

NEUTRINO GENERATED RADIATION FROM A HIGH ENERGY MUON COLLIDER

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Abstract

Muons circulating in a muon collider decay and generate neutrinos within a small solid angle, which reach the Earth's surface. One of the challenges of a high energy muon collider is to ensure that showers created by such neutrinos interacting close to the Earth's surface result in very low radiation levels. The neutrino radiation cone from a muon beam without divergence is estimated through a combination of analytical estimates and FLUKA simulations. Such neutrino cones have to be combined with the properties of the lattice to obtain the possible radiation levels at the Earth's surface. Studies of mitigation measures will be presented, combining the installation of the collider deep underground with a careful choice of the orientation, and with periodic variations of the muon beam trajectory either within the machine aperture or by deforming the whole machine in the vertical plane.

INTRODUCTION

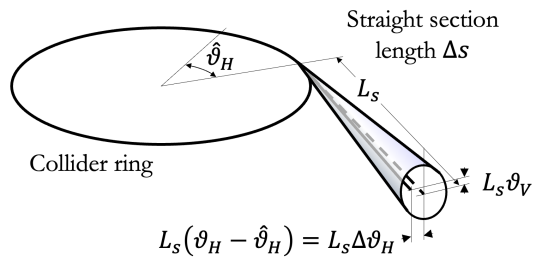


Figure 1: Muon collider and narrow radiation cone.

Neutrinos generated by decay of muons circulating in a high energy muon collider [1–3] are emitted in a narrow cone with small divergence with respect to the direction of the muon sketched in Fig. 1. Radiation doses where these neutrinos and their secondary particles reach the Earth's surface must be evaluated with care to ensure that they are negligible. An analytical estimate following closely the one given in [4], but taking care to keep the information on the angular distributions (and not only peak values in the direction of the decaying muon) results in the angular neutrino density distribution shown in Fig. 2 and given by:

$$\frac{1}{N} \frac{dN}{d\Omega} = \frac{\gamma^2}{\pi} \frac{1}{(1 + \gamma^2 \vartheta^2)^2}$$

with γ the relativistic Lorentz factor and $\vartheta = \sqrt{\Delta \vartheta_H^2 + \vartheta_V^2}$. The typical neutrino energies in the forward direction with small ϑ are larger than for larger ϑ . As a higher energy neutrino is more likely to interact with matter and deposit

more energy, the angular dose distribution per decaying muon (taking the effect of both neutrinos generated) is even more narrow and given by:

$$\Delta D \approx (1.104 \cdot 10^{-28} \text{ Gy m}^2) \frac{4\gamma^4}{\pi L_s^2 (1 + \gamma^2 (d/L_s)^2)^4}$$

with L_s the distance from the muon decay and $d = L_s \vartheta$ the lateral offset. Despite the different cross sections and fractions of energy deposited in the shower for the decay of μ^- and μ^+ , the final result after adding up all contributions is almost identical. The approximation:

$$\Delta D \approx \Delta D_G = (1.406 \cdot 10^{-28} \text{ Gy m}^2) \frac{\gamma^4}{L_s^2} e^{-3\gamma^2 \vartheta^2} \quad (1)$$

plotted together with the exact expression in Fig. 2 is sufficient for the purpose of this study and has the advantage that folding with a possible Gaussian beam divergence becomes trivial.

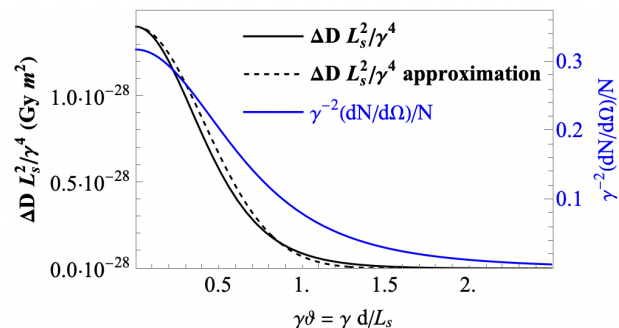


Figure 2: Neutrino angular density and absorbed dose per muon decay from analytical estimations.

