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FCC circumference studies based on RF synchronization

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ABSTRACT: As the study of the Future Circular Collider (FCC) project progresses, the choice of the exact circumference of the accelerator tunnel becomes essential. It is defined by RF-related constraints, mainly the RF frequency of the FCC-hh, as well as the revolution frequency ratios with the hadron injector. It is shown firstly how favourable circumference options assuming the present LHC RF frequency have been identified and ranked from the RF point of view. Secondly, the entire frequency range of interest has been systematically scanned, assuming the possibility of operating SPS or LHC as the potential injector of FCC-hh. Considerations for the RF synchronization between injector and collider have been taken into account. Several options are studied in detail, amongst them the particularly attractive circumferences of 90478.6 m and 90837.7 m, and compared to the present working hypothesis of 90658.2 m. The results are analysed and rated from the point of view of flexibility and longitudinal beam dynamics.

KEYWORDS: Beam dynamics; Instrumentation for particle accelerators and storage rings - high energy (linear accelerators, synchrotrons)

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1 Introduction

Layout and placement of the Future Circular Collider (FCC) have evolved since the Conceptual Design Report (CDR) [1] due to geological constraints unveiled in the framework of the feasibility study. Accordingly, the ring circumference has been reduced from 97.75 km to around 91.1 km [2], and more recently to the latest value of around 90.66 km. To progress with the further detailed study of the FCC-ee collider, the choice of the exact ring circumference becomes imperative. Considering that the FCC-ee and FCC-hh colliders will share the same tunnel, the circumference is actually determined by FCC-hh constraints in order to allow a conventional synchronized transfer from the SPS or the LHC to the FCC-hh.

For the injection of electrons into FCC-ee most of the constraints do not apply. The fixed RF frequencies in the injectors and collider can be derived from a common reference. Due to the strong synchrotron radiation damping, no beam-based control loops are required for leptons during acceleration. Hence the time from bunch generation to the final injection into the collider is deterministically defined, both in terms of RF phase and number of turns. It is therefore assumed that any circumference well suited for FCC-hh will meet the requirements for FCC-ee.

New transfer schemes may be conceivable by the time of FCC-hh, but conservative choices for the tunnel length are deliberately made to keep the flexibility for future parameter changes.

Indeed, in high-energy hadron colliders such as LHC, RHIC or the Tevatron, a chain of several synchrotrons is necessary because the ratio of extraction to injection energy for each synchrotron is restricted within the range of 10 to 20. Thus, the beam must be transferred from one synchrotron to another in order to cover larger energy ranges.

In order to rapidly accumulate a large number of bunches in the collider ring, long trains of bunches have to be transferred between accelerators for an efficient injection process. Hence the RF frequencies must be either be the same or integer multiples of each other. To ensure a successful beam transfer between two synchrotrons, the revolution frequencies or periods must be synchronized, similarly to the alignment of mechanical cogwheels. Particularly, it naturally demands RF synchronization between two hadron synchrotrons in cascade for the so-called bunch-to-bucket transfer. This specific method of bunch-to-bucket transfer is widely utilized to minimize beam loss in the longitudinal plane during transfer, which requires the synchronization of the RF systems of both synchrotrons so that the bunches injected from the source synchrotron into the target synchrotron reach the intended buckets [3, 4], as illustrated in figure 1.

The beam energy and velocity remain constant during transfer. Therefore, the circumference ratio of the two synchrotrons equals the ratio of their frequencies, which can be reduced to the ratio of two integers, n_1 and n_2 , as expressed below:

$$\frac{C_2}{C_1} = \frac{v \cdot T_{\text{rev},2}}{v \cdot T_{\text{rev},1}} = \frac{f_{\text{rev},1}}{f_{\text{rev},2}} = \frac{n_1}{n_2},$$
(1.1)

where C_1 , C_2 , $T_{rev,1}$, $T_{rev,2}$, $f_{rev,1}$, and $f_{rev,2}$ are the circumferences, revolution periods and revolution frequencies of the two synchrotrons, respectively. The relationship between the RF frequency and the revolution frequency is given by $f_{rf} = h \cdot f_{rev}$, with *h* being the harmonic number. If we assume the RF frequencies of the two synchrotrons to be the same, i.e., $f_{rf,1} = f_{rf,2}$, then the ratio of their harmonic numbers, h_1 and h_2 , equals the ratio of their circumferences:

$$\frac{h_2}{h_1} = \frac{C_2}{C_1} = \frac{n_1}{n_2}.$$
(1.2)

Therefore, beam transfer is possible with a periodicity of $n_1 \cdot T_{rev,1} = n_2 \cdot T_{rev,2}$. For example, for the SPS-to-LHC transfer, we have $h_{LHC}/h_{SPS} = C_{LHC}/C_{SPS} = 27/7$, so this transfer is possible every 7 LHC revolution periods corresponding to 27 turns in the SPS. Another example is the PS-to-SPS transfer, which can take place every SPS revolution since $h_{SPS}/h_{PS} = C_{SPS}/C_{PS} = 11/1$.

