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$1/Q^2$ power corrections to TMD factorization for Drell-Yan hadronic tensor

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Abstract: I calculate $\frac{1}{Q^2}$ power corrections to unpolarized Drell-Yan hadronic tensor for electromagnetic (EM) current at large N_c and demonstrate the EM gauge invariance at this level.

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1 Introduction

Particle production in hadron-hadron scattering with transverse momentum of produced particle(s) much smaller than the invariant mass is described in the framework of TMD factorization [1–5]. The typical factorization formula for particle production in hadron-hadron scattering looks like [1, 6]

$$\frac{d\sigma}{d\eta d^2q_\perp} = \sum_f \int d^2b_\perp e^{i(q,b)_\perp}\mathcal{D}_{f/A}(x_A,b_\perp,\eta)\mathcal{D}_{f/B}(x_B,b_\perp,\eta)\sigma(ff\to X)$$
$$+ \text{power corrections} + \text{Y} - \text{terms} \tag{1.1}$$

where η is the rapidity, $\mathcal{D}_{f/A}(x,z_\perp,\eta)$ is the TMD density of a parton f in hadron A, and $\sigma(ff\to X)$ is the cross section of production of particle(s) X of invariant mass $m_X^2 = Q^2$ by the fusion of two partons.

Typically, leading first term in Eq. (1.1) is given by quark-antiquark TMDs (or two-gluon TMDs in the case of Higgs boson production). The second term stands for the power corrections given by a series in $q_\perp^2/Q^2$ while the third describes transition to the regime $q_\perp^2 \sim Q^2$ governed by the collinear factorization.

The significance of power corrections is twofold. First, they show up to what $q_\perp^2$ the differential cross section is given by the first term in the formula (1.1) with controlled accuracy. For example, the estimate for Z-boson production in DY process gives power corrections reaching order of few per cent at $\frac{q_\perp}{Q} \sim \frac{1}{4}$ [7].

The second use of power corrections is due to the fact that for certain characteristics of a scattering the power corrections are actually the leading terms. It turns out that some angular distributions of produced particle(s) are defined by quark-quark-gluon TMDs forming power corrections $\sim \frac{q_\perp^2}{Q^2}$. For example, the symmetric DY hadronic tensor $W_{\mu\nu}$, defined as [1]

$$
\begin{aligned}
W_{\mu\nu}(q) &\stackrel{\rm def}{=} \frac{1}{(2\pi)^4}\sum_X\int d^4x\ e^{-iqx}\frac{1}{2}\Big(\langle p_A,p_B|J_\mu(x)|X\rangle\langle X|J_\nu(0)|p_A,p_B\rangle+\mu\leftrightarrow\nu\Big)\\
&= \frac{1}{(2\pi)^4}\int d^4x\ e^{-iqx}\frac{1}{2}\langle p_A,p_B|J_\mu(x)J_\nu(0)+\mu\leftrightarrow\nu|p_A,p_B\rangle, \qquad (1.2)
\end{aligned}
$$

has 4 tensor structures for unpolarized hadrons. Two of them are determined by leading-twist quark TMDs while two other ones start from terms $\frac{q_\perp}{Q}$ and $\sim\frac{q_\perp^2}{Q^2}$ described by quark-quark-gluon TMDs. Note that while $\frac{q_\perp}{Q}$ power corrections were known for more than two decades [8], there was no calculations of $\frac{q_\perp^2}{Q^2}$ until recently, starting from the paper [7].

In two previous papers [9, 10] I calculated such $\frac{q_\perp^2}{Q^2}$ power corrections and found DY angular distributions at small Bjorken x_B in the leading order in $\frac{1}{N_c}$. In this paper I generalize the results of Ref. [9] to arbitrary values of x_B. As a result, the number of relevant TMDs increases: for unpolarized protons, in addition to eight quark-antiquark TMDs, there are about twenty quark-antiquark-gluon TMDs on $\frac{1}{Q^2}$, leading-N_c level.

The paper is organized as follows. In section 2 I outline the derivation of TMD factorization by rapidity factorization of the double functional integral for a cross section of particle production. Also, I briefly remind the method of calculation of power corrections based on approximate solution of classical Yang-Mills equations [7]. In Sect. 3 I present the leading-twist result and discuss the types of $\frac{1}{Q^2}$ power corrections calculated in this paper. In the next three Sections I calculate different types of $\frac{1}{Q^2}$ power corrections using the aforementioned method. The result is presented In Sect. 7 and discussed in Sect. 8. The necessary technical formulas and parametrizations of relevant TMDs can be found in appendices.

2 TMD factorization from rapidity factorization

We use Sudakov variables $p=\alpha p_1+\beta p_2+p_\perp$, where p_1 and p_2 are light-like vectors close to p_A and p_B so that $p_A=p_1+\frac{m^2}{s}p_2$ and $p_B=p_2+\frac{m^2}{s}p_1$ with m being the proton mass. Also, we use the notations $x_\bullet\equiv x_\mu p_1^\mu$ and $x_\star\equiv x_\mu p_2^\mu$ for the dimensionless light-cone "Ioffe times" $x_\star=\sqrt{\frac{s}{2}}x_+$ and $x_\bullet=\sqrt{\frac{s}{2}}x_-$. Our metric is $g^{\mu\nu}=(1,-1,-1,-1)$ which we will frequently rewrite as a sum of longitudinal part and transverse part:

$$
g^{\mu\nu} = g_\parallel^{\mu\nu}+g_\perp^{\mu\nu} = \frac{2}{s}(p_1^\mu p_2^\nu+p_2^\mu p_1^\nu)+g_\perp^{\mu\nu} \qquad (2.1)
$$

Consequently, $p\cdot q = (\alpha_p\beta_q+\alpha_q\beta_p)\frac{s}{2}-(p,q)_\perp$ where $(p,q)_\perp\equiv -p_iq^i$. Throughout the paper, the sum over the Latin indices i, j, ... runs over two transverse components while the sum over Greek indices μ, ν, ... runs over four components as usual.

Following Ref. [11] we separate quark and gluon fields into three sectors (see figure 1): "projectile" fields A_μ,ψ_A with $|\beta|<\sigma_p$, "target" fields B_μ,ψ_B with $|\alpha|<\sigma_t$ and "central

[1] Here p_A,p_B are hadron momenta, q is the momentum of DY pair, $\sum_X$ denotes the sum over full set of "out" states and $J_\mu=\sum e_f\bar\psi^f\gamma_\mu\psi_f$ is an electromagnetic current. We take into account only u,d,s,c quarks and consider them massless. In principle, one can include "massless" b-quark for $q_\perp^2\gg m_b^2$

rapidity" fields C_μ, ψ_C with $|\alpha| > \sigma_t$ and $|\beta| > \sigma_p$, see Fig. 1. (For convenience, I call hadron A by the name "projectile" and hadron B by the name "target"). Our goal is to

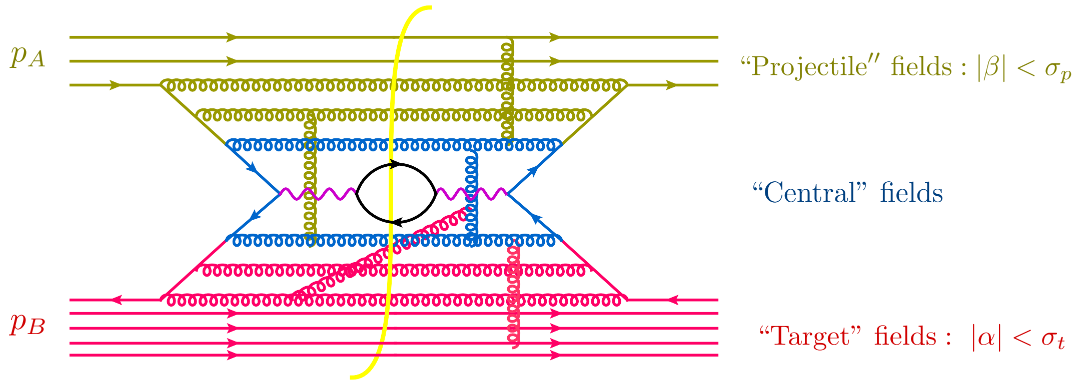

Figure 1. Rapidity factorization for DY particle production

integrate over central fields and get the amplitude in the factorized form, i.e. as a product of functional integrals over A fields representing projectile matrix elements (TMDs of the projectile) and functional integrals over B fields representing target matrix elements (TMDs of the target). In the spirit of background-field method, we "freeze" projectile and target fields and get a sum of diagrams in these external fields. As we shall see below, for the purpose of calculation of most of the power corrections we can set $\beta = 0$ for the projectile fields and $\alpha = 0$ for the target fields. The corrections to this approximation are $O\big(\frac{m^2}{\sigma_p s}\big)$ and $O\big(\frac{m^2}{\sigma_t s}\big)$ and can be neglected almost everywhere, see the discussion in Sect. 3.2.3.

In the coordinate space, the $\beta = 0$ approximation means that projectile fields do not depend on x_* and $\alpha = 0$ means that target ones do not depend on $x_\bullet$. [2] In this case, as discussed in Ref. [9], central fields at the tree level are given by a set of Feynman diagrams with *retarded* propagators in background field $A+B$ and $\psi_A+\psi_B$.The set of such "retarded" diagrams represent the solution of QCD equations of motion with sources being projectile and target fields. After summation of these diagrams the hadronic tensor (1.2) can be represented as

$$W_{\mu\nu} \;=\; \frac{1}{(2\pi)^4}\int d^4x e^{-iqx}\sum_{m,n}\int dz_m c_{m,n}(q,x)\langle p_A|\hat{\Phi}_A(z_m)|p_A\rangle\int dz'_n\langle p_B|\hat{\Phi}_B(z'_n)|p_B\rangle. \quad (2.2)$$

where $c_{m,n}$ are coefficients and Φ can be any of the background fields promoted to operators after integration over projectile and target fields.

In general, solution of classical QCD equations with projectile and target sources is a formidable task which still awaits its solution. Fortunately, as demonstrated in Ref. [9], at our kinematics we have a small parameter $\frac{q_\perp^2}{Q^2} \ll 1$ and it is possible to expand classical solution for central fields in powers of this parameter. It is convenient to choose a gauge

[2] Beyond the tree level, the integration over C fields produces logarithms of the cutoffs σ_p and σ_t which match the corresponding logs in TMDs of the projectile and the target, see the discussion in Ref. [12]

where $A_\star = 0$ for projectile fields and $B_\bullet = 0$ for target fields. [3] (The existence of such gauge was proved in appendix B of Ref. [11] by explicit construction). Also, since we are dealing with tree approximation and quark equations of motion, it is convenient to include coupling constant g in the definition of gluon fields so that $D_\mu\psi = \partial_\mu\psi - iA_\mu\psi$, $\bar\psi \overleftarrow{D}_\mu = \partial_\mu\bar\psi + i\bar\psi A_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$.

As demonstrated in Ref. [7], the expansion of classical quark fields in powers of $p_\perp^2/p_\parallel^2$ has the form [4]

$$\Psi(x) \;=\; \Psi_1(x) + \Psi_2(x) + \ldots, \tag{2.3}$$

where ($P_\perp \equiv \partial_\perp + A_\perp + B_\perp$)

$$\begin{aligned}
\Psi_1 &= \psi_A + \Upsilon_1, \quad \Upsilon_1 = \Xi_1 + \Xi_1', \quad \Xi_1 = -\frac{\not p_2}{s}\not B_\perp\frac{1}{\alpha+i\epsilon}\psi_A,\\
\Xi_1' &= -\frac{\not p_1}{s}\Big(\frac{1}{\beta+i\epsilon}\not B_\perp\Big)\psi_A + \frac{1}{s^2}\Big(\frac{\not p_1}{\beta+i\epsilon}\not P_\perp\frac{\not p_2}{\alpha+i\epsilon} + \frac{\not p_2}{\alpha+i\epsilon}\not P_\perp\frac{\not p_1}{\beta+i\epsilon}\Big)\not B_\perp\psi_A\\
\bar\Psi_1 &= \bar\psi_A + \bar\Upsilon_1, \quad \bar\Upsilon_1 = \bar\Sigma_1 + \bar\Sigma_1', \quad \bar\Sigma_1 = -\,(\bar\psi_A\frac{1}{\alpha-i\epsilon})\not B_\perp\frac{\not p_2}{s}\\
\bar\Xi_1' &= -\bar\psi_A\Big(\frac{\not B}{\beta-i\epsilon}\Big)\frac{\not p_1}{s} + \frac{1}{s^2}\bar\psi_A\not B\Big(\frac{\not p_1}{\beta-i\epsilon}\not P_\perp\frac{\not p_2}{\alpha-i\epsilon} + \frac{\not p_2}{\alpha-i\epsilon}\not P_\perp\frac{\not p_1}{\beta-i\epsilon}\Big)\\
\Psi_2 &= \psi_B + \Upsilon_2, \quad \Upsilon_2 = \Xi_2 + \Xi_2', \quad \Xi_2 = -\frac{\not p_1}{s}\not A_\perp\frac{1}{\beta+i\epsilon}\psi_B\\
\Xi_2' &= -\frac{\not p_2}{s}\Big(\frac{1}{\alpha+i\epsilon}\not A\Big)\psi_B + \frac{1}{s^2}\Big(\frac{\not p_1}{\beta+i\epsilon}\not P_\perp\frac{\not p_2}{\alpha+i\epsilon} + \frac{\not p_2}{\alpha+i\epsilon}\not P_\perp\frac{\not p_1}{\beta+i\epsilon}\Big)\not A_\perp\psi_B\\
\bar\Psi_2 &= \bar\psi_A + \bar\Upsilon_1, \quad \bar\Upsilon_2 = \bar\Sigma_2 + \bar\Sigma_2', \quad \bar\Xi_2 = -\,(\bar\psi_B\frac{1}{\beta-i\epsilon})\not A_\perp\frac{\not p_1}{s}\\
\bar\Xi_2' &= -\bar\psi_B\Big(\frac{\not A}{\alpha-i\epsilon)}\Big)\frac{\not p_2}{s} + \frac{1}{s^2}\bar\psi_B\not A\Big(\frac{\not p_1}{\beta-i\epsilon}\not P_\perp\frac{\not p_2}{\alpha-i\epsilon} + \frac{\not p_2}{\alpha-i\epsilon}\not P_\perp\frac{\not p_1}{\beta-i\epsilon}\Big)
\end{aligned}\tag{2.4}$$

and the dots stand for higher-order power corrections. [5] In the above formulas

$$\begin{aligned}
\frac{1}{\alpha+i\epsilon}\psi_A(x_\bullet, x_\perp) &\equiv -i\int_{-\infty}^{x_\bullet} dx_\bullet'\;\psi_A(x_\bullet', x_\perp),\\
\Big(\bar\psi_A\frac{1}{\alpha-i\epsilon}\Big)(x_\bullet, x_\perp) &\equiv i\int_{-\infty}^{x_\bullet} dx_\bullet'\;\bar\psi_A(x_\bullet', x_\perp)
\end{aligned}\tag{2.5}$$

and similarly for $\frac{1}{\beta\pm i\epsilon}$. For brevity, in what follows we denote $\big(\bar\psi_A\frac{1}{\alpha}\big)(x) \equiv \big(\bar\psi_A\frac{1}{\alpha-i\epsilon}\big)(x)$ and $\big(\bar\psi_B\frac{1}{\beta}\big)(x) \equiv \big(\bar\psi_B\frac{1}{\beta-i\epsilon}\big)(x)$. Similarly to Eq. (2.5), more complicated expressions for $\bar\Psi$ should be read from right to left, for example

$$\bar\psi_A\not B\Big(\frac{\not p_1}{\beta}\not P_\perp\frac{\not p_2}{\alpha} + \frac{\not p_2}{\alpha}\not P_\perp\frac{\not p_1}{\beta}\Big)(x) \;=\; \int dz\;\bar\psi_A(z)\not B(z)(z|\frac{\not p_1}{\beta}\not P_\perp\frac{\not p_2}{\alpha} + \frac{\not p_2}{\alpha}\not P_\perp\frac{\not p_1}{\beta}|x) \tag{2.6}$$

[3] Throughout the paper, we will keep different notations A_i and B_i for the projectile and target gluon fields because of different gauge choices, see e.g. Eqs. (9.11) and (9.12).

[4] The corresponding expansion of classical gluon fields is presented in Ref. [11], but we do not need it here.

[5] The relevant expressions for $\Xi_i', \bar\Xi_i'$ from Ref. [7] are more complicated than those of Eq. (2.4) but the additional terms are shown in Sect. 5 to be negligible.

with $\alpha - i\epsilon$ and $\beta - i\epsilon$ in the denominators. Here $(x|f(p)|y) \equiv (2\pi)^{-d}\int d^dp e^{-ip(x-y)}f(p)$ are Schwinger's notations for propagators.

The contributions from the terms $\Xi_i, \bar{\Xi}_i$ were calculated in Ref. [9] in the $\alpha_q, \beta_q \ll 1$ approximation and in this paper we will repeat the calculation relaxing the aforementioned condition. The contributions from the terms $\Xi_i', \bar{\Xi}_i'$ are new and will be calculated in Sect. 5.

3 Hadronic tensor at $Q^2 \gg q_\perp^2$: leading twist and power corrections

As we noted above, we take into account only hadronic tensor due to electromagnetic currents of u, d, s, c quarks and consider these quarks to be massless. It is convenient to define coordinate-space hadronic tensor multiplied by $N_c\frac{2}{s}$ (and denoted by extra "check" mark) as follows

$$\check{W}_{\mu\nu}(x) \equiv N_c\frac{1}{s}\langle A,B|J_\mu(x)J_\nu(0) + \mu \leftrightarrow \nu|A,B\rangle \tag{3.1}$$

so that

$$\begin{aligned} W_{\mu\nu}(q) &= \frac{s/2}{(2\pi)^4 N_c}\int d^4x\ e^{-iqx}\check{W}_{\mu\nu}(x). \\ &= \frac{1}{(2\pi)^4 N_c}\int dx_\bullet dx_\star d^2x_\perp\ e^{-i\alpha_q x_\bullet - i\beta_q x_\star + i(q,x)_\perp}\check{W}_{\mu\nu}(x). \end{aligned} \tag{3.2}$$

Hereafter we use notation $|A,B\rangle \equiv |p_A, p_B\rangle$ for brevity.

For future use, let us also define the hadronic tensor in mixed representation: in the momentum longitudinal space but in the transverse coordinate space

$$\begin{aligned} W_{\mu\nu}(q) &= \int d^2x_\perp\ e^{i(q,x)_\perp}W_{\mu\nu}(\alpha_q,\beta_q,x_\perp), \\ W_{\mu\nu}(\alpha_q,\beta_q,x_\perp) &\equiv \frac{1}{(2\pi)^4 s}\int dx_\bullet dx_\star\ e^{-i\alpha_q x_\bullet - i\beta_q x_\star}\langle A,B|J_\mu(x_\bullet,x_\star,x_\perp)J_\nu(0) + \mu \leftrightarrow \nu|A,B\rangle. \end{aligned} \tag{3.3}$$

After integration over central fields in the tree approximation we obtain

$$\check{W}_{\mu\nu}(x) \equiv N_c\frac{1}{s}\langle A,B|J_\mu(x_\bullet,x_\star,x_\perp)J_\nu(0) + \mu \leftrightarrow \nu|A,B\rangle \tag{3.4}$$

where

$$\begin{aligned} J^\mu &= J_A^\mu + J_B^\mu + J_{AB}^\mu + J_{BA}^\mu, \\ J_A^\mu &= \sum_f e_f\bar{\Psi}_1^f\gamma^\mu\Psi_1^f, \quad J_{AB}^\mu = \sum_f e_f\bar{\Psi}_1^f\gamma^\mu\Psi_2^f \end{aligned} \tag{3.5}$$

and similarly for J_B^μ and J_{BA}^μ. Here $\langle A,B|\mathcal{O}(\psi_A,A_\mu,\psi_B,B_\mu)|A,B\rangle$ denotes double functional integral over A and B fields which gives matrix elements between projectile and target states of Eq. (2.2) type.

The leading-twist contribution to $W_{\mu\nu}(q)$ comes only from product $J_{AB}^\mu(x)J_{BA}^\nu(0)$ (or $J_{BA}^\mu(x)J_{AB}^\nu(0)$), while power corrections may come also from other terms like $J_A^\mu(x)J_B^\nu(0)$.

However, as demonstrated in Refs. [7, 9], at leading-N_c power corrections come only from $J^\mu_{AB}(x)J^\nu_{BA}(0)$ or $J^\mu_{BA}(x)J^\nu_{AB}(0)$. Since these contributions are diagonal in flavor, we will perform the calculations for one flavor of quarks (with $J_\mu = \bar\psi\gamma_\mu\psi$) and will write down sum over flavors only in the final result (7.1).

3.1 Leading-N_c terms from $J^\mu_{AB}(x)J^\nu_{BA}(0)$

With our $\frac{1}{Q^2}$, leading-N_c accuracy we get from Eq. (2.4):

$$
\begin{aligned}
&J^\mu_{AB}(x)J^\nu_{BA}(0) \ + \ x\leftrightarrow 0 \ = \ \bar\Psi_1(x)\gamma^\mu\Psi_2(x)\bar\Psi_2(0)\gamma^\nu\Psi_1(0) \ + \ x\leftrightarrow 0 \ + \ ...\\
&= \ \big[\big(\bar\psi_A+\bar\Upsilon_1\big)(x)\gamma_\mu\big(\psi_B+\Upsilon_2\big)(x)\big]\big[\big(\bar\psi_B+\bar\Upsilon_2\big)(0)\gamma_\nu\big(\psi_A+\Upsilon_1\big)(0)\big] \ + \ x\leftrightarrow 0\\
&= \ [\bar\psi_A(x)\gamma_\mu\psi_B(x)]\,[\bar\psi_B(0)\gamma_\nu\psi_A(0)] \qquad (3.6)\\
&+ [(\bar\Xi_1+\bar\Xi'_1)(x)\gamma_\mu\psi_B(x)]\,[\bar\psi_B(0)\gamma_\nu\psi_A(0)] + [\bar\psi_A(x)\gamma_\mu\psi_B(x)]\,[\bar\psi_B(0)\gamma_\nu(\Xi_1+\Xi'_1)(0)]\\
&+ [\bar\psi_A(x)\gamma_\mu\psi_B(x)]\,[(\bar\Xi_2+\bar\Xi'_2)(0)\gamma_\nu\psi_A(0)] + [\bar\psi_A(x)\gamma_\mu(\Xi_2+\Xi'_2)(x)]\,[\bar\psi_B(0)\gamma_\nu\psi_A(0)]\\
&+ [\bar\Xi_1(x)\gamma_\mu\psi_B(x)]\,[\bar\psi_B(0)\gamma_\nu\Xi_1(0)] + [\bar\psi_A(x)\gamma_\mu\Xi_2(x)]\,[\bar\Xi_2(0)\gamma_\nu\psi_A(0)]\\
&+ [\bar\Xi_1(x)\gamma_\mu\psi_B(x)]\,[\bar\Xi_2(0)\gamma_\nu\psi_A(0)] + [\bar\psi_A(x)\gamma_\mu\Xi_2(x)]\,[\bar\psi_B(0)\gamma_\nu\Xi_1(0)]\\
&+ [\bar\Xi_1(x)\gamma_\mu\Xi_2(x)]\,[\bar\psi_B(0)\gamma_\nu\psi_A(0)] + [\bar\psi_A(x)\gamma_\mu\psi_B(x)]\,[\bar\Xi_2(0)\gamma_\nu\Xi_1(0)] \ + \ x\leftrightarrow 0.
\end{aligned}
$$

where the square brackets mean trace over Lorentz and color indices. [6]

First, let us consider the leading-twist term and power corrections coming from the first term in the r.h.s. of this equation.

3.2 Contribution of quark-antiquark TMDs

3.2.1 Leading-twist contribution

As we mentioned, the leading-twist term comes from from the first term in the r.h.s. of Eq. (3.6). Using Fierz transformation (9.1) one obtains the quark-antiquark contribution in the form

$$
\begin{aligned}
&\check W^{qq}_{\mu\nu}(x) \ = \ \frac{N_c}{s}\big([\bar\psi_A(x_\bullet,x_\perp)\gamma_\mu\psi_B(x_\star,x_\perp)]\,[\bar\psi_B(0)\gamma_\nu\psi_A(0)] + \mu\leftrightarrow\nu\big) \ + \ x\leftrightarrow 0\\
&= \ \frac{g_{\mu\nu}}{2s}\big(-[\bar\psi_A\psi_A][\bar\psi_B\psi_B) + [\bar\psi_A\gamma_5\psi_A][\bar\psi_B\gamma_5\psi_B] + [\bar\psi_A\gamma_\alpha\psi_A][\bar\psi_B\gamma^\alpha\psi_B]\\
&+ [\bar\psi_A\gamma_\alpha\gamma_5\psi_A][\bar\psi_B\gamma^\alpha\gamma_5\psi_B] - \frac12[\bar\psi_A\sigma^{\alpha\beta}\psi_A][\bar\psi_B\sigma_{\alpha\beta}\psi_B]\big)\\
&- \frac{1}{2s}\big([\bar\psi_A\gamma_\mu\psi_A][\bar\psi_B\gamma_\nu\psi_B] + \mu\leftrightarrow\nu\big) - \frac{1}{2s}\big([\bar\psi_A\gamma_\mu\gamma_5\psi_A][\bar\psi_B\gamma_\nu\gamma_5\psi_B] + \mu\leftrightarrow\nu\big)\\
&+ \frac{1}{2s}\big([\bar\psi_A\sigma_{\nu\alpha}\psi_A][\bar\psi_B\sigma_{\mu\alpha}\psi_B] + [\bar\psi_A\sigma_{\mu\alpha}\psi_A][\bar\psi_B\sigma_{\nu\alpha}\psi_B]\big) \ + \ x\leftrightarrow 0 \qquad (3.7)
\end{aligned}
$$

where the arguments of the fields are the same as in the l.h.s.. From the parametrization of two-quark operators in section 9.2.1, it is clear that the leading-twist contribution to

[6] As demonstrated in Sect. 6, the terms coming from expressions with one $\Xi_i, \bar\Xi_i$ and one $\Xi'_i, \bar\Xi'_i$ are negligible in our approximation)

$W_{\mu\nu}(q)$ comes from

$$\frac{1}{2s}(g_{\mu\nu}g^{\alpha\beta}-\delta^\alpha_\mu\delta^\beta_\nu-\delta^\alpha_\nu\delta^\beta_\mu)[\bar\psi_A(x)\gamma_\alpha\psi_A(0)]\bar\psi_B(0)\gamma_\beta\psi_B(x)]$$
$$+\frac{1}{2s}\big(\delta^\alpha_\mu\delta^\beta_\nu+\delta^\alpha_\nu\delta^\beta_\mu-\frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\big)[\bar\psi_A(x)\sigma_{\alpha\xi}\psi_A(0)][\bar\psi_B(0)\sigma_\beta^{\ \xi}\psi_B(x)]$$
$$+\frac{1}{2s}(g_{\mu\nu}g^{\alpha\beta}-\delta^\alpha_\mu\delta^\beta_\nu-\delta^\alpha_\nu\delta^\beta_\mu)[\bar\psi_A(x)\gamma_\alpha\gamma_5\psi_A(0)][\bar\psi_B(0)\gamma_\beta\gamma_5\psi_B(x)]$$
$$-\frac{g_{\mu\nu}}{2s}[\bar\psi_A(x)\psi_A(0)][\bar\psi_B(0)\psi_B(x)]\ +\ x\leftrightarrow 0 \tag{3.8}$$

With the leading-twist accuracy we can replace $\delta^\alpha_\mu\to\frac{2}{s}p_{1\mu}p_2^\alpha$, $\delta^\beta_\nu\to\frac{2}{s}p_{2\nu}p_1^\beta$, $g^{\alpha\beta}\to\frac{2}{s}p_2^\alpha p_1^\beta$, and get

$$\check W^{\rm l}_{\mu\nu}\ =\ \frac{1}{s^2}g^\perp_{\mu\nu}\big([\bar\psi_A(x)\not{p}_2\psi_A(0)][\bar\psi_B(0)\not{p}_1\psi_B(x)]+[\bar\psi_A(x)\not{p}_2\gamma_5\psi_A(0)][\bar\psi_B(0)\not{p}_1\gamma_5\psi_B(x)]\big)$$
$$+\ \frac{1}{s^2}\big(g^\perp_{\mu\xi}g^\perp_{\nu\eta}+g^\perp_{\nu\xi}g^\perp_{\mu\eta}-g^\perp_{\mu\nu}g^\perp_{\xi\eta}\big)[\bar\psi_A(x)\sigma_{\star\xi}\psi_A(0)][\bar\psi_B(0)\sigma_\bullet^{\ \xi}\psi_B(x)]\ +\ x\leftrightarrow 0 \tag{3.9}$$

where $g^\parallel_{\mu\nu}\equiv\frac{2}{s}\big(p_{1\mu}p_{2\nu}+\mu\leftrightarrow\nu\big)$ and $g^\perp_{\mu\nu}\equiv g_{\mu\nu}-g^\parallel_{\mu\nu}$.

As mentioned above, the dependence of ψ_A on $x_\star$ and ψ_B on $x_\bullet$ is very slow so at the leading-twist order we can replace $\psi_A(x)\to\psi_A(x_\bullet,x_\perp)$ and $\psi_B(x)\to\psi_B(x_\bullet,x_\perp)$ (the corrections will be considered in next Section).

Next, after integration over background fields A and B we promote A, ψ_A and B, ψ_B to operators $\hat A$, $\hat\psi$. A subtle point is that our operators are not under T-product ordering so one should be careful while changing the order of operators in formulas like Fierz transformation. Fortunately, all operators in the r.h.s of Eq. (3.9) and in similar formulas for power corrections are separated either by space-like intervals or light-like intervals so they commute with each other. We get [7]

$$\check W^{\rm lt}_{\mu\nu}\ =\ \frac{1}{s^2}g^\perp_{\mu\nu}\langle\bar\psi(x_\bullet,x_\perp)\not{p}_2\psi(0)\rangle_A\langle\bar\psi(0)\not{p}_1\psi(x_\star,x_\perp)\rangle_B+\ (\not{p}_2\otimes\not{p}_1\leftrightarrow\not{p}_2\gamma_5\otimes\not{p}_1\gamma_5) \tag{3.11}$$
$$+\ \frac{1}{s^2}\big(g^\perp_{\mu\xi}g^\perp_{\nu\eta}+g^\perp_{\nu\xi}g^\perp_{\mu\eta}-g^\perp_{\mu\nu}g^\perp_{\xi\eta}\big)\langle\bar\psi(x_\bullet,x_\perp)\sigma_\star^{\ \xi}\psi(0)\rangle_A\langle\bar\psi(0)\sigma_\bullet^{\ \eta}\psi(x_\star,x_\perp)\rangle_B\ +\ x\leftrightarrow 0$$

Hereafter we use notations $\langle\mathcal{O}\rangle_A\equiv\langle p_A|\mathcal{O}|p_A\rangle$ and $\langle\mathcal{O}\rangle_A\equiv\langle p_B|\mathcal{O}|p_B\rangle$ for brevity. The corresponding leading-twist contribution to to $W_{\mu\nu}(q)$ has the form [13]

$$W^{\rm lt}_{\mu\nu}(\alpha_q,\beta_q,q_\perp)\ =\ \frac{1}{16\pi^4N_c}\int dx_\bullet dx_\star d^2x_\perp\ e^{-i\alpha_qx_\bullet-i\beta_qx_\star+i(q,x)_\perp}\check W^{\rm lt}_{\mu\nu}(x)$$
$$=\ \frac{1}{N_c}\int d^2k_\perp\Big(-g^\perp_{\mu\nu}\big[f_1(\alpha_q,k_\perp)\bar f_1(\beta_q,q_\perp-k_\perp)+\bar f_1(\alpha_q,k_\perp)f_1(\beta_q,q_\perp-k_\perp)\big]$$
$$-\ \frac{1}{m^2}\big[k^\perp_\mu(q-k)^\perp_\nu+k^\perp_\nu(q-k)^\perp_\mu+g^\perp_{\mu\nu}(k,q-k)_\perp\big]$$
$$\times\ \big[h_1^\perp(\alpha_q,k_\perp)\bar h_1^\perp(\beta_q,q_\perp-k_\perp)+\bar h_1^\perp(\alpha_q,k_\perp)h_1^\perp(\beta_q,q_\perp-k_\perp)\big]\Big) \tag{3.12}$$

[7] In a general gauge for projectile and target fields these matrix elements read

$$\langle p_A|\bar\psi(x)\gamma_\mu\psi(0)|p_A\rangle\ =\ \langle p_A|\psi(x)\gamma_\mu[x,x-\infty p_2][x-\infty p_2,-\infty p_2][-\infty p_2,0]\psi(0)|p_A\rangle,$$
$$\langle p_B|\psi(x)\gamma_\mu\psi(0)|p_B\rangle\ =\ \langle p_B|\psi(x)\gamma_\mu[x,x-\infty p_1][x-\infty p_1,-\infty p_1][-\infty p_1,0]\psi(0)|p_B\rangle \tag{3.10}$$

where $[x,y]\equiv{\rm Pexp}\Big\{i\int_0^1du\ x^\mu A_\mu((ux+(1-u)y)\Big\}$, and similarly for other matrix elements.

To compare with general formula (1.1) we need to identify $\alpha_q \equiv x_A$ and $\beta_q \equiv x_B$. To avoid confusion with coordinates, throughout the paper we will keep notations α_q and β_q.

3.2.2 Types of $\frac{1}{Q^2}$ power corrections

Let us outline power corrections calculated in this paper. As I mentioned in the Introduction, only leading-N_c power corrections up to $\frac{1}{Q^2}$ will be taken into account. Specifically, this means the PCs

$$N_cW_{\mu\nu}(q) \ \sim\ g_\perp^{\mu\nu}\Big[1+\frac{q_\perp^2}{Q^2}\Big],\ \ \frac{q_\perp^\mu q_\perp^\nu}{m_\perp^2}\Big[1+\frac{m_\perp^2}{Q^2}\Big],\ \ g_\parallel^{\mu\nu}\Big[0+\frac{m_\perp^2}{Q^2}\Big],$$
$$\frac{1}{Q^2}\big(p_2^\mu q_\perp^\nu+\mu\leftrightarrow\nu\big),\ \ \frac{1}{Q^2}\big(p_2^\mu q_\perp^\nu+\mu\leftrightarrow\nu\big),\ \ \frac{1}{Q^2}\big(p_1^\mu q_\perp^\nu+\mu\leftrightarrow\nu\big),\ \ \frac{p_1^\mu p_1^\nu}{Q^4},\ \ \frac{p_2^\mu p_2^\nu}{Q^4} \tag{3.13}$$

Among those, corrections $\sim p_{1,2}^\mu q_\perp^\nu$ are of order $\frac{1}{Q}$ while the rest is $\sim \frac{1}{Q^2}$. Here $m_\perp^2 \sim q_\perp^2, m^2$. When counting powers of $\frac{1}{Q}$ we do not distinguish between $q_\perp^2, k_\perp^2$ and m^2 but in concrete formulas we keep them different so we can consider, for example, case $q_\perp^2 \gg m^2$. Similarly, parametrically we do not distinguish between s and $Q^2 = \alpha_q\beta_q s - q_\perp^2$ but keep track in our formulas so they are correct both at $s \sim Q^2$ and $s \gg Q^2$.

Let us also specify the terms which are not calculated in this paper. Roughly speaking, they correspond to terms in Eq. (3.13) multiplied by extra power(s) of $\frac{m_\perp}{Q}$ or by extra $\frac{1}{N_c}$. Our strategy in the next sections is to compare a certain term in $\check{W}_{\mu\nu}$ to terms in Eq. (3.13), and, if it is smaller, neglect, and if it is of the same size, calculate.

3.2.3 Power corrections due to quark-antiquark TMDs

As one can see from parametrization in Sect. 9.2.1, the r.h.s. of Eq. (3.7) contains not only the leading-twist contributions (3.12), but also a number of power corrections.

We start from the corrections obtained by expansions

$$\begin{aligned}\psi(x) &= \psi(x_\perp,x_\bullet,0)+x_\star\frac{2}{s}D_\bullet\psi(x_\perp,x_\bullet,0)\ +...,\\ \bar\psi(x) &= \bar\psi(x_\perp,x_\bullet,0)+x_\star\frac{2}{s}\bar\psi\overleftarrow{D}_\bullet(x_\perp,x_\bullet,0)\ +...\end{aligned} \tag{3.14}$$

for projectile matrix elements and

$$\begin{aligned}\psi(x) &= \psi(x_\perp,0,x_\star)+x_\bullet\frac{2}{s}D_\star\psi(x_\perp,0,x_\star)\ +...\\ \bar\psi(x) &= \bar\psi(x_\perp,0,x_\star)+x_\star\frac{2}{s}\bar\psi\overleftarrow{D}_\bullet(x_\perp,0,x_\star)\ +...\end{aligned} \tag{3.15}$$

for the target ones.

