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SPACE CHARGES IN ELECTRON STORAGE RINGS AND THE REMOVAL OF POSITIVE IONS BY A D.C. CLEARING FIELD.

Summary.

An approximation is given for the electrostatic and magnetic fields of a stacked electron beam of elliptic cross-section. The shift of the vertical betatron frequency due to space charge forces is worked out and subsequently an upper limit is given for the electron density without and with neutralisation by positive ions. Furthermore the time is calculated in which a critical positive space charge is built up by ionisation of the residual gas.

Different kinds of external electrostatic clearing fields are proposed, in order to remove the positive ions, and the perturbation of the equilibrium orbits by these clearing fields are assessed.

1. The Space Charge Field.

We assume an electron beam of elliptic cross-section and homogeneous current density, as shown in Fig. 1.

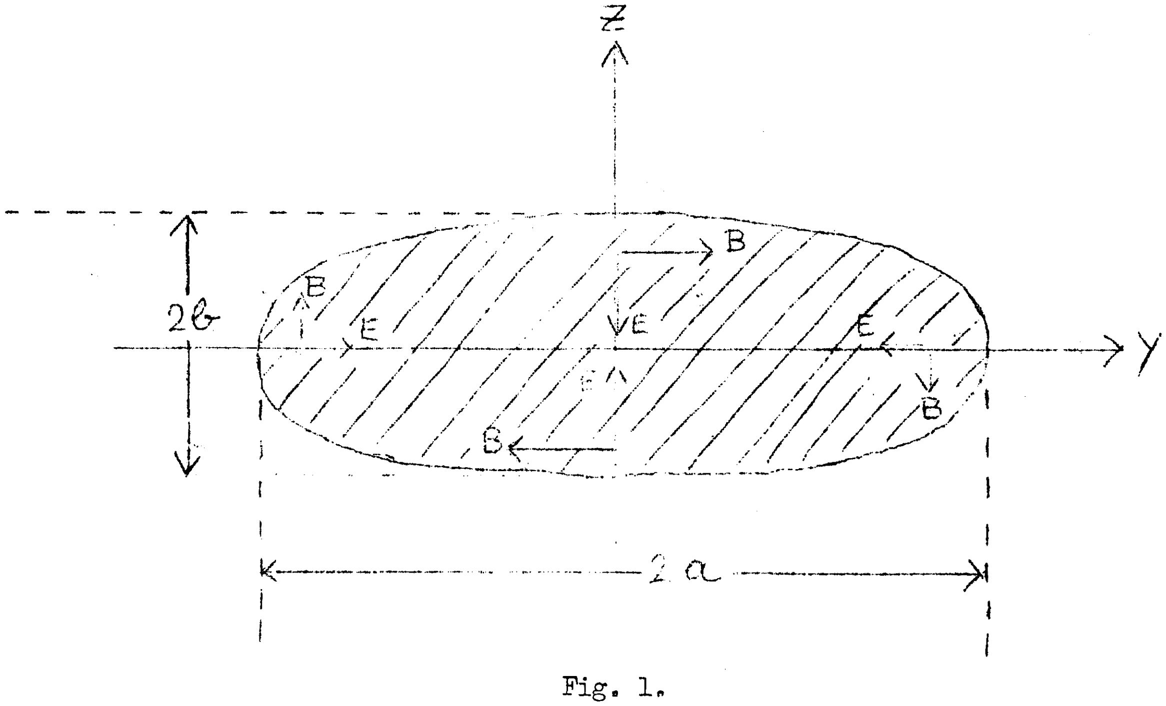

Fig. 1.

Beam cross-section with electrostatic and magnetic field strength inside the beam.
a, b semi-axes.

Assuming furthermore that the beam surface is an equipotential, we obtain the potential inside the beam:

$$\varphi = \frac{e_o n_e}{2 \varepsilon_o} \frac{b^2 y^2 + a^2 z^2}{a^2 + b^2} , \tag{1}$$

where e_o is the electron charge, n_e the electron density and ε_o the dielectric constant of free space.

Equation (1) is a solution of the Poisson equation

$$\nabla^2 \varphi = \frac{e_o n_e}{\varepsilon_o} \tag{2}$$

The two components of the field strength $\vec{E} = -\operatorname{grad} \varphi$ inside the beam, calculated from Eq. (1) are

$$E_y = - \frac{e_o n_e}{\varepsilon_o} (1 - f) y \tag{3}$$

$$E_z = - \frac{e_o n_e}{\varepsilon_o} f z , \tag{4}$$

where the factor

$$f = \frac{1}{1 + (\frac{b}{a})^2} , \tag{5}$$

is determined by the axis ratio of the cross-section. f equals $1/2$ for a circular beam and approaches 1 for a flat one. The field strength from eqs. (3), (4) is not too bad an approximation for any real beam and well suited for different kinds of estimates, because the component E_y is proportional to y and independent of z and the component E_z proportional to z and independent of y.