FLUKA SIMULATIONS TO REFINE THE SOURCE TERM

Simulations using FLUKA [5–7] have been carried out with the following aims:

- Improvement of the accuracy of the effective dose $H = W_R D$ by estimating W_R from simulation rather than assuming a radiation weighting factor $W_R = 1 \text{ Sv/Gy}$ as in [4].
- Assessment of the widening of the neutrino radiation cone due the lateral extension of the shower.
- General verification of the simplified expressions.

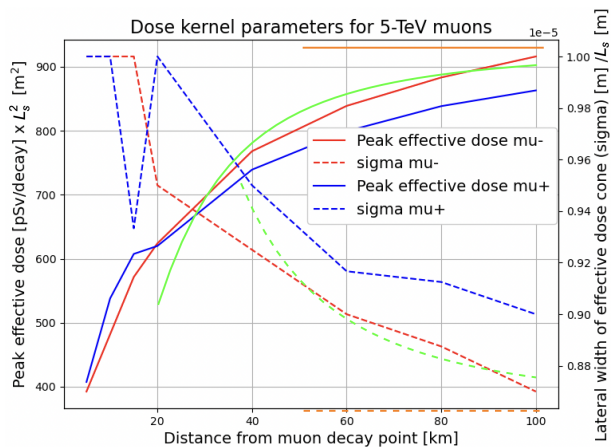


Figure 3: Summary of FLUKA simulations for 5 TeV muon decays. The peak effective dose multiplied with the square of the distance L_s^2 and the rms width divided by the distance are plotted as a function of the distance L_s .

The main results of extensive FLUKA studies [8] for the decay of 5 TeV muons (10 TeV centre of mass (com) collider) with $\gamma \approx 47320$ is summarized in Fig. 3 showing the rms transverse extension and the peak effective dose of the neutrino radiation cone as a function of the distance L_s from the muon decay. Motivated by Eq. 1 the rms of the lateral distribution is approximated by $\sigma_d = \sqrt{L_s^2/6\gamma^2 + d_s^2}$ and the peak effective dose by $\Delta\hat{H} = W_T (1.406 \cdot 10^{-28} \text{ Gy m}^2) (\gamma^4/L_s^2) / \sqrt{1 + d_s^2 6\gamma^2/L_s^2}$ with d_s the lateral extension of the shower and W_T the weighting factor to convert from absorbed dose to effective dose. The orange and green lines in Fig. 3 have been obtained setting $W_T = 1.32 \text{ Sv/Gy}$. The orange and green lines are obtained with $d_s = 0.15 \text{ m}$ and $d_s = 0 \text{ m}$, respectively. One notes that the widening of the radiation cone is moderate for the large distances of at least several tens of km relevant for the positioning of a high energy muon collider. Thus, the shower extension has been set to $d_s = 0$ leading to the following effective dose per muon decay:

$$\Delta H \approx K_H \frac{\gamma^4}{L_s^2} e^{-3\gamma^2\vartheta^2} \quad (2)$$

with $K_H = 1.855 \cdot 10^{-28} \text{ Sv m}^2$, which is sufficiently accurate for the purpose of this study.

Note that the lateral dose distributions found in FLUKA studies [8] were not perfectly Gaussian. In particular, tails at locations distant by several rms were significantly higher than expected for a Gaussian distribution. However, as the absolute values of the effective dose in these tails caused by muon (anti-)neutrinos is low, they can be neglected for further studies.

