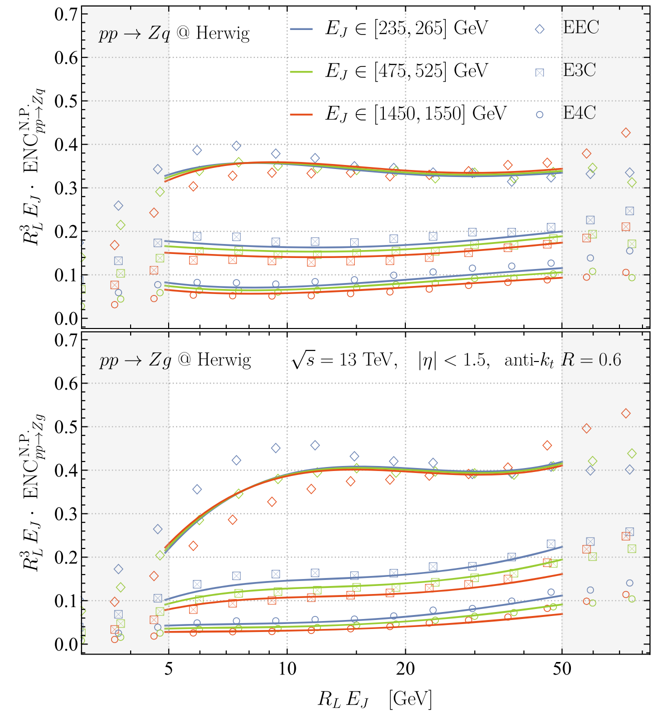

FIG. 4. Comparison of analytic and Monte Carlo predictions for the quantum scaling violation for N-point projected correlators in $pp \to Zq$ and $pp \to Zg$.

results.) The results are shown in Fig. 3, which displays a comparison of Eq. (16) to the Monte Carlo prediction obtained with Eqs. (7) and (8). We notice that the latter contains subleading power corrections not accounted for in Eq. (16). In general, we observe very good agreement, hence validating the expectation for the perturbative scaling violation presented in this Letter. From Fig. 3 we observe that in the case of the EEC the region of validity of Eq. (16) is substantially pushed towards larger angles. An explanation of this fact, particularly prominent in the gluonic case, is that subleading power corrections neglected in the OPE (3) receive a contribution from operators with $J \sim 1$, whose anomalous dimensions feature a strong enhancement due to the radiation of soft gluons [97–101]. The enhanced quadratic power corrections may be ultimately responsible for the discrepancy in the left region of the plot. This phenomenon is present only in the EEC case while for $N > 2$ the contribution of $J < 2$ operators is further power suppressed.

It is interesting to apply the same procedure to the case of ENC measured on hadronic jets at the LHC [4], in the limit in which the largest angular resolution $\sqrt{x_L} \to R_L \equiv \sqrt{\Delta y^2 + \Delta\phi^2}$ between detectors is much smaller than the jet radius R. We consider $pp \to Z + q/g$ jets LHC events at $\sqrt{s} = 13$ TeV, with jet energies in the range $E_J \in 250$–1500 GeV. Accordingly we now define $K_\perp = R_L E_J$. Jets are defined using the anti-k_t algorithm [102] with a jet radius $R = 0.6$, as implemented in FastJet [103]. We generate separately events with quark and gluon jets, that we extract from Zq and Zg final states, respectively. An analysis at higher perturbative orders, however, would require a more refined definition of quark and gluon jet fractions. The results are shown in Fig. 4, where the solid lines indicate our predictions from Eq. (16) with $Q_0 = 500$ GeV and $Q = E_J$. The effect of initial-state radiation, present in pp collisions, impacts mildly the correlators measured inside jets at the nonperturbative level (e.g., via color reconnection). While the EEC, as in Fig. 3, is affected by large subleading power corrections, the analytic prediction describes very well the simulation for the $N > 2$ correlators, confirming the validity of our results also in the hadron-collider case (cf. [77] for additional studies).

We envision that the leading power correction can be directly extracted from experimental data at a reference scale and then evolved at different scales using the results presented in this Letter. In Ref. [77] we present also a study of the effect of quantum scaling violation on ratios of energy correlators, used to measure α_s in Ref. [4]. This Letter will enhance the role of energy correlators in the precision physics programme and their use for the extraction of fundamental properties of QCD at the LHC and future colliders.

Note added—Recently, Ref. [104] presented a related study of the power corrections to the projected correlators using a renormalon analysis in the context of extractions of the strong coupling constant. The connection of their findings to our prediction from the light-ray OPE is nontrivial and deserves further investigation.

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