In a similar way an approximation can be found for the magnetic field inside the beam.

$$B_y = \mu_o G f z \tag{6}$$

$$B_z = \mu_o G (1 - f) y \tag{7}$$

where G is the electron current density and μ_o the permeability of free space.

The equations (6) and (7) are a solution of the Maxwell equations

$$\text{rot } \vec{B} = - \mu_o \vec{G} \tag{8}$$

$$\text{div } \vec{B} = 0 \tag{9}$$

Current density and electron density are connected by

$$G = e_o n_e \beta c \tag{10}$$

where βc is the velocity of the electrons. It is easy to see that

$$B_y = - \frac{\beta}{c} E_z \tag{11}$$

$$B_z = \frac{\beta}{c} E_y \tag{12}$$

2. Space Charge Forces on an Electron from the Beam and the Shift of Betatron Frequencies.

The space charge force on a moving electron is

$$\vec{F}_s = - e_o \left\{ \vec{E}_i + \vec{E}_e + c \left[\vec{\beta}\, \vec{B} \right] \right\} \tag{13}$$

where $\vec{E}_i$ and $\vec{E}_e$ are the electric fields due to ions and electrons respectively and $c\,\vec{\beta}$ is the velocity vector. For the two components of the force we work out, using Eqs. (3), (4), (6), (7), (10) and the relationship $(1 - \beta^2) = 1/\gamma^2$:

$$F_{sy} = \frac{e_o^2}{\varepsilon_o} \left(\frac{n_e}{\gamma^2} - n_i \right) (1 - f)\, y \tag{14}$$

$$F_{sz} = \frac{e_o^2}{\varepsilon_o} \left(\frac{n_e}{\gamma^2} - n_i \right) f\, z \tag{15}$$

where n_i is the ion density, which is supposed to be uniform.

The term n_e/γ^2 comes from the combination of the electrostatic and magnetic forces of the moving electron charge (second and third term in Eq. (13)) which almost cancel each other out. No such cancelling occurs for the pure electrostatic force proportional to n_i.

The space charge force $\vec{F}_s$ is in competition with the confining force $\vec{F}_f$ of the focusing field.

From the differential equation for the betatron motion in "smooth approximation"

$$\frac{d^2\xi}{dt^2} + \frac{Q_\xi^2 \beta^2 c^2}{R^2} \xi = 0 \qquad (16)$$

where ξ denotes either y or z, we obtain

$$F_{f\xi} = m_o \gamma \frac{d^2\xi}{dt^2} = - \frac{m_o \gamma \beta^2 c^2 Q_\xi^2}{R^2} \xi \qquad (17)$$

where R is the average orbit radius, m_o the electron rest mass and Q_ξ the number of betatron oscillations per revolution. The shift in betatron frequency, due to a change in the total confining force, expressed by the change ΔQ_ξ of the Q-value is

$$\frac{\Delta Q_\xi}{Q_\xi} = \frac{1}{2} \frac{\Delta F_\xi}{F_\xi} \qquad (18)$$

The more important shift occurs in the direction of the small axis of the ellipse, which is supposed to be the z-axis. We obtain

$$\Delta Q_z = \frac{Q_z F_{sz}}{F_{fz}} = - \frac{e_o^2 R^2 f(n_e - \gamma^2 n_i)}{2 \varepsilon_o m_o c^2 \beta^2 \gamma^3 Q_z} \qquad (19)$$

The minus sign means, that an electron space charge decreases the betatron frequency, and positive space charge increases it. The influence of an ion density on the frequency shift is γ^2 times greater than that of the same number of electrons per unit volume.

3. Space Charge Limits.

The maximum possible electron denisty without positive ions is determined by the maximum possible shift ΔQ_z.

$$(n_e)_{max} = \frac{2\, \varepsilon_o\, m_o\, c^2\, \beta^2\, \gamma^3\, Q_z\, (\Delta Q_z)_{max}}{e_o^2\, R^2\, f} \qquad (20)$$

Using the following parameters and approximations,

$$\begin{aligned} R &= 4 \text{ m} \\ Q_z &= 3.75 \\ (\Delta Q_z)_{max} &= 0.2 \\ f &\simeq 1 \quad \text{(flat beam)} \\ \beta &\simeq 1 \quad , \end{aligned} \qquad (21)$$

we obtain:

$$(n_e)_{max} = (2.65 \times 10^6 \text{ cm}^{-3})\, \gamma^3 \qquad (22)$$

$$G_{max} = (1.27 \times 10^{-2} \frac{\text{A}}{\text{cm}^2})\, \gamma^3 \qquad (23)$$

Table I shows the results for different energies.

