Open charm tetraquarks in broken $\mathrm{SU}(3)_F$ symmetry

L. Maiani,[1,2] A. D. Polosa,[1] and V. Riquer[1,2]
[1]*Sapienza University of Rome and INFN, Piazzale Aldo Moro 2, I-00185 Rome, Italy*
[2]*CERN, 1211 Geneva 23, Switzerland*

(Received 17 May 2024; accepted 10 July 2024; published 12 August 2024)

Prompted by a recent lattice QCD calculation, we review the SU(3) light quark flavor structure of charmed tetraquarks with spin 0 diquarks. Fermi statistics forces the three light quarks to be in the representation $\bar{\mathbf{3}}\otimes\bar{\mathbf{3}}=\mathbf{3}\oplus\bar{\mathbf{6}}$. This agrees with the weak repulsion in the $\mathbf{15}$ of the $\mathbf{3}\otimes\mathbf{8}$ in $\bar{D}K$ scattering studied on the lattice. We analyze the $\mathbf{3}\oplus\bar{\mathbf{6}}$ multiplet broken by the strange quark mass and determine the five independent masses from the known masses of diquarks. The mass of $D^*_{s0}(2317)$ is predicted within 50 MeV accuracy. The recently observed $\bar{D}_s^{--}(2900)$ and $\bar{D}_s^0(2900)$, likely part of a $I=1$ multiplet, with flavor composition $\bar{c}\bar{q}q's$, and $X_0(2900)$, an isosinglet with flavor composition $\bar{c}\bar{s}ud$, fit naturally in a $\mathbf{3}\oplus\bar{\mathbf{6}}$ structure as the first radial excitations. We discuss also the decay modes of $D^*_{s0}(2317)$, of the radial excitations and of the predicted particles.

DOI: 10.1103/PhysRevD.110.034014

I. INTRODUCTION

Charmed-strange tetraquarks are studied in a recent lattice QCD calculation [1] in connection with the $\mathrm{SU}(3)_F$ configurations of possible bound states in the $\bar{D}K$ channel. Allowed $\mathrm{SU}(3)_F$ multiplets are those appearing as irreducible components of the tensor product

$$\bar{D}K=\mathbf{3}\otimes\mathbf{8}=\mathbf{3}\oplus\bar{\mathbf{6}}\oplus\mathbf{15}. \tag{1}$$

Reference [1] finds attraction in $\mathbf{3}$ and $\bar{\mathbf{6}}$ but not in $\mathbf{15}$.

Tetraquarks of the same flavor have been considered earlier in connection with the SELEX observation of a charm-strange meson decaying into[1] $D_s^+ + \eta$ or D^0+K^+ [2].

With reference to $\mathrm{SU}(3)_F$, we consider here the antidiquark-diquark composition

$$[\bar{c}\bar{v}]_0^{\mathbf{3}_c}[q_1q_2]_0^{\bar{\mathbf{3}}_c}, \tag{2}$$

where the subscript refers to spin zero and $(v,q_1,q_2=u,d,s)$.

II. QUANTUM NUMBERS AND STATES

Fermi statistics requires the product q_1q_2 to be antisymmetric in flavor, it being already antisymmetric in spin (to get total spin 0) and color (to obtain a $\bar{\mathbf{3}}_c$). The corresponding $\mathrm{SU}(3)_F$ multiplets are in the tensor product

$$\bar{\mathbf{3}}\otimes\bar{\mathbf{3}}=\mathbf{3}\oplus\bar{\mathbf{6}}, \tag{3}$$

the same attractive channels found in [1] and no $\mathbf{15}$.

Some authors have considered diquark-antidiquark states with diquarks in color $\mathbf{6}$. Spin 0 diquarks would be antisymmetric under spin $\times$ color exchange; therefore, they would be in a $\mathbf{6}$ representation of $\mathrm{SU}(3)_F$. Uncharmed, quarks would belong then to the flavor representations $\bar{\mathbf{3}}\otimes\mathbf{6}=\mathbf{3}\oplus\overline{\mathbf{15}}$, in disagreement with [1].