Let us show that second terms in these expansions are $\sim \frac{1}{Q^2}$. To this end, note that $D_\bullet \sim \beta_{\rm proj}s \le \sigma_p s$. As discussed in Ref. [12], the natural scales for rapidity factorization outlined in Sect. 2 are $\sigma_p \sim \frac{q_\perp^2}{\alpha_q s}$ and $\sigma_t \sim \frac{q_\perp^2}{\beta_q s}$. Adding estimates $x_\bullet \sim \frac{1}{\alpha_q}$, $x_\star \sim \frac{1}{\beta_q}$ we get

$$x_\star\frac{2}{s}D_\bullet\ \sim\ \frac{1}{\alpha_q}\sigma_p s\ \sim\ \frac{q_\perp^2}{\alpha_q\beta_q s}\ \sim\ \frac{q_\perp^2}{Q^2} \tag{3.16}$$

for the projectile and

$$x_\bullet \frac{2}{s} D_\star \sim \frac{1}{\beta_q}\sigma_t s \sim \frac{q_\perp^2}{\alpha_q\beta_q s} \sim \frac{q_\perp^2}{Q^2} \tag{3.17}$$

for the target matrix elements. This means that no further terms in expansions (3.16), (3.17) are necessary and moreover, the only place where we need these corrections is the leading contributions (3.9). [8]

Before expansions (3.15) it is convenient to use translational invariance and make a shift in Eq. (3.9)

$$\begin{aligned}
\check{W}^{1}_{\mu\nu} \;=\; & \frac{1}{s^2} g^\perp_{\mu\nu} \langle \bar\psi(x_\bullet, x_\perp, \frac{x_*}{2}) \not{p}_2 \psi(-\frac{x_*}{2})\rangle_A \langle \bar\psi(-\frac{x_\bullet}{2}) \not{p}_1 \psi(x_\star, x_\perp, \frac{x_\bullet}{2})\rangle_B \\
& + (\not{p}_2 \otimes \not{p}_1 \leftrightarrow \not{p}_2\gamma_5 \otimes \not{p}_1\gamma_5) + \frac{1}{s^2}(g^\perp_{\mu\xi}g^\perp_{\nu\eta} + g^\perp_{\nu\xi}g^\perp_{\mu\eta} - g^\perp_{\mu\nu}g^\perp_{\xi\eta}) \\
& \times \langle \bar\psi(x_\bullet, x_\perp, \frac{x_*}{2}) \sigma_\star{}^\xi \psi(-\frac{x_*}{2})\rangle_A \langle \bar\psi(-\frac{x_\bullet}{2}) \sigma_\bullet{}^\eta \psi(x_\star, x_\perp, \frac{x_\bullet}{2})\rangle_B \\
& + \Big(\{x_\bullet, x_\perp, \frac{x_*}{2}\} \leftrightarrow \{-\frac{x_*}{2}\}, \{x_*, x_\perp, \frac{x_\bullet}{2}\} \leftrightarrow \{-\frac{x_\bullet}{2}\}\Big) \\
= \;& \check{W}^{\rm l.t.}_{\mu\nu} - \frac{g^\perp_{\mu\nu}}{2s^2}\Big[x_\star \langle \bar\psi(x_\bullet, x_\perp) \not{p}_2 \overset{\leftrightarrow}{D}_\bullet \psi(0)\rangle_A \langle \bar\psi(0) \not{p}_1 \psi(x_\star, x_\perp)\rangle_B \\
& + x_\bullet \langle \bar\psi(x_\bullet, x_\perp) \not{p}_2 \psi(0)\rangle_A \langle \bar\psi(0) \not{p}_1 \overset{\leftrightarrow}{D}_\star \psi(x_\star, x_\perp)\rangle_B + (\not{p}_2 \otimes \not{p}_1 \leftrightarrow \not{p}_2\gamma_5 \otimes \not{p}_1\gamma_5)\Big] \\
& - \frac{1}{2s^2}(g^\perp_{\mu\xi}g^\perp_{\nu\eta} + g^\perp_{\nu\xi}g^\perp_{\mu\eta} - g^\perp_{\mu\nu}g^\perp_{\xi\eta})\Big[x_\star \langle \bar\psi(x_\bullet, x_\perp) \sigma_\star{}^\xi \overset{\leftrightarrow}{D}_\bullet \psi(0)\rangle_A \langle \bar\psi(0) \sigma_\bullet{}^\eta \psi(x_\star, x_\perp)\rangle_B \\
& + x_\bullet \langle \bar\psi(x_\bullet, x_\perp) \sigma_\star{}^\xi \psi(0)\rangle_A \langle \bar\psi(0) \sigma_\bullet{}^\eta \overset{\leftrightarrow}{D}_\star \psi(x_\star, x_\perp)\rangle_B\Big] \;=\; \check{W}^{\rm l.t.}_{\mu\nu} + \check{W}^D_{\mu\nu}
\end{aligned} \tag{3.18}$$

Here

$$\bar\psi(x_\bullet, x_\perp) \not{p}_2 \overset{\leftrightarrow}{D}_\bullet \psi(0) \equiv \bar\psi(x_\bullet, x_\perp) \not{p}_2 D_\bullet \psi(0)\rangle_A - \bar\psi \overset{\leftarrow}{D}_\bullet (x_\bullet, x_\perp) \not{p}_2 \psi(0), \tag{3.19}$$

and similarly for other terms. Using parametrizations (9.54) and (9.55), one easily obtains

$$\begin{aligned}
W^D_{\mu\nu}(\alpha_q, \beta_q, q_\perp) \;=\; & \frac{1}{16\pi^4 N_c}\int dx_\bullet dx_\star d^2x_\perp \; e^{-i\alpha_q x_\bullet - i\beta_q x_\star + i(q,x)_\perp} \check{W}^{\rm lt}_{\mu\nu}(x) \qquad (3.20)\\
= \;& \frac{2}{sN_c}\int d^2k_\perp \Big\{ m^2 g^\perp_{\mu\nu} \big[\Re f_D(\alpha_q, k_\perp) \bar f_1'(\beta_q, q_\perp - k_\perp) + \Re \bar f_D(\alpha_q, k_\perp) f_1'(\beta_q, q_\perp - k_\perp) \\
& \qquad + f_1'(\alpha_q, k_\perp) \Re \bar f_D(\beta_q, q_\perp - k_\perp) + \bar f_1'(\alpha_q, k_\perp) \Re f_D(\beta_q, q_\perp - k_\perp)\big] \\
& + \big[k^\perp_\mu (q-k)^\perp_\nu + k^\perp_\nu (q-k)^\perp_\mu + g^\perp_{\mu\nu}(k, q-k)_\perp\big] \\
& \quad \times \big[\Re h_D(\alpha_q, k_\perp) \bar h'^\perp_1(\beta_q, q_\perp - k_\perp) + \Re \bar h_D(\alpha_q, k_\perp) h'^\perp_1(\beta_q, q_\perp - k_\perp) \\
& \qquad + h'^\perp_1(\alpha_q, k_\perp) \Re \bar h_D(\beta_q, q_\perp - k_\perp) + \bar h'^\perp_1(\alpha_q, k_\perp) \Re h_D(\beta_q, q_\perp - k_\perp)\big]\Big\}
\end{aligned}$$

where $\bar f_1'(\beta_q, q_\perp - k_\perp) \equiv \frac{\partial}{\partial \beta_q} \bar f_1(\beta_q, q_\perp - k_\perp)$ etc.

As was mentioned above, for the rest of $\frac{1}{Q^2}$ corrections one can neglect the dependence of projective fields on x_* and target ones on $x_\bullet$. Using parametrizations in Sect. 9.2.1, one

[8] The author is indebted to A. Vladimirov for clarifying this point.

obtains for quark-antiquark contribution (3.7)

$$W^{q\bar{q}}(\alpha_q,\beta_q,q_\perp) \;=\; \frac{2}{N_c s}\int d^2k_\perp\Big\{-\frac{s}{2}g^\perp_{\mu\nu}\{f_1\bar{f}_1+\bar{f}_1f_1\}\tag{3.21}$$
$$+[k^\perp_\mu(q-k)^\perp_\nu+k^\perp_\nu(q-k)^\perp_\mu+g_{\mu\nu}(k,q-k)_\perp]\{f_\perp\bar{f}_\perp+\bar{f}_\perp f_\perp\}$$
$$+\,(k^\perp_\mu p_{2\nu}+k^\perp_\nu p_{2\mu})\{f_\perp\bar{f}_1+\bar{f}_\perp f_1\}+[(q-k)^\perp_\mu p_{1\nu}+(q-k)^\perp_\nu p_{1\mu}]\{f_1\bar{f}_\perp+\bar{f}_1f_\perp)\}$$
$$+\,\frac{4m^2}{s}p_{1\mu}p_{1\nu}\{f_1\bar{f}_3+\bar{f}_1f_3\}+\frac{4m^2}{s}p_{2\mu}p_{2\nu}\{f_3\bar{f}_1+\bar{f}_3f_1\}\Big\}$$
$$+\,[(g^\parallel_{\mu\nu}-g^\perp_{\mu\nu})(k,q-k)_\perp-k^\perp_\mu(q-k)^\perp_\nu-k^\perp_\nu(q-k)^\perp_\mu]\{g^\perp\bar{g}^\perp+\bar{g}^\perp g^\perp\}-m^2g_{\mu\nu}\{e\bar{e}+\bar{e}e\}$$
$$-\frac{s}{2m^2}\big(g^\perp_{\mu\nu}+k^\perp_\mu(q-k)^\perp_\nu+k^\perp_\nu(q-k)^\perp_\mu\big)\{h^\perp_1\bar{h}^\perp_1+\bar{h}^\perp_1h^\perp_1\}+m^2\big(g^\perp_{\mu\nu}-g^\parallel_{\mu\nu}\big)\{h\bar{h}+\bar{h}h\}$$
$$+\,\big(k^\perp_\mu p_{1\nu}+k^\perp_\nu p_{1\mu}\big)\{h^\perp_1\bar{h}+\bar{h}^\perp_1h\}+\big[p_{2\mu}(q-k)^\perp_\nu+p_{2\nu}(q-k)^\perp_\mu\big]\{h\bar{h}^\perp_1+\bar{h}h^\perp_1\}$$
$$+\,\frac{4}{s}(k,q-k)_\perp\big[p_{1\mu}p_{1\nu}\{h^\perp_1\bar{h}^\perp_3+\bar{h}^\perp_1h^\perp_3\}+p_{2\mu}p_{2\nu}\{h^\perp_3\bar{h}^\perp_1+\bar{h}^\perp_3h^\perp_1\}\big]$$
$$-\,\frac{2}{s}m^2\big[(k,q-k)_\perp(g^\perp_{\mu\nu}+k^\perp_\mu(q-k)^\perp_\nu+k^\perp_\nu(q-k)^\perp_\mu\big]\{h^\perp_3\bar{h}^\perp_3+\bar{h}^\perp_3h^\perp_3\}\Big]$$

Hereafter we use the notation

$$\{f_1\bar{f}_2+\bar{f}_1f_2\} \;\equiv\; f_1(\alpha_q,k_\perp)\bar{f}_2(\beta_q,q_\perp-k_\perp)+\bar{f}_1(\alpha_q,k_\perp)f_2(\beta_q,q_\perp-k_\perp)\tag{3.22}$$

so that the argument of the first function is always $(\alpha_q,k_\perp)$ and that of the second is $(\beta_q,q_\perp-k_\perp)$, similarly to the leading-twist contribution (3.12) which we included for completeness.

4 Terms with one quark-quark-gluon operator Ξ_i or $\bar{\Xi}_i$

We separate terms in Eq. (3.6) according to the number of gluon fields (contained in Ξ's).

$$\check{W}_{\mu\nu} \;\overset{\text{sym }\mu,\nu}{=}\; \check{W}^{\rm lt}_{\mu\nu}+\check{W}^{(1)}_{\mu\nu}+\check{W}^{(1')}_{\mu\nu}+\check{W}^{(2a)}_{\mu\nu}+\check{W}^{(2b)}_{\mu\nu}+\check{W}^{(2c)}_{\mu\nu}\tag{4.1}$$

where leading-twist terms without gluons (quark-antiquark TMDs) were considered in previous Section, and

$$\check{W}^{(1)}_{\mu\nu}(x) \;=\; \frac{N_c}{s}\langle A,B|\big[\bar\psi_A(x)\gamma_\mu\psi_B(x)\big]\big[\bar\psi_B(0)\gamma_\nu\Xi_1(0)\big]$$
$$+\;\big[\bar\Xi_1(x)\gamma_\mu\psi_B(x)\big]\big[\bar\psi_B(0)\gamma_\nu\psi_A(0)\big]+\big[\bar\psi_A(x)\gamma_\mu\Xi_2(x)\big]\big[\bar\psi_B(0)\gamma_\nu\psi_A(0)\big]$$
$$+\;\big[\bar\psi_A(x)\gamma_\mu\psi_B(x)\big]\big[\bar\Xi_2(0)\gamma_\nu\psi_A(0)\big]+\mu\leftrightarrow\nu|A,B\rangle\;+\;x\leftrightarrow 0\tag{4.2}$$

$$\check{W}^{(1')}_{\mu\nu}(x) \;=\; \frac{N_c}{s}\langle A,B|\big[\bar\psi_A(x)\gamma_\mu\psi_B(x)\big]\big[\bar\psi_B(0)\gamma_\nu\Xi'_1(0)\big]$$
$$+\;\big[\bar\Xi'_1(x)\gamma_\mu\psi_B(x)\big]\big[\bar\psi_B(0)\gamma_\nu\psi_A(0)\big]+\big[\bar\psi_A(x)\gamma_\mu\Xi'_2(x)\big]\big[\bar\psi_B(0)\gamma_\nu\psi_A(0)\big]$$
$$+\;\big[\bar\psi_A(x)\gamma_\mu\psi_B(x)\big]\big[\bar\Xi'_2(0)\gamma_\nu\psi_A(0)\big]+\mu\leftrightarrow\nu|A,B\rangle\;+\;x\leftrightarrow 0\tag{4.3}$$

and

$$\check{W}^{(2a)}_{\mu\nu}(x) \;=\; \frac{N_c}{s}\langle A,B|\big[\bar\psi_A(x)\gamma_\mu\Xi_2(x)\big]\big[\bar\psi_B(0)\gamma_\nu\Xi_1(0)\big]$$
$$+\;\big[\bar\Xi_1(x)\gamma_\mu\psi_B(x)\big]\big[\bar\Xi_2(0)\gamma_\nu\psi_A(0)\big]+\mu\leftrightarrow\nu|A,B\rangle\;+\;x\leftrightarrow 0\tag{4.4}$$

$$\check W^{(2b)}_{\mu\nu}(x) \;=\; \frac{N_c}{s}\langle A,B|\big[\bar\psi_A(x)\gamma_\mu\Xi_2(x)\big]\big[\bar\Xi_2(0)\gamma_\nu\psi_A(0)\big]$$
$$+\;\big[\bar\Xi_1(x)\gamma_\mu\psi_B(x)\big]\big[\bar\psi_B(0)\gamma_\nu\Xi_1(0)\big]+\mu\leftrightarrow\nu|A,B\rangle \;+\; x\leftrightarrow 0 \tag{4.5}$$

$$\check W^{(2c)}_{\mu\nu}(x) \;=\; \frac{N_c}{s}\langle A,B|\big[\bar\Xi_1(x)\gamma_\mu\Xi_2(x)\big]\big[\bar\psi_B(0)\gamma_\nu\psi_A(0)\big]$$
$$+\;\big[\bar\psi_A(x)\gamma_\mu\psi_B(x)\big]\big[\bar\Xi_2(0)\gamma_\nu\Xi_1(0)\big]+\mu\leftrightarrow\nu|A,B\rangle \;+\; x\leftrightarrow 0 \tag{4.6}$$

The corresponding contributions to $W_{\mu\nu}(q)$ will be denoted $W^{(1)}_{\mu\nu}$, $W^{(1')}_{\mu\nu}$, $W^{(2)a}_{\mu\nu}$, $W^{(2)b}_{\mu\nu}$, and $W^{(2)c}_{\mu\nu}$, respectively. In this and next two Sections, I will consider these contributions in turn. Whenever possible, I will refer to calculations in Ref. [9] to pinpoint terms which can be safely neglected. The calculations in this paper are very similar to those of Ref. [9] but much more lengthly. The result is presented in Sect. 7.

4.1 Terms with Ξ_1

Let us start with the first term in Eq. (4.2). The Fierz transformation (9.1) yields

$$\begin{aligned}
&\frac{1}{2}[\bar\psi_A(x)\gamma_\mu\psi_B(x)]\big[\bar\psi_B(0)\gamma_\nu\Xi_1(0)\big] \;+\; \mu\leftrightarrow\nu\\
&=\;\frac{g_{\mu\nu}}{4}\big\{\big[\bar\psi^m_A(x)\frac{\not{p}_2}{s}\gamma^i\frac{1}{\alpha}\psi^k_A(0)\big]\big[\bar\psi^n_B\bar B^{nk}_i(0)\psi^m_B(x)\big] \;-\; (\psi^k_A\otimes\psi^n_B\leftrightarrow\gamma_5\psi^k_A\otimes\gamma_5\psi^n_B\big\}\\
&+\;\frac{1}{4}(\delta^\alpha_\mu\delta^\beta_\nu+\delta^\alpha_\nu\delta^\beta_\mu-g_{\mu\nu}g^{\alpha\beta})\\
&\times\;\big\{\big[\bar\psi^m_A(x)\gamma_\alpha\frac{\not{p}_2}{s}\gamma^i\frac{1}{\alpha}\psi^k_A(0)\big]\big[\bar\psi^n_B\bar B^{nk}_i(0)\gamma_\beta\psi^m_B(x)\big] \;+\; (\gamma_\alpha\otimes\gamma_\beta\leftrightarrow\gamma_\alpha\gamma_5\otimes\gamma_\beta\gamma_5)\big\}\\
&-\;\frac{1}{4}(\delta^\alpha_\mu\delta^\beta_\nu+\delta^\alpha_\nu\delta^\beta_\mu-\frac{1}{2}g_{\mu\nu}g^{\alpha\beta})\big[\bar\psi^m_A(x)\sigma_{\alpha\xi}\frac{\not{p}_2}{s}\gamma^i\frac{1}{\alpha}\psi^k_A(0)\big]\big[\bar\psi^n_B\bar B^{nk}_i(0)\sigma_\beta^{\ \xi}\psi^m_B(x)\big]
\end{aligned}\tag{4.7}$$

where we used Eq. (2.4) $\Xi_1(0)=-\frac{\not{p}_2}{s}\gamma^i\bar B_i\frac{1}{\alpha}\psi_A(0)$. To save space, from now on we use the notations $A\psi(x)\equiv A(x)\psi(x)$ and $\bar\psi A(x)\equiv\bar\psi(x)A(x)$. Note that all colors are in the fundamental representation so e.g. $B^{mn}(x)\equiv(t_a)^{mn}B^a(x)$.

Promoting A and B fields to operators and sorting out the color-singlet contributions we get

$$\check W^{(1\Xi_1)}_{\mu\nu}(x) \;=\; \frac{N_c}{s}\langle A,B|[\bar\psi_A(x)\gamma_\mu\psi_B(x)]\big[\bar\psi_B(0)\gamma_\nu\Xi_1(0)\big] \;+\; \mu\leftrightarrow\nu|A,B\rangle \;+\; x\leftrightarrow 0 \tag{4.8}$$
$$=\; g_{\mu\nu}\check U^{(1a)}(x)+(\delta^\alpha_\mu\delta^\beta_\nu+\delta^\alpha_\nu\delta^\beta_\mu-g_{\mu\nu}g^{\alpha\beta})\check U^{(1b)}_{\alpha\beta}(x)+(\delta^\alpha_\mu\delta^\beta_\nu+\delta^\alpha_\nu\delta^\beta_\mu-\frac{1}{2}g_{\mu\nu}g^{\alpha\beta})\check U^{(1c)}_{\alpha\beta}(x)$$

where

$$\begin{aligned}
\check U^{(1a)}(x) \;&=\; \frac{1}{2s^2}\big[\langle\bar\psi(x)\not{p}_2\gamma^i\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi\bar B_i(0)\psi(x)]\rangle_B \;-\; (\psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\big]\\
&\quad+\;x\leftrightarrow 0\\
\check U^{(1b)}_{\alpha\beta}(x) \;&=\; \frac{1}{4s^2}\big[\langle\bar\psi(x)\gamma_\alpha\not{p}_2\gamma_i\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi B^i(0)\gamma_\beta\psi(x)\rangle_B\\
&\quad+\;(\psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)+\alpha\leftrightarrow\beta\big] \;+\; x\leftrightarrow 0\\
\check U^{(1c)}_{\alpha\beta}(x) \;&=\; -\big[\frac{1}{4s^2}\langle\bar\psi(x)\sigma_{\alpha\xi}\not{p}_2\gamma^i\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi B^i(0)\sigma_\beta^{\ \xi}\psi(x)\rangle_B+\alpha\leftrightarrow\beta\big] \;+\; x\leftrightarrow 0
\end{aligned}\tag{4.9}$$

4.1.1 Term $\check{U}^{(1a)}$

It is easy to see that

$$\check{U}^{(1a)}(x) \;=\; \frac{1}{2s^2}\big\{\langle\bar\psi(x)\not{p}_2\gamma^i\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi\bar{B}_i(0)\psi(x)]\rangle_A$$
$$-\;(\psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\big\}\;+\;x\leftrightarrow 0$$
$$=\;-i\frac{1}{2s^2}\langle\bar\psi(x)\sigma_{\star i}\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi[B_i(0)+i\tilde{B}_i\gamma_5(0)]\psi(x)\rangle_A\;+\;x\leftrightarrow 0 \tag{4.10}$$

where we used formula

$$\sigma_{\star i}\otimes B^i-\sigma_{\star i}\gamma_5\otimes B^i\gamma_5\;=\;\sigma_{\star i}\otimes\acute{B}^i \tag{4.11}$$

Throughout the paper we will use the notations

$$\acute{A}_i\equiv A_i+i\tilde{A}_i\gamma_5,\qquad \grave{A}\equiv A_i-i\tilde{A}_i\gamma_5,\qquad \acute{B}_i\equiv B_i+i\tilde{B}_i\gamma_5,\qquad \grave{B}\equiv B_i-i\tilde{B}_i\gamma_5 \tag{4.12}$$

From formula (9.41) and parametrization (9.58) from Appendix 9.2.2 we get

$$U^{(1a)}(q)\;=\;\frac{1}{16\pi^4N_c}\int dx_\star dx_\bullet d^2x_\perp\; e^{-i\alpha_qx_\bullet-i\beta_qx_\star+i(q,x)_\perp}\check{U}^{(1a)}(x) \tag{4.13}$$
$$=\;-i\frac{1}{\alpha_qsN_c}\int d^2k_\perp\;(k,q-k)_\perp\{h_1^\perp\bar{\acute{e}}_G+\bar{h}_1^\perp\acute{e}_G\}$$

4.1.2 Term $\check{U}^{(1b)}_{\alpha\beta}$

In this section we consider

$$\check{U}^{(1b)}_{\alpha\beta}(x)\;=\;\frac{1}{4s^2}\big\{\langle\bar\psi(x)\gamma_\alpha\not{p}_2\gamma_i\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi B^i(0)\gamma_\beta\psi(x)\rangle_B \tag{4.14}$$
$$+\;\big(\psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\big)\;+\;\alpha\leftrightarrow\beta\big\}\;+\;x\leftrightarrow 0\;=\;\check{U}^{(1b)}_{1\alpha\beta}(x)+\check{U}^{(1b)}_{1\alpha\beta}(x\leftrightarrow 0)$$

Let us start from the first term in the r.h.s. of this equation. From Eq. (9.18)

$$\check{U}^{(1b)}_{1\alpha\beta}(x)\;=\;\frac{1}{4s^2}\big\{\langle\bar\psi(x)\gamma_\alpha\not{p}_2\gamma_i\frac{1}{\alpha}\psi(0)\rangle_A \tag{4.15}$$
$$\times\;\langle\bar\psi B^i(0)\gamma_\beta\psi(x)\rangle_B\;+\;(\psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\big\}\;+\;\alpha\leftrightarrow\beta$$
$$=\;\frac{1}{4s^2}\Big\{-\langle\bar\psi(x)\not{p}_2\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi\gamma_{\beta_\perp}\grave{B}_\alpha(0)\psi(x)\rangle_B-\frac{2}{s}p_{2\alpha}p_{2\beta}\langle\bar\psi(x)\gamma_i\frac{1}{\alpha}\psi(0)\rangle_A$$
$$\times\;\langle\bar\psi\not{B}(0)\not{p}_1\gamma^i\psi(x)\rangle_B-\frac{2}{s}p_{2\alpha}p_{2\beta}\langle\bar\psi(x)\gamma_i\gamma_5\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi\not{B}(0)\not{p}_1\gamma^i\gamma_5\psi(x)\rangle_B$$
$$+\;\frac{2}{s}p_{2\beta}\langle\bar\psi(x)\not{p}_2\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi\not{B}(0)\not{p}_1\gamma_{\alpha_\perp}\psi(x)\rangle_B\;+\;\alpha\leftrightarrow\beta\Big\},$$

From Eqs. (9.45) and (9.65) we get

$$
\begin{aligned}
U^{(1b)}_{1\alpha\beta}(q) \;&=\; \frac{1}{16\pi^4 N_c}\int dx_\star dx_\bullet d^2x_\perp\; e^{-i\alpha_q x_\bullet - i\beta_q x_\star + i(q,x)_\perp}\check{U}^{(1b)}_{1\alpha\beta}(x) && (4.16)\\
&= \frac{1}{2\alpha_q s}\int d^2k_\perp\Big[(q-k)_{\alpha_\perp}p_{2\beta}f_1(\alpha_q,k_\perp)[\bar{f}_1^\perp - \beta_q(\bar{f}_\perp - i\bar{g}_\perp)](\beta_q,q_\perp-k_\perp)\\
&- f_1(\alpha_q,k_\perp)\bar{\bar{f}}_{1G}(\beta_q,q_\perp-k_\perp)\big[(q-k)_\alpha(q-k)_\beta + (q-k)^2_\perp\frac{g^\perp_{\alpha\beta}}{2}\big]\\
&- \frac{g^\perp_{\alpha\beta}}{2}f_1(\alpha_q,k_\perp)\big[(q-k)^2_\perp(\bar{f}+i\bar{g}) - 2\beta_q m^2\bar{f}_3\big](\beta_q,q_\perp-k_\perp)\\
&+ \frac{2}{s}p_{2\alpha}p_{2\beta}(k,q-k)_\perp[f_\perp + ig_\perp](\alpha_q,k_\perp)\big(\bar{f}_1^\perp - \beta_q[\bar{f}_\perp - i\bar{g}_\perp]\big)(\beta_q,q_\perp-k_\perp)\Big] \;+\; \alpha\leftrightarrow\beta
\end{aligned}
$$

where for brevity

$$
[\bar{f}_1^\perp - \beta_q(\bar{f}_\perp - i\bar{g}_\perp)](\beta_q,q_\perp-k_\perp) \equiv \bar{f}_1^\perp - \beta_q\Big(\bar{f}_\perp(\beta_q,q_\perp-k_\perp) - i\bar{g}_\perp(\beta_q,q_\perp-k_\perp)\Big) \quad (4.17)
$$

and similarly for other terms here and throughout the paper.

Next,

$$
\begin{aligned}
\check{U}^{(1b)}_{2\alpha\beta}(x) \;&\equiv\; \check{U}^{(1b)}_{1\alpha\beta}(x\leftrightarrow 0) && (4.18)\\
&= \frac{1}{4s^2}\big\{\langle\bar\psi(0)\gamma_\alpha \not{p}_2\gamma_i\frac{1}{\alpha}\psi(x)\rangle_A\langle\bar\psi B^i(x)\gamma_\beta\psi(0)\rangle_B\\
&\qquad + (\psi(x)\otimes\psi(0)\leftrightarrow\gamma_5\psi(x)\otimes\gamma_5\psi(0)\big\} \;+\; \alpha\leftrightarrow\beta\\
&= \Big\{\langle\bar\psi(0)\not{p}_2\psi(x)\rangle_A\langle\bar\psi\gamma_{\beta_\perp}\grave{B}_\alpha(x)\psi(0)\rangle_B + \frac{2}{s}p_{2\alpha}p_{2\beta}\langle\bar\psi(0)\gamma_i\psi(x)\rangle_A\langle\bar\psi\not{B}(x)\not{p}_1\gamma^i\psi(0)\rangle_B\\
&\qquad + \frac{2}{s}p_{2\alpha}p_{2\beta}\langle\bar\psi(0)\gamma_i\gamma_5\psi(x)\rangle_A\langle\bar\psi\not{B}(x)\not{p}_1\gamma^i\gamma_5\psi(0)\rangle_B\\
&\qquad - \frac{2}{s}p_{2\beta}\langle\bar\psi(0)\not{p}_2\psi(x)\rangle_A\langle\bar\psi\not{B}(x)\not{p}_1\gamma_{\alpha_\perp}\psi(0)\rangle_B\;\Big\}\frac{1}{4s^2\alpha_q} \;+\; \alpha\leftrightarrow\beta
\end{aligned}
$$

Now, from Eqs. (9.65) and (9.47) we get

$$
\begin{aligned}
U^{(1b)}_{2\alpha\beta}(q) \;&\equiv\; \frac{1}{16\pi^4 N_c}\int dx_\star dx_\bullet d^2x_\perp\; e^{-i\alpha_q x_\bullet - i\beta_q x_\star + i(q,x)_\perp}\check{U}^{(1b)}_{2\alpha\beta}(x)\\
&= \frac{1}{2\alpha_q s N_c}\int d^2k_\perp\Big[-\bar{f}_1^\perp(\alpha_q,k_\perp)\grave{f}_{1G}(\beta_q,q_\perp-k_\perp)\big[(q-k)^\perp_\alpha(q-k)^\perp_\beta + (q-k)^2_\perp\frac{g^\perp_{\alpha\beta}}{2}\big]\\
&- \frac{g^\perp_{\alpha\beta}}{2}\bar{f}_1^\perp(\alpha_q,k_\perp)\big[(q-k)^2_\perp(f-ig) - 2\beta_q m^2 f_3\big](\beta_q,q_\perp-k_\perp)\\
&+ \frac{2}{s}p_{2\alpha}p_{2\beta}(k,q-k)_\perp\big(\bar{f}_\perp - i\bar{g}_\perp\big)(\alpha_q,k_\perp)\big(f_1 - \beta_q[f_\perp + ig_\perp](\beta_q,q_\perp-k_\perp)\big)\\
&+ (q-k)_{\alpha_\perp}p_{2\beta}\bar{f}_1(\alpha_q,k_\perp)[f_1 - \beta_q(f_\perp + ig_\perp)](\beta_q,q_\perp-k_\perp)\Big] \;+\; \alpha\leftrightarrow\beta && (4.19)
\end{aligned}
$$

Finally,

$$U^{(1b)}_{\alpha\beta}(q) \;=\; \frac{1}{16\pi^4 N_c}\int dx_\star dx_\bullet d^2x_\perp\; e^{-i\alpha_q x_\bullet - i\beta_q x_\star + i(q,x)_\perp}\big(\breve{U}^{(1b)}_{1\alpha\beta}(x) + \breve{U}^{(1b)}_{2\alpha\beta}(x)\big) \qquad (4.20)$$

$$= \frac{1}{2\alpha_q s N_c}\int d^2k_\perp\Big\{(q-k)_{\alpha_\perp} p_{2\beta}\{f_1[\bar{f}_1 - \beta_q(\bar{f}_\perp - i\bar{g}_\perp)] + \bar{f}_1[f_1 - \beta_q(f_\perp + ig_\perp)]\}$$

$$- \big[(q-k)_\alpha (q-k)_\beta + (q-k)^2_\perp \frac{g^\perp_{\alpha\beta}}{2}\big]\{f_1 \bar{\grave{f}}_{1G} + \bar{f}_1 \grave{f}_{1G}\}$$

$$- \frac{g^\perp_{\alpha\beta}}{2}\Big((q-k)^2_\perp\{f_1[\bar{f} + i\bar{g}] + \bar{f}_1[f - ig]\} - 2\beta_q m^2\{f_1\bar{f}_3 + \bar{f}_1 f_3\}\Big)$$

$$+ \frac{2}{s}p_{2\alpha}p_{2\beta}(k,q-k)_\perp\Big[\{[f_\perp + ig_\perp]\bar{f}_1 + [\bar{f}_\perp - i\bar{g}_\perp]f_1\}$$

$$- \beta_q\{[f_\perp + ig_\perp][\bar{f}_\perp - i\bar{g}_\perp] + [\bar{f}_\perp - i\bar{g}_\perp][f_\perp + ig_\perp]\}\Big]\Big\} \;+\; \alpha \leftrightarrow \beta$$

where we used notation (3.22).

4.1.3 Term $\breve{U}^{(1c)}_{\alpha\beta}$

Next, consider

$$\breve{U}^{(1c)}_{\alpha\beta}(x) \;=\! -\; \frac{1}{4s^2}\big[\langle\bar\psi(x)\sigma_{\alpha\xi}\not{p}_2\gamma^i\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi B_i(0)\sigma_\beta^{\ \xi}\psi(x)\rangle_B \;+\; \alpha\leftrightarrow\beta\big] \;+\; x\leftrightarrow 0 \quad (4.21)$$

From Eq. (9.8) we get

$$\breve{U}^{(1c)}_{1\alpha\beta}(x) \;=\; \frac{1}{4s^2}\langle\bar\psi(x)i\sigma_{\alpha\xi}\sigma_{\star i}\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi B^i(0)\sigma_\beta^{\ \xi}\psi(x)\rangle_B \;+\; \alpha\leftrightarrow\beta$$

$$= \frac{1}{4s^2}\Big\{\langle\bar\psi(x)\sigma_{\star j}\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi B_\alpha(0)\sigma_{\beta_\perp}^{\ j}\psi(x)\rangle_B + i\langle\bar\psi(x)\sigma_{\star\alpha_\perp}\frac{1}{\alpha}\psi(0)\rangle_A$$

$$\times\langle\bar\psi(\not{B}\gamma_{\beta_\perp} - B_\beta)(0)\psi(x)\rangle_B + \frac{2i}{s}p_{2\beta}\langle\bar\psi(x)\sigma_{\star\alpha_\perp}\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi\not{B}(0)\not{p}_1\psi(x)\rangle_B$$

$$+ \frac{2}{s}p_{2\alpha}p_{2\beta}\langle\bar\psi(x)\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi\not{B}(0)\not{p}_1\psi(x)\rangle_B - \frac{4i}{s^2}p_{2\alpha}p_{2\beta}\langle\bar\psi(x)\sigma_{\bullet *}\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi\not{B}(0)\not{p}_1\psi(x)\rangle_B$$

$$+ \frac{4}{s^2}\big(p_{1\alpha}p_{2\beta} + p_{2\alpha}p_{1\beta}\big)\langle\bar\psi(x)\sigma_{\star i}\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi B^i(0)\sigma_{\star\bullet}\psi(x)\rangle_B\Big\} \;+\; \alpha\leftrightarrow\beta \qquad (4.22)$$

where we used the fact that

$$\langle\bar\psi(0)[B_i(0)\sigma_{\bullet j} - B_j(0)\sigma_{\bullet i}]\psi(x)\rangle_A = 0 \qquad (4.23)$$

for unpolarized hadrons. Next, from Eq. (9.62) and Eq. (9.59) we obtain

$$U^{(1c)}_{1\alpha\beta}(q) \;\equiv\; \frac{1}{16\pi^4 N_c}\int dx_\star dx_\bullet d^2x_\perp\; e^{-i\alpha_q x_\bullet - i\beta_q x_\star + i(q,x)_\perp}\breve{U}^{(1c)}_{1\alpha\beta}(x) \qquad (4.24)$$

$$= \frac{1}{2\alpha_q s N_c}.\int d^2k_\perp\Big[(k,q-k)_\perp g^\perp_{\alpha\beta}h_1^\perp(\alpha_q,k_\perp)[i\bar{e} - i\bar{e}_G + \beta_q\bar{h}_3^\perp + \bar{h}_D](\beta_q,q_\perp - k_\perp)$$

$$- p_{2\beta}k^\perp_\alpha\frac{1}{m^2}h_1^\perp(\alpha_q,k_\perp)\big[(q-k)^2_\perp\bar{h}_1^\perp + m^2\beta_q(i\bar{e} + \bar{h})\big](\beta_q,q_\perp - k_\perp)$$

$$+ \frac{2}{s}p_{2\alpha}p_{2\beta}\; e(\alpha_q,k_\perp)\big[-i(q-k)^2_\perp\bar{h}_1^\perp + \beta_q m^2(\bar{e} - i\bar{h})\big](\beta_q,q_\perp - k_\perp)$$

$$+ \frac{2}{s}p_{2\alpha}p_{2\beta}h(\alpha_q,k_\perp)\big[(q-k)^2_\perp\bar{h}_1^\perp + \beta_q m^2(i\bar{e} + \bar{h})\big](\beta_q,q_\perp - k_\perp)$$

$$- \frac{2}{s}\big(p_{1\alpha}p_{2\beta} + p_{2\alpha}p_{1\beta}\big)(k,q-k)_\perp h_1^\perp(\alpha_q,k_\perp)\big[\beta\bar{h}_3^\perp - \bar{h} - \bar{h}_D - i\bar{\tilde{e}}_G\big](\beta_q,q_\perp - k_\perp) + \alpha\leftrightarrow\beta$$

Now consider

$$\check{U}^{(1c)}_{2\alpha\beta}(x) \;=\; \check{U}^{(1c)}_{1\alpha\beta}(x\leftrightarrow 0) \;=\; \frac{1}{4s^2}\langle\bar\psi(0)i\sigma_{\alpha\xi}\sigma_{\star i}\frac{1}{\alpha}\psi(x)\rangle_A\langle\bar\psi B^i(x)\sigma_\beta{}^\xi\psi(0)\rangle_B \;+\; \alpha\leftrightarrow\beta$$
$$= \frac{1}{4s^2}\Big\{\langle\bar\psi(0)\sigma_{\star j}\frac{1}{\alpha}\psi(x)\rangle_A\langle\bar\psi B_\alpha(x)\sigma_{\beta_\perp}^{\;j}\psi(0)\rangle_B + i\langle\bar\psi(0)\sigma_{\star\alpha_\perp}\frac{1}{\alpha}\psi(x)\rangle_A$$
$$\times\langle\bar\psi(\not{B}\gamma_{\beta_\perp} - B_\beta)(x)\psi(0)\rangle_B + \frac{2i}{s}p_{2\beta}\langle\bar\psi(0)\sigma_{\star\alpha_\perp}\frac{1}{\alpha}\psi(x)\rangle_A\langle\bar\psi\not{B}(x)\not{p}_1\psi(0)\rangle_B$$
$$+\frac{2}{s}p_{2\alpha}p_{2\beta}\langle\bar\psi(0)\frac{1}{\alpha}\psi(x)\rangle_A\langle\bar\psi\not{B}(x)\not{p}_1\psi(0)\rangle_B - \frac{4i}{s^2}p_{2\alpha}p_{2\beta}\langle\bar\psi(0)\sigma_{\bullet *}\frac{1}{\alpha}\psi(x)\rangle_A\langle\bar\psi\not{B}(x)\not{p}_1\psi(0)\rangle_B$$
$$+\frac{4}{s^2}\big(p_{1\alpha}p_{2\beta}+p_{2\alpha}p_{1\beta}\big)\langle\bar\psi(0)\sigma_{\star i}\frac{1}{\alpha}\psi(x)\rangle_A\langle\bar\psi B^i(x)\sigma_{\star\bullet}\psi(0)\rangle_B\Big\} \;+\; \alpha\leftrightarrow\beta$$

where we again used Eq. (4.23) for unpolarized hadrons.