NEUTRINO RADIATION DUE TO A MUON COLLIDER

The effective dose due to neutrinos from a muon collider reaching the Earth surface are estimated based on the

”source” term given by Eq. 2 combined with the properties of the lattice and taking the divergence of the beam into account. Assuming Gaussian distributions, the folding integral describing the neutrino generated dose becomes:

$$\frac{dH}{dt} = K_H \frac{N f_r \gamma^4}{C L_s^2} \frac{1/6\gamma^2}{\sigma_{\vartheta_H} \sigma_{\vartheta_V}} \int ds e^{-\frac{(\vartheta_H - \vartheta_H(s))^2}{2\sigma_{\vartheta_H}^2} - \frac{\vartheta_V^2}{\sigma_{\vartheta_V}^2}} \quad (3)$$

with s the longitudinal position around the machine, N the number of muons per injected bunch, f_r the repetition rate, C the collider circumference and the horizontal and vertical rms divergences given by $\sigma_{\vartheta_H}^2 = 1/6\gamma^2 + \varepsilon_H\gamma_H + (D'\sigma_p/p)^2$ and $\sigma_{\vartheta_V}^2 = 1/6\gamma^2 + \varepsilon_V\gamma_V$. The quantities ε_H and ε_V are the physical rms emittances, γ_H and γ_V the Twiss γ function at position s , D' the derivative of the dispersion and σ_p/p the rms relative momentum spread. The divergence due to betatron oscillations has a significant impact on the effective dose distribution from the straight sections housing the experiments, but is negligible for the arcs. The divergence caused by D' may have a significant impact on the dose caused by short straight sections in the arcs.

Numerical evaluations using Eq. 3 for half an arc cell of the latest version of the 10 TeV com collider with a circumference $C = 8667 \text{ m}$ presented with all other relevant parameters in [9] are shown in Fig. 4. The effective doses for one of the two circulating beams are given for a distance from the muon decay of $L_s = 100 \text{ km}$ and assuming operation over 5000 hours corresponding to a total number of about $1.62 \cdot 10^{20}$ muons injected per beam. The upper plot shows the Twiss parameters and the lower plot the effective dose over one year as function of ϑ_H and for $\vartheta_V = 0$. One half cell corresponds to the regions between the vertical dashed lines.

Several of the features of the plot with radiation doses can be explained with the help of the plot of the lattice functions:

- The sharp peaks are caused by short straight sections with length 0.3 m between magnets. Focusing and chromaticity correction is based on combined function magnets to avoid even higher peaks. The feasibility of such short straight section between superconducting dipoles and combine function quadrupoles and sextupoles still has to be investigated.
- Even though the length of all straight sections is identical, the height of the peaks are lower at the beginning of the cell than towards the end of the cell. The main reason is additional divergence created by the large momentum spread and D' and to a lesser extent due to the betatron oscillations in both planes. D' and the Twiss gamma functions become small towards the end of the half cell leading to larger radiation peaks.
- The dipolar magnetic field of combined function magnets is lower than the one of pure dipoles leading to slightly increased radiation levels. This is clearly visible, e.g., for the D quadrupole at $s = 0 \text{ m}$ corresponding

to $\vartheta_H = 0$ and the F quadrupole at $s \approx 15$ m corresponding to $\vartheta_H \approx 9$ mrad. The variations of dose from different positions along the quadrupoles is caused by variations of the beam divergence.