Let us find the explicit form of tetraquarks (2). We introduce the tensors T^i in the $\mathbf{3}_F$ representation and the tensors S_{ij} in the $\bar{\mathbf{6}}_F$ representation as[2]

$$T^i=\bar{v}_\alpha(q^\beta q^\gamma)\epsilon_{\beta\gamma\delta}\epsilon^{\delta\alpha i}\propto\bar{v}_\alpha q^\alpha q^i \tag{4}$$

since quark fields anticommute. The normalized vectors for triplet (T) tetraquarks are (diquark spin 0 understood)

$$S=0,\qquad T^1=\frac{[\bar{c}\bar{d}][du]+[\bar{c}\bar{s}][su]}{\sqrt{2}},$$
$$T^2=\frac{[\bar{c}\bar{u}][ud]+[\bar{c}\bar{s}][sd]}{\sqrt{2}}, \tag{5}$$

$$S=-1,\qquad T^3=\frac{[\bar{c}\bar{u}][su]+[\bar{c}\bar{d}][sd]}{\sqrt{2}}. \tag{6}$$

[1]We define $\bar{D}_s^-=(\bar{c}s),\bar{D}=(\bar{c}q),\ K=(\bar{q}s)$.

[2]The convention is that quarks (antiquarks) carry an upper (a lower) flavor index.

Similarly in the flavor sextet (S) tetraquarks

$$S_{ij} = \frac{1}{2}[\bar{v}_i(q^\beta q^\gamma)\epsilon_{j\beta\gamma} + (i \leftrightarrow j)], \tag{7}$$

and the normalized $\bar{\mathbf{6}}$ vectors are

$$S = +1, \qquad S_{33} = [\bar{c}\bar{s}][ud], \tag{8}$$

$$S = 0, \qquad S_{13} = \frac{[\bar{c}\bar{u}][ud] + [\bar{c}\bar{s}][ds]}{\sqrt{2}},$$

$$S_{23} = \frac{[\bar{c}\bar{d}][ud] + [\bar{c}\bar{s}][su]}{\sqrt{2}}, \tag{9}$$

$$S = -1, \qquad S_{11} = [\bar{c}\bar{u}][ds], \qquad S_{12} = \frac{[\bar{c}\bar{u}][su] - [\bar{c}\bar{d}][sd]}{\sqrt{2}},$$

$$S_{22} = [\bar{c}\bar{d}][su]. \tag{10}$$

In the presence of $\mathrm{SU}(3)_F$ breaking, $m_u = m_d < m_s$, we expect the mass eigenstates with $S = 0$ to correspond to the combinations

$$S_{13} \pm T^2, \qquad S_{23} \pm T^1. \tag{11}$$

Figure 1 gives the pattern of sextet and triplet states in the I_3-strangeness plane.

Following [1], we identify T^3 in Eq. (6) with the observed $D_{s0}^*(2317)$ [3] (see also the review [4]). In Sec. V we will discuss the particles observed by LHCb: $D_{s0}(2900)^0 \to D_s^+\pi^- = [cd\bar{s}\bar{u}]$, $D_{s0}(2900)^{++} \to D_s^+\pi^+ = [cu\bar{s}\bar{d}]$ [5], and $X_0(2900) \to D^-K^+ = [\bar{c}\bar{s}du]$ [6].

III. MASS FORMULAS IN BROKEN $\mathrm{SU}(3)_F$

We introduce the symmetric masses with $M_{\bar{\mathbf{6}}}, M_{\mathbf{3}}$, and add octet $\mathrm{SU}(3)_F$ breaking using the symbols $m_{\mathbf{6}}$ and $m_{\mathbf{3}}$. In the product $\bar{\mathbf{6}} \otimes \mathbf{6}$ representation $\mathbf{8}$ appears only once, so

