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THE POSSIBILITIES OF LOCAL SUPERPOSITION OF EQUILIBRIUM ORBITS IN A STORAGE RING, WITH LITTLE INCREASE OF THE APERTURE REQUIREMENTS.

1. General Remarks.

The Terwilliger proposal [*)] reduces the radial width of a stacked beam, at a number of points round the ring, by arranging that the equilibrium orbits for different momenta coincide at these points: i.e., the momentum compaction factor is locally zero. At an equal number of points the m.c.f. is, in first approximation, doubled; so that the stack requires twice the horizontal aperture.

For storage rings in which only one or two straight sections are going to be used for colliding beam experiments, it would be of considerable interest to reduce the m.c.f. to zero only in the interaction region(s), if this can be done without doubling it elsewhere.

2. First Proposal.

The simplest device that produces a localised bump in a closed orbit consists of two kicks separated by half a wavelength of the betatron oscillations. With two quadrupoles at this spacing, the closed orbits for different momenta can be made to pass through a common point; and, in smoothed apprxomation, look like Fig. I.

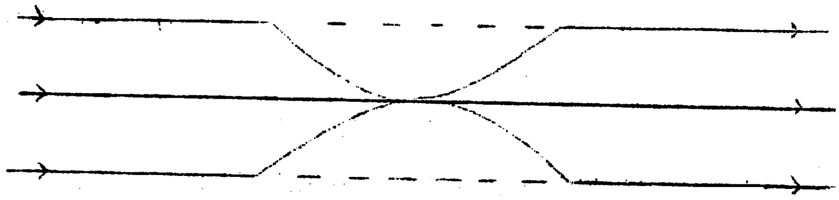

Fig. I.

*) CERN Symposium 1959, p. 53.

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The quadrupoles have to be focusing in the horizontal plane, and their strength, (reciprocal focal length, defocusing reckoned positive) must, in smoothed approximation, be $-2\pi/\lambda$ where λ is the wavelength of the betatron oscillation.

For the C.P.S. structure $\lambda/2\pi$ is 16 metres. The optical strength of a magnetic quadrupole for protons is numerically: -

$$32 \quad 10^{-6} \; \frac{1}{\beta \; \gamma} \; B'\ell \quad m^{-1} \tag{1}$$

where B' is the field gradient, ℓ the effective length, $B'\ell$ being in Gauss.

At 25 GeV we have $\beta\gamma = 27.6$, and would require a $B'\ell$ of 54 kGauss. The quadrupoles presently in the C.P.S. make about this (700 G/cm x 77 cm), but would need water cooling in order to do it on a D.C. basis.

The length over which the m.c.f. is reduced by a factor 10 or more is 14 metres.

This system, of two lenses with half a wavelength of machine between them, has a matrix for horizontal betatron oscillations: -

$$\begin{pmatrix} 1 & 0 \\ -1/\bar{\lambda} & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/\bar{\lambda} & 0 \end{pmatrix} \tag{2}$$

$$= \begin{pmatrix} -1 & 0 \\ 2/\bar{\lambda} & -1 \end{pmatrix} \tag{3}$$

If we multiply this by the matrix for the rest of the ring, 5.75 wavelengths, and take half the trace, we get -1, indicating that we are on a π-type stop-band. Presumably the added quadrupoles have just raised the Q to 6.5. We therefore adjust the rest of the ring, a little, and let its matrix be:

$$\begin{pmatrix} \cos\theta & \bar{\lambda}\sin\theta \\ -1/\bar{\lambda}\,\sin\theta & \cos\theta \end{pmatrix} \tag{4}$$

Multiplying (3) by (4) we obtain a half-trace of

$$Tr/2 \;=\; \sin\Theta - \cos\Theta \tag{5}$$

To work in the middle of the diamond we make this zero, which can conveniently be done by putting

$$\Theta \;=\; 5\tfrac{5}{8}\,.\,2\pi \tag{6}$$

Thus the focusing strength in the rest of the machine must be reduced enough to change the betatron wavelength in the ratio 5.75 : 5.625 , or about 2.2 o/o. The associated change in the m.c.f. will be about 4.5 o/o. By averaging the m.c.f. round the whole ring we find that transition energy will be raised a little less than half a percent.

For the vertical motion the added quadrupoles are defocusing, and by a similar calculation one finds that the vertical betatron wavelength in the rest of the machine must be shortened in the ratio 5.75 : 5.875.

These two added quadrupoles are a rather substantial n-type perturbation of the ring: to find out whether the working diamond is still reasonably large we write the whole machine matrix for the horizontal motion as: -

$$\begin{pmatrix} 1 & 0 \\ -1/\lambda & 1 \end{pmatrix}\begin{pmatrix} \cos\phi & \lambda\sin\phi \\ -1/\lambda\,\sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1/\lambda & 1 \end{pmatrix}\begin{pmatrix} \cos\Theta & \lambda\sin\Theta \\ -1/\lambda\,\sin\Theta & \cos\Theta \end{pmatrix} \tag{7}$$

The second matrix represents the region between the lenses, the last matrix is the rest of the machine. We then put

$$\begin{aligned} \phi &= k\,\pi \\ \Theta &= k\;5.625\;\;2\,\pi \end{aligned} \tag{8}$$

and consider values of k in the neighbourhood of one. So we are looking at the e effect of changes in the betatron w avelength, around its design value, neglecting any associated variation of λ .