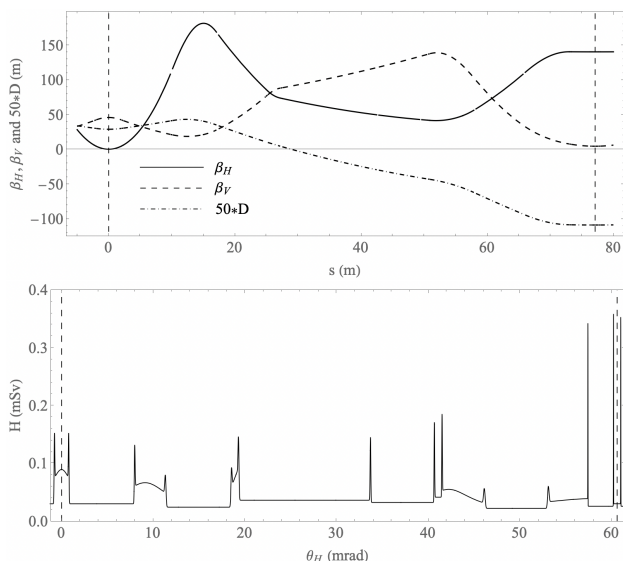


Figure 4: Lattice functions and peak annual dose at $L_s = 100$ km distance for half a 10 TeV muon collider flexible momentum compaction arc cell.

MITIGATION MEASURES

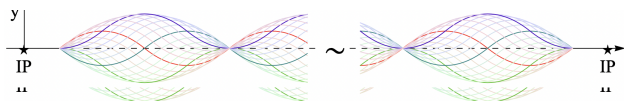


Figure 5: Wobbling, i.e. periodic deformation of the machine in the vertical plane.

Obvious mitigation measures are to minimize the length of straight sections in regions outside the long straight section housing the experiments and to install the device deep underground leading to large values of L_s . Careful lattice design avoiding straight sections without horizontal beam divergence due to D' may allow the radiation dose peaks from the arcs to be mitigated.

Higher radiation levels will occur in the prolongation of the long straight sections housing the IPs and other equipment and are unavoidable. A tool to localize these positions with somewhat higher radiation doses and to easily adjust the positioning of the machine is being developed [10]. This tool will be used to carefully position the collider ring such that higher radiation levels are localized in regions such as a rock face, which may be owned by the laboratory and fenced, or at sea.

Additional mitigation measures are required for a 10 TeV muon collider. The present plan is to implement "wobbling" of the machine outside the long straight section housing the IPs, i.e. a periodic deformation ring as sketched in Fig. 5.

This implies that the magnets and all other infrastructure are installed on a movement system. To ensure that the radiation averaged over a one year run at a given location becomes negligible, a full deformation cycle must not last longer than a few weeks.

A deformation pattern consisting of a succession of parabolas (alternating the sign of the horizontal magnetic field required for vertical deflection to ensure that the beam follows the deformed machine) with period of about 600 m and an amplitude of ± 0.15 m allows the vertical slope to be modulated by ± 1 mrad. The peak radiation doses are generated by straight sections, where the vertical (and horizontal) divergence of the beam is negligible. There the rms divergence is approximated by the one of the source term in Eq. 1 and for a 10 TeV collider given by $1/\sqrt{6}\gamma = 8.63 \mu\text{rad}$. The wobbling procedure described above allows the peak dose rates caused by neutrinos at the Earth surface to be reduced by a factor $2 \text{ mrad} / (\sqrt{2\pi} 8.63 \mu\text{rad}) \approx 100$.

SUMMARY AND OUTLOOK

A formalism based on analytical derivations and refined by extensive FLUKA simulations [8] to estimate radiation levels from neutrinos, generated by muon decays in a collider reaching the Earth's surface, has been described. Numerical evaluations for the arc cell of the latest version of a 10 TeV com muon collider [9] have been presented.

FLUKA studies have also been carried out for a beam energy of 1.5 TeV for a 3 TeV com muon collider, but not used for the present. They may be used in a similar way for future studies on neutrino generated radiation at the Earth's surface by a 3 TeV com muon collider.

For a typical 10 TeV com collider, "wobbling", i.e. the periodic deformation of the whole machine outside the long straight sections housing the experiments, is mandatory to keep the radiation at negligible values.

Further feasibility studies will be carried out (i) to refine the model around the transitions between magnets, so far assumed to be hard edge, and straight sections, (ii) to understand the impact of machine "wobbling" on beam dynamics (iii) to assess the feasibility of the required mechanical movement system and (iv) find an appropriate site and collider positioning ensuring that radiation doses remain negligible.

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