Using Eqs. (9.62), (9.63), and (9.59) we get

$$U^{(1c)}_{2\alpha\beta}(q) \;=\; \frac{1}{16\pi^4N_c}\int dx_\star dx_\bullet d^2x_\perp\; e^{-i\alpha_qx_\bullet - i\beta_qx_\star + i(q,x)_\perp}\check{W}^{(1c)}_{2\alpha\beta}(x) \tag{4.25}$$
$$= \frac{1}{2\alpha_q sN_c}\int d^2k_\perp\Big[-i(k,q-k)_\perp g^\perp_{\alpha\beta}\bar h_1^\perp(\alpha_q,k_\perp)(e+e_G+i\beta_qh_3^\perp - ih_D)(\beta_q,q_\perp-k_\perp)$$
$$-k^\perp_\alpha p_{2\beta}\frac{1}{m^2}\bar h_1^\perp(\alpha_q,k_\perp)\big[(q-k)^2_\perp h_1^\perp + \beta_qm^2(h-ie)\big](\beta_q,q_\perp-k_\perp)$$
$$+\frac{p_{2\alpha}p_{2\beta}}{s}i\bar e(\alpha_q,k_\perp)\big[(q-k)^2_\perp h_1^\perp + m^2\beta_q(h-ie)\big](\beta_q,q_\perp-k_\perp)$$
$$+\frac{2}{s}p_{2\alpha}p_{2\beta}\bar h(\alpha_q,k_\perp)\big[(q-k)^2_\perp h_1^\perp + m^2\beta_q(h-ie)\big](\beta_q,q_\perp-k_\perp)$$
$$-\frac{2}{s}\big(p_{1\alpha}p_{2\beta}+p_{2\alpha}p_{1\beta}\big)\bar h_1^\perp(\alpha_q,k_\perp)\big[\beta h_3^\perp - h - i\tilde e_G + h_D\big](\beta_q,q_\perp-k_\perp) \;+\; \alpha\leftrightarrow\beta$$

The sum of Eqs. (4.24) and (4.25) is

$$U^{(1c)}_{\alpha\beta}(q) \;=\; \frac{1}{16\pi^4N_c}\int dx_\star dx_\bullet d^2x_\perp\; e^{-i\alpha_qx_\bullet - i\beta_qx_\star + i(q,x)_\perp}\check{U}^{(1c)}_{\alpha\beta}(x) \;=\; \frac{1}{2\alpha_q sN_c} \tag{4.26}$$
$$\times\int d^2k_\perp\Big\{(k,q-k)_\perp g^\perp_{\alpha\beta}\Big(\{h_1^\perp[i\bar e - i\bar e_G + \beta_q\bar h_3^\perp + \bar h_D] + \bar h_1^\perp[-ie - ie_G + \beta_qh_3^\perp - h_D]\}\Big)$$
$$-p_{2\alpha}k^\perp_\beta\Big(\frac{(q-k)^2_\perp}{m^2}\{h_1^\perp\bar h_1^\perp + \bar h_1^\perp h_1^\perp\} + \beta_q\{h_1[\bar h + i\bar e] + \bar h_1[h-ie]\}\Big) + \frac{2}{s}p_{2\alpha}p_{2\beta}$$
$$\times\Big((q-k)^2_\perp\{[h-ie]\bar h_1^\perp + [\bar h + i\bar e]h_1^\perp\} + \beta_qm^2\{[h-ie][\bar h + i\bar e] + [\bar h + i\bar e][h-ie]\}\Big)$$
$$-g^\parallel_{\alpha\beta}(k,q-k)_\perp\{h_1^\perp[\beta\bar h_3^\perp - \bar h - i\bar{\tilde e}_G - \bar h_D] + \bar h_1^\perp[\beta h_3^\perp - h - i\tilde e_G + h_D]\}\Big\} \;+\; \alpha\leftrightarrow\beta$$

Term with $\check{W}^{(1\Xi_1)}_{\mu\nu}(q)$ is given by the sum of Eqs. (4.13), (4.20), and (4.26).

4.2 Terms with $\bar\Xi_1$, Ξ_2 and $\bar\Xi_2$

Let us consider now the second term in Eq. (4.2). from

$$\Big\{\int dx\; e^{-iqx}\big[\bar\Xi_1(x)\gamma_\mu\psi_B(x)\big]\big[\bar\psi_B(0)\gamma_\nu\psi_A(0)\big]\Big\}^*$$
$$= \int dx\; e^{-iqx}\big[\bar\psi_A(x)\gamma_\nu\psi_B(x)\big]\big[\bar\psi_B(0)\gamma_\mu\Xi_1(0)\big] \tag{4.27}$$

we see that effectively addition of the term $\check{W}^{(1\bar{\Xi}_1)}_{\mu\nu}(q)$ doubles the real part of $\check{W}^{(1\Xi_1)}_{\mu\nu}(q)$ so one obtains

$$
\begin{aligned}
W^{(1\Xi_1+1\bar{\Xi}_1)}_{\mu\nu}(q) \;&=\; \frac{1}{16\pi^4 N_c}\int dx_\star dx_\bullet d^2x_\perp \; e^{-i\alpha_q x_\bullet - i\beta_q x_\star + i(q,x)_\perp}\big[\check{W}^{(1\Xi_1)}_{\mu\nu}(x) + \check{W}^{(1\bar{\Xi}_1)}_{\mu\nu}(x)\big]\\
&=\; \frac{1}{16\pi^4 N_c}\int dx_\star dx_\bullet d^2x_\perp \; e^{-i\alpha_q x_\bullet - i\beta_q x_\star + i(q,x)_\perp}\frac{N_c}{s}\Big(\langle A,B|\big[\bar{\psi}_A(x)\gamma_\mu\psi_B(x)\big]\big[\bar{\psi}_B(0)\gamma_\nu\Xi_1(0)\big]\\
&+ \big[\bar{\Xi}_1(x)\gamma_\mu\psi_B(x)\big]\big[\bar{\psi}_B(0)\gamma_\nu\psi_A(0)\big]|A,B\rangle \;+\; \mu\leftrightarrow\nu \;+\; x\leftrightarrow 0\Big) \qquad (4.28)\\
&=\; g_{\mu\nu}\big[\check{U}^{(1a)}(q) + \check{U}^{(2a)}(q)\big] + (\delta^\alpha_\mu\delta^\beta_\nu + \delta^\alpha_\nu\delta^\beta_\mu - g_{\mu\nu}g^{\alpha\beta})\big[\check{U}^{(1b)}_{\alpha\beta}(q) + \check{U}^{(2b)}_{\alpha\beta}(q)\big]\\
&+ (\delta^\alpha_\mu\delta^\beta_\nu + \delta^\alpha_\nu\delta^\beta_\mu - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta})\big[\check{U}^{(1c)}_{\alpha\beta}(q) + \check{U}^{(2c)}_{\alpha\beta}(q)\big]\\
&=\; \frac{2}{\alpha_q s N_c}\int d^2k_\perp \; \Big\{(k,q-k)_\perp g_{\mu\nu}\{h_1^\perp \Im\bar{\acute{e}}_G + \bar{h}_1^\perp \Im\acute{e}_G\}\\
&+ [(q-k)_{\mu\perp}p_{2\nu} + \mu\leftrightarrow\nu]\big(\{f_1\bar{f}_1 + \bar{f}_1 f_1\} - \beta_q\{f_1\bar{f}_\perp + \bar{f}_1 f_\perp\}\big)\\
&- \big[(q-k)^\perp_\mu(q-k)^\perp_\nu + \mu\leftrightarrow\nu + g^\perp_{\mu\nu}(q-k)^2_\perp\big]\{f_1\Re\bar{\grave{f}}_{1G} + \bar{f}_1\Re\grave{f}_{1G}\}\\
&+ g^\parallel_{\mu\nu}\big[(q-k)^2_\perp\{f_1\bar{f}_\perp + \bar{f}_1 f_\perp\} - 2\beta_q m^2\{f_1\bar{f}_3 + \bar{f}_1 f_3\} + m^2(f_1\Re\bar{f}_D + \bar{f}_1\Re f_D)\big]\\
&+ \frac{4}{s}p_{2\mu}p_{2\nu}(k,q-k)_\perp\big(\{f_\perp\bar{f}_1 + \bar{f}_\perp f_1\} - \beta_q\{f_\perp\bar{f}_\perp + \bar{f}_\perp f_\perp\} - \beta_q\{g_\perp\bar{g}_\perp + \bar{g}_\perp g_\perp\}\big)\\
&+ [g^\perp_{\mu\nu} - g^\parallel_{\mu\nu}](k,q-k)_\perp\big(2\beta_q\{h_1^\perp\bar{h}_3^\perp + \bar{h}_1^\perp h_3^\perp\} - \{h_1^\perp\bar{h} + \bar{h}_1^\perp h\} + \{h_1^\perp\Im\bar{\acute{e}}_G + \bar{h}_1^\perp\Im\acute{e}_G\}\big)\\
&- [p_{2\mu}k^\perp_\nu + p_{2\nu}k^\perp_\mu]\Big(\frac{(q-k)^2_\perp}{m^2}\{h_1^\perp\bar{h}_1^\perp + \bar{h}_1^\perp h_1^\perp\} + \beta_q\{h_1^\perp\bar{h} + \bar{h}_1^\perp h\}\Big)\\
&+ \frac{4}{s}p_{2\mu}p_{2\nu}\big[m^2\beta_q\{e\bar{e} + \bar{e}e\} + (q-k)^2_\perp\{h\bar{h}_1^\perp + \bar{h}h_1^\perp\} + \beta_q m^2\{h\bar{h} + \bar{h}h\}\big]\Big\}
\end{aligned}
$$

where we used $e_G + \tilde{e}_G = \acute{e}_G$, see parametrizations (9.56) and (9.58)

Next, the term with Ξ_2 can be obtained by projectile$\leftrightarrow$target replacement

$$
p_1\leftrightarrow p_2, \quad x_\bullet\leftrightarrow x_\star, \quad \alpha_q\leftrightarrow\beta_q, \quad k^\perp\leftrightarrow(q-k)^\perp \qquad (4.29)
$$

so we get

$$
\begin{aligned}
W^{(1\Xi_2+1\bar\Xi_2)}_{\mu\nu}(q) \;&=\; \frac{1}{16\pi^4N_c}\int dx_\star dx_\bullet d^2x_\perp\; e^{-i\alpha_qx_\bullet-i\beta_qx_\star+i(q,x)_\perp}[\check W^{(1\Xi_2)}_{\mu\nu}(x)+\check W^{(1\bar\Xi_2)}_{\mu\nu}(x)]\\
&=\; \frac{1}{16\pi^4N_c}\int dx_\star dx_\bullet d^2x_\perp\; e^{-i\alpha_qx_\bullet-i\beta_qx_\star+i(q,x)_\perp}\frac{N_c}{s}\Big(\langle A,B|\big[\bar\psi_A(x)\gamma_\mu\Xi_2(x)\big]\big[\bar\psi_B(0)\gamma_\nu\psi_A(0)\big]\\
&+\big[\bar\psi_A(x)\gamma_\nu\psi_B(x)\big]\big[\bar\Xi_2(0)\gamma_\mu\psi_B(0)\big]|A,B\rangle \;+\; \mu\leftrightarrow\nu \;+\; x\leftrightarrow 0\Big) \qquad (4.30)\\
&=\; \frac{2}{\beta_qsN_c}\int d^2k_\perp\;\Big[(k,q-k)_\perp g_{\mu\nu}\{\Im\bar{\acute e}_Gh_1^\perp+\Im\acute e_G\bar h_1^\perp\}\\
&+\Big\{[k_{\mu_\perp}p_{1\nu}+\mu\leftrightarrow\nu]\big(\{f_1\bar f_1+\bar f_1f_1\}-\alpha_q\{f_\perp\bar f_1+\bar f_\perp f_1\}\big)\\
&-\big[k^\perp_\mu k^\perp_\nu+\mu\leftrightarrow\nu+g^\perp_{\mu\nu}k_\perp^2\big]\{\Re\bar{\grave f}_{1G}f_1+\Re\grave f_{1G}\bar f_1\}\\
&+g^{\parallel}_{\mu\nu}\big[k_\perp^2\{\bar f_\perp f_1+f_\perp\bar f_1\}-2\alpha_qm^2\{\bar f_3f_1+f_3\bar f_1\}+m^2\{\Re\bar f_Df_1+\Re f_D\bar f_1\}\big]\\
&+\frac{4}{s}p_{1\mu}p_{1\nu}(k,q-k)_\perp\big[\{\bar f_1f_\perp+f_1\bar f_\perp\}-\alpha_q\{\bar f_\perp f_\perp+f_\perp\bar f_\perp\}-\alpha_q\{g_\perp\bar g_\perp+\bar g_\perp g_\perp\}\big]\\
&+[g^\perp_{\mu\nu}-g^{\parallel}_{\mu\nu}](k,q-k)_\perp\big(2\alpha_q\{\bar h_3^\perp h_1^\perp+h_3^\perp\bar h_1^\perp\}-\{\bar hh_1^\perp+h\bar h_1^\perp\}+\{\Im\bar{\acute e}_Gh_1^\perp+\Im\acute e_G\bar h_1^\perp\}\big)\\
&-[p_{1\mu}(q-k)^\perp_\nu+p_{1\nu}(q-k)^\perp_\mu]\Big(\frac{k_\perp^2}{m^2}\{h_1^\perp\bar h_1^\perp+\bar h_1^\perp h_1^\perp\}+\alpha_q\{\bar hh_1^\perp+h\bar h_1^\perp\}\Big)\\
&+\frac{4}{s}p_{1\mu}p_{1\nu}\big[m^2\alpha_q\{e\bar e+\bar ee\}+k_\perp^2\{\bar h_1^\perp h+h_1^\perp\bar h\}+\alpha_qm^2\{h\bar h+\bar hh\}\big]\Big\}
\end{aligned}
$$

5 Terms with Ξ'_i or $\bar\Xi'_i$

These terms were not considered in Ref. [9] so the analysis below will be more detailed than in previous Section.

5.1 Terms with Ξ'_1

We start from

$$\check W^{(1\Xi'_1)}_{\mu\nu}(x) \;=\; \frac{N_c}{s}\langle A,B|[\bar\psi_A(x)\gamma_\mu\psi_B(x)\big]\big[\bar\psi_B(0)\gamma_\nu\Xi'_1(0)\big] \;+\; \mu\leftrightarrow\nu|A,B\rangle \;+\; x\leftrightarrow 0 \qquad (5.1)$$

where Ξ'_1 can be taken from Ref. [7]

$$
\begin{aligned}
\Xi'_1 \;&=\; -\frac{\not p_1}{s}\gamma^i\frac{B_i}{\beta}\psi_A+\frac{1}{s^2}\Big(\frac{\not p_1}{\beta}\not P_\perp\frac{\not p_2}{\alpha}+\frac{\not p_2}{\alpha}\not P_\perp\frac{\not p_1}{\beta}\Big)\gamma_iB^i\psi_A\\
&-\frac{2}{s^2}\not p_2\not p_1B_\star\frac{1}{\alpha}\psi_A+\frac{2}{s^2}\Big(B_\star\frac{\not p_2}{\alpha^2}+A_\bullet\frac{\not p_1}{\beta^2}\Big)\not B_\perp\psi_A \qquad (5.2)
\end{aligned}
$$

First, note that the term $\not{A}_\perp$ in $\not{P}_\perp$ and $A_\bullet$ in the last term can be neglected at large N_c. Indeed, after separation of color singlet contributions

$$\begin{aligned}
&\langle A,B|(\bar\psi_A^k\psi_B^k)(\bar\psi_B^m A_\mu^{ml}B_\nu^{ln}\psi_A^n)|A,B\rangle \;=\; \langle A,B|(\bar\psi_A^k A_\mu^{ml}\psi_A^n)(\bar\psi_B^m B_\nu^{ln}\psi_B^k)|A,B\rangle\\
&=\; \langle A,B|(\bar\psi_A^k A_\mu^{ln}\psi_A^n)(\bar\psi_B^m B_\nu^{ml}\psi_B^k)|A,B\rangle + if^{abc}\langle A,B|A_\mu^a B_\nu^b(\bar\psi_A^k\psi_A^n)(\bar\psi_B^m(t^c)_{mn}\psi_B^k)|A,B\rangle\\
&=\; \frac{1}{N_c}\langle\bar\psi_A A_\mu\psi_A\rangle_A\langle\bar\psi_B B_\nu\psi_B\rangle_B + 2if^{abc}\langle\bar\psi_A^k t_{kn}^d A_\mu^a\psi_A^n\rangle_A\langle\bar\psi_B^m t_{ms}^c t_{sl}^d B_\nu^b\psi_B^l\rangle_B\\
&+\; \frac{i}{N_c}f^{abc}\langle\bar\psi_A^k A_\mu^a\psi_A^k\rangle_A\langle\bar\psi_B^m t_{ml}^c B_\nu^b\psi_B^l\rangle_B\\
&=\; \frac{1}{N_c}\langle\bar\psi_A A_\mu\psi_A\rangle_A\langle\bar\psi_B B_\nu\psi_B\rangle_B + 2if^{abc}\langle A_m^a u(\bar\psi_A t^d\psi_A)\rangle_A\langle B_\nu^b(\bar\psi_B t^c t^d\psi_B)\rangle_B\\
&\qquad\qquad + \frac{i}{N_c}f^{abc}\langle A_\mu^a(\bar\psi_A\psi_A)\rangle_A\langle B_\nu^b(\bar\psi_B t^c\psi_B)\rangle_B\\
&=\; \frac{1}{N_c}\langle\bar\psi_A A_\mu\psi_A\rangle_A\langle\bar\psi_B B_\nu\psi_B\rangle_A + 2i\frac{f^{abc}}{N_c^2-1}\langle(\bar\psi_A A_j\psi_A)\rangle_A\langle\bar\psi_B t^c t^a B_j^b\psi_B\rangle_B\\
&=\; -\frac{1}{N_c(N_c^2-1)}\langle\bar\psi_A A_\mu\psi_A\rangle_A\langle\bar\psi_B B_\nu\psi_B\rangle_B
\end{aligned}\tag{5.3}$$

so effectively Ξ_1' reduces to

$$\begin{aligned}
&\Xi_1'\\
&=\; -\frac{\not{p}_1}{s}\gamma^i\frac{B_i}{\beta}\psi_A + \frac{1}{s^2}\Big(\frac{\not{p}_1}{\beta}\not{P}_\perp^B\frac{\not{p}_2}{\alpha} + \frac{\not{p}_2}{\alpha}\not{P}_\perp^B\frac{\not{p}_1}{\beta}\Big)\gamma_i B^i\psi_A - \frac{2}{s^2}\not{p}_2\not{p}_1 B_\star\psi_A + \frac{2}{s^2}\not{p}_2 B_\star\not{B}_\perp\frac{1}{\alpha^2}\psi_A\\
&\qquad\qquad =\; -\frac{\not{p}_1}{s}\gamma^i\frac{B_i}{\beta}\psi_A - \frac{1}{s\beta}\not{P}_\perp^B\not{B}_\perp\frac{1}{\alpha}\psi_A + \frac{2}{s^2}\not{p}_2 B_\star\not{B}_\perp\frac{1}{\alpha^2}\psi_A\\
&\qquad\qquad\qquad + \frac{1}{s^2}\not{p}_2\not{p}_1\big[\frac{1}{\beta},\not{P}_\perp^B\big]_-\frac{1}{\alpha}\psi_A - \frac{2}{s^2}\not{p}_2\not{p}_1 B_\star\psi_A
\end{aligned}\tag{5.4}$$

where $\big[\frac{1}{\beta},\not{P}_\perp^B\big]_-$ is a commutator and $P_i^B = i\partial_i + B_i$.

The Fierz transformation (9.1) yields (cf. Eq. (4.8))

$$\begin{aligned}
\check W_{\mu\nu}^{(1\Xi_1')}(x) \;&=\; \frac{N_c}{s}\langle A,B|[\bar\psi_A(x)\gamma_\mu\psi_B(x)]\,[\bar\psi_B(0)\gamma_\nu\Xi_1'(0)] \;+\; \mu\leftrightarrow\nu|A,B\rangle \;+\; x\leftrightarrow 0 \quad (5.5)\\
&=\; g_{\mu\nu}\check U^{(1'a)}(x) + (\delta_\mu^\alpha\delta_\nu^\beta + \delta_\nu^\alpha\delta_\mu^\beta - g_{\mu\nu}g^{\alpha\beta})\check U_{\alpha\beta}^{(1'b)}(x) + (\delta_\mu^\alpha\delta_\nu^\beta + \delta_\nu^\alpha\delta_\mu^\beta - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta})\check U_{\alpha\beta}^{(1'c)}(x)
\end{aligned}$$

where

$$\begin{aligned}
\check U^{(1'a)}(x) \;&=\; \langle A,B| -\frac{N_c}{2s}\big[\bar\psi_A(x)\Xi_1'(0)\big]\big[\bar\psi_B(0)\psi_B(x)\big] \qquad\qquad (5.6)\\
&\qquad + \frac{1}{2s}\big[\bar\psi_A(x)\gamma_5\Xi_1'(0)\big]\big[\bar\psi_B(0)\gamma_5\psi_B(x)\big]|A,B\rangle \;+\; x\leftrightarrow 0\\
\check U_{\alpha\beta}^{(1'b)}(x) \;&=-\; \frac{N_c}{4s}\langle A,B|\Big(\big[\bar\psi_A(x)\gamma_\alpha\Xi_1'(0)\big]\big[\bar\psi_B(0)\gamma_\beta\psi_B(x)\big]\\
&\qquad + \big[\bar\psi_A(x)\gamma_\alpha\gamma_5\Xi_1'(0)\big]\big[\bar\psi_B(0)\gamma_\beta\gamma_5\psi_B(x)\big]\Big)|A,B\rangle \;+\; \alpha\leftrightarrow\beta \;+\; x\leftrightarrow 0\\
\check U_{\alpha\beta}^{(1'c)}(x) \;&=\; \frac{N_c}{4s}\langle A,B|\big[\bar\psi_A(x)\sigma_{\alpha\xi}\Xi_1'(0)\big]\big[\bar\psi_B(0)\sigma_\beta^{\ \xi}\psi_B(x)\big]|A,B\rangle \;+\; \alpha\leftrightarrow\beta \;+\; x\leftrightarrow 0
\end{aligned}$$

First, it is easy to see that $\check U^{(1'a)}(x)$ is $\sim g_{\mu\nu}\frac{m_\perp^4}{s^2}$ or less.

5.1.1 Term $\check{U}^{(1'b)}$

From Eq. (5.4) we get

$$\begin{aligned}
\check{U}^{(1'b)}_{\alpha\beta} \;=\;& -\frac{N_c}{4s}\langle A,B|\big[\bar\psi_A(x)\gamma_\alpha\Xi_1(0)\big]\big[\bar\psi_B(0)\gamma_\beta\psi_B(x)\big] \\
&+\big(\psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\big)|A,B\rangle+\alpha\leftrightarrow\beta+x\leftrightarrow 0 \\
=\;& \frac{N_c}{4s^2}\langle A,B|\big[\bar\psi_A(x)\gamma_\alpha\Big(\not{p}_1\gamma^i\frac{B_i}{\beta}+\frac{1}{\alpha_q}\frac{1}{\beta}\not{P}_\perp\not{B} \\
&-\frac{1}{\alpha_q s}\not{p}_2\not{p}_1\big([\frac{1}{\beta},\not{P}^B_\perp]_-\not{B}-2B_\star\big)-\frac{2}{\alpha_q^2 s}\not{p}_2B_\star\not{B}\Big)(0)\psi_A(0)\big]\big[\bar\psi_B(0)\gamma_\beta\psi_B(x)\big]\big]|A,B\rangle \\
&+\big(\psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\big)+\alpha\leftrightarrow\beta+\big(x\leftrightarrow 0,\alpha_q\leftrightarrow-\alpha_q\big)
\end{aligned} \tag{5.7}$$

(in the few fromulas involving $B_\star$ the notation $\not{B}$ still means $B^i\gamma_i$). Let us demonstrate that the terms in the fourth line of the r.h.s are negligible. First,

$$\begin{aligned}
&\frac{1}{s^3}\big[\bar\psi_A(x)\gamma_\alpha\not{p}_2\not{p}_1\big([\frac{1}{\beta},\not{P}^B_\perp]_-\not{B}-2B_\star\big)(0)\psi_A(0)\big]\big[\bar\psi_B(0)\gamma_\beta\psi_B(x)\big] \\
&=\;\frac{1}{2\alpha_q s^3}\Big(\big[\bar\psi_A(x)\gamma_{\alpha_\perp}\not{p}_2\not{p}_1\big([\frac{1}{\beta},\not{P}^B_\perp]_-\not{B}-2B_\star\big)(0)\psi_A(0)\big] \\
&+2p_{2\alpha}\big[\bar\psi_A(x)\not{p}_1\big([\frac{1}{\beta},\not{P}^B_\perp]_-\not{B}-2B_\star\big)(0)\psi_A(0)\big]\Big)\big[\bar\psi_B(0)\gamma_\beta\psi_B(x)\big]
\end{aligned} \tag{5.8}$$

If index β is transverse, the first term in the r.h.s. is $\sim q^\perp_\alpha q^\perp_\beta\frac{m_\perp^2}{s^2}$ (after Fourier transformation (3.2)) and the second is $\sim p_{2\alpha}q^\perp_\beta\frac{m_\perp^4}{s^3}$. If the index β is longitudinal, the first term is $\sim q^\perp_\alpha p_{2\beta}\frac{m_\perp^2}{s^2}$ and the second is $\sim p_{2\alpha}p_{2\beta}\frac{m_\perp^4}{s^3}$ so all these terms are negligible in comparison to those in Eq. (3.13). Second,

$$\begin{aligned}
&\frac{1}{s^3}\big[\bar\psi_A(x)\gamma_\alpha\not{p}_2B_\star\not{B}(0)\psi_A(0)\big]\big[\bar\psi_B(0)\gamma_\beta\psi_B(x)\big] \;=\; \frac{1}{2\alpha_q s^3}\Big(\big[\bar\psi_A(x)\gamma_{\alpha_\perp}\not{p}_2B_\star\not{B}(0)\psi_A(0)\big] \\
&+\frac{2}{s}p_{2\alpha}\big[\bar\psi_A(x)\not{p}_1\not{p}_2B_\star\not{B}(0)\psi_A(0)\big]\Big)\big[\bar\psi_B(0)\gamma_\beta\psi_B(x)\big]
\end{aligned} \tag{5.9}$$

Again, if index β is transverse, the first term in the r.h.s. is $\sim q^\perp_\alpha q^\perp_\beta\frac{m_\perp^2}{s^2}$ and the second is $\sim p_{2\alpha}q^\perp_\beta\frac{m_\perp^4}{s^3}$, and if the index β is longitudinal, the first term is $\sim q^\perp_\alpha p_{2\beta}\frac{m_\perp^2}{s^2}$ and the second is $\sim p_{2\alpha}p_{2\beta}\frac{m_\perp^4}{s^3}$, so all these terms are negligible in comparison to those in Eq. (3.13).

We get

$$\begin{aligned}
\check{U}^{(1'b)}_{\alpha\beta}(x) \;=\;& \frac{N_c}{4s^2}\langle A,B|\Big(\big[\bar\psi_A(x)\gamma_\alpha\not{p}_1\gamma^i\frac{1}{\beta}B_i\psi_A(0)\big] \\
&+\frac{1}{\alpha_q}\big[\bar\psi_A(x)\gamma_\alpha\frac{1}{\beta}\not{P}_\perp\not{B}\psi_A(0)\big]\Big)\big[\bar\psi_B(0)\gamma_\beta\psi_B(x)\big]|A,B\rangle \\
&+\big(\psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\big)+\alpha\leftrightarrow\beta+\big(x\leftrightarrow 0,\alpha_q\leftrightarrow-\alpha_q\big)
\end{aligned} \tag{5.10}$$

Similar analysis shows that the only non-negligible contribution is $\sim \frac{p_{1\alpha}p_{2\beta}}{s^2}$ so separating color-singlet contributions we obtain

$$\begin{aligned}
\breve{U}^{(1'b)}_{\alpha\beta} &= \frac{p_{1\alpha}p_{2\beta}}{s^4}\Big\{\Big(\langle\bar\psi_A(x)\not{p}_2\not{p}_1\gamma^i\psi_A(0)\rangle\langle\bar\psi_B(\frac{1}{\beta}B_i)(0)\not{p}_1\psi_B(x)\rangle \\
&+ \frac{1}{\alpha_q}\langle\bar\psi_A(x)\not{p}_2\not{p}_\perp\gamma^i\psi_A(0)\rangle\Big)\langle\bar\psi_B(\frac{1}{\beta}B_i)(0)\not{p}_1\psi_B(x)\rangle \\
&+ \frac{1}{\alpha_q}\langle\bar\psi_A(x)\not{p}_2\gamma^j\gamma^i\psi_A(0)\rangle \times \langle\bar\psi_B(\frac{1}{\beta}P^B_jB_i)(0)\not{p}_1\psi_B(x)\rangle\Big\} \\
&+ \big(\psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\big) + \alpha\leftrightarrow\beta + \big(x\leftrightarrow 0, \alpha_q\leftrightarrow -\alpha_q\big)
\end{aligned} \tag{5.11}$$

Using Eqs. (9.16) and (9.17), we get

$$\begin{aligned}
\breve{U}^{(1'b)}_{\alpha\beta} &= \frac{p_{1\alpha}p_{2\beta}}{s^4}\Big\{\frac{s}{2}\langle\bar\psi(x)\gamma^i\psi(0)\rangle_A\langle\bar\psi\grave{\mathcal{B}}_i(0)\not{p}_1\psi(x)\rangle_B + \frac{1}{\alpha_q}\langle\bar\psi(x)\not{p}_2p_i\psi(0)\rangle_A \\
&\times \langle\bar\psi\acute{\mathcal{B}}_i(0)\not{p}_1\psi(x)\rangle_B + \frac{1}{\alpha_q}\langle\bar\psi(x)\not{p}_2\psi(0)\rangle_A\langle\bar\psi\not{p}_1\big(\mathcal{PB} + \frac{\epsilon^{ij}}{2}\mathcal{B}_{ij}\gamma_5\big)(0)\psi(x)\rangle_B\Big\} \\
&+ \big(\psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\big) + \alpha\leftrightarrow\beta + \big(x\leftrightarrow 0, \alpha_q\leftrightarrow -\alpha_q\big)
\end{aligned} \tag{5.12}$$

where we introduced the notations

$$\begin{aligned}
\mathcal{B}_i(x_\star,x_\perp) &\equiv \frac{1}{\beta+i\epsilon}B_i(x_\star,x_\perp) \equiv -i\int_{-\infty}^{x_\star}dx'_\star\ B_i(x'_\star,x_\perp) \\
\mathcal{PB}(x_\star,x_\perp) &\equiv \frac{1}{\beta+i\epsilon}iD^iB_i(x_\star,x_\perp) = \int_{-\infty}^{x_\star}dx'_\star\ (\partial^i - iB^i)B_i(x'_\star,x_\perp) \\
\mathcal{B}_{ij}(x_\star,x_\perp) &\equiv \frac{1}{\beta+i\epsilon}F^{(B)}_{ij}(x_\star,x_\perp) = -i\int_{-\infty}^{x_\star}dx'_\star\ F^{(B)}_{ij}(x'_\star,x_\perp)
\end{aligned} \tag{5.13}$$

and $\grave{\mathcal{B}}_i \equiv \mathcal{B}_i - i\gamma_5\tilde{\mathcal{B}}_i$, $\acute{\mathcal{B}}_i \equiv \mathcal{B}_i + i\gamma_5\tilde{\mathcal{B}}_i$ similarly to Eq. (4.12). Using parametizations (9.70), (9.71), (9.72) and formula $\grave{B}_i\gamma_5 = -i\epsilon_{ij}\grave{B}^j$ we get

$$\begin{aligned}
\breve{U}^{(1'b)}_{\alpha\beta} &= \frac{g^{\parallel}_{\alpha\beta}}{2\alpha_q sN_c}\int d^2k_\perp\Big[(k,q-k)_\perp\Big(\{f_1\bar{\acute{f}}_{1\mathcal{G}} + \bar f_1\acute{f}_{1\mathcal{G}}\}\bar{\grave{f}}_{1\mathcal{G}} \\
&+\alpha_q\{[f_\perp - ig_\perp]\bar{\grave{f}}_{1\mathcal{G}} + [\bar f_\perp + i\bar g_\perp)]\grave{f}_{1\mathcal{G}}\}\Big) + m^2\{f_1(\alpha_q,k_\perp) + \bar f_1[f_{2\mathcal{G}} + f_{3\mathcal{G}}]\}\Big],
\end{aligned} \tag{5.14}$$

cf. Eq. (4.20).

5.1.2 Term $\breve{U}^{(1'c)}$

Next,

$$\begin{aligned}
\breve{U}^{(1'c)}(x) &= \frac{N_c}{4s}\langle A,B|\big[\bar\psi_A(x)\sigma_{\alpha\xi}\Xi'_1(0)\big]\big[\bar\psi_B(0)\sigma_\beta^{\ \xi}\psi_B(x)|A,B\rangle + \alpha\leftrightarrow\beta + x\leftrightarrow 0 \\
&= -\frac{N_c}{4s^2}\langle A,B|\big[\bar\psi_A(x)\sigma_{\alpha\xi}\Big(\not{p}_1\gamma^i\frac{B_i}{\beta} + \frac{1}{\alpha_q}\frac{1}{\beta}\not{P}_\perp\not{B} - \frac{1}{\alpha_q s}\not{p}_2\not{p}_1\big([\frac{1}{\beta},\not{P}^B_\perp]_-\not{B} - 2B_\star\big) \\
&- \frac{2}{\alpha_q^2 s}\not{p}_2B_\star\not{B}_\perp\Big)(0)\psi_A(0)\big]\big[\bar\psi_B(0)\sigma_\beta^{\ \xi}\psi_B(x)\big]|A,B\rangle + \alpha\leftrightarrow\beta + \big(x\leftrightarrow 0, \alpha_q\leftrightarrow -\alpha_q\big)
\end{aligned} \tag{5.15}$$

Let us again demonstrate that the last two terms in curly brackets are negligible. Ignoring the transverse factors p_i, B_i and $B_\star$ which cannot produce factor s, we obtain the following estimates

$$
\begin{aligned}
&\frac{1}{s^3}\sigma_{\alpha_\perp\xi}\not{p}_2(\not{p}_1)\otimes\sigma_{\beta_\perp}^{\ \xi} \ = \ O\Big(\frac{g^\perp_{\alpha\beta}}{s^2}\Big), && (5.16)\\
&\frac{p_{2\beta}}{s^4}\sigma_{\alpha_\perp\xi}\not{p}_2(\not{p}_1)\otimes\sigma_\bullet^{\ \xi} \ = \ O\Big(\frac{p_{2\beta}q^\perp_\alpha}{s^2}\Big), \quad \frac{p_{1\beta}}{s^4}\sigma_{\alpha_\perp\xi}\not{p}_2(\not{p}_1)\otimes\sigma_\star^{\ \xi} \ = \ O\Big(\frac{p_{1\beta}q^\perp_\alpha}{s^3}\Big)\\
&\frac{p_{1\alpha}}{s^4}\sigma_{\star\xi}\not{p}_2(\not{p}_1)\otimes\sigma_{\beta_\perp}^{\ \xi} \ = \ O\Big(\frac{p_{1\alpha}q^\perp_\beta}{s^2}\Big), \quad \frac{p_{2\alpha}}{s^4}\sigma_{\bullet\xi}\not{p}_2(\not{p}_1)\otimes\sigma_{\beta_\perp}^{\ \xi} \ = \ O\Big(\frac{p_{2\alpha}q^\perp_\beta}{s^2}\Big),\\
&\frac{p_{1\alpha}p_{1\beta}}{s^5}\sigma_{\star\xi}\not{p}_2(\not{p}_1)\otimes\sigma_\star^{\ \xi} \ = \ 0, \quad \frac{p_{1\alpha}p_{2\beta}}{s^5}\sigma_{\star\xi}\not{p}_2(\not{p}_1)\otimes\sigma_\bullet^{\ \xi} \ = \ O\Big(\frac{p_{1\alpha}p_{2\beta}}{s^3}\Big),\\
&\frac{p_{2\alpha}p_{1\beta}}{s^5}\sigma_{\bullet\xi}\not{p}_2(\not{p}_1)\otimes\sigma_\star^{\ \xi} \ = \ O\Big(\frac{p_{2\alpha}p_{1\beta}}{s^3}\Big), \quad \frac{p_{2\alpha}p_{2\beta}}{s^5}\sigma_{\bullet\xi}\not{p}_2(\not{p}_1)\otimes\sigma_\bullet^{\ \xi} \ = \ O\Big(\frac{p_{2\alpha}p_{2\beta}}{s^3}\Big)
\end{aligned}
$$

where the factors $(\not{p}_1)$ means that inclusion (or non-inclusion) of $\not{p}_1$ does not change the power of s in the projectile TMDs. Similarly to Eq. (5.10) we get

$$
\begin{aligned}
&\check{U}^{(1'c)}_{\alpha\beta}(x)\\
&= \ -\frac{N_c}{4s^2}\langle A,B|\Big(\big[\bar\psi_A(x)\sigma_{\alpha\xi}\not{p}_1\gamma^i\mathcal{B}_i\psi_A(0)\big]+\frac{1}{\alpha_q}\big[\bar\psi_A(x)\sigma_{\alpha\xi}\frac{1}{\beta}\not{P}_\perp\not{B}\psi_A(0)\big]\Big)\\
&\times\ \big[\bar\psi_B(0)\sigma_\beta^{\ \xi}\psi_B(x)\big]|A,B\rangle+\alpha\leftrightarrow\beta+\big(x\leftrightarrow 0,\alpha_q\leftrightarrow-\alpha_q\big) \qquad (5.17)
\end{aligned}
$$

For the first term, from Eq. (9.9) we get

$$
\begin{aligned}
&\frac{1}{4s^2}i\sigma_{\alpha\xi}\sigma_{\bullet i}\otimes\sigma_\beta^{\ \xi}\mathcal{B}^i \ + \ \alpha\leftrightarrow\beta \qquad (5.18)\\
&= \ \frac{1}{4s^2}\Big[-\frac{2}{s}\sigma_{\bullet *}\otimes\big(\sigma_{\bullet\beta_\perp}\mathcal{B}_\alpha-\frac{g^\perp_{\alpha\beta}}{2}\sigma_{\bullet i}\mathcal{B}^i\big)+\frac{i}{2}g_{\alpha\beta}\otimes\sigma_{\bullet i}\mathcal{B}^i+\frac{g_{\alpha\beta}}{s}\sigma_{\star\bullet}\otimes\sigma_{\bullet i}\mathcal{B}^i\\
&\qquad +\ i\otimes\big(\sigma_{\bullet\beta_\perp}\mathcal{B}_\alpha-\frac{g^\perp_{\alpha\beta}}{2}\sigma_{\bullet i}\mathcal{B}^i\big) \ + \ \alpha\leftrightarrow\beta\Big]
\end{aligned}
$$

Next, due to the definition (5.5), $U^{1'c}_{\alpha\beta}$ will be multiplied by the traceless tensor $(\delta^\alpha_\mu\delta^\beta_\nu+\delta^\alpha_\nu\delta^\beta_\mu-\frac{1}{2}g_{\mu\nu}g^{\alpha\beta})$ so the contributions to $U^{1'c}_{\alpha\beta}$ proportional to $g_{\alpha\beta}$ can be ignored. Effectively,

$$
\begin{aligned}
&\frac{1}{4s^2}i\sigma_{\alpha\xi}\sigma_{\bullet i}\otimes\sigma_\beta^{\ \xi}\mathcal{B}^i \ + \ \alpha\leftrightarrow\beta \qquad (5.19)\\
&= \ \frac{1}{4s^2}\Big[-\frac{2}{s}\sigma_{\bullet *}\otimes\big(\sigma_{\bullet\beta_\perp}\mathcal{B}_\alpha-\frac{g^\perp_{\alpha\beta}}{2}\sigma_{\bullet i}\mathcal{B}^i\big)+i\otimes\big(\sigma_{\bullet\beta_\perp}\mathcal{B}_\alpha-\frac{g^\perp_{\alpha\beta}}{2}\sigma_{\bullet i}\mathcal{B}^i\big) \ + \ \alpha\leftrightarrow\beta\Big]
\end{aligned}
$$

and we get the first term in Eq. (5.17) in the form

$$
\begin{aligned}
\check{U}^{(1'c)}_{\alpha\beta(1)}(x) \ &\equiv \ -\frac{N_c}{4s^2}\langle A,B|\big[\bar\psi_A(x)\sigma_{\alpha\xi}\not{p}_1\gamma^i\frac{B_i}{\beta}\psi_A(0)\big] \qquad (5.20)\\
&\times\ \big[\bar\psi_B(0)\sigma_\beta^{\ \xi}\psi_B(x)\big]|A,B\rangle+\alpha\leftrightarrow\beta+x\leftrightarrow 0,+\big(x\leftrightarrow 0,\alpha_q\leftrightarrow-\alpha_q\big)\\
&= \ \Big(\frac{2}{s^3}\langle\bar\psi(x)\sigma_{\star\bullet}\psi(0)\rangle_A+\frac{i}{4s^2}\langle\bar\psi(x)\psi(0)\rangle_A\Big)\\
&\times\ \langle\bar\psi\big(\sigma_{\bullet\beta_\perp}\mathcal{B}_\alpha-\frac{g^\perp_{\alpha\beta}}{2}\sigma_{\bullet i}\mathcal{B}^i\big)(0)\psi(x)\rangle_B+\alpha\leftrightarrow\beta+x\leftrightarrow 0
\end{aligned}
$$