2 Main constraints

Three main constraints have initially been identified to determine the exact FCC circumference from the RF-related constraints on the FCC-hh. Firstly, we consider the possibility of keeping either SPS or LHC as potential injector of FCC-hh. Secondly, the FCC circumference should be around 91 km, compatible with the latest geographical and geological survey. Thirdly, the LHC RF frequency of 400.79 MHz is the initial baseline option for the FCC-hh RF system.

With these considerations, and taking into account the circumferences of LHC and SPS, i.e., 26658.883 m and 6911.562 m (= $C_{LHC} \times 7/27$), and their respective harmonic numbers of 35640 (= $2 \times 3 \times 4 \times 5 \times 11 \times 27$) and 9240 (= $2 \times 3 \times 4 \times 5 \times 11 \times 7$) at the baseline RF frequency of 400.79 MHz, the corresponding harmonic number of FCC-hh for the intended circumference can be evaluated using eq. (1.2). For instance, if we were to naively pick a circumference of 91 km, the corresponding harmonic number of 121657 (= $19 \times 19 \times 337$) would not be a good option as it is not divisible by 10 (which is necessary to provide a continuous bunch clock for the nominal bunch spacing of about 25 ns). Additionally, it is not suited for a synchronous transfer as the waiting time would be very long due to the large denominator of the harmonic number ratio



Figure 1. Bunch-to-bucket transfer between two hadron synchrotrons in cascade. Synchrotron 1: the source synchrotron; Synchrotron 2: the target synchrotron. The coloured dots represent RF bucket locations. Cyan indicates RF buckets occupied by bunches, while red and grey represent unoccupied ones. On this schematic, $h_2/h_1 = C_2/C_1 = 4/1$, so beam transfer is possible every turn of Synchrotron 2, corresponding to four turns in Synchrotron 1.

 $h_{\text{FCC}}/h_{\text{LHC}} = 121657/35640$. It is worth noting that the ratio of harmonic numbers of LHC and SPS is exactly 27/7, which comes from the ratio of circumferences of the two accelerators. A factor 11 in the harmonic numbers of SPS and LHC comes from the ratio of circumferences of SPS and PS, which would not be a constraint assuming a replacement of the ageing PS as the FCC-hh pre-injector.

Other constraints on the FCC-hh harmonic number have also been identified to obtain possible optimum options. Firstly, the FCC-hh harmonic number must be divisible by 2 considering 4 interaction points (IPs) in the latest FCC layout. Note that a factor of 2 (instead of 4) in the FCC-hh harmonic number is sufficient to serve 4 symmetrically distributed IPs [5]. Although a factor of 4 would enable beam collisions to occur simultaneously in the 4 IPs, this is no imperative. If the continuous bunch clock operation remains adopted for the FCC-hh, its harmonic number after dividing by 2 should be divisible by the number of RF buckets, *n*, corresponding to the desired bunch spacing. The present baseline assumes about 25 ns bunch spacing, i.e., *n* equals 10, while alternative options with 5 ns, 12.5 ns and 15 ns are also under consideration. Thus, it is important that the FCC-hh harmonic number provides more flexible bunch spacing options in case of future needs, so it should contain as many integer factors from 1 to *n* as possible. Moreover, the largest prime factor in the factorization of the harmonic numbers of FCC-hh, LHC, and SPS should be as small as possible to be capable of generating signals at intermediate frequencies by division from the RF frequency. Finally, we need to check all harmonic ratios of h_{FCC}/h_{LHC} and h_{FCC}/h_{SPS} to keep also the denominators as small as possible, like less than about 300, to minimize the waiting time for the transfer.

For convenience, all of the constraints that will be imposed in the following analysis are summarized in table 1. Constraints that must be satisfied are marked as "compulsory", while those that can be relaxed are marked as "optional".

 Table 1. Constraints on the choice of FCC circumference and RF frequency.