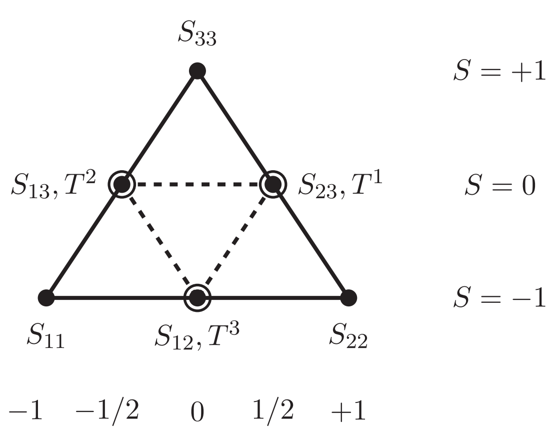

FIG. 1. The $\mathbf{3} \oplus \bar{\mathbf{6}}$ representation in the I_3- trangeness plane. Electric charges are as follow: $Q(S_{11}) = -2$, $Q(S_{13}) = Q(S_{12}) = -1$, and $Q(S_{33}) = Q(S_{23}) = Q(S_{22}) = 0$.

there is only one operator to describe the symmetry breaking, namely the hypercharge of the light quarks, given by the formula

$$Q_\ell = I_3 + \frac{1}{2}Y_\ell, \tag{12}$$

and suffix ℓ means that we ignore the charm antiquark. For the representation $\bar{\mathbf{6}}$

$$Y_{\ell,\bar{\mathbf{6}}} = \mathrm{diag}\left(\frac{4}{3}, \frac{1}{3}, -\frac{2}{3}\right) \text{ for } (S_{33}, S_{12}, S_{11}) \text{ and } \mathrm{Tr}(Y_{\ell,\bar{\mathbf{6}}}) = 0. \tag{13}$$

The symmetry breaking mass in the representation $\bar{\mathbf{6}}$ is

$$m_{\bar{\mathbf{6}}} = \beta_{\bar{\mathbf{6}}}\frac{1}{2}\left(Y_{\ell,\bar{\mathbf{6}}} + \frac{2}{3}\right), \tag{14}$$

explicitly

$$m_{\bar{\mathbf{6}}} = \beta_{\bar{\mathbf{6}}}\mathrm{diag}\left(1, \frac{1}{2}, 0\right) \quad \text{for } (S_{33}, S_{12}, S_{11}). \tag{15}$$

Similarly, for the $\mathbf{3}$ representation

$$Y_{\ell,\mathbf{3}} = \mathrm{diag}\left(\frac{1}{3}, -\frac{2}{3}\right) \text{ for } (T^1, T^3) \text{ and } \mathrm{Tr}(Y_{\ell,\bar{\mathbf{3}}}) = 0 \tag{16}$$

with the symmetry breaking

$$m_{\mathbf{3}} = \beta_{\mathbf{3}}\left(Y_{\ell,\mathbf{3}} + \frac{2}{3}\right) = \beta_{\mathbf{3}}\mathrm{diag}(1, 0) \quad \text{for } (T_1, T_3). \tag{17}$$

Mixing $\mathbf{3} - \bar{\mathbf{6}}$ is described by the matrix

$$m_{\mathrm{mix}} \propto \lambda_8 = \mathrm{diag}(1, 1, -2) \tag{18}$$

and the matrix $\mathcal{M}$ mixing T^1, S_{23} or equivalently T^2, S_{13} is

$$\mathcal{M} = \begin{pmatrix} M_{\mathbf{3}} + \beta_{\mathbf{3}} & \delta \\ \delta & M_{\bar{\mathbf{6}}} + \frac{\beta_{\bar{\mathbf{6}}}}{2} \end{pmatrix}. \tag{19}$$