Using parametrization (9.73) we get

$$\begin{aligned} U^{(1'c)}_{\alpha\beta(1)}(q) &= \frac{1}{16\pi^4N_c}\int dx_\star dx_\bullet d^2x_\perp\ e^{-i\alpha_q x_\bullet - i\beta_q x_\star + i(q,x)_\perp} U^{(1'c)}_{\alpha\beta(1)} \\ &= -\frac{1}{sN_c}\int d^2k_\perp\ \big[(q-k)^\perp_\alpha (q-k)^\perp_\beta + \frac{g^\perp_{\alpha\beta}}{2}(q-k)^2_\perp\big]\{[h+ie]\bar{h}_{1\mathcal{G}} + [\bar{h}-i\bar{e}]h_{1\mathcal{G}}\} \end{aligned} \tag{5.21}$$

To get the second term in Eq. (5.17) we need a table of estimates similar to Eq. (5.16)

$$\begin{aligned}
&\frac{1}{s^2}\sigma_{\alpha_\perp\xi}\otimes\sigma_{\beta_\perp}^{\ \xi} = \frac{2}{s^3}\sigma_{\alpha_\perp *}\otimes\sigma_{\beta_\perp\bullet}, \\
&\frac{p_{2\beta}}{s^4}\sigma_{\alpha_\perp\xi}\not{p}_1\not{p}_2\otimes\sigma_\bullet^{\ \xi} = O\big(\frac{p_{2\beta}q^\perp_\alpha}{s^2}\big), \qquad \frac{p_{1\beta}}{s^4}\sigma_{\alpha_\perp\xi}\not{p}_1\not{p}_2\otimes\sigma_\star^{\ \xi} = O\big(\frac{p_{1\beta}q^\perp_\alpha}{s^2}\big) \\
&\frac{p_{1\alpha}}{s^4}\sigma_{\star\xi}\not{p}_1\not{p}_2\otimes\sigma_{\beta_\perp}^{\ \xi} = O\big(\frac{p_{1\alpha}q^\perp_\beta}{s^2}\big), \qquad \frac{p_{2\alpha}}{s^4}\sigma_{\bullet\xi}\not{p}_1\not{p}_2\otimes\sigma_{\beta_\perp}^{\ \xi} = O\big(\frac{p_{2\alpha}q^\perp_\beta}{s^2}\big), \\
&\frac{p_{1\alpha}p_{1\beta}}{s^5}\sigma_{\star\xi}\not{p}_1\not{p}_2\otimes\sigma_\star^{\ \xi} = O\big(\frac{p_{1\alpha}p_{1\beta}}{s^3}\big), \qquad \frac{p_{1\alpha}p_{2\beta}}{s^5}\sigma_{\star\xi}\not{p}_1\not{p}_2\otimes\sigma_\bullet^{\ \xi} = \frac{p_{1\alpha}p_{2\beta}}{s^4}\sigma_{\star i}\otimes\sigma_\bullet^{\ i}, \\
&\frac{p_{2\alpha}p_{1\beta}}{s^5}\sigma_{\bullet\xi}\not{p}_1\not{p}_2\otimes\sigma_\star^{\ \xi} = O\big(\frac{p_{2\alpha}p_{1\beta}}{s^3}\big), \qquad \frac{p_{2\alpha}p_{2\beta}}{s^5}\sigma_{\bullet\xi}\not{p}_1\not{p}_2\otimes\sigma_\bullet^{\ \xi} = O\big(\frac{p_{2\alpha}p_{2\beta}}{s^3}\big)
\end{aligned} \tag{5.22}$$

Combining these equations and equations from table (5.16) (with $\not{p}_1$ included) we see that

$$\begin{aligned} \frac{1}{s^2}\sigma_{\alpha\xi}\otimes\sigma_\beta^{\ \xi} + \alpha\leftrightarrow\beta &= \frac{2}{s^3}\sigma_{\alpha_\perp *}\otimes\sigma_{\beta_\perp\bullet} + \frac{4p_{1\alpha}p_{2\beta}}{s^4}\sigma_{\star i}\otimes\sigma_\bullet^{\ i} + \alpha\leftrightarrow\beta \\ &= \frac{2}{s^3}\sigma_\star^{\ i}\otimes\sigma_\bullet^{\ j}\Big(g^\perp_{\alpha i}g^\perp_{\beta j} + g^\perp_{\beta i}g^\perp_{\alpha j} - g^\perp_{\alpha\beta}g_{ij} + g^\perp_{\alpha\beta}g_{ij} + \frac{2p_{1\alpha}p_{2\beta}}{s}g_{ij} + \frac{2p_{2\alpha}p_{1\beta}}{s}g_{ij}\Big) \\ &= \frac{2}{s^3}\sigma_\star^{\ i}\otimes\sigma_\bullet^{\ j}\big(g^\perp_{\alpha i}g^\perp_{\beta j} + g^\perp_{\beta i}g^\perp_{\alpha j} - g^\perp_{\alpha\beta}g_{ij} + g_{\alpha\beta}g_{ij}\big) = -\frac{2}{s^3}\sigma_\star^{\ i}\otimes\sigma_\bullet^{\ j}P_{\alpha\beta;ij} \end{aligned} \tag{5.23}$$

where

$$P_{\alpha\beta;ij} \equiv g^\perp_{\alpha\beta}g_{ij} - g^\perp_{\alpha i}g^\perp_{\beta j} - g^\perp_{\beta i}g^\perp_{\alpha j} \tag{5.24}$$

Note that in the last line in Eq. (5.23) we dropped term $\sim g_{\alpha\beta}$ since it vanishes after multiplication by $(\delta^\alpha_\mu\delta^\beta_\nu + \delta^\alpha_\nu\delta^\beta_\mu - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta})$. Thus, the second term in Eq. (5.17) takes the form

$$\begin{aligned} \breve{U}^{(1'c)}_{\alpha\beta(2)}(x) &= -\frac{N_c}{4s^2\alpha_q}\langle A,B|\big[\bar\psi_A(x)\sigma_{\alpha\xi}\frac{1}{\beta}\not{P}_\perp\not{B}\psi_A(0)\big] \\ &\quad\times\big[\bar\psi_B(0)\sigma_\beta^{\ \xi}\psi_B(x)\big]|A,B\rangle + \alpha\leftrightarrow\beta + \big(x\leftrightarrow 0, \alpha_q\leftrightarrow -\alpha_q\big) \\ &= \frac{N_c}{2s^3\alpha_q}P_{\alpha\beta;ij}\langle A,B|\big[\bar\psi_A(x)\sigma_\star^{\ i}\frac{1}{\beta}\not{P}_\perp\not{B}\psi_A(0)\big] \\ &\quad\times\big[\bar\psi_B(0)\sigma_\bullet^{\ j}\psi_B(x)\big]|A,B\rangle + \big(x\leftrightarrow 0, \alpha_q\leftrightarrow -\alpha_q\big) \\ &= \frac{N_c}{2s^3\alpha_q}P_{\alpha\beta;ij}\Big(\langle\bar\psi(x)\sigma_\star^{\ i}\not{p}_\perp\gamma_l\bar\psi(0)\rangle_A\langle\bar\psi(0)\sigma_\bullet^{\ j}\mathcal{B}^l\psi(x)\rangle_B \\ &\quad + \langle\bar\psi(x)\sigma_\star^{\ i}\gamma_k\gamma_l\psi(0)\rangle_A\langle\bar\psi\sigma_\bullet^{\ j}\big(\frac{1}{\beta}iD_kB^l\big)(0)\psi(x)\rangle_B\Big) + \big(x\leftrightarrow 0, \alpha_q\leftrightarrow -\alpha_q\big) \end{aligned} \tag{5.25}$$

Using formula (9.3) we get

$$\sigma_{\star i}\gamma_k\gamma_l = g_{kl}\sigma_{\star i} + g_{ik}\sigma_{\star l} - g_{il}\sigma_{\star k} \tag{5.26}$$

so the second contribution to Eq. (5.15) is

$$\begin{aligned}\breve{U}^{(1'c)}_{\alpha\beta(2)}(x) &= \frac{N_c}{2s^3\alpha_q}P_{\alpha\beta;ij}\langle\bar\psi(x)\sigma_\star{}^i\not{p}_\perp\gamma_l\psi(0)\rangle_A\langle\bar\psi(0)\sigma_\bullet{}^j\mathcal{B}^l(0)\psi(x)\rangle_B\\ &= \frac{N_c}{2s^3\alpha_q}P_{\alpha\beta;ij}\langle\bar\psi(x)\big(\sigma_{\star i}p^l+\sigma_{\star l}p_i-g_{il}\sigma_{\star k}p^k\big)\psi(0)\rangle_A\langle\bar\psi\mathcal{B}^l(0)\sigma_\bullet{}^j\psi(x)\rangle_B\\ &+\big(x\leftrightarrow 0,\alpha_q\leftrightarrow-\alpha_q\big)\end{aligned}\tag{5.27}$$

and the corresponding Fourier transformation yields

$$\begin{aligned}U^{(1'c)}_{\alpha\beta(2)}(q) &= \frac{1}{2\alpha_q sN_c}P_{\alpha\beta;ij}\int d^2k_\perp(2k_ik_l+k_\perp^2 g_{il})\\ &\times\Big(h_1^\perp(\alpha_q,k_\perp)\big[(q-k)^j(q-k)^l\bar h_{1\mathcal{G}}+\frac{g^{jl}}{2}(q-k)_\perp^2(\bar h_{1\mathcal{G}}+\bar h_{2\mathcal{G}})(\beta_q,q_\perp-k_\perp)\big]\\ &+\bar h_1^\perp(\alpha_q,k_\perp)\big[(q-k)^j(q-k)^l h_{1\mathcal{G}}(\beta_q,q_\perp-k_\perp)+\frac{g^{jl}}{2}(h_{1\mathcal{G}}+h_{2\mathcal{G}})(\beta_q,q_\perp-k_\perp)\big]\Big)\\ &= \frac{1}{2\alpha_q sN_c m^2}\int d^2k_\perp\Big(2\mathcal{W}^\perp_{\alpha\beta}(q,k_\perp)\{h_1^\perp\bar h_{1\mathcal{G}}+\bar h_1^\perp h_{1\mathcal{G}}\}\\ &-[g^\perp_{\alpha\beta}k_\perp^2(q-k)_\perp^2+2k_\alpha k_\beta(q-k)_\perp^2]\{h_1^\perp\bar h_{2\mathcal{G}}+\bar h_1^\perp h_{2\mathcal{G}}\}\Big)\end{aligned}\tag{5.28}$$

where we used parametrization (9.73) and defined the notation

$$\begin{aligned}\mathcal{W}^\perp_{\mu\nu}(q_\perp,k_\perp) &\equiv g^\perp_{\mu\nu}(k,q-k)_\perp^2-g^\perp_{\mu\nu}k_\perp^2(q-k_\perp)^2\\ &+[k^\perp_\mu(q-k)^\perp_\nu+\mu\leftrightarrow\nu](k,q-k)_\perp-k_\perp^2(q-k)^\perp_\mu(q-k)^\perp_\nu-\ (q-k_\perp)^2k^\perp_\mu k^\perp_\nu\end{aligned}\tag{5.29}$$

It is easy to see that $q^\mu\mathcal{W}^\perp_{\mu\nu}(q,k_\perp)=0$ and $\mathcal{W}_i^{\perp i}(q,k_\perp)=0$.

The second term in Eq. (5.25) is

$$\begin{aligned}\breve{U}^{(1'c)}_{\alpha\beta(3)}(x) &= \frac{N_c}{2s^3\alpha_q}P_{\alpha\beta;ij}\langle\bar\psi(x)\sigma_\star{}^i\gamma_k\gamma_l\psi(0)\rangle_A\langle\bar\psi\big(\frac{1}{\beta}iD_kB^l\big)(0)\sigma_\bullet{}^j\psi(x)\rangle_B\\ &= \frac{N_c}{2s^3\alpha_q}P_{\alpha\beta;ij}\Big(\langle\bar\psi(x)\sigma_\star{}^i\psi(0)\rangle_A\langle\bar\psi\mathcal{P}\mathcal{B}(0)\sigma_\bullet{}^j\psi(x)\rangle_B\\ &+i\langle\bar\psi(x)\sigma_{\star l}\psi(0)\rangle_A\langle\bar\psi\mathcal{B}^{il}(0)\sigma_\bullet{}^j\psi(x)\rangle_B\Big)+\big(x\leftrightarrow 0,\alpha_q\leftrightarrow-\alpha_q\big)\end{aligned}\tag{5.30}$$

where we used Eq. (5.26) and the notations (5.13).

For unpolarized hadrons, Eq. (5.30) can re rewritten as

$$\begin{aligned}\breve{U}^{(1'c)}_{\alpha\beta(3)}(x) &= \frac{N_c}{2s^3\alpha_q}P_{\alpha\beta;ij}\langle\bar\psi(x)\sigma_\star{}^i\psi(0)\rangle_A\\ &\times\Big(\langle\bar\psi(0)\sigma_\bullet{}^j\mathcal{P}\mathcal{B}(0)\psi(x)\rangle_B-i\bar\psi(0)\sigma_{\bullet m}\mathcal{B}^{mj}(0)\psi(x)\rangle_B\Big)+\big(x\leftrightarrow 0,\alpha_q\leftrightarrow-\alpha_q\big)\end{aligned}\tag{5.31}$$

and, using parametrizations (9.74) and (9.75), we obtain

$$\begin{aligned}U^{(1'c)}_{\alpha\beta(3)}(q) &= -\frac{1}{2\alpha_q sN_c}P_{\alpha\beta;ij}\int d^2k_\perp k^i(q-k)^j\\ &\times\Big(h_1^\perp(\alpha_q,k_\perp)[\bar h_{3\mathcal{G}}-i\bar h_{4\mathcal{G}}](\beta_q,q_\perp-k_\perp)+\ \bar h_1^\perp(\alpha_q,k_\perp)[h_{3\mathcal{G}}-ih_{4\mathcal{G}}](\beta_q,q_\perp-k_\perp)\Big)\\ &= \frac{1}{2\alpha_q sN_c}\int d^2k_\perp\big[k^\perp_\alpha(q-k)^\perp_\beta+k^\perp_\beta(q-k)^\perp_\alpha+(k,q-k)_\perp g^\perp_{\alpha\beta}\big]\\ &\times\Big(h_1^\perp(\alpha_q,k_\perp)[\bar h_{3\mathcal{G}}-i\bar h_{4\mathcal{G}}](\beta_q,q_\perp-k_\perp)+\ \bar h_1^\perp(\alpha_q,k_\perp)[h_{3\mathcal{G}}-ih_{4\mathcal{G}}](\beta_q,q_\perp-k_\perp)\Big)\end{aligned}\tag{5.32}$$

Thus, from Eqs. (5.5), (5.14), (5.21), (5.28), and (5.32) we get

$$\begin{aligned}
W_{\mu\nu}^{(1\Xi_1')}(q) = &\ \frac{1}{(2\pi)^4N_c}\int dx_\bullet dx_\star d^2x_\perp\ e^{-i\alpha_q x_\bullet - i\beta_q x_\star + i(q,x)_\perp}\check{W}_{\mu\nu}^{(1')}(x)\ =\ \frac{1}{\alpha_q sN_c}\\
&\times\int d^2k_\perp\Big[-g_{\mu\nu}^\perp(k,q-k)_\perp\Big(\{f_1\bar{\acute{f}}_{1\mathcal{G}}+\bar{f}_1\acute{f}_{1\mathcal{G}}\}+\alpha_q\{[f_\perp-ig_\perp]\bar{\grave{f}}_{1\mathcal{G}}+[\bar{f}_\perp+i\bar{g}_\perp)]\grave{f}_{1\mathcal{G}}\}\Big)\\
&-g_{\mu\nu}^\perp m^2\{f_1[\bar{f}_{2\mathcal{G}}+\bar{f}_{3\mathcal{G}}]+\bar{f}_1[f_{2\mathcal{G}}+f_{3\mathcal{G}}]\}\\
&-\alpha_q\big[2(q-k)_\mu^\perp(q-k)_\nu^\perp+g_{\mu\nu}^\perp(q-k)_\perp^2\big]\{[h+ie]\bar{h}_{1\mathcal{G}}+[\bar{h}-i\bar{e}]h_{1\mathcal{G}}\}\\
&+\frac{1}{m^2}\Big(2\mathcal{W}_{\mu\nu}^\perp(q,k_\perp)\{h_1^\perp\bar{h}_{1\mathcal{G}}+\bar{h}_1^\perp h_{1\mathcal{G}}\}\\
&-[g_{\mu\nu}^\perp k_\perp^2(q-k)_\perp^2+2k_\mu k_\nu(q-k)_\perp^2]\{h_1^\perp\bar{h}_{2\mathcal{G}}+\bar{h}_1^\perp h_{2\mathcal{G}}\}\Big)\\
&+\big[k_\mu^\perp(q-k)_\nu^\perp+k_\nu^\perp(q-k)_\mu^\perp+(k,q-k)_\perp g_{\mu\nu}^\perp\big]\{h_1^\perp[\bar{h}_{3\mathcal{G}}-i\bar{h}_{4\mathcal{G}}]+\ \bar{h}_1^\perp[h_{3\mathcal{G}}-ih_{4\mathcal{G}}]\}\Big]
\end{aligned}\tag{5.33}$$

5.2 Terms coming from $\bar{\Xi}_1'$, Ξ_2', and $\bar{\Xi}_2'$

Replacing $\Xi_1\to\Xi_1'$ and $\bar{\Xi}_1\to\bar{\Xi}_1'$ in Eq. (4.27) we see that the contribution of terms with $\bar{\Xi}_1'(x)$ is a complex conjugate of the contribution (5.33) of $\Xi_1'(0)$ so

$$\begin{aligned}
W_{\mu\nu}^{(1\Xi_1'+1\bar{\Xi}_1')}(q)\ =&\ \frac{1}{(2\pi)^4N_c}\int dx_\bullet dx_\star d^2x_\perp\ e^{-i\alpha_q x_\bullet - i\beta_q x_\star + i(q,x)_\perp}\big[\check{W}_{\mu\nu}^{(1\Xi_1')}(x)+\check{W}_{\mu\nu}^{(1\bar{\Xi}_1')}(x)\big]\\
=&\ g_{\mu\nu}\big[\check{U}^{(1'a)}(q)+\check{U}^{(2'a)}(q)\big]+(\delta_\mu^\alpha\delta_\nu^\beta+\delta_\nu^\alpha\delta_\mu^\beta-g_{\mu\nu}g^{\alpha\beta})\big[\check{U}_{\alpha\beta}^{(1'b)}(q)+\check{U}_{\alpha\beta}^{(2'b)}(q)\big]\\
&+(\delta_\mu^\alpha\delta_\nu^\beta+\delta_\nu^\alpha\delta_\mu^\beta-\frac{1}{2}g_{\mu\nu}g^{\alpha\beta})\big[\check{U}_{\alpha\beta}^{(1'c)}(q)+\check{U}_{\alpha\beta}^{(2'c)}(q)\big]\\
=&\ \frac{2}{\alpha_q sN_c}\int d^2k_\perp\Big\{-g_{\mu\nu}^\perp\Big[(k,q-k)_\perp\big(\{f_1\Re\bar{\acute{f}}_{1\mathcal{G}}+\bar{f}_1\Re\acute{f}_{1\mathcal{G}}\}\\
&\qquad+\alpha_q\{f_\perp\Re\bar{\grave{f}}_{1\mathcal{G}}+\bar{f}_\perp\Re\grave{f}_{1\mathcal{G}}\}+\alpha_q\{g_\perp\Im\bar{\grave{f}}_{1\mathcal{G}}-\bar{g}_\perp\Im\grave{f}_{1\mathcal{G}}\}\big)\\
&+m^2\big(\{f_1\Re\bar{f}_{2\mathcal{G}}+\bar{f}_1\Re f_{2\mathcal{G}}\}+\{f_1\Re\bar{f}_{3\mathcal{G}}+\bar{f}_1\Re f_{3\mathcal{G}}\}\Big]\\
&-\alpha_q\big[2(q-k)_\mu^\perp(q-k)_\nu^\perp+g_{\mu\nu}^\perp(q-k)_\perp^2\big]\big(\{h\Re\bar{h}_{1\mathcal{G}}+\bar{h}\Re h_{1\mathcal{G}}\}+\{\bar{e}\Im h_{1\mathcal{G}}-e\Im\bar{h}_{1\mathcal{G}}\}\big)\\
&+\Big(\frac{2}{m^2}\mathcal{W}_{\mu\nu}^\perp\{h_1^\perp\Re\bar{h}_{1\mathcal{G}}+\bar{h}_1^\perp\Re h_{1\mathcal{G}}\}-[g_{\mu\nu}^\perp k_\perp^2+2k_\mu k_\nu]\frac{(q-k)_\perp^2}{m^2}\{h_1^\perp\Re\bar{h}_{2\mathcal{G}}+\bar{h}_1^\perp\Re h_{2\mathcal{G}}\}\Big)\\
&+\big[(k,q-k)_\perp g_{\mu\nu}^\perp+\ k_\mu^\perp(q-k)_\nu^\perp+k_\nu^\perp(q-k)_\mu^\perp\big]\\
&\qquad\times\ \big(\{h_1^\perp\Re\bar{h}_{3\mathcal{G}}+\bar{h}_1^\perp\Re h_{3\mathcal{G}}\}+\{h_1^\perp\Im\bar{h}_{4\mathcal{G}}+\bar{h}_1^\perp\Im h_{4\mathcal{G}}\}\big)\Big\}
\end{aligned}\tag{5.34}$$

where $\mathcal{W}_{\mu\nu}^\perp(q,k_\perp)$ is defined in Eq. (5.29).

The corresponding contribution of terms coming from Ξ_2' and $\bar{\Xi}_2'$

$$\begin{aligned}
\check{W}_{\mu\nu}^{(1\Xi_2')}(x)+\check{W}_{\mu\nu}^{1(\bar{\Xi}_2')}(x)\ =&\ \frac{N_c}{s}\langle A,B|[\bar{\psi}_A(x)\gamma_\mu\psi_B(x)]\,[\bar{\Xi}_2'(0)\gamma_\nu\psi_A(0)]\\
&+[\bar{\psi}_A(x)\gamma_\mu\Xi_2'(x)]\,[\bar{\psi}_B(0)\gamma_\nu\psi_A(0)]|A,B\rangle\ +\ x\leftrightarrow 0
\end{aligned}\tag{5.35}$$

is obtained from Ea. (5.34) by the projectile↔target replacement (4.29)

$$
\begin{aligned}
W^{(1\Xi_2'+1\bar\Xi_2')}_{\mu\nu}(q) \;&=\; \frac{1}{(2\pi)^4N_c}\int dx_\bullet dx_\star d^2x_\perp\; e^{-i\alpha_q x_\bullet - i\beta_q x_\star + i(q,x)_\perp}\big[\check W^{(1\Xi_2')}_{\mu\nu}(x)+\check W^{(1\bar\Xi_2')}_{\mu\nu}(x)\big]\\
=\; & \frac{2}{\beta_q sN_c}\int d^2k_\perp\Big\{-g^\perp_{\mu\nu}\Big[(k,q-k)_\perp\big(\{\Re\bar{\acute f}_{1\mathcal{G}}f_1+\Re\acute f_{1\mathcal{G}}\bar f_1\}+\;\beta_q\{\Re\bar{\grave f}_{1\mathcal{G}}f_\perp+\Re\grave f_{1\mathcal{G}}\bar f_\perp\}\\
&\qquad +\beta_q\{\Im\bar{\grave f}_{1\mathcal{G}}g_\perp-\Im\grave f_{1\mathcal{G}}\bar g_\perp\}\big)+\;m^2\big(\{\Re\bar f_{2\mathcal{G}}f_1+\Re f_{2\mathcal{G}}\bar f_1\}+\{\Re\bar f_{3\mathcal{G}}f_1+\Re f_{3\mathcal{G}}\bar f_1\}\Big]\\
&-\beta_q\big[2k^\perp_\mu k^\perp_\nu+g^\perp_{\mu\nu}k^2_\perp\big]\big(\{\Re h_{1\mathcal{G}}\bar h\Re\bar h_{1\mathcal{G}}h\}+\{\Im h_{1\mathcal{G}}\bar e-\Im\bar h_{1\mathcal{G}}e\}\big)\\
&+\frac{1}{m^2}\Big(2\mathcal{W}^\perp_{\mu\nu}(q,k_\perp)\{\Re h_{1\mathcal{G}}\bar h^\perp_1+\Re\bar h_{1\mathcal{G}}h^\perp_1\}\\
&\qquad -[g^\perp_{\mu\nu}k^2_\perp(q-k)^2_\perp+2(q-k)_\mu(q-k)_\nu k^2_\perp]\{\Re h_{2\mathcal{G}}\bar h^\perp_1+\Re\bar h_{2\mathcal{G}}h^\perp_1\}\Big)\\
&+\big[(k,q-k)_\perp g^\perp_{\mu\nu}+\;k^\perp_\mu(q-k)^\perp_\nu+k^\perp_\nu(q-k)^\perp_\mu\big]\\
&\qquad\times\big(\{\Re\bar h_{3\mathcal{G}}h^\perp_1+\Re h_{3\mathcal{G}}\bar h^\perp_1\}+\{\Im\bar h_{4\mathcal{G}}h^\perp_1+\Im h_{4\mathcal{G}}\bar h^\perp_1\}\big)\Big\}
\end{aligned}
\tag{5.36}
$$

The final contribution of Ξ' terms is a sum of Eq. (5.34) and (5.36)

$$
\begin{aligned}
W^{(1\Xi_i'+1\bar\Xi_i')}_{\mu\nu}(q)\;&\equiv\; W^{(1\Xi_1'+1\bar\Xi_1')}_{\mu\nu}(q)+W^{(1\Xi_2'+1\bar\Xi_2')}_{\mu\nu}(q) \qquad (5.37)\\
=\;&\frac{2}{sN_c}\int d^2k_\perp\Big\{-g^\perp_{\alpha\beta}\Big[(k,q-k)_\perp\Big(\frac{1}{\alpha_q}\{f_1\Re\bar{\acute f}_{1\mathcal{G}}+\bar f_1\Re\acute f_{1\mathcal{G}}\}+\frac{1}{\beta_q}\{\Re\bar{\acute f}_{1\mathcal{G}}f_1+\Re\acute f_{1\mathcal{G}}\bar f_1\}\\
&+\{f_\perp\Re\bar{\grave f}_{1\mathcal{G}}+\bar f_\perp\Re\grave f_{1\mathcal{G}}\}+\{g_\perp\Im\bar{\grave f}_{1\mathcal{G}}-\bar g_\perp\Im\grave f_{1\mathcal{G}}\}+\;\{\Re\bar{\grave f}_{1\mathcal{G}}f_\perp+\Re\grave f_{1\mathcal{G}}\bar f_\perp\}\\
&+\{\Im\bar{\grave f}_{1\mathcal{G}}g_\perp-\Im\grave f_{1\mathcal{G}}\bar g_\perp\}\Big)+\;\frac{m^2}{\alpha_q}\big(\{f_1\Re\bar f_{2\mathcal{G}}+\bar f_1\Re f_{2\mathcal{G}}\}+\{f_1\Re\bar f_{3\mathcal{G}}+\bar f_1\Re f_{3\mathcal{G}}\}\\
&\qquad\qquad+\;\frac{m^2}{\beta_q}\big(\{\Re\bar f_{2\mathcal{G}}f_1+\Re f_{2\mathcal{G}}\bar f_1\}+\{\Re\bar f_{3\mathcal{G}}f_1+\Re f_{3\mathcal{G}}\bar f_1\}\Big]\\
&-\big[2(q-k)^\perp_\mu(q-k)^\perp_\nu+g^\perp_{\mu\nu}(q-k)^2_\perp\big]\big(\{h\Re\bar h_{1\mathcal{G}}+\bar h\Re h_{1\mathcal{G}}\}+\{\bar e\Im h_{1\mathcal{G}}-e\Im\bar h_{1\mathcal{G}}\}\big)\\
&-\big[2k^\perp_\mu k^\perp_\nu+g^\perp_{\mu\nu}k^2_\perp\big]\big(\{\Re h_{1\mathcal{G}}\bar h+\Re\bar h_{1\mathcal{G}}h\}+\{\Im h_{1\mathcal{G}}\bar e-\Im\bar h_{1\mathcal{G}}e\}\big)\\
&+\frac{2}{m^2}\mathcal{W}^\perp_{\mu\nu}(q,k_\perp)\Big(\frac{1}{\alpha_q}\{h^\perp_1\Re\bar h_{1\mathcal{G}}+\bar h^\perp_1\Re h_{1\mathcal{G}}\}+\frac{1}{\beta_q}\{\Re h_{1\mathcal{G}}\bar h^\perp_1+\Re\bar h_{1\mathcal{G}}h^\perp_1\}\Big)\\
&-\frac{(q-k)^2_\perp}{\alpha_q m^2}[g^\perp_{\mu\nu}k^2_\perp+2k_\mu k_\nu]\{h^\perp_1\Re\bar h_{2\mathcal{G}}+\bar h^\perp_1\Re h_{2\mathcal{G}}\}\\
&-\frac{k^2_\perp}{\beta_q m^2}[g^\perp_{\mu\nu}(q-k)^2_\perp+2(q-k)_\mu(q-k)_\nu]\{\Re h_{2\mathcal{G}}\bar h^\perp_1+\Re\bar h_{2\mathcal{G}}h^\perp_1\}\\
&+\big[(k,q-k)_\perp g^\perp_{\alpha\beta}+\;k^\perp_\alpha(q-k)^\perp_\beta+k^\perp_\beta(q-k)^\perp_\alpha\big]\Big(\frac{1}{\alpha_q}\{h^\perp_1\Re\bar h_{3\mathcal{G}}+\bar h^\perp_1\Re h_{3\mathcal{G}}\}\\
&+\frac{1}{\alpha_q}\{h^\perp_1\Im\bar h_{4\mathcal{G}}+\bar h^\perp_1\Im h_{4\mathcal{G}}\}+\frac{1}{\beta_q}\big(\{\Re\bar h_{3\mathcal{G}}h^\perp_1+\Re h_{3\mathcal{G}}\bar h^\perp_1\}+\frac{1}{\beta_q}\{\Im\bar h_{4\mathcal{G}}h^\perp_1+\Im h_{4\mathcal{G}}\bar h^\perp_1\}\big)\Big\}
\end{aligned}
$$

6 Terms with two quark-quark-gluon operators

First, in Ref. [9] it was demonstrated that after sorting out color-singlet matrix elements the contribution $W^{(2c)}_{\mu\nu}$ is $O\big(\frac{1}{N_c^2}\big)$ in comparison to $W^{(2a)}_{\mu\nu}$ (and $W^{(2b)}_{\mu\nu}$) so it will be neglected in accordance with our leading-N_c accuracy.

6.1 Terms with two quark-quark-gluon operators coming from Ξ_1 and Ξ_2

Let us start with the first term in the r.h.s. of Eq. (4.4). Performing Fierz transformation (9.1) we obtain

$$\frac{N_c}{s}\langle A,B|(\bar\psi_A^m(x)\gamma_\mu\Xi_2^m(x))(\bar\psi_B^n(0)\gamma_\nu\Xi_1^n(0))+\mu\leftrightarrow\nu|A,B\rangle+x\leftrightarrow 0 \;=\; g_{\mu\nu}\check V_1+\check V_{2\mu\nu}+\check V_{3\mu\nu} \tag{6.1}$$

where

$$\begin{aligned}\check V^{(1)} \;=\; & \frac{N_c}{2s}\langle A,B| -[\bar\psi_A^n(x)\Xi_1^m(0)][\bar\psi_B^n(0)\Xi_2^m(x)]+[\bar\psi_A^m(x)\gamma_5\Xi_1^n(0)][\bar\psi_B^n(0)\gamma_5\Xi_2^m(x)] \\ & +[\bar\psi_A^m(x)\gamma_\alpha\Xi_1^m(0)][\bar\psi_B^n(0)\gamma^\alpha\Xi_2^n(x)]+[\bar\psi_A^m(x)\gamma_\alpha\gamma_5\Xi_1^m(0)][\bar\psi_B^n(0)\gamma^\alpha\gamma_5\Xi_2^n(x)]|A,B\rangle+x\leftrightarrow 0,\end{aligned} \tag{6.2}$$

$$\begin{aligned}\check V^{(2)}_{\mu\nu} \;=\; & \frac{N_c}{2s}\langle A,B| -\; [\bar\psi_A^m(x)\gamma_\mu\Xi_1^n(0)][\bar\psi_B^n(0)\gamma_\nu\Xi_2^m(x)] \\ & -[\bar\psi_A^m(x)\gamma_\mu\gamma_5\Xi_1^n(0)][\bar\psi_B^n(0)\gamma_\nu\gamma_5\Xi_2^m(x)]+\mu\leftrightarrow\nu|A,B\rangle+x\leftrightarrow 0,\end{aligned} \tag{6.3}$$

and

$$\begin{aligned}\check V^{(3)}_{\mu\nu} \;=\; & \frac{N_c}{2s}\langle A,B|[(\bar\psi_A^m(x)\sigma_{\mu\alpha}\Xi_1^n(0)][\bar\psi_B^n(0)\sigma_\nu{}^\alpha\Xi_2^m(x)]+\mu\leftrightarrow\nu \\ & -\frac{g_{\mu\nu}}{2}[\bar\psi_A^m(x)\sigma^{\alpha\beta}\Xi_1^n(0)][\bar\psi_B^n(0)\sigma_{\alpha\beta}\Xi_2^m(x)|A,B\rangle+x\leftrightarrow 0\end{aligned} \tag{6.4}$$

It is convenient to define $\check V^{(3)}_{\mu\nu}$ to be traceless. In next Sections, we will consider these terms in turn.

6.1.1 Term propotional to $g_{\mu\nu}$

Using $\Xi_1 = -\frac{g\not{p}_2}{s}\gamma^i B_i\frac{1}{\alpha+i\epsilon}\psi_A$ and $\Xi_2 = -\frac{g\not{p}_1}{s}\gamma^i A_i\frac{1}{\beta+i\epsilon}\psi_B$ from Eq. (2.4) and extracting color-singlet contributions one obtains

$$\begin{aligned}\check V^{(1)} \;=\; & \frac{1}{2s^3} \\ \times \Big\{ & -\Big[\langle\bar\psi A_i(x)\not{p}_2\gamma^j\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi B_j(0)\not{p}_1\gamma^i\frac{1}{\beta}\psi(x)\rangle_B-\psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\Big] \\ & +\Big[\langle\bar\psi A_i(x)\gamma_k\not{p}_2\gamma^j\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi B_j(0)\gamma^k\not{p}_1\gamma^i\frac{1}{\beta}\psi(x)\rangle_B+\;\psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\Big] \\ & \quad +\frac{2}{s}\Big[\langle\bar\psi A_i(x)\not{p}_1\not{p}_2\gamma^j\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi B_j(0)\not{p}_2\not{p}_1\gamma^i\frac{1}{\beta}\psi(x)\rangle_B \\ & \qquad +\psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\Big]\Big\}\;+\;x\leftrightarrow 0\end{aligned} \tag{6.5}$$

From the power counting it is clear that the third term is $\sim\frac{m_\perp^2}{s}$ with respect to the first two ones. By the same token, if we replace Ξ_1 by Ξ_1' or Ξ_2 by Ξ_2' the contribution will be small.

Let us start with the first term in Eq. (6.5). Using Eq. (9.28) and the fact that $\langle\bar\psi(x)\big[A_k\sigma_{\star j}-A_j(x)\sigma_{\star k}\big]\psi(0)\rangle_A=0$ (cf. Eq (4.23)), we obtain

$$\begin{aligned}\check V^{(1)}_{1a} &= -\frac{1}{2s^3}\langle\bar\psi \not{A}(x)\not{p}_2\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi \not{B}(0)\not{p}_1\frac{1}{\beta}\psi(x)\rangle_B-\frac{1}{4s^2}\langle\bar\psi \not{A}(x)\gamma_i\frac{1}{\alpha}\psi(0)\rangle_A\\ &\times\langle\bar\psi \not{B}(0)\gamma^i\frac{1}{\beta}\psi(x)\rangle_B-\frac{1}{8s^2}\langle\bar\psi A^i(x)\sigma_{jk}\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi B_i(0)\sigma^{jk}\frac{1}{\beta}\psi(x)\rangle_B\\ &= -\frac{1}{2s^3}\langle\bar\psi \not{A}(x)\not{p}_2\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi \not{B}(0)\not{p}_1\frac{1}{\beta}\psi(x)\rangle_B\Big[1+O\big(\frac{q_\perp^2}{s}\big)\Big]\end{aligned}\tag{6.6}$$

where we used the fact that projectile and target matrix elements in the two last terms in the l.h.s. cannot produce factor of s. The corresponding contribution to $V^{(1)}(q)$ has the form

$$\begin{aligned}&-\frac{1}{\alpha_q\beta_q sN_c}\frac{1}{2m^2}\int d^2k_\perp\,[k_\perp^2h_1^\perp+m^2\alpha_q(h-ie)](\alpha_q,k_\perp)\\ &\qquad\times\,\big[(q-k)_\perp^2\bar h_1^\perp+m^2\beta_q(\bar h+i\bar e)\big](\beta_q,q_\perp-k_\perp)\end{aligned}\tag{6.7}$$

due to EOMs (9.42), (9.43).

Next, consider second term in Eq. (6.5). Using Eqs. (9.24) and (9.28), one can rewrite is as

$$\begin{aligned}\check V^{(1)}_{1b} &= \frac{1}{2s^3}\Big[\langle\bar\psi A_i(x)\gamma_k\not{p}_2\gamma^j\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi B_j(0)\gamma^k\not{p}_1\gamma^i\frac{1}{\beta}\psi(x)\rangle_B\end{aligned}\tag{6.8}$$

$$+\ \psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\Big] = \frac{1}{s^3}\langle\bar\psi(x)\not{A}(x)\not{p}_2\gamma_i\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi(0)\not{B}(0)\not{p}_1\gamma^i\frac{1}{\beta}\psi(x)\rangle_B$$

The corresponding contribution to $V^{(1)}(q)$ has the form

$$\frac{1}{\alpha_q\beta_q sN_c}\int d^2k_\perp(k,q-k)_\perp\big[f_1-\alpha_q(f_\perp+ig_\perp)\big](\alpha_q,k_\perp)\big[\bar f_1-\beta_q(\bar f_\perp-i\bar g_\perp)\big](\beta_q,q_\perp-k_\perp)\tag{6.9}$$

where again we used E)Ms (9.42), (9.43).