The half-trace of (7) is

$$\cos \phi \cos \theta - \sin \phi \cos \theta - \cos \phi \sin \theta - 1/2 \sin \phi \sin \theta \qquad (9)$$

We have already arranged that this is zero for $k = 1$. By tabulation one finds that it becomes one and minus one respectively at k's of 0.980 and 1.021 , so we have a diamond width of about 4 o/o instead of the old value, without the added lenses, of $0.5/6.25 = 8$ o/o.

One would probably not like to build a ring with so small a diamond width, but Q-values are fairly easy to measure accurately, and are relatively stable at energies where stray magnetic fields are negligible, so one could consider running-in a ring without the added quadrupoles and then bringing them into action in stages, checking the Q's at each stage.

3. Second Proposal.

We consider next the possibility of obtaining an m.c.f. locally zero, without this big effect on the size of the working diamond.

One can regard the two quadrupoles used in the first proposal as a Terwilliger scheme in which the single localized reduction in m.c.f. is obtained by the simultaneous use of many Fourier terms, with different values of M , in the added quadrupole field. Relatively strong quadrupoles are needed because no other values of M are so efficient as those near to Q . To reduce the adverse effect on the working diamond, we must try to get rid of the terms with M-values of 12 and 13, because in first approximation these are the terms that broaden the stop-bands at $Q = 6.0$ and $Q = 6.5$. From this point of view a pair of quadrupoles separated by half a wavelength are approximately additive: to get some cancellation one would like to separate them by a quarter wavelength.

We could add another horizontally focusing quadrupole in the middle of Fig. I: at this point it has no effect on the equilibrium orbits. But it would have to be split into two (two on each beam) in the colliding-beam case, because a focusing quadrupole magnet is defocusing for the beam travelling in the opposite direction.

Alternatively we could consider replacing each lens in our first proposal by a pair at $\lambda/4$ separation. Each would have to be 70.7 o/o of the old strength, but one would expect a substantial improvement in the diamond width. With four lenses spread evenly over a wavelength, one begins to think of departing completely from a point-lens philosophy and working rather in terms of a whole section of the ring with modified focusing properties.

The closed orbits for one such scheme are sketched in Fig. II.

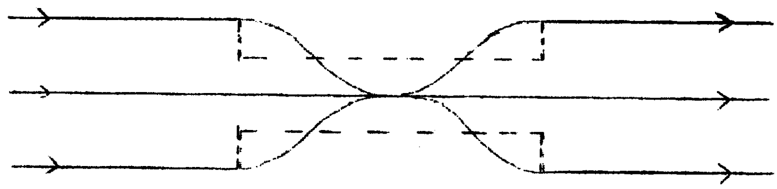

Fig. II.

We have one wavelength in which the local equilibrium orbits for $\pm \Delta P$ particles are brought twice as close together, so that the closed orbits touch in the middle. This "special wavelength" is made of a focusing structure which must be characterised by an m.c.f. of half the normal value, so one can expect that within it the betatron wavelength will be a factor 0.707 less than in the rest of the ring, for

$$\text{m.c.f.} \;=\; Q^{-2} \tag{10}$$

is a good approximation for any practical A.G. structure.

For the betatron oscillations, in smoothed approximation, the "special wavelength" has a matrix:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{11}$$

and all that we need do to the rest of the ring is to tune it a little so that we have 5.25 wavelengths in it. This requires an increase in the betatron wavelength by 4.9 o/o, and the associated increase in the m.c.f. will be about 10 o/o. Averaging the m.c.f. round the machine we find that the transition energy will be raised about 1.7 o/o

To investigate the dimaond width we write the matrix for the whole machine as: -

$$\begin{pmatrix} \cos\phi & \frac{\lambda}{\sqrt{2}}\sin\phi \\ -\frac{\sqrt{2}}{\lambda}\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \cos\theta & \lambda\sin\theta \\ -\frac{1}{\lambda}\sin\theta & \cos\theta \end{pmatrix} \qquad (12)$$

the first matrix being the "special wavelength" and the second being the rest of the machine. We then put

$$\begin{aligned} \phi &= k\,2\pi \\ \theta &= k\,5.25\,2\pi \end{aligned} \qquad (13)$$

The half-trace of (12) is

$$\cos(\theta + \phi) - 0.06066 \sin\theta \sin\phi \qquad (14)$$

and by tabulating this we find that the diamond width is only 6.6 o/o less than the unperturbed value.

In a ring like the C.P.S. one would make the structure for this "special wavelength" by adding quadrupoles in the straight sections, rather than increasing the gradient in the magnet units. In a structure with separate bending and focusing it would be convenient to energize the quadrupoles in this region more strongly than the rest of the ring. We shall estimate the strength required of the extra lenses for the first case.