- 1. Keep either SPS or LHC as the potential injector of FCC-hh (compulsory).
- 2. Propose FCC circumference as close as possible to 90.66 km (compulsory).
- 3. RF frequency should remain 400.79 MHz for FCC-hh (optional).
- 4. h_{FCC} must divisible by 2 (compulsory).
- 5. h_{FCC} should keep a continuous bunch clock for at least 25 ns bunch spacing (compulsory).
- 6. h_{FCC} should provide as many bunch spacings as possible (optional).
- 7. Largest prime factor in the factorization of h_{FCC} , h_{LHC} , and h_{SPS} should be smaller than 300 (optional).
- 8. Denominators in $h_{\text{FCC}}/h_{\text{LHC}}$ and $h_{\text{FCC}}/h_{\text{SPS}}$ should less than 300 (optional).

3 Options with the current LHC RF frequency

We first consider the FCC circumference options at the current LHC RF frequency of 400.79 MHz. If we were to only consider the constraints 1–4 in table 1, the possible harmonic numbers would be too numerous to list. We first lay out the compulsory constraints and corresponding options, and then narrow them down by adding additional constraints.

3.1 Most attractive options

First of all, we impose the constraints from the continuous bunch clock and the flexibility of bunch spacing, i.e., the constraints 5 and 6 in table 1. If we intend to include all bunch spacings up to 25 ns at 400.79 MHz, the FCC-hh harmonic number must be divisible by 5040, as shown in figure 2. With the RF frequency of 400.79 MHz, the closest harmonic number to 121657 is 120960, along with the corresponding circumference of 90478.6 m. It seems an excellent option with very short transfer waiting time and a favourably small largest prime factor, as listed in the first row in table 2. This option is later referred to as option 1, and it meets all the constraints in table 1.

In order to provide alternative circumferences in case the above circumference cannot satisfy the geological requirement in a further placement survey, the constraint on the flexibility of bunch spacing can be relaxed, which will exclude some bunch spacing options.

In fact, inherited from the common factor of SPS and LHC harmonic numbers, the FCC-hh harmonic number should ideally be divisible by $120 (= 2 \times 3 \times 4 \times 5)$. Note that the factor of 11 is unnecessary for bunch spacings up to 25 ns. Under this constraint, every 1, 2, 3, 4, 5, 6, 10th RF bucket can be occupied to provide relatively flexible bunch spacing options. Furthermore, it features a continuous bunch clock for the nominal bunch spacing of 25 ns since the FCC-hh harmonic number can be divided by 10, which meets the constraint 5 in table 1. Three options become available with this additional requirement, as listed from the second row to the fourth row in table 2, which have respective advantages and disadvantages. The option with the FCC-hh harmonic number 121200, referred to as option 2, is relatively favourable with moderate transfer waiting time and a medium largest prime factor. The option with the FCC-hh harmonic number 121440, referred to as option 3, seems again extremely attractive, as it has very short transfer waiting time and a small largest prime



Figure 2. Constraint on the FCC-hh harmonic number from the flexibility of bunch spacing up to 25 ns at the baseline frequency of 400.79 MHz.

factor. Interestingly, the denominators of the harmonic number ratios of the FCC with respect to LHC and SPS are directly the circumference ratio of these two injectors. Besides, the option with the FCC-hh harmonic number 121320, referred to as option 4, has relatively short transfer waiting time, but a large largest prime factor slightly above constraint 7.

Option number	$h_{ m FCC}$	<i>C</i> _{FCC} [m]	$h_{\rm FCC}/h_{ m LHC}$	$h_{\rm FCC}/h_{ m SPS}$	Largest prime factor
1	$120960 = 2^7 \times 3^3 \times 5 \times 7$	90478.6	112/33	144/11	7
2	$121200 = 2^4 \times 3 \times 5^2 \times 101$	90658.2	1010/297	1010/77	101
3	$121440 =$ $2^5 \times 3 \times 5 \times 11 \times 23$	90837.7	92/27	92/7	23
4	$121320 = 2^3 \times 3^2 \times 5 \times 337$	90747.9	337/99	1011/77	337
5	$121220 = 2^2 \times 5 \times 11 \times 19 \times 29$	90673.1	551/162	551/42	29
6	$121260 =$ $2^2 \times 3 \times 5 \times 43 \times 47$	90703.0	2031/594	2021/154	47
7	$121360 = 2^4 \times 5 \times 37 \times 41$	90777.8	3034/891	3034/231	41
8	$121380 =$ $2^2 \times 3 \times 5 \times 7 \times 17^2$	90792.8	2023/594	289/22	17

Table 2. Some favourable options for the fixed LHC RF frequency of 400.79 MHz.