Similarly, from Eq. (9.28) we get the third term in the form

$$\begin{aligned}&\frac{1}{s^4}\Big[\langle\bar\psi A_i(x)\not{p}_1\not{p}_2\gamma^j\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi B_j(0)\not{p}_2\not{p}_1\gamma^i\frac{1}{\beta}\psi(x)\rangle_B+\ \psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\Big]\\ &= \frac{1}{4s^2}\Big[\langle\bar\psi\grave A_i(x)\gamma^j\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi\grave B_j(0)\gamma^i\frac{1}{\beta}\psi(x)\rangle_B+\ \psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\Big]\end{aligned}\tag{6.10}$$

Since both projectile and target matrix elements cannot give factor s this contribution is $O\big(\frac{q_\perp^2}{s}\big)$ in comparison to that of the two first terms. Using QCD equations of motion (9.42), (9.43), we obtain the contribution to $W_{\mu\nu}$ in the form

$$g_{\mu\nu}V_1^{(1)}(q) = \frac{g_{\mu\nu}}{16\pi^4N_c}\int dx_\bullet dx_\star d^2x_\perp\ e^{-i\alpha_qx_\bullet-i\beta_qx_\star+i(q,x)_\perp}\check V^{(1)}(x)\tag{6.11}$$

$$\begin{aligned}&= \frac{g_{\mu\nu}}{Q^2N_c}\int d^2k_\perp\Big[(k,q-k)_\perp\big[f_1-\alpha_q(f_\perp+ig_\perp)\big](\alpha_q,k_\perp)\big[\bar f_1-\beta_q(\bar f_\perp-i\bar g_\perp)\big](\beta_q,q_\perp-k_\perp)\\ &-\frac{1}{2m^2}\big[k_\perp^2h_1^\perp+m^2\alpha_q(h-ie)](\alpha_q,k_\perp\big]\big[(q-k)_\perp^2\bar h_1^\perp+m^2\beta_q(\bar h+i\bar e)\big](\beta_q,q_\perp-k_\perp)\Big]\end{aligned}$$

Next, the $x \leftrightarrow 0$ term is

$$\check{V}_2^{(1)} = -\frac{1}{2s^3}\langle\bar\psi \not{A}(0)\not{p}_2\frac{1}{\alpha}\psi(x)\rangle_A\langle\bar\psi \not{B}(x)\not{p}_1\frac{1}{\beta}\psi(0)\rangle_B + \frac{1}{s^3}\langle\bar\psi \not{A}(0)\not{p}_2\gamma_i\frac{1}{\alpha}\psi(x)\rangle_A\langle\bar\psi \not{B}(x)\not{p}_1\gamma^{\ i}\frac{1}{\beta}\psi(0)\rangle_B \tag{6.12}$$

Similarly to Eq. (6.11), we get

$$g_{\mu\nu}V_2^{(1)}(q) = \frac{g_{\mu\nu}}{16\pi^4 N_c}\int dx_\bullet dx_\star d^2x_\perp\ e^{-i\alpha_q x_\bullet - i\beta_q x_\star + i(q,x)_\perp}\check{V}_2^{(1)}(x) \tag{6.13}$$

$$= \frac{g_{\mu\nu}}{Q^2N_c}\int d^2k_\perp\Big[(k,q-k)_\perp\big[\bar f_1 - \alpha_q(\bar f_\perp - i\bar g_\perp)\big](\alpha_q,k_\perp)\big[f_1-\beta_q(f_\perp+ig_\perp)\big](\beta_q,q_\perp-k_\perp)$$
$$- \frac{1}{2m^2}\big[k_\perp^2\bar h_1^\perp + m^2\alpha_q(\bar h+i\bar e)\big](\alpha_q,k_\perp)\big[(q-k)_\perp^2 h_1^\perp + m^2\beta_q(h-ie)\big](\beta_q,q_\perp-k_\perp)\Big]$$

Sum of Eq. (6.11) and (6.13) is

$$g_{\mu\nu}V^{(1)}(q) = \frac{g_{\mu\nu}}{16\pi^4 N_c}\int dx_\bullet dx_\star d^2x_\perp\ e^{-i\alpha_q x_\bullet - i\beta_q x_\star + i(q,x)_\perp}\check{V}^{(1)}(x) \tag{6.14}$$

$$= \frac{g_{\mu\nu}}{Q^2N_c}\int d^2k_\perp\Big\{(k,q-k)_\perp\Big(\{f_1\bar f_1 + \bar f_1 f_1\} - \alpha_q\{[f_\perp+ig_\perp]\bar f_1 + [\bar f_\perp - i\bar g_\perp]f_1\}$$
$$- \beta_q\{f_1[\bar f_\perp - i\bar g_\perp] + \bar f_1[f_\perp+ig_\perp]\} + \alpha_q\beta_q\{[f_\perp+ig_\perp][\bar f_\perp - i\bar g_\perp] + [\bar f_\perp - i\bar g_\perp][f_\perp+ig_\perp]\}\Big)$$
$$- \frac{1}{2}\Big(\frac{k_\perp^2(q-k)_\perp^2}{m^2}\{h_1^\perp\bar h_1^\perp + \bar h_1^\perp h_1^\perp\} + \alpha_q(q-k)_\perp^2\{[(h-ie)]\bar h_1^\perp + [\bar h + i\bar e]h_1^\perp\}$$
$$+ \beta_q k_\perp^2\{h_1^\perp[\bar h+i\bar e] + \bar h_1^\perp[h-ie]\} + m^2\alpha_q\beta_q\{[h-ie][\bar h+i\bar e] + [\bar h+i\bar e][h-ie]\}\Big)$$

6.1.2 Term $\check{V}_{\mu\nu}^{(2)}$

Separating color-singlet contributions one can rewrite Eq. (6.3) as

$$\check{V}_{\mu\nu}^{(2)} = -\frac{1}{2s^3}\{\langle\bar\psi A_i(x)\gamma_\mu\not{p}_2\gamma^j\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi B_j(0)\gamma_\nu\not{p}_1\gamma^i\frac{1}{\beta}\psi(x)\rangle_B + \psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x) + \mu\leftrightarrow\nu\} + x\leftrightarrow 0 \tag{6.15}$$

As demonstrated in Ref. [9], only transverse μ and ν contribute at $\frac{1}{Q^2}$ level. In this case we can use formula (9.25) and get

$$\check{V}_{1\mu_\perp\nu_\perp}^{(2)} = -\frac{1}{2s^3}\{\langle\bar\psi A_i(x)\gamma_{\mu_\perp}\not{p}_2\gamma^j\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi B_j(0)\gamma_{\nu_\perp}\not{p}_1\gamma^i\frac{1}{\beta}\psi(x)\rangle_B + \psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x) + \mu\leftrightarrow\nu\}$$
$$= -\frac{g_{\mu\nu}^\perp}{s^3}\langle\bar\psi \not{A}(x)\not{p}_2\gamma_i\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar\psi \not{B}(0)\not{p}_1\gamma^i\frac{1}{\beta}\psi(x)\rangle_B \tag{6.16}$$

which gives the contribution to $W_{\mu\nu}$ in the form

$$V_{1\mu\nu}^{(2)}(q) = \frac{1}{16\pi^4N_c}\int dx_\bullet dx_\star d^2x_\perp\ e^{-i\alpha_q x_\bullet - i\beta_q x_\star + i(q,x)_\perp}\check{V}_{1\mu_\perp\nu_\perp}^{(2)}(x) = -\frac{g_{\mu\nu}^\perp}{\alpha_q\beta_q sN_c}$$
$$\times\int d^2k_\perp(k,q-k)_\perp\big[f_1 - \alpha_q(f^\perp + ig^\perp)\big](\alpha_q,k_\perp)\big[\bar f_1 - \beta_q(\bar f^\perp - i\bar g^\perp)\big](\beta_q,q_\perp-k_\perp) \tag{6.17}$$

where we again used EOMs (9.42), (9.43)

The corresponding $x \leftrightarrow 0$ contribution is

$$\check{V}^{(2)}_{2\mu_\perp\nu_\perp} = -\frac{g^\perp_{\mu\nu}}{s^3}\langle\bar{\psi}\not{A}(0)\not{p}_2\gamma_i\frac{1}{\alpha}\psi(x)\rangle_A\langle\bar{\psi}\not{B}(x)\not{p}_1\gamma^i\frac{1}{\beta}\psi(0)\rangle_A \tag{6.18}$$

which gives

$$\begin{aligned} V^{(2)}_{2\mu\nu}(q) &= -\frac{g^\perp_{\mu\nu}}{Q^2N_c}\int d^2k_\perp(k,q-k)_\perp \\ &\times \big[\bar{f}_1-\alpha_q(\bar{f}^\perp-i\bar{g}^\perp)\big](\alpha_q,k_\perp)\big[f_1-\beta_q(f^\perp+ig^\perp)\big](\beta_q,q_\perp-k_\perp) \end{aligned} \tag{6.19}$$

The sum of Eq. (6.17) and (6.19) has the form

$$\begin{aligned} &V^{(2)}_{\mu\nu}(q) \\ &= -\frac{g^\perp_{\mu\nu}}{Q^2N_c}\int d^2k_\perp(k,q-k)_\perp\Big(\{f_1\bar{f}_1+\bar{f}_1f_1\}-\alpha_q\{[f^\perp+ig^\perp]\bar{f}_1+[\bar{f}^\perp-i\bar{g}^\perp]f_1\} \\ &-\beta_q\{f_1[\bar{f}^\perp-i\bar{g}^\perp]+\bar{f}_1[f^\perp+ig^\perp]\}+\alpha_q\beta_q\{[f_\perp+ig_\perp][\bar{f}_\perp-i\bar{g}_\perp]+[\bar{f}_\perp-i\bar{g}_\perp][f_\perp+ig_\perp]\}\Big) \end{aligned} \tag{6.20}$$

6.1.3 Term $\check{V}^{(3)}_{\mu\nu}$

Let us consider now

$$\check{V}'_{3\mu\nu} = \frac{N_c}{2s}\langle A,B|[(\bar{\psi}^m_A(x)\sigma_{\mu\alpha}\Xi^m_1(0)][\bar{\psi}^n_B(0)\sigma_\nu{}^\alpha\Xi^n_2(x)]+\mu\leftrightarrow\nu+x\leftrightarrow 0 \tag{6.21}$$

(the trace will be subtracted after the calculation). Separating color-singlet contributions, we get

$$\check{V}'_{3\mu\nu} = \frac{1}{2s^3}\langle\bar{\psi}A_i(x)\sigma_{\mu\alpha}\not{p}_2\gamma^j\frac{1}{\alpha}\psi(0)\rangle_A\langle\bar{\psi}B_j(0)\sigma_\nu{}^\alpha\not{p}_1\gamma^i\frac{1}{\beta}\psi(x)\rangle_A+\mu\leftrightarrow\nu+x\leftrightarrow 0 \tag{6.22}$$

From Eq. (9.10)

$$\begin{aligned} A^j\sigma_{\mu\xi}\sigma_{\star i}\otimes B^i\sigma_\nu{}^\xi\sigma_{\bullet j} &= -\big(A_\nu\sigma_{\star k}-\frac{g_{\nu k}}{2}A^j\sigma_{\star j}\big)\otimes\big(B_\mu\sigma_\bullet{}^k-\frac{1}{2}\delta^k_\mu B^j\sigma_{\bullet j}\big) \\ &-\big(A_k\sigma_{\star\mu_\perp}-\frac{g_{\mu k}}{2}A^j\sigma_{\star j}\big)\otimes\big(B^k\sigma_{\bullet\nu_\perp}-\frac{1}{2}\delta^k_\nu B^j\sigma_{\bullet j}\big) \\ &-\frac{2}{s}(p_{1\mu}p_{2\nu}+p_{2\mu}p_{1\nu})\big(A^j\sigma_{\star i}-\frac{\delta^j_i}{2}A^k\sigma_{\star k}\big)\otimes\big(B^i\sigma_{\bullet j}-\frac{\delta^j_i}{2}B^l\sigma_{\bullet l}\big) \\ &-\frac{1}{s}(p_{1\mu}p_{2\nu}+p_{2\mu}p_{1\nu})\sigma_{\star i}A^i\otimes\sigma_{\bullet j}B^j+\frac{g^\perp_{\mu\nu}}{2}A^j\sigma_{\star j}\otimes B^k\sigma_{\bullet k} \end{aligned} \tag{6.23}$$

Using EOMs (9.42), (9.43) and parametrizations (9.49), (9.64) and similar one for projectile matrix elements, we get

$$\begin{aligned} V'_{3\mu\nu} &= \frac{g^\perp_{\mu\nu}-g^\parallel_{\mu\nu}}{2\alpha_q\beta_q sN_c}\int d^2k_\perp\Big(\frac{1}{m^2}\big[k_\perp^2h_1^\perp+\alpha_q m^2(h-ie)\big]\big[(q-k)_\perp^2\bar h_1^\perp+\beta_q m^2(\bar h+i\bar e)\big] \\ &\quad+\frac{1}{m^2}\big[k_\perp^2\bar h_1^\perp+\alpha_q m^2(\bar h+i\bar e)\big]\big[(q-k)_\perp^2 h_1^\perp+\beta_q m^2(h-ie)\big]\Big) \\ &\quad+\frac{1}{\alpha_q\beta_q sN_c}\int d^2k_\perp\frac{1}{m^2}\{[k^\perp_\mu(q-k)^\perp_\nu+\mu\leftrightarrow\nu](k,q-k)_\perp-k_\perp^2(q-k)^\perp_\mu(q-k)^\perp_\nu \\ &\quad-(q-k_\perp)^2k^\perp_\mu k^\perp_\nu-\frac{g^\perp_{\mu\nu}}{2}k_\perp^2(q-k_\perp)^2-g^\parallel_{\mu\nu}\big[(k,q-k)_\perp^2-\frac12 k_\perp^2(q-k)_\perp^2\big]\} \\ &\quad\times\big[h^\perp_{1G}(\alpha_q,k_\perp)\bar h^\perp_{1G}(\beta_q,q_\perp-k_\perp)+\bar h^\perp_{1G}(\alpha_q,k_\perp)h^\perp_{1G}(\beta_q,q_\perp-k_\perp)\big] \end{aligned} \tag{6.24}$$

After subtracting trace we obtain

$$\begin{aligned} V^{(3)}_{\mu\nu} &= V'_{3\mu\nu}-\frac{g_{\mu\nu}}{4}V'^{\ \xi}_{3\xi} \\ &= \frac{g^\perp_{\mu\nu}-g^\parallel_{\mu\nu}}{2Q^2N_c}\int d^2k_\perp\Big(\frac{k_\perp^2(q-k)_\perp^2}{m^2}\{h_1^\perp\bar h_1^\perp+\bar h_1^\perp h_1^\perp\}+\alpha_q(q-k)_\perp^2\{[h-ie]\bar h_1^\perp+[\bar h+i\bar e]h_1^\perp\} \\ &\quad+\beta_q k_\perp^2\{h_1^\perp[\bar h+i\bar e]+\bar h_1^\perp[h-ie]\}+\alpha_q\beta_q m^2\{[h-ie][\bar h+i\bar e]+[\bar h+i\bar e][h-ie]\}\Big) \\ &\quad+\frac{1}{\alpha_q\beta_q sN_c}\int d^2k_\perp\frac{1}{m^2}\mathcal{W}^\perp_{\mu\nu}(q_\perp,k_\perp)\{h^\perp_{1G}\bar h^\perp_{1G}+\bar h^\perp_{1G}h^\perp_{1G}\} \end{aligned} \tag{6.25}$$

where $\mathcal{W}^\perp_{\mu\nu}(q_\perp,k)$ is defined in Eq. (5.29).

Let us now assemble the contribution of terms (6.1) to $W_{\mu\nu}$. Summing Eqs. (6.14), (6.20), and (6.25) we get

$$\begin{aligned} g_{\mu\nu}\check V_1+\check V_{2\mu\nu}+\check V_{3\mu\nu} &= \frac{1}{32\pi^4}\int dx_\bullet dx_\star d^2x_\perp\ e^{-i\alpha x_\bullet-i\beta x_\star+i(q,x)_\perp} \\ &\quad\big[\langle A,B|(\bar\psi^m_A(x)\gamma_\mu\Xi_2^m(x))(\bar\psi^n_B(0)\gamma_\nu\Xi_1^n(0))+\mu\leftrightarrow\nu|A,B\rangle+x\leftrightarrow 0\big] \\ &= \frac{g^\parallel_{\mu\nu}}{Q^2N_c}\int d^2k_\perp\Big\{(k,q-k)_\perp\Big[\{f_1\bar f_1+\bar f_1 f_1\}-\alpha_q\{[f^\perp+ig^\perp]\bar f_1+[\bar f^\perp-i\bar g^\perp]f_1\} \\ &\quad-\beta_q\{f_1[\bar f^\perp-i\bar g^\perp]+\bar f_1[f^\perp+ig^\perp]\}+\alpha_q\beta_q\{[f_\perp+ig_\perp][\bar f_\perp-i\bar g_\perp]+[\bar f_\perp-i\bar g_\perp][f_\perp+ig_\perp]\}\Big] \\ &\quad-\Big[\frac{k_\perp^2(q-k)_\perp^2}{m^2}\{h_1^\perp\bar h_1^\perp+\bar h_1^\perp h_1^\perp\}+\alpha_q(q-k)_\perp^2\{[h-ie]\bar h_1^\perp+[\bar h+i\bar e]h_1^\perp\} \\ &\quad+\beta_q k_\perp^2\{h_1^\perp[\bar h+i\bar e]+\bar h_1^\perp[h-ie]\}+\alpha_q\beta_q m^2\{[h-ie][\bar h+i\bar e]+[\bar h+i\bar e][h-ie]\}\Big]\Big\} \\ &\quad+\frac{1}{\alpha_q\beta_q sN_c}\int d^2k_\perp\frac{1}{m^2}\mathcal{W}^\perp_{\mu\nu}(q_\perp,k)\{h^\perp_{1G}\bar h^\perp_{1G}+\bar h^\perp_{1G}h^\perp_{1G}\} \end{aligned} \tag{6.26}$$

Finally, to get $W^{(2a)}_{\mu\nu}(q)$ of Eq. (4.4) we need to add the contribution of the term $\big[\bar\Xi_1(x)\gamma_\mu\psi_B(x)\big]\big[\bar\Xi_2(0)\gamma_\nu\psi_A(0)\big]$. Similarly to the case of one quark-quark-gluon operator considered in Sect. 4, it can be demonstrated that this contribution doubles the real part of the result (6.26) so we get

$$
\begin{aligned}
W^{(2a)}_{\mu\nu}(q) &= \frac{1}{32\pi^4}\int dx_\bullet dx_\star d^2x_\perp\ e^{-i\alpha x_\bullet - i\beta x_\star + i(q,x)_\perp}\big[\langle A,B|(\bar\psi^m_A(x)\gamma_\mu\Xi^m_2(x))\\
&\times(\bar\psi^n_B(0)\gamma_\nu\Xi^n_1(0)) + [\bar\Xi_1(x)\gamma_\mu\psi_B(x)]\,[\bar\Xi_2(0)\gamma_\nu\psi_A(0)] + \mu\leftrightarrow\nu|A,B\rangle + x\leftrightarrow 0\big]\\
&= \frac{2g^{\parallel}_{\mu\nu}}{\alpha_q\beta_q sN_c}\int d^2k_\perp\Big\{(k,q-k)_\perp\Big(\{f_1\bar f_1 + \bar f_1 f_1\} - \alpha_q\{f_\perp\bar f_1 + \bar f_\perp f_1\} - \beta_q\{f_1\bar f_\perp + \bar f_1 f_\perp\}\\
&+ \alpha_q\beta_q\{f_\perp\bar f_\perp + \bar f_\perp f_\perp\} + \alpha_q\beta_q\{g_\perp\bar g_\perp + \bar g_\perp g_\perp\}\Big)\\
&- \big[\frac{1}{m^2}k^2_\perp(q-k)^2_\perp\{h^\perp_1\bar h^\perp_1 + \bar h^\perp_1 h^\perp_1\} + \alpha_q(q-k)^2_\perp\{h\bar h^\perp_1 + \bar h h^\perp_1\} + \beta_q k^2_\perp\{h^\perp_1\bar h + \bar h^\perp_1 h\}\\
&+ \alpha_q\beta_q m^2\{h\bar h + \bar h h\} + \alpha_q\beta_q m^2\{e\bar e + \bar e e\}\big]\Big\}\\
&+ \frac{2}{\alpha_q\beta_q sN_c}\int d^2k_\perp\frac{1}{m^2}\mathcal{W}^\perp_{\mu\nu}(q_\perp,k)\Re\big(\{h^\perp_{1G}\bar h^\perp_{1G} + \bar h^\perp_{1G}h^\perp_{1G}\}\big)
\end{aligned}
\tag{6.27}
$$

6.2 Terms with two quark-quark-gluon operators coming from $\bar\Xi_2$ and Ξ_2

Let us start with the first term in Eq. (4.5).

$$
\check W^{(2b)}_{1\mu\nu} = \frac{N_c}{s}\langle A,B|\big[\bar\psi_A(x)\gamma_\mu\Xi_2(x)\big]\big[\bar\Xi_2(0)\gamma_\nu\psi_A(0)\big] + \mu\leftrightarrow\nu|A,B\rangle\ +\ x\leftrightarrow 0 \tag{6.28}
$$

After Fierz transformation (9.1) we obtain

$$
\begin{aligned}
\check W^{(2b)}_{1\mu\nu} &= -\frac{N_c}{2s}(\delta^\alpha_\mu\delta^\beta_\nu + \delta^\alpha_\nu\delta^\beta_\mu - g_{\mu\nu}g^{\alpha\beta})\langle A,B|\big\{[\bar\psi^m_A(x)\gamma_\alpha\psi^n_A(0)][\bar\Xi^n_2(0)\gamma_\beta\Xi^m_2(x)] \qquad (6.29)\\
&\qquad + \gamma_\alpha\otimes\gamma_\beta\leftrightarrow\gamma_\alpha\gamma_5\otimes\gamma_\beta\gamma_5\big\}|A,B\rangle\\
&+ \frac{N_c}{2s}(\delta^\alpha_\mu\delta^\beta_\nu + \delta^\alpha_\nu\delta^\beta_\mu - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta})\langle A,B|[\bar\psi^m_A(x)\sigma_{\alpha\xi}\psi^n_A(0)][\bar\Xi^n_2(0)\sigma_\beta{}^\xi\Xi^m_2(x)]|A,B\rangle\ +\ x\leftrightarrow 0
\end{aligned}
$$

(note that $\bar\Xi_2\Xi_2 = \bar\Xi_2\gamma_5\Xi_2 = 0$). Using explicit expressions (2.4) for quark fields and separating color-singlet terms we get

$$
\check W^{(2b)}_{1\mu\nu} = \check V^4_{\mu\nu} + \check V^5_{\mu\nu} \tag{6.30}
$$

where

$$
\begin{aligned}
\check V^4_{\mu\nu} &= -\frac{1}{s^3}(\delta^\alpha_\mu p_{1\nu} + \delta^\alpha_\nu p_{1\mu} - g_{\mu\nu}p^\alpha_1)\Big(\langle\bar\psi(x)A_j(x)\gamma_\alpha A_i(0)\psi(0)\rangle_A\\
&\times\langle(\bar\psi\frac{1}{\beta})(0)\gamma^i\not p_1\gamma^j\frac{1}{\beta}\psi(x)\rangle_A + \psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\Big)\ +\ x\leftrightarrow 0
\end{aligned}
\tag{6.31}
$$

and

$$
\begin{aligned}
\check V^5_{\mu\nu} &= \frac{1}{s^3}(\delta^\alpha_\mu\delta^\beta_\nu + \delta^\alpha_\nu\delta^\beta_\mu - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta})\\
&\times\Big\{-p_{1\beta}\langle\bar\psi(x)A_j(x)\sigma_{\alpha k}A_i(0)\psi(0)\rangle_A\langle(\bar\psi\frac{1}{\beta})(0)\gamma^i\sigma_\bullet{}^k\gamma^j\frac{1}{\beta}\psi(x)\rangle_A\\
&+ \langle\bar\psi(x)A_j(x)\sigma_{\alpha\bullet}A_i(0)\psi(0)\rangle_A\langle(\bar\psi\frac{1}{\beta})(0)\gamma^i\sigma_{\bullet\beta_\perp}\gamma^j\frac{1}{\beta}\psi(x)\rangle_A\Big\}\ +\ x\leftrightarrow 0
\end{aligned}
\tag{6.32}
$$

We will consider them in turn.

6.2.1 Term $\check V^4_{\mu\nu}$

First, as demonstrated in Ref. [9], the term $\sim g_{\mu\nu}$ is small, so

$$\begin{aligned}\check V^4_{\mu\nu} &= -\frac{p_{1\mu}}{s^3}\Big(\langle\bar\psi A_j(x)\gamma_\nu A_i\psi(0)\rangle_A\langle(\bar\psi\frac{1}{\beta})(0)\gamma^i\not{p}_1\gamma^j\frac{1}{\beta}\psi(x)\rangle_A\\ &+ \psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\Big) + \mu\leftrightarrow\nu + x\leftrightarrow 0 \end{aligned}\tag{6.33}$$

Also, it is demonstrated there that only longitudinal index ν gives $\frac{1}{Q^2}$ power correction, so

$$\begin{aligned}\check V^4_{\mu\nu} &= -\frac{4p_{1\mu}p_{1\nu}}{s^4}\Big(\langle\bar\psi A_j(x)\not{p}_2A_i\psi(0)\rangle_A\langle(\bar\psi\frac{1}{\beta})(0)\gamma^i\not{p}_1\gamma^j\frac{1}{\beta}\psi(x)\rangle_A\\ &+ \psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\Big)\\ &- \frac{g^{\parallel}_{\mu\nu}}{s^3}\Big(\langle\bar\psi A_j(x)\not{p}_1A_i\psi(0)\rangle_A\langle(\bar\psi\frac{1}{\beta})(0)\gamma^i\not{p}_1\gamma^j\frac{1}{\beta}\psi(x)\rangle_A\\ &+ \psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\Big) + x\leftrightarrow 0\end{aligned}\tag{6.34}$$

Moreover, the contribution of the second term to $W_{\mu\nu}$ is small [9], so we are left with the first term in the r.h.s. of Eq. (6.34). Using Eq. (9.7) it can be rewritten as

$$\begin{aligned}\check V^4_{\mu\nu} &= -\frac{4p_{1\mu}p_{1\nu}}{s^4}\Big(\langle\bar\psi\not{A}(x)\not{p}_2\not{A}\psi(0)\rangle_A\langle(\bar\psi\frac{1}{\beta})(0)\not{p}_1\frac{1}{\beta}\psi(x)\rangle_A\\ &+ \psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\Big) + x\leftrightarrow 0\end{aligned}\tag{6.35}$$

The corresponding contribution to $W_{\mu\nu}$ is obtained from formulas (9.40) and (9.52)

$$V^4_{\mu\nu}(q) = \frac{4p_{1\mu}p_{1\nu}}{\beta_q^2s^2N_c}\int d^2k_\perp\Big[k_\perp^2\{f_1\bar f_1+\bar f_1f_1\} - 2\alpha_q\{f_\perp\bar f_1+\bar f_\perp f_1\} + 2\alpha_q^2m^2\{f_3\bar f_1+\bar f_3f_1\}\Big]\tag{6.36}$$

6.2.2 Term $\check V^5_{\mu\nu}$

Again, as demonstrated in Ref. [9], the term $\sim g_{\mu\nu}$ is small, so

$$\begin{aligned}\check V^5_{\mu\nu} = \frac{1}{s^3}\Big\{&-p_{1\mu}\langle\bar\psi A_j(x)\sigma_{\nu k}A_i\psi(0)\rangle_A\langle(\bar\psi\frac{1}{\beta})(0)\gamma^i\sigma_\bullet{}^k\gamma^j\frac{1}{\beta}\psi(x)\rangle_A\\ &+ \langle\bar\psi A_j(x)\sigma_{\mu\bullet}A_i\psi(0)\rangle_A\langle(\bar\psi\frac{1}{\beta})(0)\gamma^i\sigma_{\bullet\nu_\perp}\gamma^j\frac{1}{\beta}\psi(x)\rangle_A\Big\} + \mu\leftrightarrow\nu + x\leftrightarrow 0\end{aligned}\tag{6.37}$$

Moreover, the second term in the r.h.s. is also small [9], and therefore

$$\begin{aligned}\check V^5_{\mu\nu} &= -\frac{p_{1\mu}}{s^3}\langle\bar\psi A_j(x)\sigma_{\nu k}A_i\psi(0)\rangle_A\langle(\bar\psi\frac{1}{\beta})(0)\gamma^i\sigma_\bullet{}^k\gamma^j\frac{1}{\beta}\psi(x)\rangle_A + \mu\leftrightarrow\nu + x\leftrightarrow 0\\ &= -\frac{4p_{1\mu}p_{1\nu}}{s^4}\langle\bar\psi A_j(x)\sigma_{\star k}A_i\psi(0)\rangle_A\langle(\bar\psi\frac{1}{\beta})(0)\gamma^i\sigma_\bullet{}^k\gamma^j\frac{1}{\beta}\psi(x)\rangle_A\\ &- \frac{g^{\parallel}_{\mu\nu}}{s^4}\langle\bar\psi A_j(x)\sigma_{\bullet k}A_i\psi(0)\rangle_A\langle(\bar\psi\frac{1}{\beta})(0)\gamma^i\sigma_\bullet{}^k\gamma^j\frac{1}{\beta}\psi(x)\rangle_A\\ &- \Big(\frac{p_{1\mu}}{s^3}\langle\bar\psi A_j(x)\sigma_{\nu k}A_i\psi(0)\rangle_A\langle(\bar\psi\frac{1}{\beta})(0)\gamma^i\sigma_\bullet{}^k\gamma^j\frac{1}{\beta}\psi(x)\rangle_A + \mu\leftrightarrow\nu\Big) + x\leftrightarrow 0\end{aligned}\tag{6.38}$$

As demonstrated in [9], the two last lines in the above equation are small. As to the first term in r.h.s. of Eq. (6.38), using Eq. (9.6) it can be rewritten as

$$\check{V}^5_{\mu\nu} \;=\; -\frac{4p_{1\mu}p_{1\nu}}{s^4}\langle\bar{\psi}\not{A}(x)\sigma_{\star j}\not{A}\psi(0)\rangle_A\langle(\bar{\psi}\frac{1}{\beta})(0)\sigma_{\bullet}{}^{j}\frac{1}{\beta}\psi(x)\rangle_A \;+\; x\leftrightarrow 0 \tag{6.39}$$

so the corresponding contribution to $W_{\mu\nu}$ takes the form

$$V^5_{\mu\nu} = -\frac{4p_{1\mu}p_{1\nu}}{\beta_q^2 s^2 N_c} \tag{6.40}$$
$$\times\int d^2k_\perp (k,q-k)_\perp\Big(\frac{k_\perp^2}{m^2}\{h_1^\perp\bar{h}_1^\perp+\bar{h}_1^\perp h_1^\perp\}+2\alpha_q\{h\bar{h}_1^\perp+\bar{h}h_1^\perp\}-2\alpha_q^2\{h_3^\perp\bar{h}_1^\perp+\bar{h}_3^\perp h_1^\perp\}\Big)$$

where we used Eqs. (9.40) and (9.52). The full result for $W^{(2b)}_{\mu\nu}$ is given by the sum of Eqs. (6.36) and (6.40)

$$W^{(2b)}_{1\mu\nu} \;=\; \frac{4p_{1\mu}p_{1\nu}}{\beta_q^2 s^2 N_c}\int d^2k_\perp\Big(k_\perp^2\{f_1\bar{f}_1+\bar{f}_1 f_1\}-2\alpha_q\{f_\perp\bar{f}_1+\bar{f}_\perp f_1\}+2\alpha_q^2 m^2\{f_3\bar{f}_1+\bar{f}_3 f_1\}\Big)$$
$$-\,(k,q-k)_\perp\Big(\frac{k_\perp^2}{m^2}\{h_1^\perp\bar{h}_1^\perp+\bar{h}_1^\perp h_1^\perp\}+2\alpha_q\{h\bar{h}_1^\perp+\bar{h}h_1^\perp\}-2\alpha_q^2\{h_3^\perp\bar{h}_1^\perp+\bar{h}_3^\perp h_1^\perp\}\Big)\Big] \tag{6.41}$$

6.2.3 Second term in Eq. (4.5)

Consider the second term in Eq. (4.5).

$$\check{W}^{(2b)}_{2\mu\nu} \;=\; \frac{N_c}{s}\langle A,B|\big[\bar{\Xi}_1(x)\gamma_\mu\psi_B(x)\big]\big[\bar{\psi}_B(0)\gamma_\nu\Xi_1(0)\big]+\mu\leftrightarrow\nu|A,B\rangle \;+\; x\leftrightarrow 0 \tag{6.42}$$

After Fierz transformation (9.1) we obtain

$$\check{W}^{(2b)}_{2\mu\nu} \;=\; -\frac{N_c}{2s}(\delta_\mu^\alpha\delta_\nu^\beta+\delta_\nu^\alpha\delta_\mu^\beta-g_{\mu\nu}g^{\alpha\beta})\langle A,B|\big\{[\bar{\Xi}^m_1(x)\gamma_\alpha\Xi^n_1(0)][\bar{\psi}^n_B(0)\gamma_\beta\psi^m_B(x)] \tag{6.43}$$
$$+\;\gamma_\alpha\otimes\gamma_\beta\leftrightarrow\gamma_\alpha\gamma_5\otimes\gamma_\beta\gamma_5\big\}|A,B\rangle$$
$$+\;\frac{N_c}{2s}(\delta_\mu^\alpha\delta_\nu^\beta+\delta_\nu^\alpha\delta_\mu^\beta-\frac{1}{2}g_{\mu\nu}g^{\alpha\beta})\langle A,B|[\bar{\Xi}^m_1(x)\sigma_{\alpha\xi}\Xi^n_1(0)][\psi^n_{1B}(0)\sigma_\beta{}^\xi\psi^m_{1B}(x)]|A,B\rangle \;+\; x\leftrightarrow 0$$

Sorting out color-singlet terms, we get similarly to sum of Eqs. (6.31) and (6.32)

$$\check{W}^{(2b)}_{2\mu\nu} \;=\; -\frac{1}{s^3}(\delta_\mu^\alpha p_{2\nu}+\delta_\nu^\alpha p_{2\mu}-g_{\mu\nu}p_2^\alpha)\Big(\langle\bar{\psi}B_j(x)\gamma_\alpha B_i\psi(0)\rangle_A$$
$$\times\;\langle(\bar{\psi}\frac{1}{\alpha})(0)\gamma^i\not{p}_2\gamma^j\frac{1}{\alpha}\psi(x)\rangle_A+\psi(0)\otimes\psi(x)\leftrightarrow\gamma_5\psi(0)\otimes\gamma_5\psi(x)\Big) \;+\; x\leftrightarrow 0$$
$$+\;\frac{1}{s^3}(\delta_\mu^\alpha\delta_\nu^\beta+\delta_\nu^\alpha\delta_\mu^\beta-\frac{1}{2}g_{\mu\nu}g^{\alpha\beta})$$
$$\times\;\Big\{-p_{2\alpha}\langle(\bar{\psi}\frac{1}{\alpha})(0)\gamma^i\sigma_\star{}^k\gamma^j\frac{1}{\alpha}\psi(x)\rangle_A\langle\bar{\psi}B_j(x)\sigma_{\beta k}B_i\psi(0)\rangle_A$$
$$+\;\langle(\bar{\psi}\frac{1}{\alpha})(0)\gamma^i\sigma_{\star\alpha_\perp}\gamma^j\frac{1}{\alpha}\psi(x)\rangle_A\langle\bar{\psi}B_j(x)\sigma_{\beta *}B_i\psi(0)\rangle_A\Big\} \;+\; x\leftrightarrow 0 \tag{6.44}$$

Starting from this point, all calculations repeat those of Sections 6.2.1 and 6.2.2 with replacements of $p_1\leftrightarrow p_2$, $\alpha_q\leftrightarrow\beta_q$ and exchange of projectile matrix elements and the target ones. The result is Eq. (6.41) with these replacements so we get

$$W^{(2b)}_{2\mu\nu} \tag{6.45}$$
$$= \frac{4p_{1\mu}p_{1\nu}}{\beta_q^2 s^2 N_c}\int d^2k_\perp\Big((q-k)^2_\perp\{f_1\bar f_1+\bar f_1 f_1\}-2\beta_q\{f_1\bar f_\perp+\bar f_1 f_\perp\}+2\beta_q^2 m^2\{f_1\bar f_3+\bar f_1 f_3\}\Big)$$
$$-\,(k,q-k)_\perp\Big(\frac{(q-k)^2_\perp}{m^2}\{h_1^\perp\bar h_1^\perp+\bar h_1^\perp h_1^\perp\}+2\beta_q\{h_1^\perp\bar h+\bar h_1^\perp h\}-2\beta_q^2\{h_1^\perp\bar h_3^\perp+\bar h_1^\perp h_3^\perp\}\Big)\Big]$$

and therefore the contribution of Eq. (4.5) takes the form

$$W^{(2b)}_{\mu\nu} \tag{6.46}$$
$$= \frac{2}{sN_c}\int d^2k_\perp\Big\{\frac{2p_{1\mu}p_{1\nu}}{\beta_q^2 s}\Big(k_\perp^2\{f_1\bar f_1+\bar f_1 f_1\}-2\alpha_q k_\perp^2\{f_\perp\bar f_1+\bar f_\perp f_1\}+2\alpha_q^2 m^2\{f_3\bar f_1+\bar f_3 f_1\}$$
$$-\,\frac{(k,q-k)_\perp}{m^2}\big[k_\perp^2\{h_1^\perp\bar h_1^\perp+\bar h_1^\perp h_1^\perp\}+2\alpha_q m^2\{h\bar h_1^\perp+\bar h h_1^\perp\}-2\alpha_q^2 m^2\{h_3^\perp\bar h_1^\perp+\bar h_3^\perp h_1^\perp\}\big]\Big)$$
$$+\,\frac{2p_{2\mu}p_{2\nu}}{\alpha_q^2 s}\Big((q-k)^2_\perp\{f_1\bar f_1+\bar f_1 f_1\}-2\beta_q(q-k)^2_\perp\{f_1\bar f_\perp+\bar f_1 f_\perp\}+2\beta_q^2 m^2\{f_1\bar f_3+\bar f_1 f_3\}$$
$$-\,(k,q-k)_\perp\big[\frac{(q-k)^2_\perp}{m^2}\{h_1^\perp\bar h_1^\perp+\bar h_1^\perp h_1^\perp\}+2\beta_q\{h_1^\perp\bar h+\bar h_1^\perp h\}-2\beta_q^2\{h_1^\perp\bar h_3^\perp+\bar h_1^\perp h_3^\perp\}\big]\Big)$$

7 Result

The resulting power correction with $\frac{1}{Q^2}$, $\frac{1}{N_c}$ accuracy is a sum of equations (3.21), (4.28), (4.30), (6.27), (6.46), and (5.37). It is convenient to present it as a sum of three terms

$$W_{\mu\nu}(q) \;=\; \frac{1}{N_c}\sum_f e_f^2\;[W^{1f}_{\mu\nu}(q)+W^{2f}_{\mu\nu}(q)+W^{3f}_{\mu\nu}(q)]\;+\;O\Big(\frac{1}{Q^3}\Big)\;+\;O\Big(\frac{1}{N_c^2}\Big) \tag{7.1}$$

The first part was calculated in Ref. [9] while the second and third parts are the result of this paper.