In total we have five states and four independent physical masses: (i) $M(S_{33})$; (ii) and (iii) corresponding to the masses $M_\pm$ [see Eq. (22)] of the two $S = 0$ states arising from the mixing matrix (19), and (iv) $M(S_{11}) = M(T^3)$, since they have the same flavor composition. Enforcing the latter condition gives the relation

$$M_{\mathbf{3}} = M_{\bar{\mathbf{6}}}, \tag{20}$$

and we remain with four parameters, $M_{\bar{\mathbf{6}}} = M, \beta_{\mathbf{3}}, \beta_{\bar{\mathbf{6}}}, \delta$. The magic mixing in (11) is obtained for equal diagonal terms in Eq. (19), that is,

$$\beta_{\mathbf{3}} = \frac{\beta_{\bar{\mathbf{6}}}}{2}. \tag{21}$$

To first order in $\beta_{\mathbf{3}}$ and $\beta_{\bar{\mathbf{6}}}$, eigenvalues and eigenstates of the mixing matrix (11) with the substitution (20) are given by

$$M_{\pm} = M + \frac{2\beta_{\mathbf{3}} + \beta_{\bar{\mathbf{6}}}}{4} \pm \delta. \tag{22}$$

In addition to the equality of $M(S_{11})$ and $M(T^3)$, the quark composition of the $\mathbf{3} \oplus \bar{\mathbf{6}}$ suggests an interesting regularity, namely that $\beta_{\mathbf{3}}$ and $\beta_{\bar{\mathbf{6}}}$ have to be very small, if not vanishing at all. Indeed, according to (7), the lower indices in S_{11} correspond to the quark-diquark antisymmetric configuration $\bar{u} \otimes [ds]_{\mathrm{A}}$ while the lower indices in S_{33} correspond to $\bar{s} \otimes [ud]_{\mathrm{A}}$ which have obviously the same content in quark masses, two light and one heavy.

Exact equality of the bound states masses corresponds to $\beta_{\mathbf{3}} = \beta_{\bar{\mathbf{6}}} = 0$: the same masses at the upper vertex and lower corners of the triangle in Fig. 1. In this case, symmetry breaking is restricted to the mass difference between the two $S = 0, I = 1/2$ multiplets induced by $\mathbf{3} - \bar{\mathbf{6}}$ mixing and of order $\mu \sim 2(m_s - m_q)$, with all other masses degenerate at M.

Small values of $\beta_{\mathbf{3}}$ and $\beta_{\bar{\mathbf{6}}}$ could result from differences in the hyperfine interactions, which are between different pairs in the two cases [see below, Eq. (33)].

The situation can be compared to the case of charmed baryons, where the two light quarks in spin one are also in a flavor symmetric **6** representation. In this case indices 1 or 3 univocally correspond to u or s quarks, and the top and bottom particles (Σ_c and Ω_c) differ in mass by 240 MeV,[3] of the order of $2(m_s - m_q)$.

Group theory is effective at disentangling the ambiguity in these two cases by making use of the parameters allowed by the Wigner-Eckart theorem.

Another interesting case is that of hidden charm $SU(3)_F$ tetraquarks where a lower or upper index 3 is unequivocally associated with a strange quark or antiquark and, correspondingly, the octet obeys the equal spacing rule of vector mesons, with spacing $\sim(m_s - m_q)$, well satisfied by the masses of $X(3872) - Z_{cs}(4003) - X(4140)$ [8].

IV. COMPARING WITH THE DIQUARK-ANTIDIQUARK MODEL

Mass formulas for tetraquarks in terms of diquark masses and hyperfine interactions have been spelled out in Ref. [9], with reference to hidden charm tetraquarks.

For hyperfine interactions, the formula proposed in Ref. [9] is

$$(H_{\mathrm{h.f.}})_{ij} = 2\kappa_{ij}(\boldsymbol{s}_i \cdot \boldsymbol{s}_j) = \kappa_{ij}\left[s(s+1) - \frac{3}{2}\right],$$
$$\kappa_{ij} = \frac{|\Psi(0)|^2}{m_i m_j}, \tag{23}$$

where s is the total spin of the ij pair belonging to the same diquark, under the assumption that the overlap probability for quarks in different diquarks is negligible. This hypothesis reproduces the observed mass ordering: $X(3872)$, $Z(3900) < Z(4020)$.