Before we can do this it is necessary to decide what we are going to do with the vertical focusing properties in this region. If one merely added horizontally focusing quadrupoles, either spread uniformly, or equally in both types of straight section, a strength sufficient to double the horizontal Q^2 would approximately reduce the vertical Q to zero in this region, and the vertical diamond width for the whole machine would be much reduced. There are two reasonable ways of obtaining a good vertical diamond width: -

(a) We can hold the vertical betatron wavelength unchanged within the "special wavelength". It will then behave, in smoothed approximation, exactly like the rest of the structure.

(b) We can make the vertical betatron wavelength in this region about $\sqrt{2}$ times the normal value. It will then contain about half a wavelength of betatron oscillations, and the effect on the working diamond will be small.

Of these, (b) is more economical in added lens strengths, but it results in an increased amplitude of vertical betatron oscillations in the interaction region, so we prefer (a).

We shall calculate the (point) lens strengths that are required in the straight sections of the C.P.S. structure, to raise the horizontal betatron frequency by a factor $\sqrt{2}$, holding the vertical frequency unchanged. This is not exactly what is required in the "special wavelength", but near enough to use for a preliminary estimate.

The matrix for a half-period of the C.P.S., from mid D to mid F, is

$$\begin{pmatrix} 1.35 \cos \pi/8 & \lambda \sin \pi/8 \\ -1/\lambda \sin \pi/8 & \frac{1}{1.35} \cos \pi/8 \end{pmatrix} \tag{15}$$

Here $\pi/8$ is half the μ value of a period, 1.35 is the betatron-oscillation wiggle factor, and λ is the geometric mean of the mid-D and mid-F axis ratios $(\hat{x}/\hat{x}')$.

We put a lens of strength $2D_1$ in each horizontal mid-F, and $2D_2$ in each horizontal mid-D. The horizontal matrix for a half period then becomes:

$$\begin{pmatrix} 1 & 0 \\ D_1 & 1 \end{pmatrix} \begin{pmatrix} 1.35 \cos \pi/8 & \lambda \sin \pi/8 \\ -1/\lambda \sin \pi/8 & \frac{1}{1.35} \cos \pi/8 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ D_2 & 1 \end{pmatrix} \tag{16}$$

To meet our requirements on the horizontal betatron frequency, the product of the diagonal terms of this must be

$$\cos^2 \sqrt{2}\, \pi/8 \tag{17}$$

For the vertical oscillations, a half period has the matrix:

$$\begin{pmatrix} 1 & 0 \\ -D_2 & 1 \end{pmatrix} \begin{pmatrix} 1.35 \cos \pi/8 & \lambda \sin \pi/8 \\ -1/\lambda \sin \pi/8 & \frac{1}{1.35} \cos \pi/8 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -D_1 & 1 \end{pmatrix} \tag{18}$$

and for this we require the product of the diagonal terms to be

$$\cos^2 \pi/8 \tag{19}$$

Hence we find

$$\begin{aligned} 2\,D_1 &= -\,0.705/\lambda \\ 2\,D_2 &= +\,0.349/\lambda \end{aligned} \tag{20}$$

Thus the quadrupoles in hor.-F must be hor.-F , and need $B'\ell$ of 38 kGauss, the other being about half this strength. Theoretically our "special wavelength" has a length of 11.87 magnet units, so one would probably need 6 quadrupoles of each type, on each ring.

4. Limitations of the Smoothed Approximation.

The errors introduced into these calculations by the use of the smoothed approximation are not worth discussing at any length, because it will be necessary to do much more detailed calculations when we design the interaction region of a storage ring pair to take account of the presence of two beams and of experimental requirements.

It is, however, worth mentioning that by putting the lenses of section 2 in hor.-F straight sections their strength could be reduced by a factor $1/1.35$ compared with the smoothed value quoted, and then their effect on the vertical motion would be less by a factor $1/1.35^2$

The unwanted mismatches caused by the discontinuity in wiggle-factor at the ends of our "special wavelength" in section 3 will be largely self-cancelling in the horizontal plane. If they are enough to be troublesome in the vertical plane, it is probable that an arrangement of 13 lenses (instead of 12) with the end ones reduced in strength would effectively smooth out the discontinuity.

5. Summary.

There are at least two ways of modifying a ring like the C.P.S. to produce zero m.c.f. at one point and an order of magnitude reduction of m.c.f. over a length of several metres. The quadrupoles required are perfectly feasible.

One scheme requires two quadrupoles per ring and increases the m.c.f. round the rest of the ring by about 4.5 o/o, but suffers from the disadvantage of reducing the horizontal width of the working diamond by about a factor 2. Another scheme almost eliminates this diamond-reduction, but requires about 12 lenses per ring and increases the m.c.f. elsewhere by 10 o/o.

There are, no doubt, many other possible schemes, but it seems probable that most of them are, in their advantages and disadvantages, between the two considered.

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