The first part has the form [9]

$$W^1_{\mu\nu}(q) \;=\; \frac{1}{N_c}\int d^2k_\perp\Big(\mathcal{W}^F_{\mu\nu}(q,k_\perp)\{f_1\bar f_1+\bar f_1 f_1\}+\;\mathcal{W}^H_{\mu\nu}(q,k_\perp)\{h_1^\perp\bar h_1^\perp+\bar h_1^\perp h_1^\perp\}\Big) \tag{7.2}$$

where

$$\mathcal{W}^F_{\mu\nu}(q,k_\perp) \;=\; -\,g^\perp_{\mu\nu}+\frac{1}{Q^2}(q^\parallel_\mu q^\perp_\nu+q^\parallel_\nu q^\perp_\mu)+\frac{q_\perp^2}{Q^4}q^\parallel_\mu q^\parallel_\nu+\frac{\tilde q_\mu\tilde q_\nu}{Q^4}[q_\perp^2-4(k,q-k)_\perp]$$
$$-\,\Big[\frac{\tilde q_\mu}{Q^2}\Big(g^\perp_{\nu i}-\frac{q^\parallel_\nu q_i}{Q^2}\Big)(q-2k)^i_\perp+\mu\leftrightarrow\nu\Big] \tag{7.3}$$

$$\mathcal{W}^H_{\mu\nu}(q,k_\perp) \tag{7.4}$$
$$= \;-\,\frac{1}{m^2}\big[k^\perp_\mu(q-k)^\perp_\nu-k^\perp_\nu(q-k)^\perp_\mu-g^\perp_{\mu\nu}(k,q-k)_\perp\big]+2\frac{\tilde q_\mu\tilde q_\nu-q^\parallel_\mu q^\parallel_\nu}{Q^4m^2}k_\perp^2(q-k)^2_\perp$$
$$-\,\frac{1}{m^2Q^2}\big(q^\parallel_\mu\big[k_\perp^2(q-k)^\perp_\nu+k^\perp_\nu(q-k)^2_\perp\big]+\;\tilde q_\mu\big[k_\perp^2(q-k)^\perp_\nu-k^\perp_\nu(q-k)^2_\perp\big]+\mu\leftrightarrow\nu\big)$$
$$-\,\frac{\tilde q_\mu\tilde q_\nu+q^\parallel_\mu q^\parallel_\nu}{Q^4m^2}\big[q_\perp^2-2(k,q-k)_\perp\big](k,q-k)_\perp-\;\frac{q^\parallel_\mu\tilde q_\nu+\tilde q_\mu q^\parallel_\nu}{Q^4m^2}(2k-q,q)_\perp(k,q-k)_\perp$$

Here $q^{\parallel}_\mu \equiv \alpha_q p_1 + \beta_q p_2$ and $\tilde q_\mu \equiv \alpha_q p_1 - \beta_q p_2$. Note also that $\alpha_q \equiv x_A$ and $\beta_q \equiv x_B$ in the notations of conventional TMD factorization formula (1.1).

In Eq. (7.1) we need to sum over flavors. From the Fierz transformation (9.1) it is easy to see that power corrections calculated in this paper are diagonal in flavor so the final result is a sum of e_f^2 multiplied by TMDs of corresponding flavor, for example $\{f_1\bar f_1 + \bar f_1 f_1\}$ in Eq. (7.2) should be replaced by $\sum_f e_f^2\{f_1^f \bar f_1^f + \bar f_1^f f_1^f\}$. To avoid cluttering of formulas, this summation over flavors is written only once in Eq. (7.1).

It is easy to see that $q^\mu \mathcal{W}^F_{\mu\nu} = q^\mu \mathcal{W}^H_{\mu\nu}$ so the first part is EM gauge invariant. Note that gauge invariance of the leading-twist part $\sim O(1)$ is restored by adding $\sim O\big(1/Q\big)$ and $\sim O\big(1/Q^2\big)$ contributions in Eqs. (7.3) and (7.4). One may call W^F a "gauge-invariant completion" of the leading-twist result (3.12). It worth noting that if one takes only the "f_1" part of the result (7.3) [9] and performs back-of-the-envelope estimation of Z-boson angular distribution coefficients one gets reasonable agreement with LHC data with expected $\frac{1}{N_c}$ accuracy. Needless to say, the angular coefficients are determined by "gauge completion" terms in Eq. (7.3) rather than the leading-twist term.

The second part of the result (7.1) is

$$
\begin{aligned}
W^2_{\mu\nu}(q) \;=\;& \frac{2}{N_c Q^2}\int d^2k_\perp \\
\times \Big\{&\Big[\tilde q_\mu (q-k)_\nu + \frac{2}{\beta_q s}\tilde q_\mu p_{1\nu}(k,q-k)_\perp + \frac{2}{\alpha_q s}\tilde q_\mu p_{2\nu}(q-k)^2_\perp \;+\; \mu\leftrightarrow\nu\Big] \\
&\qquad\times\Big(\beta_q\{f_1\bar f_\perp + \bar f_1 f_\perp\} - \alpha_q\{h\bar h_1^\perp + \bar h h_1^\perp\}\Big) \\
+ &\Big[\tilde q_\mu k^\perp_\nu + \frac{2}{s\beta_q}k_\perp^2\tilde q_\mu p_{1\nu} + \frac{2}{s\alpha_q}(k,q-k)_\perp \tilde q_\mu p_{2\nu} \;+\; \mu\leftrightarrow\nu\Big] \\
&\qquad\times\Big(-\alpha_q\{f_\perp\bar f_1 + \bar f_\perp f_1\} + \beta_q\{h_1^\perp\bar h + \bar h_1^\perp h\}\Big) \\
+ &\frac{4\tilde q_\mu\tilde q_\nu}{Q^2}\Big[m^2\Big(\alpha_q^2\{f_3\bar f_1 + \bar f_3 f_1\} + \beta_q^2\{f_1\bar f_3 + \bar f_1 f_3\} + \alpha_q\beta_q\big[\{e\bar e + \bar e e\} + \{h\bar h + \bar h h\}\big]\Big) \\
&\qquad + (k,q-k)_\perp\Big(-\alpha_q\beta_q\big[\{f_\perp\bar f_\perp + \bar f_\perp f_\perp\} + \{g_\perp\bar g_\perp + \bar g_\perp g_\perp\}\big] \\
&\qquad + \beta_q^2\{h_1^\perp\bar h_3^\perp + \bar h_1^\perp h_3^\perp\} + \alpha_q^2\{h_3^\perp\bar h_1^\perp + \bar h_3^\perp h_1^\perp\}\Big)\Big] \\
+ &\frac{1}{m^2}\mathcal{W}^\perp_{\mu\nu}(q,k_\perp)\Big[\frac{2}{\alpha_q}\{h_1^\perp\Re\bar h_{1\mathcal{G}} + \bar h_1^\perp\Re h_{1\mathcal{G}}\} + \frac{2}{\beta_q}\{\Re h_{1\mathcal{G}}\bar h_1^\perp + \Re\bar h_{1\mathcal{G}} h_1^\perp\} \\
&\qquad + \Re\big(\{h_{1G}^\perp\bar h_{1G}^\perp + \bar h_{1G}^\perp h_{1G}^\perp\}\big)\Big]
\end{aligned}
\tag{7.5}
$$

where the transverse structure $\mathcal{W}^\perp_{\mu\nu}(q,k_\perp)$ was defined in Eq. (5.29):

$$
\begin{aligned}
\mathcal{W}^\perp_{\mu\nu}(q_\perp,k_\perp) \;\equiv\;& g^\perp_{\mu\nu}(k,q-k)^2_\perp - g^\perp_{\mu\nu}k_\perp^2(q-k_\perp)^2 \\
&+ [k^\perp_\mu(q-k)^\perp_\nu + \mu\leftrightarrow\nu](k,q-k)_\perp - k_\perp^2(q-k)^\perp_\mu(q-k)^\perp_\nu - \;(q-k_\perp)^2 k^\perp_\mu k^\perp_\nu
\end{aligned}
\tag{7.6}
$$

[9] Because the Z-boson current has an additional weak current component there are additional power corrections to Z-boson cross sections calculated in Ref. [10] so the "f_1" part will be slightly more complicated than Eq. (7.3)

This EM-gauge invariant part consists of two types of contributions: $\sim \frac{1}{Q}$ (first terms in the second and fourth lines) and $\sim \frac{1}{Q^2}$ (all other terms). Note that, except for the last term, it is determined by quark-antiquark TMDs.

The remaining third part of the result (7.1) has the form

$$
\begin{aligned}
W^3_{\mu\nu}(q) \;=\; & \frac{2}{N_c Q^2}\int d^2k_\perp \Big[g^\perp_{\mu\nu}\Big\{ m^2\alpha_q\beta_q\big(\{h\bar{h}+\bar{h}h\} - \{e\bar{e}+\bar{e}e\}\big) \\
& + \alpha_q\beta_q m^2\big[\Re f_D(\alpha_q,k_\perp)\bar{f}'_1(\beta_q,q_\perp-k_\perp) + \Re\bar{f}_D(\alpha_q,k_\perp)f'_1(\beta_q,q_\perp-k_\perp) \\
& + f'_1(\alpha_q,k_\perp)\Re\bar{f}_D(\beta_q,q_\perp-k_\perp) + \bar{f}'_1(\alpha_q,k_\perp)\Re f_D(\beta_q,q_\perp-k_\perp)\big] \\
& - m^2\beta_q\big(\{f_1\Re\bar{f}_{2\mathcal{G}} + \bar{f}_1\Re f_{2\mathcal{G}}\} + \{f_1\Re\bar{f}_{3\mathcal{G}} + \bar{f}_1\Re f_{3\mathcal{G}}\}\big) \\
& - m^2\alpha_q\big(\{\Re\bar{f}_{2\mathcal{G}}f_1 + \Re f_{2\mathcal{G}}\bar{f}_1\} + \{\Re\bar{f}_{3\mathcal{G}}f_1 + \Re f_{3\mathcal{G}}\bar{f}_1\}\big) \\
+ & (k,q-k)_\perp\Big(-\beta_q\{h_1^\perp\bar{h} + \bar{h}_1^\perp h\} - \alpha_q\{h\bar{h}_1^\perp + \bar{h}h_1^\perp\} + 2\beta_q^2\{h_1^\perp\bar{h}_3^\perp + \bar{h}_1^\perp h_3^\perp\} \\
+ & 2\alpha_q^2\{h_3^\perp\bar{h}_1^\perp + \bar{h}_3^\perp h_1^\perp\} + 2\beta_q\{\Im\acute{e}_G\bar{h}_1^\perp + \Im\bar{\acute{e}}_G h_1^\perp\} + 2\alpha_q\{h_1^\perp\Im\bar{\acute{e}}_G + \bar{h}_1^\perp\Im\acute{e}_G\} \\
- & \beta_q\{f_1\Re\bar{\acute{f}}_{1\mathcal{G}} + \bar{f}_1\Re\acute{f}_{1\mathcal{G}}\} - \alpha_q\{\Re\bar{\acute{f}}_{1\mathcal{G}}f_1 + \Re\acute{f}_{1\mathcal{G}}\bar{f}_1\} - \alpha_q\beta_q\big[\{f_\perp\Re\bar{\grave{f}}_{1\mathcal{G}} + \bar{f}_\perp\Re\grave{f}_{1\mathcal{G}}\} \\
+ & \{g_\perp\Im\bar{\grave{f}}_{1\mathcal{G}} - \bar{g}_\perp\Im\grave{f}_{1\mathcal{G}}\} + \{\Re\bar{\grave{f}}_{1\mathcal{G}}f_\perp + \Re\grave{f}_{1\mathcal{G}}\bar{f}_\perp\} + \{\Im\bar{\grave{f}}_{1\mathcal{G}}g_\perp - \Im\grave{f}_{1\mathcal{G}}\bar{g}_\perp\}\big]\Big\} \\
+ & \big[g^\perp_{\mu\nu}(k,q-k)_\perp + k^\perp_\mu(q-k)^\perp_\nu + k^\perp_\nu(q-k)^\perp_\mu\big] \\
\times & \Big\{\alpha_q\beta_q\big[\{f_\perp\bar{f}_\perp + \bar{f}_\perp f_\perp\} - \{g_\perp\bar{g}_\perp + \bar{g}_\perp g_\perp\}\big] \\
+ & \alpha_q\beta_q\big[\Re h_D(\alpha_q,k_\perp)\bar{h}'^\perp_1(\beta_q,q_\perp-k_\perp) + \Re\bar{h}_D(\alpha_q,k_\perp)h'^\perp_1(\beta_q,q_\perp-k_\perp) \\
& \qquad + h'^\perp_1(\alpha_q,k_\perp)\Re\bar{h}_D(\beta_q,q_\perp-k_\perp) + \bar{h}'^\perp_1(\alpha_q,k_\perp)\Re h_D(\beta_q,q_\perp-k_\perp)\big] \\
+ & \beta_q\{h_1^\perp\Re\bar{h}_{3\mathcal{G}} + \bar{h}_1^\perp\Re h_{3\mathcal{G}}\} + \beta_q\{h_1^\perp\Im\bar{h}_{4\mathcal{G}} + \bar{h}_1^\perp\Im h_{4\mathcal{G}}\} \\
& \qquad + \alpha_q\{\Re\bar{h}_{3\mathcal{G}}h_1^\perp + \Re h_{3\mathcal{G}}\bar{h}_1^\perp\} + \alpha_q\{\Im\bar{h}_{4\mathcal{G}}h_1^\perp + \Im h_{4\mathcal{G}}\bar{h}_1^\perp\}\Big\} \\
- & [g^\perp_{\mu\nu}k_\perp^2 + 2k_\mu k_\nu] \\
\times & \Big(\frac{(q-k)_\perp^2}{\alpha_q m^2}\{h_1^\perp\Re\bar{h}_{2\mathcal{G}} + \bar{h}_1^\perp\Re h_{2\mathcal{G}}\} + \{\Re h_{1\mathcal{G}}\bar{h} + \Re\bar{h}_{1\mathcal{G}}h\} + \{\Im h_{1\mathcal{G}}\bar{e} - \Im\bar{h}_{1\mathcal{G}}e\}\Big) \\
- & \big[2(q-k)^\perp_\mu(q-k)^\perp_\nu + g^\perp_{\mu\nu}(q-k)_\perp^2\big] \\
\times & \Big(\frac{k_\perp^2}{\beta_q m^2}\{\Re h_{2\mathcal{G}}\bar{h}_1^\perp + \Re\bar{h}_{2\mathcal{G}}h_1^\perp\} + (\{h\Re\bar{h}_{1\mathcal{G}} + \bar{h}\Re h_{1\mathcal{G}}\} + \{\bar{e}\Im h_{1\mathcal{G}} - e\Im\bar{h}_{1\mathcal{G}}\}\Big]
\end{aligned}
\tag{7.7}
$$

This contribution is not gauge invariant: $q^\mu W^3_{\mu\nu}(q) \neq 0$. This is hardly surprising since from DVCS studies we know that check of EM gauge invariance sometimes involves cancellation of contributions of different twists (see e.g. Refs. [14–20]). Still, the non-gauge-invariant contribution (7.7) is proportional to transverse structures so the violation of gauge invariance is

$$
q^\mu W_{\mu\nu}(q) \;=\; q^\mu W^3_{\mu\nu}(q) \;=\; O\Big(\frac{q_\perp}{Q^2}\Big)
$$

Note that if, for example, we would have $g^\parallel_{\mu\nu}$ instead of $g^\perp_{\mu\nu}$ in Eq. (7.7), the violation of gauge invariance would be $\sim \frac{1}{Q}$. The absence of such terms is a result of many cancellations involving QCD equations of motion. Thus, the EM gauge invariance of $W^3_{\mu\nu}(q)$ is restored by $\sim \frac{1}{Q^3}$ and $\sim \frac{1}{Q^4}$ corrections so one may say that at the $\sim \frac{1}{Q^2}$ level our result (7.1) satisfies the requirement of EM gauge invariance.

Last but not least, let us discuss the choice of basis of operators for $\frac{1}{Q^2}$ corrections. Unlike $\frac{1}{Q}$ corrections which are unique, one can represent $\frac{1}{Q^2}$ corrections in many different ways using QCD equations of motion [10]

$$\begin{aligned}
\bar\psi(x)\not{A}_\perp(x) &= i\partial_i\bar\psi(x)\gamma_i + i\frac{2}{s}\partial_\star\bar\psi(x)\not{p}_1 + i\bar\psi \overleftarrow{D}_\bullet \frac{2}{s}\not{p}_2, \\
\not{A}_\perp(x)\psi(x) &= -i\not{\partial}_\perp\psi(x) - i\frac{2}{s}\not{p}_1\partial_\star\psi(x) - i\frac{2}{s}\not{p}_2 D_\bullet\psi(x)
\end{aligned} \tag{7.8}$$

for the projectile operators in $A_\star = 0$ gauge and

$$\begin{aligned}
\bar\psi(x)\not{B}_\perp(x) &= i\partial_i\bar\psi(x)\gamma_i + i\frac{2}{s}\partial_\bullet\bar\psi(x)\not{p}_2 + i\bar\psi \overleftarrow{D}_\star(x)\frac{2}{s}\not{p}_1, \\
\not{B}_\perp(x)\psi(x) &= -i\not{\partial}_\perp\psi(x) - i\frac{2}{s}\not{p}_2\partial_\bullet\psi(x) - i\frac{2}{s}\not{p}_1 D_\star\psi(x)
\end{aligned} \tag{7.9}$$

for operators in target matrix elements (in $B_\bullet = 0$ gauge). The choice in this paper is to reduce the l.h.s.'s of these EOMs to the r.h.s.'s whenever possible. This choice leads to the "gauge completion" (7.2) of the leading-twist result and helps with sorting out the $\frac{1}{Q^2}$ corrections. It should be mentioned that there is a different choice of basis of operators in the literature: in Refs. [21], [22], [23] the quark-antiquark TMDs of non-leading twist are expressed in terms of quark-antiquark-gluon TMDs using EOMs like Eq. (9.42). That choice is motivated by the fact that the evolution of $\bar q q$ TMDs of non-leading twist involves $\bar q q G$ TMDs anyway [22]. Ideally, to find the optimal basis of operators one should diagonalize the matrix of evolution equations of twist-4 TMDs and find those combinations which evolve like leading-twist TMDs which is a formidable task. [11] In my opinion, it is useful first to try to assemble the power corrections in gauge-invariant blocks like (7.2) and (7.5) using QCD equations of motion. Of course, it is quite probable that among the "leftovers" entering Eq. (7.7) there will be TMD combinations which evolve like, say, f_1 but this is in agreement with our earlier statement that the EM gauge invariance of $W^3_{\mu\nu}(q)$ is restored by $\sim \frac{1}{Q^3}$ and $\sim \frac{1}{Q^4}$ corrections.

8 Conclusions and outlook

The result of this paper is a complete set of $\frac{1}{Q^2}$ power corrections to TMD factorization for Drell-Yan process at the leading N_c order. Let me emphasize that tree-level formulas of this paper are valid at both moderate and small Bjorken x. The difference between these two cases comes from different evolution of TMDs in the moderate-x and small-x regions,

[10] As was mentioned in footnote 2, in this paper QCD coupling constant g is included in the definition of gluon field A_μ.

[11] In the case of DVCS governed by the GPD evolution, there is a byway to find light-ray operators which evolve like the leading-twist ones using conformal $SL(2,R)$ invariance [24–26]. Unfortunately, this method is not applicable to TMD operators that are not $SL(2,R)$ invariant.

see the discussion in Ref. [12]. Moreover, while the results of this paper were obtained using rapidity-only factorization, at the tree level they should be the same as obtained by conventional CSS approach. Indeed, $\frac{1}{Q}$ corrections are the same as in Ref. [8] and parts of $\frac{1}{Q^2}$ corrections coincide with Ref. [23] after taking into account different choice of operator basis, see the discussion in previous Section.

One may wonder what can be a possible way to compare the result (7.1) with experimental data on DY process. There are phenomenological estimates of leading-twist TMDs [27–30], but due to the large number of quark-antiquark-gluon TMDs involved, similar extraction of $\bar{q}qG$ TMDs from experiment seems nearly impossible. On the other hand, there are attempts to calculate quark-antiquark TMDs on the lattice [31–33] (see also the review [34]) and one may expect to get lattice estimates of quark-quark-gluon TMDs in the future. It is well known that lattice calculations are not reliable at small x, so the moderate-x result of this paper (7.1) may serve as a bridge between lattice calculations and experimental data.

An obvious outlook is to extend these results to the semi-inclusive deep inelastic scattering (SIDIS) at EIC and elsewhere. The study is in progress.

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9 Appendix

9.1 Formulas with Dirac matrices

9.1.1 Fierz transformation

First, let us write down Fierz transformation for symmetric hadronic tensor

$$\begin{aligned}
&\frac{1}{2}[(\bar{\psi}\gamma_\mu\chi)(\bar{\chi}\gamma_\nu\psi) + \mu\leftrightarrow\nu] \qquad (9.1)\\
&= \;-\frac{1}{4}\big(\delta^\alpha_\mu\delta^\beta_\nu + \delta^\alpha_\nu\delta^\beta_\mu - g_{\mu\nu}g^{\alpha\beta}\big)\big[(\bar{\psi}\gamma_\alpha\psi)(\bar{\chi}\gamma_\beta\chi) + (\bar{\psi}\gamma_\alpha\gamma_5\psi)(\bar{\chi}\gamma_\beta\gamma_5\chi)\big]\\
&+\frac{1}{4}\big(\delta^\alpha_\mu\delta^\beta_\nu + \delta^\alpha_\nu\delta^\beta_\mu - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\big)(\bar{\psi}\sigma_{\alpha\xi}\psi)(\bar{\chi}\sigma_\beta{}^\xi\chi) - \frac{g_{\mu\nu}}{4}(\bar{\psi}\psi)(\bar{\chi}\chi) + \frac{g_{\mu\nu}}{4}(\bar{\psi}\gamma_5\psi)(\bar{\chi}\gamma_5\chi)
\end{aligned}$$

9.1.2 Formulas with σ-matrices

It is convenient to define [12]

$$\epsilon_{ij} \;\equiv\; \frac{2}{s}\epsilon_{\star\bullet ij} \;=\; \frac{2}{s}p_2^\mu p_1^\nu\epsilon_{\mu\nu ij} \qquad (9.2)$$

such that $\epsilon_{12} = 1$ and $\epsilon_{ij}\epsilon_{kl} = g_{ik}g_{jl} - g_{il}g_{jk}$. The frequently used formula is

$$\sigma_{\mu\nu}\sigma_{\alpha\beta} \;=\; (g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}) - i\epsilon_{\mu\nu\alpha\beta}\gamma_5 - i(g_{\mu\alpha}\sigma_{\nu\beta} - g_{\mu\beta}\sigma_{\nu\alpha} - g_{\nu\alpha}\sigma_{\mu\beta} + g_{\nu\beta}\sigma_{\mu\alpha}) \qquad (9.3)$$

[12]We use conventions from *Bjorken & Drell* where $\epsilon^{0123} = -1$ and $\gamma^\mu\gamma^\nu\gamma^\lambda = g^{\mu\nu}\gamma^\lambda + g^{\nu\lambda}\gamma^\mu - g^{\mu\lambda}\gamma^\nu - i\epsilon^{\mu\nu\lambda\rho}\gamma_\rho\gamma_5$. Also, with this convention $\tilde{\sigma}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}\sigma^{\lambda\rho} = i\sigma_{\mu\nu}\gamma_5$.

We need also the following formulas with σ-matrices in different matrix elements

$$\tilde\sigma_{\mu\nu}\otimes\tilde\sigma_{\alpha\beta} = -\frac{1}{2}(g_{\mu\alpha}g_{\nu\beta}-g_{\nu\alpha}g_{\mu\beta})\sigma_{\xi\eta}\otimes\sigma^{\xi\eta}$$
$$+ g_{\mu\alpha}\sigma_{\beta\xi}\otimes\sigma_\nu^{\ \xi} - g_{\nu\alpha}\sigma_{\beta\xi}\otimes\sigma_\mu^{\ \xi} - g_{\mu\beta}\sigma_{\alpha\xi}\otimes\sigma_\nu^{\ \xi} + g_{\nu\beta}\sigma_{\alpha\xi}\otimes\sigma_\mu^{\ \xi} - \sigma_{\alpha\beta}\otimes\sigma_{\mu\nu} \qquad (9.4)$$

and

$$\tilde\sigma_{\mu\xi}\otimes\tilde\sigma_\nu^{\ \xi} = -\frac{g_{\mu\nu}}{2}\sigma_{\xi\eta}\otimes\sigma^{\xi\eta}+\sigma_{\nu\xi}\otimes\sigma_\mu^{\ \xi}, \qquad \sigma_{\xi\eta}\otimes\tilde\sigma^{\xi\eta} = \tilde\sigma_{\xi\eta}\otimes\sigma^{\xi\eta} \qquad (9.5)$$
$$\sigma_{\mu\xi}\gamma_5\otimes\sigma_\nu^{\ \xi}\gamma_5+\mu\leftrightarrow\nu-\frac{g_{\mu\nu}}{2}\sigma_{\xi\eta}\gamma_5\otimes\sigma^{\xi\eta}\gamma_5 = -\left[\sigma_{\mu\xi}\otimes\sigma_\nu^{\ \xi}+\mu\leftrightarrow\nu-\frac{g_{\mu\nu}}{2}\sigma_{\xi\eta}\otimes\sigma^{\xi\eta}\right]$$

$$\sigma_\star^{\ k}\otimes\gamma_i\sigma_{\bullet k}\gamma_j = \hat p_2\gamma^k\otimes\not p_1\gamma_i\gamma_k\gamma_j = \hat p_2\gamma^k\otimes\not p_1(g_{ik}\gamma_j+g_{jk}\gamma_i-g_{ij}\gamma_k)$$
$$= \hat p_2(g_{ik}\gamma_j+g_{jk}\gamma_i-g_{ij}\gamma_k)\otimes\not p_1\gamma^k = (\gamma_j\sigma_\star^{\ k}\gamma_i)\otimes\sigma_{\bullet k} \qquad (9.6)$$

We will need also

$$\not p_2\otimes\gamma_i\not p_1\gamma_j+\not p_2\gamma_5\otimes\gamma_i\not p_1\gamma_j\gamma_5 = \gamma_j\not p_2\gamma_i\otimes\not p_1+\gamma_j\not p_2\gamma_i\gamma_5\otimes\not p_1\gamma_5 \qquad (9.7)$$

and

$$i\sigma_{\alpha\xi}\sigma_{\star i}\otimes\sigma_\beta^{\ \xi}$$
$$= g_{\alpha i}\sigma_{\star j}\otimes\sigma_{\beta_\perp}^{\ j}+\sigma_{\alpha_\perp *}\otimes\sigma_{\beta_\perp i}-\frac{2}{s}p_{2\alpha}\sigma_{\star i}\otimes\sigma_{\bullet\beta_\perp}+\frac{2}{s}p_{2\beta}g_{\alpha i}\sigma_{\star j}\otimes\sigma_\bullet^{\ j}+\frac{2}{s}p_{2\beta}\sigma_{\alpha_\perp *}\otimes\sigma_{\bullet i}$$
$$+\frac{2}{s}p_{2\alpha}p_{2\beta}\left[i\otimes\sigma_{\bullet i}-\sigma_{ij}\otimes\sigma_\bullet^{\ j}\right]+\frac{4}{s^2}p_{2\alpha}p_{2\beta}\sigma_{\bullet *}\otimes\sigma_{\bullet i}-\frac{2}{s}g^{\parallel}_{\alpha\beta}\sigma_{\star i}\otimes\sigma_{\bullet *}+...,$$
$$i\sigma_{\star i}\sigma_{\alpha\xi}\otimes\sigma_\beta^{\ \xi} \qquad (9.8)$$
$$= \frac{2}{s}p_{2\alpha}\sigma_{\star i}\otimes\sigma_{\bullet\beta_\perp}-g_{i\alpha}\sigma_{\star j}\otimes\sigma_{\beta_\perp}^{\ j}+\sigma_{\star\alpha_\perp}\otimes\sigma_{\beta_\perp i}+\frac{2}{s}p_{2\beta}\sigma_{\star\alpha_\perp}\otimes\sigma_{\bullet i}-\frac{2}{s}g_{i\alpha}p_{2\beta}\sigma_{\star j}\otimes\sigma_\bullet^{\ j}$$
$$+\frac{4}{s^2}p_{2\alpha}p_{2\beta}\sigma_{\star\bullet}\otimes\sigma_{\bullet i}+\frac{2}{s}p_{2\alpha}p_{2\beta}\left[i\otimes\sigma_{\bullet i}+\sigma_{ij}\otimes\sigma_\bullet^{\ j}\right]-\frac{2}{s}g^{\parallel}_{\alpha\beta}\sigma_{\star i}\otimes\sigma_{\star\bullet} + ...$$

and

$$i\sigma_{\bullet i}\sigma_{\alpha\xi}\otimes\sigma_\beta^{\ \xi}$$
$$= \frac{2}{s}p_{1\alpha}\sigma_{\bullet i}\otimes\sigma_{\star\beta_\perp}-g_{i\alpha}\sigma_{\bullet j}\otimes\sigma_{\beta_\perp}^{\ j}+\sigma_{\bullet\alpha_\perp}\otimes\sigma_{\beta_\perp i}+\frac{2}{s}p_{1\beta}\sigma_{\bullet\alpha_\perp}\otimes\sigma_{\star i}-\frac{2}{s}g_{i\alpha}p_{1\beta}\sigma_{\bullet j}\otimes\sigma_\star^{\ j}$$
$$+\frac{4}{s^2}p_{1\alpha}p_{1\beta}\sigma_{\bullet *}\otimes\sigma_{\star i}+\frac{2}{s}p_{1\alpha}p_{1\beta}\left[i\otimes\sigma_{\star i}+\sigma_{ij}\otimes\sigma_\star^{\ j}\right]-\frac{2}{s}g^{\parallel}_{\alpha\beta})\sigma_{\bullet i}\otimes\sigma_{\bullet *}+...,$$
$$i\sigma_{\alpha\xi}\sigma_{\bullet i}\otimes\sigma_\beta^{\ \xi} \qquad (9.9)$$
$$= g_{\alpha i}\sigma_{\bullet j}\otimes\sigma_{\beta_\perp}^{\ j}+\sigma_{\alpha_\perp\bullet}\otimes\sigma_{\beta_\perp i}-\frac{2}{s}p_{1\alpha}\sigma_{\bullet i}\otimes\sigma_{\star\beta_\perp}+\frac{2}{s}p_{1\beta}g_{\alpha i}\sigma_{\bullet j}\otimes\sigma_\star^{\ j}+\frac{2}{s}p_{1\beta}\sigma_{\alpha_\perp *}\otimes\sigma_{\star i}$$
$$+\frac{2}{s}p_{1\alpha}p_{1\beta}\left[i\otimes\sigma_{\star i}+\sigma_{ij}\otimes\sigma_\star^{\ j}\right]+\frac{4}{s^2}p_{1\alpha}p_{1\beta}\sigma_{\star\bullet}\otimes\sigma_{\star i}-\frac{2}{s}g^{\parallel}_{\alpha\beta}\sigma_{\bullet i}\otimes\sigma_{\star\bullet}+...$$

From these equations after some algebra one obtains

$$\sigma_{\mu\xi}\sigma_{\star i}\otimes\sigma_\nu^{\ \xi}\sigma_{\bullet j} = -g_{\mu i}g_{\nu j}\sigma_{\star k}\otimes\sigma_\bullet^{\ k}+g_{\mu i}\sigma_{\star j}\otimes\sigma_{\bullet\nu_\perp} \qquad (9.10)$$
$$+ g_{\nu_\perp j}\sigma_{\star\mu_\perp}\otimes\sigma_{\bullet i}-g_{ij}\sigma_{\star\mu_\perp}\otimes\sigma_{\bullet\nu_\perp}-g^{\parallel}_{\mu\nu}\sigma_{\star i}\otimes\sigma_{\bullet j} + ...$$

The dots in the above formulas stand for the terms leading to contributions to $W_{\mu\nu}$ exceeding our $1/Q^2$accuracy.

9.1.3 Formulas with γ-matrices and one gluon field

In the gauge $A_\bullet = 0$ the field A_i can be represented as

$$A_i(x_\bullet, x_\perp) \;=\; \frac{2}{s}\int_{-\infty}^{x_\bullet} dx'_\bullet\; F^{(A)}_{\star i}(x'_\bullet, x_\perp) \tag{9.11}$$

Similarly, in the $B_\star = 0$ gauge

$$B_i(x_\bullet, x_\perp) \;=\; \frac{2}{s}\int_{-\infty}^{x_\star} dx'_\star\; F^{(B)}_{\bullet i}(x'_\star, x_\perp) \tag{9.12}$$

We define "dual" fields by

$$\tilde{A}_i(x_\bullet, x_\perp) \;=\; \frac{2}{s}\int_{-\infty}^{x_\bullet} dx'_\bullet\; \tilde{F}^{(A)}_{\star i}(x'_\bullet, x_\perp), \quad \tilde{B}_i(x_\star, x_\perp) \;=\; \frac{2}{s}\int_{-\infty}^{x_\star} dx'_\star\; \tilde{F}^{(B)}_{\bullet i}(x'_\star, x_\perp), \tag{9.13}$$

where $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$ as usual. With this definition we have

$$\tilde{A}_i = -\epsilon_{ij}A^j, \quad \tilde{B}_i = \epsilon_{ij}B^j, \quad \epsilon_{ij}\tilde{A}^j = A_i, \quad \epsilon_{ij}\tilde{B}^j = -B_i \tag{9.14}$$

and

$$\not{p}_2\grave{A}_i \;=\; -\not{A}\not{p}_2\gamma_i, \quad \grave{A}_i\not{p}_2 \;=\; -\gamma_i\not{p}_2\not{A}, \quad \not{p}_1\grave{B}_i \;=\; -\not{B}\not{p}_1\gamma_i, \quad \grave{B}_i\not{p}_1 \;=\; -\gamma_i\not{p}_1\not{B} \tag{9.15}$$

where $\grave{A}_i, \grave{B}_i$ are defined in Eq. (4.12).

We also use

$$\begin{aligned}
A^i\not{p}_2\otimes\gamma_n\not{p}_1\gamma_i + \; A^i\not{p}_2\gamma_5\otimes\gamma_n\not{p}_1\gamma_i\gamma_5 \;&=\; -\not{p}_2\grave{A}_n\otimes\not{p}_1 - \not{p}_2\grave{A}_n\gamma_5\otimes\not{p}_1\gamma_5\\
A^i\not{p}_2\otimes\gamma_i\not{p}_1\gamma_n + \; A^i\not{p}_2\gamma_5\otimes\gamma_i\not{p}_1\gamma_n\gamma_5 \;&=\; -\grave{A}_n\not{p}_2\otimes\not{p}_1 - \grave{A}_n\not{p}_2\gamma_5\otimes\not{p}_1\gamma_5\\
\gamma_n\not{p}_2\gamma^i\otimes\not{p}_1 B_i \;+\; \gamma_n\not{p}_2\gamma^i\gamma_5\otimes\not{p}_1\gamma_5 B_i \;&=\; -\not{p}_2\otimes\not{p}_1\grave{B}_n - \not{p}_2\gamma_5\otimes\not{p}_1\grave{B}_n\gamma_5\\
\gamma^i\not{p}_2\gamma_n\otimes\not{p}_1 B_i \;+\; \gamma^i\not{p}_2\gamma_n\gamma_5\otimes\not{p}_1\gamma_5 B_i \;&=\; -\not{p}_2\otimes\grave{B}_n\not{p}_1 - \not{p}_2\gamma_5\otimes\grave{B}_n\not{p}_1\gamma_5
\end{aligned} \tag{9.16}$$

and

$$\begin{aligned}
\frac{2}{s}\big[\not{p}_1\not{p}_2\gamma_i\otimes B^i\gamma_\nu + \not{p}_1\not{p}_2\gamma_i\gamma_5\otimes B^i\gamma_\nu\gamma_5\big] \;&=\; \gamma_i\otimes\gamma_\nu\grave{B}_i + \gamma_i\gamma_5\otimes\gamma_\nu\grave{B}_i\gamma_5\\
\frac{2}{s}\big[\gamma_i\not{p}_2\not{p}_1\otimes B^i\gamma_\nu + \gamma_i\not{p}_2\not{p}_1\gamma_5\otimes B^i\gamma_\nu\gamma_5\big] \;&=\; \gamma_i\otimes\grave{B}_i\gamma_\nu + \gamma_i\gamma_5\otimes\grave{B}_i\gamma_\nu\gamma_5\\
\frac{2}{s}\big[\not{p}_2\not{p}_1\gamma_i\otimes B^i\not{p}_1 + \not{p}_2\not{p}_1\gamma_i\gamma_5\otimes B^i\not{p}_1\gamma_5\big] \;&=\; \gamma_i\otimes\grave{B}_i\not{p}_1 + \gamma_i\gamma_5\otimes\grave{B}_i\not{p}_1\gamma_5\\
\frac{2}{s}\big[\not{p}_1\not{p}_2\gamma_i\otimes B^i\not{p}_1 + \not{p}_1\not{p}_2\gamma_i\gamma_5\otimes B^i\not{p}_1\gamma_5\big] \;&=\; \gamma_i\otimes\not{p}_1\grave{B}_i + \gamma_i\gamma_5\otimes\not{p}_1\grave{B}_i\gamma_5
\end{aligned} \tag{9.17}$$

and

$$\begin{aligned}
&\gamma_\alpha\not{p}_2\gamma_i\otimes B_i\gamma_\beta + \gamma_\alpha\not{p}_2\gamma_i\gamma_5\otimes B_i\gamma_\beta\gamma_5\\
&=\; -\not{p}_2\otimes\gamma_{\beta_\perp}\grave{B}_\alpha + \frac{2}{s}p_{2\beta}\not{p}_2\otimes\not{B}\not{p}_1\gamma_{\alpha_\perp} - \frac{2}{s}p_{2\alpha}p_{2\beta}\big[\gamma_i\otimes\not{B}\not{p}_1\gamma_i + \gamma_i\gamma_5\otimes\not{B}\not{p}_1\gamma_i\gamma_5\big] \;+\; ...,\\
&\gamma_i\not{p}_2\gamma_\alpha\otimes B_i\gamma_\beta + \gamma_i\not{p}_2\gamma_\alpha\gamma_5\otimes B_i\gamma_\beta\gamma_5\\
&=\; -\not{p}_2\otimes\grave{B}_\alpha\gamma_{\beta_\perp} + \frac{2}{s}p_{2\beta}\not{p}_2\otimes\gamma_\alpha\not{p}_1\not{B} - \frac{2}{s}p_{2\alpha}p_{2\beta}\big[\gamma_i\otimes\gamma_i\not{p}_1\not{B} + \gamma_i\gamma_5\otimes\gamma_i\not{p}_1\not{B}\gamma_5\big] \;+\; ...
\end{aligned} \tag{9.18}$$

where the dots stand for the negligible terms as usual. Let us illustrate the derivation of the first of these equations. After some algebra one obtains

$$\begin{aligned}
&\gamma_\mu \not{p}_2\gamma_i \otimes B_i\gamma_\nu + \gamma_\mu \not{p}_2\gamma_i\gamma_5 \otimes B_i\gamma_\nu\gamma_5 \qquad (9.19)\\
&= \ -\not{p}_2 \otimes \gamma_{\nu_\perp}\grave{B}_\mu - \frac{2}{s}p_{2\mu}p_{2\nu}\big[\gamma_i \otimes \not{B}\not{p}_1\gamma_i + \gamma_i\gamma_5 \otimes \not{B}\not{p}_1\gamma_i\gamma_5\big] + \frac{2}{s}p_{2\nu}\not{p}_2 \otimes \not{B}\not{p}_1\gamma_{\mu_\perp}\\
&- \not{p}_2\gamma_5 \otimes \gamma_\nu\grave{B}_\mu\gamma_5 + p_{2\mu}\gamma_i \otimes \gamma_{\nu_\perp}\grave{B}_i + p_{2\mu}\gamma_i\gamma_5 \otimes \gamma_{\nu_\perp}\grave{B}_i\gamma_5 - \frac{2}{s}p_{1\nu}\not{p}_2 \otimes \not{p}_2\grave{B}_\mu\\
&+ \frac{2}{s}p_{2\mu}p_{1\nu}\big[\gamma_i \otimes \not{p}_2\grave{B}_i + \gamma_i\gamma_5 \otimes \not{p}_2\grave{B}_i\gamma_5\big]
\end{aligned}$$

Now, looking at Eq. (4.14) it is easy to see that the last two lines in the r.h.s. do not give contributions of order of (3.13).