To simplify the notation, we define "complete diquark masses" which include the hyperfine interaction appropriate to diquarks with spin $= 0$, e.g.,

$$\bar{M}_{cq} = M_{cq} - \frac{3}{2}\kappa_{cq}, \qquad \text{etc.} \tag{24}$$

Numerical values are reported in Table I. Charmed diquark masses and hyperfine interactions are taken from Refs. [9,10] and complete masses for uncharmed, spin 0 diquarks from the, not so well determined, masses of the light scalar mesons [11], $f_0(500)$ and $f_0(980)$ (see the errors in Table I),

$$\bar{M}_{qq} = \frac{1}{2}M(f_0(500)), \qquad \bar{M}_{sq} = \frac{1}{2}M(a_0(980)). \tag{25}$$

With reference to Eqs. (8) and (10) one has

$$M(S_{33}) = \bar{M}_{cs} + \bar{M}_{qq} = M + \beta_{\bar{\mathbf{6}}}, \tag{26}$$

$$M(S_{11}) = M(T^3) = \bar{M}_{cq} + \bar{M}_{sq} = M, \tag{27}$$

where we used the first and third entries, respectively, of $m_{\bar{\mathbf{6}}}$ in Eq. (15). Here and in the following, we assume $\bar{M}_{cs} = \bar{M}_{\bar{c}\bar{s}}$, etc., and $q = u, d$. Mixed states

$$M(S_{13}) = M(S_{23}) = M + \frac{1}{2}\beta_{\bar{\mathbf{6}}},$$
$$M(T^1) = M(T^2) = M + \beta_{\mathbf{3}}, \tag{28}$$

where we used the second entry of $m_{\bar{\mathbf{6}}}$ in (15) and the first entry of $m_{\mathbf{3}}$ in (17). The sum of these two quantities is the trace of the matrix (19). Using (11) and (22)

TABLE I. Complete diquark masses, $\bar{M}_{ij}$, in MeV.

Quark	q	s	c
q	300 ± 100	490 ± 10	1877
s	490 ± 10	$\cdots$	2035
c	1877	2035	$\cdots$

[3]Baryon and meson spectroscopy suggests a value: $m_s - m_q \sim$ 170 MeV (see, e.g., [7]); however, the difference $M(\Omega_c) - M(\Sigma_c)$ receives a contribution of an opposite sign from the hyperfine, spin-spin interaction.

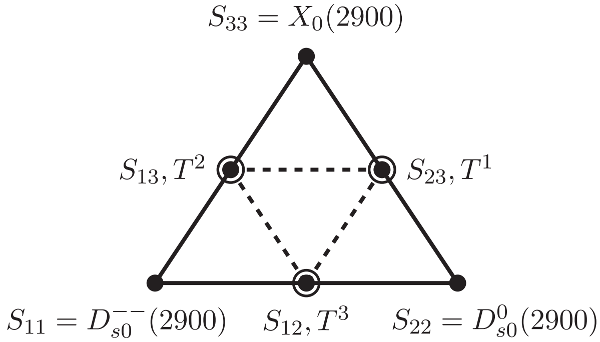

FIG. 2. The $n=2$ multiplet. $\bar{D}_s\pi, S=-1$ and $\bar{D}K, S=+1$ resonances observed by LHCb [5,6] in the $n=2$ multiplet.

$$M_+ + M_- = 2M + \frac{2\beta_{\mathbf{3}} + \beta_{\bar{\mathbf{6}}}}{2}, \tag{29}$$

that is,

$$\frac{2\beta_{\mathbf{3}} + \beta_{\bar{\mathbf{6}}}}{2} = M_+ + M_- - 2M = [(\bar{M}_{cs} - \bar{M}_{cq}) - (\bar{M}_{sq} - \bar{M}_{qq})], \tag{30}$$

given that [from (9)] $M_+ + M_- = \bar{M}_{cs} + \bar{M}_{sq} + \bar{M}_{cq} + \bar{M}_{qq}$. Similarly,

$$M(S_{33}) - M(T^3) = \beta_{\bar{\mathbf{6}}} = [(\bar{M}_{cs} - \bar{M}_{cq}) - (\bar{M}_{sq} - \bar{M}_{qq})], \tag{31}$$