3.2 Alternative options

Some additional circumferences are found by relaxing the optional constraints on h_{FCC} . The FCC-hh harmonic number must at least be divisible by 20 (= 2 × 10) to include the nominal bunch spacing of 25 ns and to serve 4 symmetrically distributed IPs (constraints 4 and 5). Four more options become available, referred to as options 5 to 8 in table 2, while still satisfying conditions 6 and 7. Amongst them only option 5 also fulfills constraint 8, making it the strongest candidate. The harmonic number of 121220 offers a moderate waiting time for transfer, but allows only four bunch spacings of 2.5, 5, 12.5 and 25 ns.

If constraint 5 in table 1 is dropped, which means that the FCC-hh harmonic number does not need to be divisible by 10 for the desired bunch spacing of 25 ns in 400.79 MHz, but the other constraints are satisfied, more interesting options with small largest prime factor and short waiting time for transfer can be found. However, if the nominal bunch spacing of 25 ns was still adopted, the non-continuous bunch clock would result in additional technical difficulties for timing and experiment triggers. Therefore these alternatives are not considered any further.

In summary, the option 2 with the FCC circumference of 90658.2 m and the corresponding harmonic number of 121200 has been retained as the present working hypothesis, with $h_{\text{FCC}}/h_{\text{LHC}} = 1010/297$ and $h_{\text{FCC}}/h_{\text{SPS}} = 1010/77$. In addition to keeping the same RF frequency as LHC, this choice can provide a continuous bunch clock for experiments with the bunch spacings of 2.5, **5.0**, 7.5, 10, **12.5**, **15**, 20, **25** ns. The bunch spacings proposed in the FCC-hh CDR [1] are marked in bold face, as well as the quite beneficial 15 ns bunch spacing that helps to avoid e-cloud instabilities in FCC-ee [6].

However, this working hypothesis shows some weaknesses. The LHC-to-FCC transfer requires 297 turns in the FCC per transfer, corresponding to 90.2 ms per transfer. Also, the largest prime number in the factorization of the harmonic number is 101, which is relatively large. As alternatives, option 1 with an FCC circumference of 90478.6 m is highly recommended, placement permitting, as its corresponding harmonic number 120960 can provide all bunch spacings up to 25 ns. It also benefits from a very small largest prime factor and a short transfer waiting time of only 112 turns when injecting from the LHC or 144 turns with the SPS. Option 3, with an FCC circumference of 90837.7 m and a corresponding harmonic number of 121440, is another extremely powerful and flexible candidate.

4 Options with alternative RF frequencies

4.1 Impact of RF frequency

RF systems operating at higher frequencies than the current LHC RF frequency of 400.79 MHz, for instance, 500 MHz, have numerous advantages, such as smaller cavity sizes and higher electric field gradients, and thus the 500 MHz frequency has been utilized in many particle accelerators (CESR, KEKB, BEPCII, etc.). The CEPC (Circular Electron Positron Collider) project under study in China has adopted a 650 MHz RF system [7]. Thus, it seems worth considering circumference options which allow alternative RF frequency choices. Here, we will take into account the impact of RF frequency on flexibility of bunch spacing and on longitudinal beam dynamics.

The RF frequency determines the granularity of the obtainable bunch spacings, and hence higher RF frequencies naturally allow more flexibility. To avoid such a bias towards higher frequencies, the

number of possible bunch spacings is normalized. The detailed study [8] shows that the frequency range from around 400 MHz to 600 MHz is most interesting.

From the longitudinal beam dynamics point of view, the RF frequency critically impacts the maximum bunch intensity. A preliminary analysis [8] indicates that the RF frequency must be a compromise between the bucket area, which suggests a lower frequency, and the stability, which requires a higher frequency. Based on these considerations a frequency range of 400 MHz to 600 MHz is again found to be reasonably advantageous.

4.2 Alternative RF frequencies

Accordingly, we scanned all LHC harmonic numbers between 30000 and 76000, applying the scaling law $h_{LHC} = 35640 + 27 \cdot n$, where *n* is an integer, to cover a large frequency range from 337 MHz to 854 MHz. The corresponding SPS harmonic number is scaled as $h_{SPS} = 2 \times 4620 + 7 \cdot n$. Note that the integers 27 and 7 in the LHC and SPS harmonic numbers come from their circumference ratios.