We need also

$$\begin{aligned}
&\gamma_\mu \not{p}_1\gamma_i \otimes B_i\gamma_\nu + \gamma_\mu \not{p}_1\gamma_i\gamma_5 \otimes B_i\gamma_\nu\gamma_5\\
&= \ p_{1\mu}\gamma^i \otimes \grave{B}_i\gamma_\nu + p_{1\mu}\gamma^i\gamma_5 \otimes \grave{B}_i\gamma_\nu\gamma_5 - \not{p}_1 \otimes \grave{B}_\mu\gamma_\nu - \not{p}_1\gamma_5 \otimes \grave{B}_\mu\gamma_\nu\gamma_5\\
&\gamma_i \not{p}_1\gamma_\mu \otimes B_i\gamma_\nu + \gamma_i \not{p}_1\gamma_\mu\gamma_5 \otimes B_i\gamma_\nu\gamma_5\\
&= \ p_{1\mu}\gamma^i \otimes \gamma_\nu\grave{B}_i + p_{1\mu}\gamma^i\gamma_5 \otimes \gamma_\nu\grave{B}_i\gamma_5 - \not{p}_1 \otimes \gamma_\nu\grave{B}_\mu - \not{p}_1\gamma_5 \otimes \gamma_\nu\grave{B}_\mu\gamma_5 \qquad (9.20)
\end{aligned}$$

9.1.4 Formulas with γ-matrices and two gluon fields

With definition (9.13), we have the following formulas

$$\begin{aligned}
&A_i \otimes \tilde{B}_j = g_{ij}\tilde{A}_k \otimes B^k - \tilde{A}_j \otimes B_i, \quad \tilde{A}_i \otimes B_j = g_{ij}A_k \otimes \tilde{B}^k - A_j \otimes \tilde{B}_i \qquad (9.21)\\
&\tilde{A}_i \otimes \tilde{B}_j = -g_{ij}A_k \otimes B^k + A_j \otimes B_i, \quad \Rightarrow \quad \tilde{A}_i \otimes \tilde{B}^i = -A_i \otimes B^i, \quad \tilde{A}_i \otimes B^i = A_i \otimes \tilde{B}^i
\end{aligned}$$

Using these formulas, after some algebra one obtains

$$\begin{aligned}
&\gamma_m \not{p}_2\gamma_j A^i \otimes \gamma_n \not{p}_1\gamma_i B^j + \gamma_m \not{p}_2\gamma_j A^i\gamma_5 \otimes \gamma_n \not{p}_1\gamma_i B^j\gamma_5 = \not{p}_2\grave{A}_n \otimes \not{p}_1\grave{B}_m + \not{p}_2\grave{A}_n\gamma_5 \otimes \not{p}_1\grave{B}_m\gamma_5\\
&\gamma_j \not{p}_2\gamma_m A^i \otimes \gamma_n \not{p}_1\gamma_i B^j + \gamma_j \not{p}_2\gamma_m A^i\gamma_5 \otimes \gamma_n \not{p}_1\gamma_i B^j\gamma_5 = \not{p}_2\grave{A}_n \otimes \grave{B}_m\not{p}_1 + \not{p}_2\grave{A}_n\gamma_5 \otimes \grave{B}_m\not{p}_1\gamma_5\\
&\gamma_m \not{p}_2\gamma_j A^i \otimes \gamma_i \not{p}_1\gamma_n B^j + \gamma_m \not{p}_2\gamma_j A^i\gamma_5 \otimes \gamma_i \not{p}_1\gamma_n B^j\gamma_5 = \grave{A}_n\not{p}_2 \otimes \not{p}_1\grave{B}_m + \grave{A}_n\not{p}_2\gamma_5 \otimes \not{p}_1\grave{B}_m\gamma_5\\
&\gamma_j \not{p}_2\gamma_m A^i \otimes \gamma_i \not{p}_1\gamma_n B^j + \gamma_j \not{p}_2\gamma_m A^i\gamma_5 \otimes \gamma_i \not{p}_1\gamma_n B^j\gamma_5 = \grave{A}_n\not{p}_2 \otimes \grave{B}_m\not{p}_1 + \grave{A}_n\not{p}_2\gamma_5 \otimes \grave{B}_m\not{p}_1\gamma_5\\
&\qquad (9.22)
\end{aligned}$$

and

$$\begin{aligned}
\not{p}_2\grave{A}_m \otimes \not{p}_1\grave{B}_n + \not{p}_2\grave{A}_n\gamma_5 \otimes \not{p}_1\grave{B}_m\gamma_5 \ &= \ g_{mn}\not{p}_2\grave{A}_k \otimes \not{p}_1\grave{B}^k\\
\not{p}_2\grave{A}_m \otimes \grave{B}_n\not{p}_1 + \not{p}_2\grave{A}_n\gamma_5 \otimes \gamma_5\grave{B}_m\not{p}_1 \ &\overset{\text{OK}}{=} \ g_{mn}\not{p}_2\grave{A}_k \otimes \grave{B}^k\not{p}_1\\
\grave{A}_m\not{p}_2 \otimes \not{p}_1\grave{B}_n + \gamma_5\grave{A}_n\not{p}_2 \otimes \not{p}_1\grave{B}_m\gamma_5 \ &= \ g_{mn}\grave{A}_k\not{p}_2 \otimes \not{p}_1\grave{B}^k\\
\grave{A}_m\not{p}_2 \otimes \grave{B}_n\not{p}_1 + \gamma_5\grave{A}_n\not{p}_2 \otimes \gamma_5\grave{B}_m\not{p}_1 \ &= \ g_{mn}\grave{A}_k\not{p}_2 \otimes \grave{B}^k\not{p}_1 \qquad (9.23)
\end{aligned}$$

The corollary of Eq. (9.23) is

$$\begin{aligned}
\not{p}_2\grave{A}_k\gamma_5 \otimes \not{p}_1\grave{B}^k\gamma_5 \ &= \ \not{p}_2\grave{A}_k \otimes \not{p}_1\grave{B}^k, \qquad \not{p}_2\grave{A}_k\gamma_5 \otimes \gamma_5\grave{B}^k\not{p}_1 \ = \ \not{p}_2\grave{A}_k \otimes \grave{B}^k\not{p}_1\\
\gamma_5\grave{A}_k\not{p}_2 \otimes \not{p}_1\grave{B}^k\gamma_5 \ &= \ \grave{A}_k\not{p}_2 \otimes \not{p}_1\grave{B}^k, \qquad \gamma_5\grave{A}_k\not{p}_2 \otimes \gamma_5\grave{B}^k\not{p}_1 \ = \ \grave{A}_k\not{p}_2 \otimes \grave{B}^k\not{p}_1 \qquad (9.24)
\end{aligned}$$

From Eqs. (9.22) and (9.23) one easily obtains

$$\gamma_m \not{p}_2 \gamma_j A^i \otimes \gamma_n \not{p}_1 \gamma_i B^j + \gamma_m \not{p}_2 \gamma_j A^i \gamma_5 \otimes \gamma_n \not{p}_1 \gamma_i B^j \gamma_5 \; + \; m \leftrightarrow n \; = \; 2 g_{mn} \not{p}_2 \grave{A}_k \otimes \not{p}_1 \grave{B}^k \quad (9.25)$$

and

$$\begin{aligned} & \gamma_m \not{p}_2 \gamma_j A^i \otimes \gamma_n \not{p}_1 \gamma_i B^j + \gamma_m \not{p}_2 \gamma_j A^i \gamma_5 \otimes \gamma_n \not{p}_1 \gamma_i B^j \gamma_5 \; - \; m \leftrightarrow n \\ & = \; 2 \not{p}_2 \grave{A}_n \otimes \not{p}_1 \grave{B}_m \; - \; m \leftrightarrow n, \\ & \gamma_j \not{p}_2 \gamma_m A^i \otimes \gamma_i \not{p}_1 \gamma_n B^j + \gamma_j \not{p}_2 \gamma_m A^i \gamma_5 \otimes \gamma_i \not{p}_1 \gamma_n B^j \gamma_5 \; - \; m \leftrightarrow n \\ & = \; 2 \grave{A}_n \not{p}_2 \otimes \grave{B}_m \not{p}_1 \; - \; m \leftrightarrow n \end{aligned} \quad (9.26)$$

We need also formulas

$$\begin{aligned} & \frac{4}{s^2} A^i \not{p}_1 \not{p}_2 \gamma_j \otimes B^j \not{p}_1 \not{p}_2 \gamma_i \\ & \qquad = \; A^i \gamma_j \otimes B^j \gamma_i - i A^i \gamma_j \gamma_5 \otimes \tilde{B}^j \gamma_i + i \tilde{A}^i \gamma_j \otimes B^j \gamma_i \gamma_5 + \tilde{A}^i \gamma_j \gamma_5 \otimes \tilde{B}^j \gamma_i \gamma_5, \\ & \frac{4}{s^2} \big(A^i \not{p}_1 \not{p}_2 \gamma_j \otimes B^j \not{p}_1 \not{p}_2 \gamma_i + A^i \not{p}_1 \not{p}_2 \gamma_j \gamma_5 \otimes B^j \not{p}_1 \not{p}_2 \gamma_i \gamma_5 \big) \\ & \qquad = \; \gamma^j \grave{A}_i \otimes \gamma^i \grave{B}_j + \gamma^j \grave{A}_i \gamma_5 \otimes \gamma^i \grave{B}_j \gamma_5, \\ & \gamma_i \grave{A}_j \gamma_5 \otimes \gamma_j \grave{A}_i \gamma_5 \; = \; \gamma_i \grave{A}_j \otimes \gamma^i \grave{B}^j - \gamma_i \grave{A}^i \otimes \gamma_j \grave{B}^j \end{aligned} \quad (9.27)$$

and

$$\begin{aligned} A_k \gamma_i \not{p}_2 \gamma^j \otimes B_j \gamma^i \not{p}_1 \gamma^k \; &= \; \not{p}_2 \grave{A}_i \otimes \not{p}_1 \grave{B}^i \; = \; \not{A} \not{p}_2 \gamma_i \otimes \not{B} \not{p}_1 \gamma^i, \\ A_k \gamma^j \not{p}_2 \gamma_i \otimes B_j \gamma^k \not{p}_1 \gamma^i \; &= \; \grave{A}_i \not{p}_2 \otimes \grave{B}_i \not{p}_1 \; = \; \gamma_i \not{p}_2 \not{A} \otimes \gamma^i \not{p}_1 \not{B}, \\ A_k \gamma_i \not{p}_2 \gamma_j \otimes B^j \gamma^k \not{p}_1 \gamma^i \; &= \; \not{p}_2 \grave{A}_i \otimes \grave{B}^i \not{p}_1 \; = \; \not{A} \not{p}_2 \gamma_i \otimes \gamma^i \not{p}_1 \not{B}, \\ A_k \gamma_j \not{p}_2 \gamma_i \otimes B^j \gamma^i \not{p}_1 \gamma^k \; &= \; \grave{A}_i \not{p}_2 \otimes \not{p}_1 \grave{B}^i \; = \; \gamma_i \not{p}_2 \not{A} \otimes \not{B} \not{p}_1 \gamma^i, \end{aligned} \quad (9.28)$$

$$\begin{aligned} & A^k \gamma_m \not{p}_2 \gamma_j \otimes B^j \gamma_n \not{p}_1 \gamma_k + m \leftrightarrow n - g_{mn} A^k \gamma_i \not{p}_2 \gamma_j \otimes B^j \gamma^i \not{p}_1 \gamma_k \\ & = \; \grave{A}_m \not{p}_2 \otimes \grave{B}_n \not{p}_1 + m \leftrightarrow n - g_{mn} \grave{A}_k \not{p}_2 \otimes \grave{B}^k \not{p}_1, \\ & A^k \gamma_j \not{p}_2 \gamma_m \otimes B^j \gamma_k \not{p}_1 \gamma_n + m \leftrightarrow n - g_{mn} A^k \gamma_j \not{p}_2 \gamma_i \otimes B^j \gamma_k \not{p}_1 \gamma^i \\ & = \; \not{p}_2 \grave{A}_m \otimes \not{p}_1 \grave{B}_n + m \leftrightarrow n - g_{mn} \not{p}_2 \grave{A}_k \otimes \not{p}_1 \grave{B}^k, \\ & A^k \gamma_m \not{p}_2 \gamma_j \otimes B^j \gamma_k \not{p}_1 \gamma_n + m \leftrightarrow n - g_{mn} A^k \gamma_j \not{p}_2 \gamma_i \otimes B^j \gamma_k \not{p}_1 \gamma^i \\ & = \; \grave{A}_m \not{p}_2 \otimes \not{p}_1 \grave{B}_n + m \leftrightarrow n - g_{mn} \grave{A}_k \not{p}_2 \otimes \not{p}_1 \grave{B}^k, \\ & A^k \gamma_j \not{p}_2 \gamma_m \otimes B^j \gamma_n \not{p}_1 \gamma_k + m \leftrightarrow n - g_{mn} A^k \gamma_j \not{p}_2 \gamma_i \otimes B^j \gamma_k \not{p}_1 \gamma^i \; = \\ & = \; \not{p}_2 \grave{A}_m \otimes \grave{B}_n \not{p}_1 + m \leftrightarrow n - g_{mn} \not{p}_2 \grave{A}_k \otimes \grave{B}^k \not{p}_1, \end{aligned} \quad (9.29)$$

$$\begin{aligned} & \frac{2}{s} \big[A_i \not{p}_1 \not{p}_2 \gamma^j \otimes B_j \gamma_n \not{p}_1 \gamma^i + A_i \not{p}_1 \not{p}_2 \gamma^j \gamma_5 \otimes B_j \gamma_{\nu_\perp} \not{p}_1 \gamma^i \gamma_5 \big] \quad (9.30) \\ & = \; - \gamma_i \grave{A}_n \otimes \not{p}_1 \grave{B}^i - \gamma_i \grave{A}_n \gamma_5 \otimes \not{p}_1 \grave{B}^i \gamma_5 \; = \; \gamma_i \grave{A}_n \otimes \not{B} \not{p}_1 \gamma^i + \gamma_i \grave{A}_n \gamma_5 \otimes \not{B} \not{p}_1 \gamma^i \gamma_5, \\ & \frac{2}{s} \big[A_i \gamma_n \not{p}_2 \gamma^j \otimes B_j \not{p}_2 \not{p}_1 \gamma^i + A_i \gamma_n \not{p}_2 \gamma^j \gamma_5 \otimes B_j \not{p}_2 \not{p}_1 \gamma^i \gamma_5 \big] \\ & = - \not{p}_2 \grave{A}_i \otimes \gamma^i \grave{B}_n - \not{p}_2 \grave{A}_i \gamma_5 \otimes \gamma^i \grave{B}_n \gamma_5 \; = \; \not{A} \not{p}_2 \gamma_i \otimes \gamma^i \grave{B}_n + \not{A} \not{p}_2 \gamma_i \gamma_5 \otimes \gamma^i \grave{B}_n \gamma_5. \end{aligned}$$

9.2 TMD matrix elements

9.2.1 Parametrization of leading-twist matrix elements

Let us first consider matrix elements of operators without γ_5. The standard parametrization of quark TMDs reads (see e.g. Ref. [35, 36]) [13]

$$\begin{aligned}
&\frac{1}{16\pi^3}\int dx_\bullet d^2x_\perp\ e^{-i\alpha x_\bullet+i(k,x)_\perp}\ \langle A|\bar\psi(x_\bullet,x_\perp)\gamma^\mu\psi(0)|A\rangle \\
&\qquad\qquad = p_1^\mu f_1(\alpha,k_\perp)+k_\perp^\mu f_\perp(\alpha,k_\perp)+p_2^\mu\frac{2m_N^2}{s}f_3(\alpha,k_\perp), \\
&\frac{1}{16\pi^3}\int dx_\bullet d^2x_\perp\ e^{-i\alpha x_\bullet+i(k,x)_\perp}\ \langle A|\bar\psi(x_\bullet,x_\perp)\psi(0)|A\rangle\ =\ me(\alpha,k_\perp)
\end{aligned}\tag{9.31}$$

for quark distributions in the projectile and

$$\begin{aligned}
&\frac{1}{16\pi^3}\int dx_\bullet d^2x_\perp\ e^{-i\alpha x_\bullet+i(k,x)_\perp}\ \langle A|\bar\psi(0)\gamma^\mu\psi(x_\bullet,x_\perp)|A\rangle \\
&\qquad\qquad = -p_1^\mu \bar f_1(\alpha,k_\perp)-k_\perp^\mu \bar f_\perp(\alpha,k_\perp)-p_2^\mu\frac{2m_N^2}{s}\bar f_3(\alpha,k_\perp), \\
&\frac{1}{16\pi^3}\int dx_\bullet d^2x_\perp\ e^{-i\alpha x_\bullet+i(k,x)_\perp}\ \langle A|\bar\psi(0)\psi(x_\bullet,x_\perp)|A\rangle\ =\ m\bar e(\alpha,k_\perp)
\end{aligned}\tag{9.32}$$

for the antiquark distributions. [14]

The corresponding matrix elements for the target are obtained by trivial replacements $p_1\leftrightarrow p_2$, $x_\bullet\leftrightarrow x_\star$ and $\alpha\leftrightarrow\beta$:

$$\begin{aligned}
&\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x_\star,x_\perp)\gamma^\mu\psi(0)|B\rangle \\
&\qquad\qquad = p_2^\mu f_1(\beta,k_\perp)+k_\perp^\mu f_\perp(\beta,k_\perp)+p_1^\mu\frac{2m_N^2}{s}f_3(\beta,k_\perp), \\
&\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x_\star,x_\perp)\psi(0)|B\rangle\ =\ me(\beta,k_\perp),
\end{aligned}\tag{9.33}$$

and

$$\begin{aligned}
&\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)\gamma^\mu\psi(x_\star,x_\perp)|B\rangle \\
&\qquad\qquad = -p_2^\mu \bar f_1(\beta,k_\perp)-k_\perp^\mu \bar f_\perp(\beta,k_\perp)-p_1^\mu\frac{2m_N^2}{s}\bar f_3(\beta,k_\perp), \\
&\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)\psi(x_\star,x_\perp)|B\rangle\ =\ m\bar e(\beta,k_\perp).
\end{aligned}\tag{9.34}$$

Matrix elements of operators with γ_5 are parametrized as follows:

$$\begin{aligned}
&\frac{1}{16\pi^3}\int dx_\bullet d^2x_\perp\ e^{-i\alpha x_\bullet+i(k,x)_\perp}\ \langle A|\bar\psi(x_\bullet,x_\perp)\gamma_\mu\gamma_5\psi(0)|A\rangle\ =\ -\,\epsilon_{\mu_\perp i}k^i g^\perp(\alpha,k_\perp), \\
&\frac{1}{16\pi^3}\int dx_\bullet d^2x_\perp\ e^{-i\alpha x_\bullet+i(k,x)_\perp}\ \langle A|\bar\psi(0)\gamma_\mu\gamma_5\psi(x_\bullet,x_\perp)|A\rangle\ =\ -\,\epsilon_{\mu_\perp i}k^i \bar g^\perp(\beta,k_\perp)
\end{aligned}\tag{9.35}$$

[13] Our notations differ from "TMD handbook" [36]: $g^{\rm QCD}_{\rm our}=-g^{\rm QCD}_{\rm hbook}$, $\epsilon^{\mu\nu\lambda\rho}_{\rm our}=-\epsilon^{\mu\nu\lambda\rho}_{\rm hbook}$, but $\epsilon^{ij}_{\rm our}=\epsilon^{ij}_{\rm hbook}$

[14] In an arbitrary gauge, there are gauge links to $-\infty$ as displayed in Eq. (3.10).

and

$$\frac{1}{16\pi^3}\int dx_\bullet d^2x_\perp\ e^{-i\alpha x_\bullet+i(k,x)_\perp}\ \langle B|\bar\psi(x_\star,x_\perp)\gamma_\mu\gamma_5\psi(0)|B\rangle\ =\ \epsilon_{\mu_\perp i}k^i g^\perp(\beta,k_\perp),$$
$$\frac{1}{16\pi^3}\int dx_\bullet d^2x_\perp\ e^{-i\alpha x_\bullet+i(k,x)_\perp}\ \langle B|\bar\psi(0)\gamma_\mu\gamma_5\psi(x_\star,x_\perp)|B\rangle\ =\ \epsilon_{\mu_\perp i}k^i\bar g^\perp(\beta,k_\perp) \tag{9.36}$$

The parametrizations of time-odd Boer-Mulders TMDs are

$$\begin{aligned}
&\frac{1}{16\pi^3}\int dx_\bullet d^2x_\perp\ e^{-i\alpha x_\bullet+i(k,x)_\perp}\ \langle A|\bar\psi(x_\bullet,x_\perp)\sigma^{\mu\nu}\psi(0)|A\rangle\\
&\quad=\ \frac{1}{m}(k_\perp^\mu p_1^\nu-\mu\leftrightarrow\nu)h_1^\perp(\alpha,k_\perp)+\frac{2m}{s}(p_1^\mu p_2^\nu-\mu\leftrightarrow\nu)h(\alpha,k_\perp)\\
&\qquad\qquad+\ \frac{2m}{s}(k_\perp^\mu p_2^\nu-\mu\leftrightarrow\nu)h_3^\perp(\alpha,k_\perp),\\
&\frac{1}{16\pi^3}\int dx_\bullet d^2x_\perp\ e^{-i\alpha x_\bullet+i(k,x)_\perp}\ \langle A|\bar\psi(0)\sigma^{\mu\nu}\psi(x_\bullet,x_\perp)|A\rangle\\
&\quad=\ -\frac{1}{m}(k_\perp^\mu p_1^\nu-\mu\leftrightarrow\nu)\bar h_1^\perp(\alpha,k_\perp)-\frac{2m}{s}(p_1^\mu p_2^\nu-\mu\leftrightarrow\nu)\bar h(\alpha,k_\perp)\\
&\qquad\qquad-\ \frac{2m}{s}(k_\perp^\mu p_2^\nu-\mu\leftrightarrow\nu)\bar h_3^\perp(\alpha,k_\perp)
\end{aligned}\tag{9.37}$$

and similarly for the target with usual replacements $p_1\leftrightarrow p_2$, $x_\bullet\leftrightarrow x_\star$ and $\alpha\leftrightarrow\beta$:

$$\begin{aligned}
&\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x_\star,x_\perp)\sigma^{\mu\nu}\psi(0)|B\rangle\\
&\quad=\ \frac{1}{m}(k_\perp^\mu p_2^\nu-\mu\leftrightarrow\nu)h_1^\perp(\beta,k_\perp)+\frac{2m}{s}(p_2^\mu p_1^\nu-\mu\leftrightarrow\nu)h(\beta,k_\perp)\\
&\qquad\qquad+\ \frac{2m}{s}(k_\perp^\mu p_1^\nu-\mu\leftrightarrow\nu)h_3^\perp(\beta,k_\perp),\\
&\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)\sigma^{\mu\nu}\psi(x_\star,x_\perp)|B\rangle\\
&\quad=\ -\frac{1}{m}(k_\perp^\mu p_2^\nu-\mu\leftrightarrow\nu)\bar h_1^\perp(\beta,k_\perp)-\frac{2m}{s}(p_2^\mu p_1^\nu-\mu\leftrightarrow\nu)\bar h(\beta,k_\perp)\\
&\qquad\qquad-\ \frac{2m}{s}(k_\perp^\mu p_1^\nu-\mu\leftrightarrow\nu)\bar h_3^\perp(\beta,k_\perp)
\end{aligned}\tag{9.38}$$

Note that the coefficients in front of f_3, $g^\perp$, h and $h_3^\perp$ in eqs. (9.31), (9.33), (9.35), (9.36), (9.37), and (9.38) contain an extra $\frac{1}{s}$ since p_2^μ enters only through the direction of gauge link so the result should not depend on rescaling $p_2\to\lambda p_2$.

9.2.2 Matrix elements of quark-quark-gluon operators

First, let us demonstrate that operators $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ in Eqs. (2.5) are replaced by $\pm\frac{1}{\alpha_q}$ and $\pm\frac{1}{\beta_q}$ in forward matrix elements. Indeed,

$$\int dx_\bullet\ e^{-i\alpha_q x_\bullet}\langle\bar\Phi(x_\bullet,x_\perp)\Gamma\frac{1}{\alpha+i\epsilon}\psi(0)\rangle_A \tag{9.39}$$
$$=\ \frac{1}{i}\int dx_\bullet\int_{-\infty}^0 dx'_\bullet\ e^{-i\alpha_q x_\bullet}\langle\bar\Phi(x_\bullet,x_\perp)\Gamma\psi(x'_\bullet,0_\perp)\rangle_A=\ \frac{1}{\alpha_q}\int dx_\bullet\ e^{-i\alpha x_\bullet}\langle\bar\Phi(x_\bullet,x_\perp)\Gamma\psi(0)\rangle_A$$

where $\bar{\Phi}(x_\bullet, x_\perp)$ can be $\bar{\psi}(x_\bullet, x_\perp)$ or $\bar{\psi}(x_\bullet, x_\perp)A_i(x_\bullet, x_\perp)$ and Γ can be any γ-matrix. Similarly,

$$\int dx_\bullet\ e^{-i\alpha_q x_\bullet}\langle(\bar{\psi}\frac{1}{\alpha - i\epsilon})(x_\bullet, x_\perp)\Gamma\Phi(0)\rangle_A = \frac{1}{\alpha_q}\int dx_\bullet\ e^{-i\alpha x_\bullet}\langle\bar{\psi}(x_\bullet, x_\perp)\Gamma\Phi(0)\rangle_A \tag{9.40}$$
$$\int dx_\bullet\ e^{-i\alpha_q x_\bullet}\langle(\bar{\psi}\frac{1}{\alpha - i\epsilon})(x_\bullet, x_\perp)\Gamma\frac{1}{\alpha + i\epsilon}\psi(0)\rangle_A = \frac{1}{\alpha_q^2}\int dx_\bullet\ e^{-i\alpha_q x_\bullet}\langle\bar{\psi}(x_\bullet, x_\perp)\Gamma\psi(0)\rangle_A$$

where $\Phi(x_\bullet, x_\perp)$ can be $\psi(x_\bullet, x_\perp)$ or $A_i(x_\bullet, x_\perp)\psi(x_\bullet, x_\perp)$. We need also

$$\int dx_\bullet\ e^{-i\alpha_q x_\bullet}\langle(\bar{\psi}\frac{1}{\alpha - i\epsilon})(0)\Gamma\Phi(x_\bullet, x_\perp)\rangle_A = -\frac{1}{\alpha_q}\int dx_\bullet\ e^{-i\alpha x_\bullet}\langle\bar{\psi}(0)\Gamma\Phi(x_\bullet, x_\perp)\rangle_A$$
$$\int dx_\bullet\ e^{-i\alpha_q x_\bullet}\langle\bar{\Phi}(0)\Gamma\frac{1}{\alpha + i\epsilon}\psi(x_\bullet, x_\perp)\rangle_A = -\frac{1}{\alpha_q}\int dx_\bullet\ e^{-i\alpha x_\bullet}\langle\bar{\Phi}(0)\Gamma\psi(x_\bullet, x_\perp)\rangle_A \tag{9.41}$$

The corresponding formulas for target matrix elements are obtained by substitution $\alpha \leftrightarrow \beta$ (and $x_\bullet \leftrightarrow x_\star$).

9.2.3 Matrix elements of quark-quark-gluon operators related to quark-antiquark TMDs by QCD equations of motion

Next, we will use QCD equation of motion to reduce quark-quark-gluon TMDs to leading-twist TMDs (see Ref. [8]). For our quark fields QCD equations read [15]

$$\begin{aligned}
\bar{\psi}(x)\not{A}_\perp(x) &= i\partial_i\bar{\psi}(x)\gamma_i + i\frac{2}{s}\partial_\star\bar{\psi}(x)\not{p}_1 + i\bar{\psi}\overleftarrow{D}_\bullet\frac{2}{s}\not{p}_2, \\
\not{A}_\perp(x)\psi(x) &= -i\not{\partial}_\perp\psi(x) - i\frac{2}{s}\not{p}_1\partial_\star\psi(x) - i\frac{2}{s}\not{p}_2 D_\bullet\psi(x)
\end{aligned} \tag{9.42}$$

for the projectile operators in $A_\star = 0$ gauge and

$$\begin{aligned}
\bar{\psi}(x)\not{B}_\perp(x) &= i\partial_i\bar{\psi}(x)\gamma_i + i\frac{2}{s}\partial_\bullet\bar{\psi}(x)\not{p}_2 + i\bar{\psi}\overleftarrow{D}_\star(x)\frac{2}{s}\not{p}_1, \\
\not{B}_\perp(x)\psi(x) &= -i\not{\partial}_\perp\psi(x) - i\frac{2}{s}\not{p}_2\partial_\bullet\psi(x) - i\frac{2}{s}\not{p}_1 D_\star\psi(x)
\end{aligned} \tag{9.43}$$

for operators in target matrix elements (in $B_\bullet = 0$ gauge). Our strategy is as follows: when we see an operator as in the left-hand sides of these equations, we rewrite it in terms of the corresponding right-hand sides. For most of the matrix elements listed in this Section, the result can be represented through quark-antiquark TMDs. Sometimes, however, one needs the last terms in the r.h.s.'s parametrized in the next Section.

Let us present the list of formulas derived in Ref. [9]

[15] As was mentioned in footnote 2, in this paper QCD coupling constant g is included in the definition of gluon field A_μ.

$$
\begin{aligned}
\frac{1}{8\pi^3 s}\int dx_\bullet dx_\perp\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\langle A|\bar\psi\not{A}(x)\not{p}_2\gamma_i\psi(0)|A\rangle &= k_i[f_1-\alpha(f_\perp+ig^\perp)](\alpha,k_\perp)\\
\frac{1}{8\pi^3 s}\int dx_\bullet dx_\perp\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\langle A|\bar\psi(x)\gamma_i\not{p}_2\not{A}\psi(0)|A\rangle &= k_i\big[f_1-\alpha(f_\perp-ig^\perp)\big](\alpha,k_\perp)\\
\frac{1}{8\pi^3 s}\int dx_\perp dx_\bullet\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\langle A|\bar\psi(0)\gamma_i\not{p}_2\not{A}\psi(x)|A\rangle &= k_i\big[\bar f_1-\alpha(\bar f_\perp+i\bar g^\perp)\big](\alpha,k_\perp)\\
\frac{1}{8\pi^3 s}\int dx_\perp dx_\bullet\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\langle A|\bar\psi\not{A}(0)\not{p}_2\gamma_i\psi(x)|A\rangle &= k_i\big[\bar f_1-\alpha(\bar f_\perp-i\bar g_\perp)\big](\alpha_q,k_\perp)
\end{aligned}
\tag{9.44}
$$

For brevity, hereafter in the projectile matrix elements $x=(x_\bullet,0_\star,x_\perp)$

The target matrix elements are obtained by usual replacements (4.29):

$$
\begin{aligned}
\frac{1}{8\pi^3 s}\int dx_\star dx_\perp\; e^{-i\beta x_\star+i(k,x)_\perp}\langle B|\bar\psi\not{B}(x)\not{p}_1\gamma_i\psi(0)|B\rangle &= k_i[f_1-\beta(f_\perp+ig^\perp)](\beta,k_\perp)\\
\frac{1}{8\pi^3 s}\int dx_\star dx_\perp\; e^{-i\beta x_\star+i(k,x)_\perp}\langle B|\bar\psi(x)\gamma_i\not{p}_1\not{B}\psi(0)|B\rangle &= k_i\big[f_1-\beta(f_\perp-ig^\perp)\big](\beta,k_\perp)\\
\frac{1}{8\pi^3 s}\int dx_\perp dx_\star\; e^{-i\beta x_\star+i(k,x)_\perp}\langle B|\bar\psi(0)\gamma_i\not{p}_1\not{B}\psi(x)|B\rangle &= k_i\big[\bar f_1-\beta(\bar f_\perp+i\bar g^\perp)\big](\beta,k_\perp)\\
\frac{1}{8\pi^3 s}\int dx_\perp dx_\star\; e^{-i\beta x_\star+i(k,x)_\perp}\langle B|\bar\psi\not{B}(0)\not{p}_1\gamma_i\psi(x)|B\rangle &= k_i\big[\bar f_1-\beta(\bar f_\perp-i\bar g_\perp)\big](\beta_q,k_\perp)
\end{aligned}
\tag{9.45}
$$

Similarly, in the target matrix elements $x\equiv(0_\bullet,x_\star,x_\perp)$

Next, for the projectile matrix elements with an extra γ_5 one obtains

$$
\begin{aligned}
\int\frac{dx_\bullet dx_\perp}{8\pi^3 s}\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\langle A|\bar\psi\not{A}(x)\not{p}_2\gamma_i\gamma_5\psi(0)|A\rangle &= i\epsilon_{ij}k^j\big[f_1-\alpha(f_\perp+ig^\perp)\big](\alpha,k_\perp),\\
\int\frac{dx_\bullet dx_\perp}{8\pi^3 s}\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\langle A|\bar\psi(x)\gamma_i\gamma_5\not{p}_2\not{A}\psi(0)|A\rangle &= -i\epsilon_{ij}k^j\big[f_1-\alpha(f_\perp-ig^\perp)\big](\alpha,k_\perp)\\
\int\frac{dx_\bullet dx_\perp}{8\pi^3 s}\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\langle A|\bar\psi\not{A}(0)\not{p}_2\gamma_i\gamma_5\psi(x)|A\rangle &= i\epsilon_{ij}k^j\big[\bar f_1-\alpha(\bar f_\perp-i\bar g_\perp)\big](\alpha,k_\perp)\\
\int\frac{dx_\bullet dx_\perp}{8\pi^3 s}\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\langle A|\bar\psi_f(0)\gamma_i\gamma_5\not{p}_2\not{A}\psi(x)|A\rangle &= -i\epsilon_{ij}k^j\big[\bar f_1-\alpha(\bar f_\perp+i\bar g_\perp)\big](\alpha,k_\perp)
\end{aligned}
\tag{9.46}
$$

and for the target

$$
\begin{aligned}
\int\frac{dx_\star dx_\perp}{8\pi^3 s}\; e^{-i\beta x_\star+i(k,x)_\perp}\langle B|\bar\psi\not{B}(x)\not{p}_1\gamma_i\gamma_5\psi(0)|B\rangle &= -i\epsilon_{ij}k^j\big[f_1-\beta(f_\perp+ig^\perp)\big](\beta,k_\perp)\\
\int\frac{dx_\star dx_\perp}{8\pi^3 s}\; e^{-i\beta x_\star+i(k,x)_\perp}\langle B|\bar\psi(x)\gamma_i\gamma_5\not{p}_1\not{B}(0)\psi(0)|B\rangle &= i\epsilon_{ij}k^j\big[f_1-\beta(f_\perp-ig_\perp)\big](\beta,k_\perp)\\
\int\frac{dx_\star dx_\perp}{8\pi^3 s}\; e^{-i\beta x_\star+i(k,x)_\perp}\langle B|\bar\psi\not{B}(0)\not{p}_1\gamma_i\gamma_5\psi(x)|B\rangle &= -i\epsilon_{ij}k^j\big[\bar f_1-\beta(\bar f_\perp-i\bar g_\perp)\big](\beta,k_\perp)\\
\int\frac{dx_\star dx_\perp}{8\pi^3 s}\; e^{-i\beta x_\star+i(k,x)_\perp}\langle B|\bar\psi_f(0)\gamma_i\gamma_5\not{p}_1\not{B}\psi(x)|B\rangle &= i\epsilon_{ij}k^j\big[\bar f_1-\beta(\bar f_\perp+i\bar g_\perp)\big](\beta,k_\perp)
\end{aligned}
\tag{9.47}
$$

The different sign in projectile↔target replacement of matrix elements with γ_5 is due to the difference in the definitions (9.14).

$$\begin{aligned}
&\int \frac{dx_\bullet dx_\perp}{8\pi^3 s}\ e^{-i\alpha_q x_\bullet + i(k,x)_\perp}\langle A|\bar\psi \not{A}(x)\not{p}_2\psi(0)|A\rangle = \big[-i\frac{k_\perp^2}{m}h_1^\perp - \alpha m(e+ih)\big](\alpha,k_\perp)\\
&\int \frac{dx_\bullet dx_\perp}{8\pi^3 s}\ e^{-i\alpha x_\bullet + i(k,x)_\perp}\langle A|\bar\psi(x)\not{p}_2\not{A}\psi(0)|A\rangle = \big[i\frac{k_\perp^2}{m}h_1^\perp - \alpha m(e-ih)\big](\alpha,k_\perp)\\
&\int \frac{dx_\bullet dx_\perp}{8\pi^3 s}\ e^{-i\alpha x_\bullet + i(k,x)_\perp}\langle A|\bar\psi(0)\not{p}_2\not{A}\psi(x_\bullet,x_\perp)|A\rangle = \big[i\frac{k_\perp^2}{m}\bar h_1^\perp + \alpha m(\bar e+i\bar h)\big](\alpha,k_\perp),\\
&\int \frac{dx_\bullet dx_\perp}{8\pi^3 s}\ e^{-i\alpha x_\bullet + i(k,x)_\perp}\langle A|\bar\psi \not{A}(0)\not{p}_2\psi(x_\bullet,x_\perp)|A\rangle = \big[-i\frac{k_\perp^2}{m}\bar h_1^\perp + \alpha m(\bar e-i\bar h)\big](\alpha,k_\perp)
\end{aligned} \tag{9.48}$$

The target matrix elements are obtained by usual replacements (4.29) (without sign change).

$$\begin{aligned}
&\int \frac{dx_\star dx_\perp}{8\pi^3 s}\ e^{-i\beta_q x_\star + i(k,x)_\perp}\langle B|\bar\psi \not{B}(x)\not{p}_1\psi(0)|B\rangle = \big[-i\frac{k_\perp^2}{m}h_1^\perp - \beta_q m(e+ih)\big](\beta,k_\perp),\\
&\int \frac{dx_\star dx_\perp}{8\pi^3 s}\ e^{-i\beta x_\star + i(k,x)_\perp}\langle B|\bar\psi \not{B}(0)\not{p}_1\psi(x)|B\rangle = \big[-i\frac{k_\perp^2}{m}\bar h_1^\perp + \beta m(\bar e-i\bar h)\big](\beta,k_\perp).\\
&\int \frac{dx_\star dx_\perp}{8\pi^3 s}\ e^{-i\beta x_\star + i(k,x)_\perp}\langle B|\bar\psi(0)\not{p}_1\not{B}\psi(x)|B\rangle = \big[i\frac{k_\perp^2}{m}\bar h_1^\perp + \beta m(\bar e+i\bar h)\big](\beta,k_\perp),\\
&\int \frac{dx_\star dx_\perp}{8\pi^3 s}\ e^{-i\beta x_\star + i(k,x)_\perp}\langle B|\bar\psi \not{B}(0)\not{p}_2\psi(x_\star,x_\perp)|B\rangle = \big[-i\frac{k_\perp^2}{m}\bar h_1^\perp + \beta m(\bar e-i\bar h)\big](\beta,k_\perp)
\end{aligned} \tag{9.49}$$

Next, we need

$$\begin{aligned}
&\int \frac{dx_\bullet dx_\perp}{16\pi^3}\ e^{-i\alpha x_\bullet + i(k,x)_\perp}\langle A|\bar\psi \not{A}(x)\gamma_i\psi(0)|A\rangle = k_i m\big[-e - i\alpha h_3 + ih_D\big](\alpha,k_\perp),\\
&\int \frac{dx_\bullet dx_\perp}{16\pi^3}\ e^{-i\alpha x_\bullet + i(k,x)_\perp}\langle A|\bar\psi(x)\gamma_i\not{A}\psi(0)|A\rangle = k_i m\big[-e + i\alpha h_3 + ih_D^*\big](\alpha,k_\perp),\\
&\int \frac{dx_\bullet dx_\perp}{16\pi^3}\ e^{-i\alpha x_\bullet + i(k,x)_\perp}\langle A|\bar\psi \not{A}(0)\gamma_i\psi(x)|A\rangle = mk_i\big[\bar e - i\alpha\bar h_3 - i\bar h_D\big](\alpha,k_\perp),\\
&\int \frac{dx_\bullet dx_\perp}{16\pi^3}\ e^{-i\alpha x_\bullet + i(k,x)_\perp}\langle A|\bar\psi(0)\gamma_i\not{A}\psi(x)|A\rangle = mk_i\big[\bar e + i\alpha\bar h_3 + i\bar h_D^*\big](\alpha,k_\perp)
\end{aligned} \tag{9.50}$$

The matrix elements h_D are $\bar qq$ TMDs with an extra longitudinal derivative of the quark field. They are defined in the next Section. It is worth noting that contributions of these terms in the r.h.s.'s cancel in the final result.