and we find

$$\beta_{\bar{\mathbf{6}}} = -32 \pm 100 \text{ MeV}. \tag{32}$$

Predicted masses of $\mathbf{3}$ and $\bar{\mathbf{6}}$ are (use values in Table I)

$$\begin{aligned} M(S_{11}) &= M(T_3) = \bar{M}_{cu} + \bar{M}_{sd} = 2367 \pm 10 \text{ MeV}, \\ M(T_-) &= \bar{M}_{cu} + \bar{M}_{ud} = 2177 \pm 100 \text{ MeV}, \\ M(T_+) &= \bar{M}_{cs} + \bar{M}_{sd} = 2525 \pm 10 \text{ MeV}, \\ M(S_{33}) &= \bar{M}_{cs} + \bar{M}_{ud} = 2335 \pm 100 \text{ MeV}. \end{aligned} \tag{33}$$

The first value compares favorably with the mass of the observed $D_{s0}^{*-}(2317)$, with a difference of 50 ± 10 MeV.

V. THE MULTIPLET OF RADIAL EXCITATIONS

The particles $D_{s0}^{0}(2900) \to D_s^+\pi^- = [cd\bar{s}\bar{u}]$, $D_{s0}^{++}(2900) \to D_s^+\pi^+ = [cu\bar{s}\bar{d}]$, with common mass 2908 ± 25 MeV, recently observed by LHCb [5], and $X_0(2900) \to D^-K^+ = [\bar{c}\bar{s}du]$, with mass 2866 ± 7 MeV [6], are too heavy to be included in the basic $\mathbf{3} \oplus \bar{\mathbf{6}}$ together with $D_{s0}^*(2317)$. The mass difference

$$M(2900) - M(2317) = 583 \text{ MeV} \tag{34}$$

is similar to the mass gap between $\psi(2S)$ and J/ψ ($\Delta = 590$ MeV) or between $X(3872)$ and $Z(4430)$ ($\Delta = 558$ MeV), and we shall similarly interpret the LHCb resonances as the first radial excitations ($n=2$) of the basic multiplet the $D_{s0}^*(2317)$ belongs to.

We have to fit in the same multiplet $X_0(2900)$ with $D_{s0}^{--,0}(2900)$, antiparticles of the resonances observed in [5], to have the same charm quantum number; see Fig. 2. The expected $n=1$ multiplet is shown in Fig. 3.

The positive strangeness $X_0(2900)$ mass close to the masses of the negative strangeness particle $D_{s0}^{--,0}(2900)$ is a remarkable confirmation of the regularity noted in Sec. III, a real footprint of the tetraquark compositions: $[\bar{c}\bar{s}]_0[ud]_0$ and $[\bar{c}\bar{u}]_0[sd]_0$.

VI. DECAYS

The case of $D_{s0}^-(2317)$. As shown by Eq. (6), T^3 has $I=0$, and it should decay into $D_s^-\eta$, which, however, is forbidden by phase space. We can consider two independent

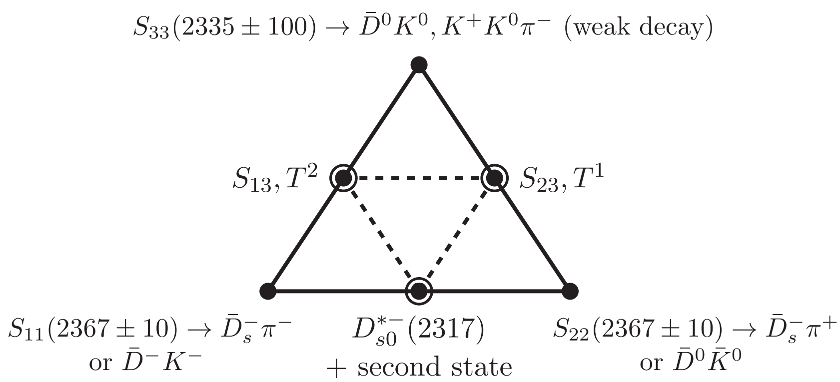

FIG. 3. The $n=1$ multiplet. The diquarks in S_{23} are $[\bar{c}\bar{s}][su](2525 \pm 10) \to \bar{D}_s^-K^0, \bar{D}^0\eta$ and $[\bar{c}\bar{d}][ud](2177 \pm 100) \to \bar{D}^0\pi^0$.

mechanisms for the observed, isospin violating, $D_s^-\pi^0$ decay, both related to the $m_d - m_u$ mass difference: mixing of T^3 with S_{12} ($I = 1, I_3 = 0$), or $\eta - \pi^0$ mixing.