All constraints listed in table 2 (except constraint 3) are imposed and the largest prime factor in constraint 7 is modified to be less than 200 to limit the number of options. In addition, the maximum bunch spacing is required to be less than or close to 25 ns. The options obtained under this new set of constraints are summarized in figure 3, which shows the FCC circumference versus RF frequency. The colour of the points indicates the number of possible bunch spacings, and a darker colour designates fewer bunch spacing options. As expected, higher RF frequencies generally allow more bunch spacing options.

One can see that the two most favourable circumferences of 90478.6 m and 90837.7 m, as suggested in the previous section, support a variety of RF frequencies (especially the main RF frequencies of interest such as 400 MHz, 500 MHz, 600 MHz, and 800 MHz). Furthermore, at these two circumferences many RF frequency choices feature a large number of possible bunch spacings (figure 3, light colour) combined with a small largest prime factor (small point size). Another point is that they reduce advantageously the harmonic ratios of FCC-hh and LHC or SPS and thus allow short transfer waiting time. Not only are options 1 and 3 very attractive at the baseline frequency, but they also guarantee full flexibility with respect to the choice of the RF frequency. In comparison, the circumference of 90658.2 m does not support as many RF frequencies as the other two options. Additionally, the corresponding harmonic numbers contain unfavourably large prime factors.

The exact bunch spacings like 25 ns proposed in the FCC-hh CDR will slightly vary when new RF systems with RF frequencies different from 400 MHz are adopted. Interestingly, the PS could theoretically be retained as the FCC-hh pre-injector for these three circumferences, as the LHC or SPS harmonic number is still divisible by 11.

5 Conclusion

Based on constraints for RF synchronization, three favourable FCC circumferences have been identified: option 1 with C = 90478.6 m, option 2 with C = 90658.2 m (currently the working hypothesis) and option 3 with C = 90837.7 m. Their key strengths and differences are summarized in table 3. All three schemes provide flexible bunch spacings, and include the ones considered in the FCC-hh CDR [1]. All of them would even allow the PS to serve as a hadron pre-injector, regardless of RF frequencies. Option 2 meets the requirements recommended by the FCC-hh



Figure 3. The distribution of the options with the FCC circumference versus RF frequency under the constraints mentioned in subsection 4.2. The colour of the points indicates the number of possible bunch spacings. Their point size represents the largest prime factor of the FCC-hh harmonic number, a larger point size means a larger prime factor, hence the smaller the better. The three stars correspond to three favourable options at the RF frequency $f_{\rm rf} = 400.79$ MHz (options 1, 2 and 3 in table 2). The dashed lines indicate different RF frequencies for these three favourable circumferences.

Table 3. Comparison between the three most favourable schemes.	Option 2 constitutes the current working
hypothesis.	

Scheme	Option 1	Option 2 (current working hypothesis)	Option 3	
$C_{\rm FCC}$ [m]	90478.633	90658.154	90837.675	
<i>f</i> _{rf} [MHz]	400.790	400.790	400.790	
h _{FCC}	120960	121200	121440	
Bunch	2.5, 5.0, 7.5, 10, 12.5,	2.5, 5.0, 7.5, 10, 12.5,	2.5, 5.0, 7.5, 10, 12.5,	
spacing [ns]	15, 17.5, 20, 22.5, 25	15, 20, 25	15, 20, 25	
LHC-to-FCC	33 revolutions in FCC	297 revolutions in FCC	27 revolutions in ECC	
transfer		2)7 revolutions in rec	27 revolutions in rec	
SPS-to-FCC	11 revolutions in ECC	77 revolutions in ECC	7 revolutions in ECC	
transfer	11 revolutions in FCC		/ levolutions in FCC	
Largest prime	7	101	23	
factor		101		

CDR [1] as well as the geographical constraints, and thus has for now been adopted as the working hypothesis. However, it cannot cover a wide range of RF frequencies. Instead, options 1 and 3 have

been favourably identified and confirmed from the perspectives of longitudinal beam dynamics and flexibility analysis of bunch spacing. If the geological conditions permit, these two circumferences are much preferred from the RF point of view.

It is worth noting that the final choice of the RF frequency at the suggested circumference also depends on beam dynamics constraints for FCC-ee, such as beam-loading, higher-order-modes power losses, coupled-bunch and coherent beam-beam instabilities [9], which are subject to ongoing study.

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