For the target we get

$$\begin{aligned}
&\int \frac{dx_\star dx_\perp}{16\pi^3}\ e^{-i\beta x_\star + i(k,x)_\perp}\langle B|\bar\psi \not{B}(x)\gamma_i\psi(0)|B\rangle = mk_i[-e - i\beta h_3^\perp + ih_D](\beta,k_\perp),\\
&\int \frac{dx_\star dx_\perp}{16\pi^3}\ e^{-i\beta x_\star + i(k,x)_\perp}\langle B|\bar\psi(x)\gamma_i\not{B}\psi(0)|B\rangle = mk_i[-e + i\beta h_3^\perp - ih_D^\star](\beta,k_\perp),\\
&\int \frac{dx_\star dx_\perp}{16\pi^3}\ e^{-i\beta_q x_\star + i(k,x)_\perp}\langle B|\bar\psi \not{B}(0)\gamma_i\psi(x)|B\rangle = mk_i[\bar e - i\beta\bar h_3^\perp - i\bar h_D](\beta,k_\perp),\\
&\int \frac{dx_\star dx_\perp}{16\pi^3}\ e^{-i\beta x_\star + i(k,x)_\perp}\langle B|\bar\psi(0)\gamma_i\not{B}\psi(x))|B\rangle = mk_i[e + i\beta\bar h_3^\perp + i\bar h_D^\star](\beta,k_\perp).
\end{aligned} \tag{9.51}$$

Finally, we need

$$\begin{aligned}
\int\frac{dx_\bullet dx_\perp}{8\pi^3 s}\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\langle A|\bar\psi \not{A}(x)\not{p}_2\not{A}\psi(0)|A\rangle \;&=\; \big[k_\perp^2(f_1-2\alpha f_\perp)+2\alpha^2m^2f_3\big](\alpha,k_\perp)\\
\int\frac{dx_\bullet dx_\perp}{8\pi^3 s}\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\langle A|\bar\psi \not{A}(0)\not{p}_2\not{A}\psi(x)|A\rangle \;&=\; -\big[k_\perp^2(\bar f_1-2\alpha \bar f_\perp)+2\alpha^2m^2\bar f_3\big](\alpha,k_\perp)\\
\int\frac{dx_\bullet dx_\perp}{8\pi^3 s}\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\langle A|\bar\psi \not{A}(x)\sigma_{\star i}\not{A}\psi(0)|A\rangle &= -k_i\big[\frac{k_\perp^2}{m}h_1^\perp+2\alpha m h-2\alpha^2 m h_3^\perp\big](\alpha,k_\perp)\\
\int\frac{dx_\bullet dx_\perp}{8\pi^3 s}\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\langle A|\bar\psi \not{A}(x)\sigma_{\star i}\not{A}\psi(0)|A\rangle &= \frac{k_i}{m}\big[k_\perp^2\bar h_1^\perp+2\alpha m^2\bar h-2\alpha^2 m^2\bar h_3^\perp\big](\alpha,k_\perp)
\end{aligned}\tag{9.52}$$

and for the target

$$\begin{aligned}
\int\frac{dx_\star dx_\perp}{8\pi^3 s}\; e^{-i\beta x_\star+i(k,x)_\perp}\langle B|\bar\psi \not{B}(x)\not{p}_2\not{B}\psi(0)|B\rangle \;&=\; \big[k_\perp^2(f_1-2\beta f_\perp)+2\beta^2m^2f_3\big](\beta,k_\perp)\\
\int\frac{dx_\star dx_\perp}{8\pi^3 s}\; e^{-i\beta x_\star+i(k,x)_\perp}\langle B|\bar\psi \not{B}(0)\not{p}_2\not{B}\psi(x)|B\rangle \;&=-\big[k_\perp^2(\bar f_1-2\beta \bar f_\perp)+2\beta^2m^2\bar f_3\big](\beta,k_\perp)\\
\int\frac{dx_\star dx_\perp}{8\pi^3 s}\; e^{-i\beta x_\star+i(k,x)_\perp}\langle B|\bar\psi \not{B}(x)\sigma_{\bullet i}\not{B}\psi(0)|B\rangle &= -k_i\big[\frac{k_\perp^2}{m}h_1^\perp+2\beta m h-2\beta^2 m h_3^\perp\big](\beta,k_\perp)\\
\int\frac{dx_\star dx_\perp}{8\pi^3 s}\; e^{-i\beta x_\star+i(k,x)_\perp}\langle B|\bar\psi \not{B}(x)\sigma_{\bullet i}\not{B}\psi(0)|B\rangle &= \frac{k_i}{m}\big[k_\perp^2\bar h_1^\perp+2\beta m^2\bar h-2\beta^2 m^2\bar h_3^\perp\big](\beta,k_\perp)
\end{aligned}\tag{9.53}$$

9.2.4 Parametrization of other quark-quark-gluon TMDs

First, let us parametrize matrix elements from Sect. 3.2.3.

$$\begin{aligned}
\frac{2}{8\pi^3 s}\int dx_\bullet d^2x_\perp\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\;\langle A|\bar\psi i\overleftarrow{D}_\bullet(x_\bullet,x_\perp)\not{p}_2\psi(0)|A\rangle \;&=\; -m^2 f_D(\alpha,k_\perp), \qquad (9.54)\\
\frac{2}{8\pi^3 s}\int dx_\bullet d^2x_\perp\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\;\langle A|\bar\psi(x_\bullet,x_\perp)\not{p}_2 iD_\bullet\psi(0)|A\rangle \;&=\; m^2 f_D^\star(\alpha,k_\perp),\\
\frac{2}{8\pi^3 s}\int dx_\bullet d^2x_\perp\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\;\langle A|\bar\psi i\overleftarrow{D}_\bullet(0)\not{p}_2\psi(x_\bullet,x_\perp)|A\rangle \;&=\; m^2 \bar f_D(\alpha,k_\perp),\\
\frac{2}{8\pi^3 s}\int dx_\bullet d^2x_\perp\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\;\langle A|\bar\psi(0)\not{p}_2 iD_\bullet\psi(x_\bullet,x_\perp)|A\rangle \;&=\; -m^2 \bar f_D^\star(\alpha,k_\perp),\\
\frac{2}{8\pi^3 s}\int dx_\star d^2x_\perp\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\;\langle A|\bar\psi i\overleftarrow{D}_\bullet(x_\bullet,x_\perp)\sigma_{\star i}\psi(0)|A\rangle \;&=\; -mk_ih_D(\alpha,k_\perp),\\
\frac{2}{8\pi^3 s}\int dx_\star d^2x_\perp\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\;\langle A|\bar\psi(x_\bullet,x_\perp)\sigma_{\star i}iD_\bullet\psi(0)|A\rangle \;&=\; mk_ih_D^\star(\alpha,k_\perp)\\
\frac{2}{8\pi^3 s}\int dx_\star d^2x_\perp\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\;\langle A|\bar\psi i\overleftarrow{D}_\bullet(0)\sigma_{\star i}\psi(x_\bullet,x_\perp)|A\rangle \;&=\; mk_i\bar h_D(\alpha,k_\perp),\\
\frac{2}{8\pi^3 s}\int dx_\star d^2x_\perp\; e^{-i\alpha x_\bullet+i(k,x)_\perp}\;\langle A|\bar\psi(0)\sigma_{\star i}iD_\bullet\psi(x_\bullet,x_\perp)|A\rangle \;&=\; -mk_i\bar h_D^\star(\alpha,k_\perp)
\end{aligned}$$

and

$$\frac{2}{8\pi^3 s}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi i \overleftarrow{D}_\star (x_\star,x_\perp)\not{p}_1\psi(0)|B\rangle\ =\ -m^2 f_D(\beta,k_\perp), \tag{9.55}$$
$$\frac{2}{8\pi^3 s}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x_\star,x_\perp)\not{p}_1 iD_\star\psi(0)|B\rangle\ =\ m^2 f^*_D(\beta,k_\perp),$$
$$\frac{2}{8\pi^3 s}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi i \overleftarrow{D}_\star (0)\not{p}_1\psi(x_\star,x_\perp)|B\rangle\ =\ m^2 \bar f_D(\beta,k_\perp),$$
$$\frac{2}{8\pi^3 s}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)\not{p}_1 iD_\star\psi(x_\star,x_\perp)|B\rangle\ =\ -m^2 \bar f^*_D(\beta,k_\perp),$$
$$\frac{2}{8\pi^3 s}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi i \overleftarrow{D}_\star (x_\star,x_\perp)\sigma_{\bullet i}\psi(0)|A\rangle\ =\ -mk_i h_D(\beta,k_\perp),$$
$$\frac{2}{8\pi^3 s}\int dx_\star d^2x_\perp\ e^{-i\beta x_\bullet+i(k,x)_\perp}\ \langle B|\bar\psi(x_\star,x_\perp)\sigma_{\bullet i} iD_\star\psi(0)|B\rangle\ =\ mk_i h^*_D(\alpha,k_\perp)$$
$$\frac{2}{8\pi^3 s}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi i \overleftarrow{D}_\star (0)\sigma_{\bullet i}\psi(x_\star,x_\perp)|B\rangle\ =\ mk_i \bar h_D(\beta,k_\perp),$$
$$\frac{2}{8\pi^3 s}\int dx_\star d^2x_\perp\ e^{-i\beta x_\bullet+i(k,x)_\perp}\ \langle B|\bar\psi(0)\sigma_{\bullet i} iD_\star\psi(x_\star,x_\perp)|B\rangle\ =\ -mk_i \bar h^*_D(\alpha,k_\perp)$$

Next, in Sect. 4 and 5 we calculate target $\bar q G q$ matrix elements and restore the corresponding contributions with projectile $\bar q G q$ ones by trivial replacements (4.29). Consequently, we will list only parametrizations of target $\bar q G q$ matrix elements. The projectile matrix elements can be obtained by the usual (4.29) replacements. [16]

We parametrize the quark-quark-gluon TMDs with matrices 1 or γ_5 as follows

$$\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x_\star,x_\perp)B_i(x_\star,x_\perp)\psi(0)|B\rangle\ =\ k_i m e_G(\beta,k_\perp)$$
$$\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x_\star,x_\perp)B_i(0)\psi(0)|B\rangle\ =\ k_i m e^*_G(\beta,k_\perp)$$
$$\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)B_i(0)\psi(x_\star,x_\perp)|B\rangle\ =\ k_i \bar e_G m(\beta,k_\perp) \tag{9.56}$$
$$\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)B_i(x)\psi(x_\star,x_\perp)|B\rangle\ =\ k_i \bar e^*_G m(\beta,k_\perp)$$

$$\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x_\star,x_\perp)i\tilde B_i(x_\star,x_\perp)\gamma_5\psi(0)|B\rangle\ =\ k_i m\tilde e_G(\beta,k_\perp)$$
$$\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x_\star,x_\perp)i\tilde B_i(0)\gamma_5\psi(0)|B\rangle\ =\ k_i m\tilde e^*_G(\beta,k_\perp)$$
$$\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)i\tilde B_i(0)\gamma_5\psi(x_\star,x_\perp)|B\rangle\ =\ k_i \bar{\tilde e}_G m(\beta,k_\perp) \tag{9.57}$$
$$\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)i\tilde B_i(x)\gamma_5\psi(x_\star,x_\perp)|B\rangle\ =\ k_i \bar{\tilde e}^*_G m(\beta,k_\perp)$$

[16] One shold be careful with $\tilde A_i \leftrightarrow \tilde B_i$ replacement due to Eq. (9.14), see e.g. Eq. (9.47).

and accordingly

$$\begin{aligned}
\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x_\star,x_\perp)\acute{B}_i(x_\star,x_\perp)\psi(0)|B\rangle\ &=\ k_im\acute{e}_G(\beta,k_\perp)\\
\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x_\star,x_\perp)\acute{B}_i(0)\psi(0)|B\rangle\ &=\ k_im\acute{e}^*_G(\beta,k_\perp)\\
\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)\acute{B}_i(0)\psi(x_\star,x_\perp)|B\rangle\ &=\ k_im\bar{\acute{e}}_G(\beta,k_\perp)\\
\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)\acute{B}_i(x)\psi(x_\star,x_\perp)|B\rangle\ &=\ k_im\bar{\acute{e}}^*_B(\beta,k_\perp) \qquad (9.58)
\end{aligned}$$

Next, we turn to quark-quark-gluon TMDs with matrices $\sigma_{\mu\nu}$. First, consider the case of $\sigma_{\bullet *}$. We get

$$\begin{aligned}
\int\frac{dx_\star d^2x_\perp}{8\pi^3 s}\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi B_i(x)\sigma_{\bullet *}\psi(0)|B\rangle\ &=\ k_im[\beta h_3^\perp - h + h_D - i\tilde{e}_G](\beta,k_\perp)\\
\int\frac{dx_\star d^2x_\perp}{8\pi^3 s}\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x)\sigma_{\bullet *}B_i\psi(0)|B\rangle\ &=\ k_im[\beta h_3^\perp - h + h^*_D + i\tilde{e}^*_G](\beta,k_\perp)\\
\int\frac{dx_\star d^2x_\perp}{8\pi^3 s}\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)\sigma_{\bullet *}\psi B_i(x)|B\rangle\ &=\ k_im[\beta \bar h_3^\perp - \bar h - \bar h^*_D + i\bar{\tilde{e}}^*_G](\beta,k_\perp)\\
\int\frac{dx_\star d^2x_\perp}{8\pi^3 s}\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi B_i(0)\sigma_{\bullet *}\psi(x)|B\rangle\ &=\ k_im[\beta \bar h_3^\perp - \bar h - \bar h_D - i\bar{\tilde{e}}_G](\beta,k_\perp)
\end{aligned} \qquad (9.59)$$

Let us illustrate the derivation of these equations. From equations of motion (9.43) and Eq. (9.3) we see that

$$\begin{aligned}
&\frac{1}{8\pi^3 s}\int dx_\star dx_\perp\ e^{-i\beta_q x_\star+i(k,x)_\perp}\langle B|\bar\psi \not{B}(x)\gamma_i\sigma_{\star\bullet}\psi(0)|B\rangle\ =\ \frac{1}{16\pi^3}\int dx_\star dx_\perp\ e^{-i\beta_q x_\star+i(k,x)_\perp}\\
&\times\ \langle B|\bar\psi(x)[\beta\sigma_{\star i} - \frac{2}{s}k_i\sigma_{\star\bullet} - \epsilon_{ij}k^j\gamma_5 + i\overleftarrow{D}_*\ \sigma_{\bullet i}]\psi(0)|B\rangle\ =\ -mk_i[\beta h_3^\perp - h + h_D](\beta,k_\perp)
\end{aligned} \qquad (9.60)$$

On the other hand

$$\begin{aligned}
\frac{2}{s}\langle B|\bar\psi\not{B}(x)\gamma_i\sigma_{\star\bullet}\psi(0)|B\rangle\ &=\ \frac{2}{s}\langle B|\bar\psi B^j(x)(\delta_{ij}+i\sigma_{ij})\sigma_{\star\bullet}\psi(0)|B\rangle \qquad (9.61)\\
&=\ \frac{2}{s}\langle B|\bar\psi B_i(x)\sigma_{\star\bullet}\psi(0)|B\rangle + \langle B|\bar\psi\tilde{B}_i(x)\gamma_5\psi(0)|B\rangle\\
\Rightarrow\ \frac{1}{8\pi^3 s}\int dx_\star dx_\perp\ e^{-i\beta_q x_\star+i(k,x)_\perp}&\langle B|\bar\psi B_i(x)\sigma_{\star\bullet}\psi(0)|B\rangle\\
=\ \frac{1}{8\pi^3 s}\int dx_\star dx_\perp\ e^{-i\beta_q x_\star+i(k,x)_\perp}&\langle B|\bar\psi\not{B}(x)\gamma_i\sigma_{\star\bullet}\psi(0)|B\rangle - i\tilde{e}_G(\beta,k_\perp)
\end{aligned}$$

where we used parametrization (9.56). From the above two equations we easily get the first of Eqs (9.59). In a similar way one can obtain the rest of formulas (9.59).

Second, for transverse σ's we get

$$\int \frac{dx_\star dx_\perp}{16\pi^3}\; e^{-i\beta x_\star+i(k,x)_\perp}\langle B|\bar\psi B^\mu(x)\sigma_{\nu_\perp j}\psi(0)|B\rangle \tag{9.62}$$
$$= m(\delta^{\mu_\perp}_{\nu_\perp}k_j-\delta^\mu_j k^\perp_\nu)\big[-ie(\beta,k_\perp)-ie_G(\beta,k_\perp)+\beta h_3^\perp(\beta,k_\perp)-h_D(\beta,k_\perp)\big],$$
$$\int \frac{dx_\star dx_\perp}{16\pi^3}\; e^{-i\beta x_\star+i(k,x)_\perp}\langle B|\bar\psi(x)B^\mu(0)\sigma_{\nu_\perp j}\psi(0)|B\rangle$$
$$= m(\delta^{\mu_\perp}_{\nu_\perp}k_j-\delta^\mu_j k^\perp_\nu)\big[ie(\beta,k_\perp)+ie^*_G(\beta,k_\perp)+\beta h_3^\perp(\beta,k_\perp)-h^*_D(\beta,k_\perp)\big],$$
$$\int \frac{dx_\star dx_\perp}{16\pi^3}\; e^{-i\beta x_\star+i(k,x)_\perp}\langle B|\bar\psi B^\mu(0)\sigma_{\nu_\perp j}\psi(x)|B\rangle$$
$$= m(\delta^{\mu_\perp}_{\nu_\perp}k_j-\delta^\mu_j k^\perp_\nu)\big[i\bar e(\beta,k_\perp)-i\bar e_G(\beta,k_\perp+\beta\bar h_3^\perp(\beta,k_\perp)+\bar h_D(\beta,k_\perp)\big],$$
$$\int \frac{dx_\star dx_\perp}{16\pi^3}\; e^{-i\beta x_\star+i(k,x)_\perp}\langle B|\bar\psi(0)\sigma_{\nu_\perp j}B^\mu\psi(x)|B\rangle$$
$$= m(\delta^{\mu_\perp}_{\nu_\perp}k_j-\delta^\mu_j k^\perp_\nu)\big[-i\bar e(\beta,k_\perp)+i\bar e^*_G(\beta,k_\perp)+\beta\bar h_3^\perp(\beta,k_\perp)+\bar h^*_D(\beta,k_\perp)\big]$$

Again, let us illustrate the derivation of the first of the above equation. After convoluting μ and ν, we need to prove that

$$\frac{1}{16\pi^3}\int dx_\star dx_\perp\; e^{-i\beta_q x_\star+i(k,x)_\perp}\langle B|\bar\psi B^\nu(x)\sigma_{\nu_\perp j}\psi(0)|B\rangle$$
$$= \frac{1}{16\pi^3}\int dx_\star dx_\perp\; e^{-i\beta_q x_\star+i(k,x)_\perp}\langle B|\bar\psi[i\not{B}_\perp(x)\gamma_j-iB_j(x)\psi(0)|B\rangle$$
$$= -imk_j\big[e(\beta_q,k_\perp)+e_G(\beta_q,k_\perp)+i\beta_q h_3^\perp(\beta,k_\perp)\big],$$
$$= mk_j\big[-ie-ie_G+\beta_q h_3^\perp-h_D\big](\beta,k_\perp) \tag{9.63}$$

which easily follows from equations of motion (9.43) and parametrizations (9.56).The rest of the equations (9.62) is proved in a similar way.

For $\sigma_{\mu\nu}$ with one longitudinal and one transverse indices we define

$$\frac{1}{16\pi^3}\frac{2}{s}\int dx_\star d^2x_\perp\; e^{-i\beta x_\star+i(k,x)_\perp}\;\langle B|\bar\psi(x_\star,x_\perp)\big[B_i(x)\sigma_{\bullet j}-\frac12 g_{ij}B^k\sigma_{\bullet k}(x)\big]\psi(0)|B\rangle$$
$$= -(k_ik_j+\frac12 g_{ij}k_\perp^2)\frac1m h_{1G}^{\perp f}(\beta,k_\perp)$$
$$\frac{1}{16\pi^3}\frac{2}{s}\int dx_\star d^2x_\perp\; e^{-i\beta x_\star+i(k,x)_\perp}\;\langle B|\bar\psi(0)[B_i(0)\sigma_{\bullet j}-\frac12 g_{ij}B^k\sigma_{\bullet k}(0)]\psi(x_\star,x_\perp)|B\rangle$$
$$= -(k_ik_j+\frac12 g_{ij}k_\perp^2)\frac1m \bar h_{1G}^{\perp f}(\beta,k_\perp) \tag{9.64}$$

Next, we parametrize

$$\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\; e^{-i\beta x_\star+i(k,x)_\perp}\;\langle B|\bar\psi(x_\star,x_\perp)\gamma_j\grave{B}_i(x_\star,x_\perp)\psi(0)|B\rangle \tag{9.65}$$
$$= \big[k_ik_j+k_\perp^2\frac{g_{ij}}{2}\big]\grave{f}_{1G}(\beta,k_\perp)+\frac{g_{ij}}{2}[k_\perp^2(f_\perp-ig_\perp)-2\beta m^2f_3](\beta,k_\perp),$$
$$\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\; e^{-i\beta x_\star+i(k,x)_\perp}\;\langle B|\bar\psi(x_\star,x_\perp)\grave{B}_i(0)\gamma_j\psi(0)|B\rangle$$
$$= \big[k_ik_j+k_\perp^2\frac{g_{ij}}{2}\big]\grave{f}^*_{1G}(\beta,k_\perp)+\frac{g_{ij}}{2}[k_\perp^2(f_\perp+ig_\perp)-2\beta m^2f_3t](\beta,k_\perp),$$
$$\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\; e^{-i\beta x_\star+i(k,x)_\perp}\;\langle B|\bar\psi(0)\gamma_j\grave{B}_i(0)\psi(x_\star,x_\perp)|B\rangle$$
$$= \big[k_ik_j+k_\perp^2\frac{g_{ij}}{2}\big]\grave{\bar{f}}_{1G}(\beta,k_\perp)+\frac{g_{ij}}{2}[k_\perp^2(\bar f_\perp+i\bar g_\perp)-2\beta m^2\bar f_3](\beta,k_\perp),$$
$$\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\; e^{-i\beta x_\star+i(k,x)_\perp}\;\langle B|\bar\psi(0)\grave{B}_i(x_\star,x_\perp)\gamma_j\psi(x_\star,x_\perp)|B\rangle$$
$$= \big[k_ik_j+k_\perp^2\frac{g_{ij}}{2}\big]\grave{\bar{f}}^*_{1G}(\beta,k_\perp)+\frac{g_{ij}}{2}[k_\perp^2(\bar f_\perp-i\bar g_\perp)-2m^2\bar f_3](\beta,k_\perp)$$

Let us prove the first of the above equations. Consider

$$\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\; e^{-i\beta x_\star+i(k,x)_\perp}\frac{2}{s}\langle B|\bar\psi(x)\not{B}(x)\not{p}_2\not{p}_1\psi(0)|B\rangle$$
$$= \frac{k_i}{16\pi^3}\int dx_\star d^2x_\perp\; e^{-i\beta x_\star+i(k,x)_\perp}\langle B|\bar\psi(x)\big[-\gamma_i+i\epsilon_{ij}\gamma^j\gamma_5+2i\overleftarrow{D}_*\not{p}_1x\big]\psi(0)|B\rangle$$
$$= k_\perp^2[f_\perp(\beta,k_\perp)+ig_\perp(\beta,k_\perp)]-2m^2f_D(\beta,k_\perp) \tag{9.66}$$

where we again used QCD equations (9.43)

On the other hand,

$$\frac{2}{s}\langle B|\bar\psi(x)\not{B}(x)\not{p}_2\not{p}_1\psi(0)|B\rangle \;=\; \bar\psi(x)\gamma^i(B_i+i\tilde{B}_i\gamma_5)(x)\psi(0)\rangle \;=\; \bar\psi(x)\grave{B}^i(x)\gamma_i\psi(0)\rangle \;\Rightarrow$$
$$\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\; e^{-i\beta x_\star+i(k,x)_\perp}\;\langle B|\bar\psi\grave{B}^i(x)\gamma_i\psi(0)|B\rangle \;=\; [k_\perp^2(f_\perp+ig_\perp)-2m^2f_D](\beta,k_\perp) \tag{9.67}$$

Next, $\gamma_i\grave{B}^i=2\not{B}-\grave{B}^i\gamma_i$ so from the equation of motion (9.43)

$$\frac{1}{16\pi^3}\int dx_\star d^2x_\perp\; e^{-i\beta x_\star+i(k,x)_\perp}\;\langle B|\bar\psi(x)\not{B}(x)\psi(0)|B\rangle \;=\; [k_\perp^2f_\perp-\beta m^2f_3-m^2f_D](\beta,k_\perp) \tag{9.68}$$

and we easily get

$$\int\frac{dx_\star d^2x_\perp}{16\pi^3}\; e^{-i\beta x_\star+i(k,x)_\perp}\;\langle B|\bar\psi(x)\gamma_i\grave{B}^i(x)\psi(0)|B\rangle \;=\; [k_\perp^2(f_\perp-ig_\perp)-2\beta m^2f_3](\beta,k_\perp) \tag{9.69}$$

The rest of the convolutions in Eqs. (9.65) are obtained in a similar way.

Finally, we parametrize TMDs with integrated gluon fields as in Eq. (5.13) as follows

$$\begin{aligned}
\frac{1}{16\pi^3}\frac{2}{s}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x)\grave{\mathcal{B}}_i(x_\star,x_\perp)\not{p}_1\psi(0)|B\rangle\ &=\ k_i\grave{f}_{1\mathcal{G}}(\beta,k_\perp)\\
\frac{1}{16\pi^3}\frac{2}{s}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)\grave{\mathcal{B}}_i(0)\not{p}_1\psi(x_\star,x_\perp)|B\rangle\ &=\ -\,k_i\bar{\grave{f}}_{1\mathcal{G}}(\beta,k_\perp)\\
\frac{1}{16\pi^3}\frac{2}{s}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x)\not{p}_1\grave{\mathcal{B}}_i(0)\psi(0)|B\rangle\ &=\ -\,k_i\grave{f}^*_{1\mathcal{G}}(\beta,k_\perp)\\
\frac{1}{16\pi^3}\frac{2}{s}\int dx_\star d^2x_\perp\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)\not{p}_1\grave{\mathcal{B}}_i(x)\psi(x_\star,x_\perp)|B\rangle\ &=\ k_i\bar{\grave{f}}^*_{1\mathcal{G}}(\beta,k_\perp)
\end{aligned}\tag{9.70}$$

$$\begin{aligned}
\int\frac{dx_\star d^2x_\perp}{8\pi^3 s}\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x)\not{p}_1\not{\mathcal{P}}\mathcal{B}(x_\star,x_\perp)\psi(0)|B\rangle\ &=\ m^2f_{2\mathcal{G}}(\beta,k_\perp)\\
\int\frac{dx_\star d^2x_\perp}{8\pi^3 s}\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)\not{p}_1\not{\mathcal{P}}\mathcal{B}(0)\psi(x_\star,x_\perp)|B\rangle\ &=\ m^2\bar{f}_{2\mathcal{G}}(\beta,k_\perp)\\
\int\frac{dx_\star d^2x_\perp}{8\pi^3 s}\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x)(\not{\mathcal{P}}\mathcal{B})^*(0)\not{p}_1\psi(0)|B\rangle\ &=\ m^2f^*_{2\mathcal{G}}(\beta,k_\perp)\\
\int\frac{dx_\star d^2x_\perp}{8\pi^3 s}\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)(\not{\mathcal{P}}\mathcal{B})^*(x)\not{p}_1\psi(x_\star,x_\perp)|B\rangle\ &=\ m^2\bar{f}^*_{2\mathcal{G}}(\beta,k_\perp)
\end{aligned}\tag{9.71}$$

$$\begin{aligned}
\int\frac{dx_\star d^2x_\perp}{8\pi^3 s}\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x)\not{p}_1\gamma_5\frac{\epsilon^{ij}}{2}\mathcal{B}_{ij}(x_\star,x_\perp)\psi(0)|B\rangle\ &=\ m^2f_{3\mathcal{G}}(\beta,k_\perp)\\
.\int\frac{dx_\star d^2x_\perp}{8\pi^3 s}\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)\not{p}_1\gamma_5\frac{\epsilon^{ij}}{2}\mathcal{B}_{ij}(0)\psi(x_\star,x_\perp)|B\rangle\ &=\ m^2\bar{f}_{3\mathcal{G}}(\beta,k_\perp)\\
\int\frac{dx_\star d^2x_\perp}{8\pi^3 s}\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x)\not{p}_1\gamma_5\frac{\epsilon^{ij}}{2}\mathcal{B}_{ij}(0)\psi(0)|B\rangle\ &=\ m^2f^*_{3\mathcal{G}}(\beta,k_\perp)\\
\int\frac{dx_\star d^2x_\perp}{8\pi^3 s}\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)\not{p}_1\gamma_5\frac{\epsilon^{ij}}{2}\mathcal{B}_{ij}(x_\star,x_\perp)\psi(x_\star,x_\perp)|B\rangle\ &=\ m^2\bar{f}^*_{3\mathcal{G}}(\beta,k_\perp)
\end{aligned}\tag{9.72}$$

$$\begin{aligned}
&\int\frac{dx_\star d^2x_\perp}{8\pi^3 s}\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x)\ \sigma_{\bullet\beta_\perp}\mathcal{B}_\alpha(x)\psi(0)|B\rangle\\
&\qquad=\ k^\perp_\alpha k^\perp_\beta\frac{1}{m}h_{1\mathcal{G}}(\beta,k_\perp)+\frac{k_\perp^2}{m}\frac{g^\perp_{\alpha\beta}}{2}[h_{1\mathcal{G}}+h_{2\mathcal{G}}](\beta,k_\perp),\\
&\int\frac{dx_\star d^2x_\perp}{8\pi^3 s}\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)\sigma_{\bullet\beta_\perp}\mathcal{B}_\alpha(0)\psi(x)|B\rangle\\
&\qquad=\ -\,k^\perp_\alpha k^\perp_\beta\frac{1}{m}\bar{h}_{1\mathcal{G}}(\beta,k_\perp)-\frac{k_\perp^2}{m}\frac{g^\perp_{\alpha\beta}}{2}[\bar{h}_{1\mathcal{G}}+\bar{h}_{2\mathcal{G}}](\beta,k_\perp)\\
&\int\frac{dx_\star d^2x_\perp}{8\pi^3 s}\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(x)\ \sigma_{\bullet\beta_\perp}\mathcal{B}_\alpha(0)\psi(0)|B\rangle\\
&\qquad=\ k^\perp_\alpha k^\perp_\beta\frac{1}{m}h^*_{1\mathcal{G}}(\beta,k_\perp)+\frac{k_\perp^2}{m}\frac{g^\perp_{\alpha\beta}}{2}[h^*_{1\mathcal{G}}+h^*_{2\mathcal{G}}](\beta,k_\perp),\\
&\int\frac{dx_\star d^2x_\perp}{8\pi^3 s}\ e^{-i\beta x_\star+i(k,x)_\perp}\ \langle B|\bar\psi(0)\sigma_{\bullet\beta_\perp}\mathcal{B}_\alpha(x)\psi(x)|B\rangle\\
&\qquad=\ -\,k^\perp_\alpha k^\perp_\beta\frac{1}{m}\bar{h}^*_{1\mathcal{G}}(\beta,k_\perp)-\frac{k_\perp^2}{m}\frac{g^\perp_{\alpha\beta}}{2}[\bar{h}^*_{1\mathcal{G}}+\bar{h}^*_{2\mathcal{G}}](\beta,k_\perp)
\end{aligned}\tag{9.73}$$

and

$$\begin{aligned}
\int \frac{dx_\star d^2x_\perp}{8\pi^3 s}\; e^{-i\beta x_\star + i(k,x)_\perp}\; \langle B|\bar\psi(x)\sigma_{\bullet i}\mathcal{P}\mathcal{B}(x_\star,x_\perp)\psi(0)|B\rangle \;&=\; k_i m h_{3\mathcal{G}}(\beta,k_\perp) \\
.\int \frac{dx_\star d^2x_\perp}{8\pi^3 s}\; e^{-i\beta x_\star + i(k,x)_\perp}\; \langle B|\bar\psi(0)\sigma_{\bullet i}\mathcal{P}\mathcal{B}(0)\psi(x_\star,x_\perp)|B\rangle \;&=\; k_i m \bar h_{3\mathcal{G}}(\beta,k_\perp) \\
\int \frac{dx_\star d^2x_\perp}{8\pi^3 s}\; e^{-i\beta x_\star + i(k,x)_\perp}\; \langle B|\bar\psi(x)\sigma_{\bullet i}\mathcal{B}\mathcal{P}(0)\psi(0)|B\rangle \;&=\; k_i m h^*_{3\mathcal{G}}(\beta,k_\perp) \qquad (9.74)\\
\int \frac{dx_\star d^2x_\perp}{8\pi^3 s}\; e^{-i\beta x_\star + i(k,x)_\perp}\; \langle B|\bar\psi(0)\sigma_{\bullet i}\mathcal{B}\mathcal{P}(x_\star,x_\perp)\psi(x_\star,x_\perp)|B\rangle \;&=\; k_i m \bar h^*_{3\mathcal{G}}(\beta,k_\perp)
\end{aligned}$$

$$\begin{aligned}
\int \frac{dx_\star d^2x_\perp}{8\pi^3 s}\; e^{-i\beta x_\star + i(k,x)_\perp}\; \langle B|\bar\psi(x)\sigma_{\bullet i}\mathcal{B}_{ij}(x_\star,x_\perp)\psi(0)|B\rangle \;&=\; k_j m h_{4\mathcal{G}}(\beta,k_\perp) \\
.\int \frac{dx_\star d^2x_\perp}{8\pi^3 s}\; e^{-i\beta x_\star + i(k,x)_\perp}\; \langle B|\bar\psi(0)\sigma_{\bullet i}\mathcal{B}_{ij}(0)\psi(x_\star,x_\perp)|B\rangle \;&=\; k_j m \bar h_{4\mathcal{G}}(\beta,k_\perp) \\
\int \frac{dx_\star d^2x_\perp}{8\pi^3 s}\; e^{-i\beta x_\star + i(k,x)_\perp}\; \langle B|\bar\psi(x)\sigma_{\bullet i}\mathcal{B}_{ij}(0)\psi(0)|B\rangle \;&=\; k_j m h^*_{4\mathcal{G}}\beta,k_\perp) \qquad (9.75)\\
\int \frac{dx_\star d^2x_\perp}{8\pi^3 s}\; e^{-i\beta x_\star + i(k,x)_\perp}\; \langle B|\bar\psi(0)\sigma_{\bullet i}\mathcal{B}_{ij}(x_\star,x_\perp)\psi(x_\star,x_\perp)|B\rangle \;&=\; k_j m \bar h^*_{4\mathcal{G}}(\beta,k_\perp)
\end{aligned}$$

As usual, the corresponding matrix elements for the projectile are obtained by trivial replacements $x_\star \leftrightarrow x_\bullet$, $\alpha_q \leftrightarrow \beta_q$ and $\not{p}_2 \leftrightarrow \not{p}_1$.

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