In both cases, mixing allows the decay $D_{s0}^* \to D_s\pi^0$ with a small width ($\Gamma < 3.8$ MeV is reported in [3]). It would be interesting to observe the decay $D_{s0}^* \to D_s\gamma\gamma$, quoted in [3] with an upper bound to the branching ratio $B(\gamma\gamma) < 0.18$, to compare with $D_{s0}^*(2317) \to D_s^-\eta^* \to D_s^-\gamma\gamma$ via the virtual η.

The missing partners of $D_{s0}^-(2317)$. With reference to Fig. 3, the bottom corners must be filled by two isovector mesons in the channels $D_s^-\pi^\pm$, in all similar to those found at mass 2900 MeV in [5]. In addition, a companion of $D_{s0}^*(2317)$ is needed, close in mass and in the same channels, $\bar{D}_s^-\pi^0$ or $\gamma\gamma$, most likely with a larger width.

The lighter, zero strangeness state, predicted at 2177, could be identified with the lower pole under $D^*(2300)$ reported in PDG [3] at mass 2105.

The most intriguing case is the particle in the upper vertex, which is predicted to be very close to the $\bar{D}K$ threshold, the channel where $X_0(2900)$ is seen. If it is below the threshold of this channel, it has to decay weakly into $K^+K^0\pi^-$.

Radial excitations. With the larger mass of the radial excitations shown in Fig. 2 all possible two body decays are open:

$$(S_{12}, T^3)_{(n=2)} \to D_s^-\pi^0, \qquad D_s^-\eta,$$
$$(S_{12}, T^3)_{(n=2)} \to \bar{D}^0K^-, \qquad \bar{D}^-\bar{K}^0. \qquad (35)$$

The mixing of $n = 2$ states S_{12} and T^3 can be determined from the decay rates as in Ref. [2].

For zero strangeness states, we expect the Okubo–Zweig–Iizuka (OZI) rule mixing to produce tetraquarks with and without one $s\bar{s}$ pair:

$$[\bar{c}\bar{s}][sd]_{(n=2)} \to \bar{D}^-\eta, \qquad \bar{D}_s^-K^0,$$
$$[\bar{c}\bar{u}][ud]_{(n=2)}, \qquad [\bar{c}\bar{d}][ud]_{(n=2)} \to \bar{D}\pi. \qquad (36)$$

VII. THE ROLE OF FERMI STATISTICS IN SINGLE CHARM TETRAQUARKS

In Ref. [12] the authors utilize the so-called light quark spin symmetry in the static quark approximation [13] to classify spin states of hidden charm molecules of quark composition $(\bar{c}q)(\bar{q}'c)$, with fixed isospin I. Calling $S_{\ell,I}$ and $S_{c\bar{c}}(=1,0)$ the light quarks and $c\bar{c}$ total spin, the possible combinations of light and heavy spin generate six states with definite isospin, total angular momentum, and charge conjugation: $J_I^{PC} = 0_I^{++}, 1_I^{+-}, 1_I'^{+-}, 1^{++}, 0_I'^{++}, 2_I^{++}$. No surprise, these are the same six J_I^{PC} states produced by diquark-antidiquark color singlet tetraquarks of the form $[cq]^{\bar{\mathbf{3}}}[\bar{c}\bar{q}']^{\mathbf{3}}$, considered in [2,9].

The situation is different in the case considered in Eq. (2) of the present paper. Assuming diquark ⊗ antidiquark colors to be $\bar{\mathbf{3}} \otimes \mathbf{3} \to \mathbf{1}$, there is a correlation between total spin and isospin [or SU$(3)_F$] of the light quarks pair q_1q_2 induced by Fermi statistics. The latter requires either (a) $\bar{\mathbf{3}}_F \leftrightarrow (S_{12} = 0)$ or (b) $\mathbf{6}_F \leftrightarrow (S_{12} = 1)$. Therefore, in the case at hand we are led univocally to flavor $\bar{\mathbf{3}}_F$ and to the $\mathbf{3}_F \oplus \bar{\mathbf{6}}_F$ composition of the tetraquark structure studied in this paper.

The situation is different for the molecular structure $(\bar{c}q_1)(\bar{v}q_2)$, in that the colors of q_1 and q_2 are not correlated and there are no apparent reasons for spin 0 molecules not to display all flavors in the representations appearing in the SU$(3)_F$ decomposition of the product $\bar{D}K$, Eq. (1). For $J^P = 0^+$ single charm exotics, the suppression of the 15, in the molecular case, was derived in Ref. [14] with an explicit calculation using chiral dynamics along the lines described in [15].

VIII. CONCLUSIONS

We show how the resonance $D_{s0}^{--,0}(2900)$ and $X_0(2900)$ nicely fit in a $\bar{\mathbf{6}}$ representation of SU$(3)_F$ with the prediction of a few more states in the sextet, in addition to the very likely $D_{s0}^-(2900)$ to fill an isotriplet with $D_{s0}^{--,0}(2900)$. The observation that $M(2900) - M(2317) = 583$ MeV $\simeq M(\psi(2S)) - M(\psi(1S))$ suggests that the sextet we discuss could be a radial excitation of a lower sextet containing the $D_{s0}^*(2317)$, in a similar way in which $Z(4430)$ can be interpreted as a radial excitation of $X(3872)$ [9]. Using SU$(3)_F$ symmetry breaking we obtain mass predictions for the missing states. Our results are in agreement with a recent lattice calculation [1] showing that in the $\bar{D}K$ scattering there are no bound states in the **15** representation, something that is expected in the quark model description we present here.

ACKNOWLEDGMENTS

We are grateful to Vanya Belyaev and Tim Gershon for a very useful discussion of LHCb results on the single charm, single strange mesons discussed here. We are indebted to Christofer Hanhart for pointing us the lattice result in [1] that has been at the origin of the present article.

[1] J. D. E. Yeo, C. E. Thomas, and D. J. Wilson, J. High Energy Phys. 07 (2024) 012.
[2] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, Phys. Rev. D **70**, 054009 (2004).
[3] R. L. Workman *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).
[4] S. Eidelman *et al.*, Heavy Non-$q\bar{q}$ Mesons, RPP (2022); arXiv:2212.02716.
[5] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **131**, 041902 (2023).
[6] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. D **102**, 112003 (2020).
[7] A. Ali, L. Maiani, and A. D. Polosa, *Multiquark Hadrons* (Cambridge University Press, Cambridge, England, 2019), ISBN 978-1-316-76146-5, 978-1-107-17158-9, 978-1-316-77419-9.
[8] L. Maiani, A. D. Polosa, and V. Riquer, Sci. Bull. **66**, 1616 (2021).
[9] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, Phys. Rev. D **89**, 114010 (2014).
[10] L. Maiani, A. D. Polosa, and V. Riquer, Phys. Rev. D **94**, 054026 (2016).
[11] R. L. Jaffe, Phys. Rev. D **15**, 281 (1977); L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, Phys. Rev. Lett. **93**, 212002 (2004).
[12] Z. H. Zhang, T. Ji, X. K. Dong, F. K. Guo, C. Hanhart, U. G. Meißner, and A. Rusetsky, arXiv:2404.11215.
[13] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989).
[14] M. Albaladejo, P. Fernandez-Soler, F. K. Guo, and J. Nieves, Phys. Lett. B **767**, 465 (2017).
[15] E. E. Kolomeitsev and M. F. M. Lutz, Phys. Lett. B **582**, 39 (2004).