Table I

Space charge limits without positive ions

γ	$(n_e)_{max}$	G_{max}
	cm^{-3}	$A\ cm^{-2}$
4	1.7×10^8	0.81
10	2.7×10^9	13
20	2.1×10^{10}	100
40	1.7×10^{11}	810
100	2.7×10^{12}	1300
200	2.1×10^{13}	10,000

If the positive ions are not removed by a clearing field, they accumulate until n_i equals n_e. The shift of the betatron frequency then will be in opposite direction as for electron space charge alone.

$$\Delta Q_z = \frac{e_o^2 R^2 f n_e (\gamma^2 - 1)}{2 \varepsilon_o m_o c^2 \beta^2 \gamma^3 Q_z} \qquad (24)$$

Hence the space charge limit for a completely neutralized beam will be

$$(n_e)^*_{max} = \frac{2 \varepsilon_o m_o c^2 \beta^2 \gamma^3 Q_z (\Delta Q_z)_{max}}{e_o^2 R^2 f (\gamma^2 - 1)} \qquad (25)$$

This space charge limit is γ^2 times smaller than that for a pure electron beam. Table II shows the maximum electron densities and current densities for a neutralized beam, using the same parameters and approximations as listed in Eqs. (21).

Table II

Space charge limits for a neutralized beam ($n_i = n_e$)

γ	$(n_e)^*_{max}$	G^*_{max}
	cm^{-3}	$A\ cm^{-2}$
4	1.1×10^7	0.051
10	2.7×10^7	0.13
20	5.3×10^7	0.25
40	1.1×10^8	0.51
100	2.7×10^8	1.3
200	5.3×10^8	2.5

4. Build-up Time for the Critical Positive Space Charge.

A critical time for the build-up of the positive space charge is the time in which so many ions are produced and trapped that they just compensate the influence of the electron space charge on the shift of the betatron frequency. This happens when

$$n_i = \frac{1}{\gamma^2} n_e \qquad (26)$$

From that moment on the space charge limit drops down from the values listed in Table I towards the values of Table II.

The number of ions produced per unit path length during the time interval τ is

$$\frac{dn_i}{dx} = \frac{I\, s\, p\, \tau}{e_o} \qquad (27)$$

where s is the specific ionisation, that is the number of ion pairs produced by a single electron per time unit and density unit, and p the gas density in Torr.

The specific ionisation for 2 MeV electrons in air is 0.06 cm^{-1} $Torr^{-1}$ and rises for higher energies slowly to 0.14 at 500 MeV. [1), 2)] As an average between 2 and 100 MeV we can take 0.08 cm^{-1} $Torr^{-1}$. From

$$\frac{dn_i}{dx} = \frac{1}{\gamma^2} \frac{d\, n_e}{dx} = \frac{I}{\gamma^2\, e_o\, \beta\, c} \qquad (28)$$

we obtain the critical time

$$\tau_{crit} = \frac{1}{\gamma^2\, s\, p\, \beta\, c} \quad , \qquad (29)$$

Numerically for $p = 10^{-9}$ Torr and $\beta \simeq 1$

$$\tau_{crit} = \frac{0.4 \text{ sec}}{\gamma^2} \qquad (30)$$

In Table III are listed the critical times for different energies.

Table III

Build-up time for a critical positive space charge

γ	τ_{crit}
4	25 ms
10	4 ms
20	1 ms
40	250 µs
100	40 µs
200	10 µs

The time necessary for complete neutralisation ($n_i = n_e$) is

$$\tau_n = 0.4 \text{ sec} \qquad (31)$$

and depends only on the gas density.

The positive ions have to be removed from the beam, if stacking experiments are to be carried out for a time longer than the critical time given by Table III. Otherwise the current density will be seriously limited, or other complications will arise from the positive space charge.

5. Constant Clearing Field.

The easiest way to remove the ions would be a homogeneous electrical D.C. field all around the orbit. The direction of the field should be axial, that means parallel to the magnetic field of the bending magnets, because only in this direction can the ions easily be withdrawn.

The strength of the clearing field must be greater than the axial component of the electrostatic field of the beam. The maximum value of this component can be obtained from Eq. (4).

$$(E_{cf})_{min} = (E_z)_{max} = \frac{e_o n_e f z}{\varepsilon_o} \qquad (32)$$

where E_{cf} is the strength of the clearing field.

For a relativistic beam of flat cross-section ($\beta \simeq 1$, $f \simeq 1$) and of current I we obtain

$$(E_{cf})_{min} \doteq \frac{I}{\varepsilon_o c \pi a} \qquad (33)$$

or

$$(E_{cf})_{min} = (24 \frac{V}{A}) \frac{I}{2a} \qquad (34)$$

The minimum clearing field strength depends only on the ratio of the current to the radial width of the beam. For instance, a clearing field of 1000 V/cm allows a stacked electron beam of 40 A for each centimetre of radial beam width.

While the beam current gives a lower limit for the clearing field, an upper limit is given from the fact that the clearing field applies a constant force to the electrons and shifts the plane of the equilibrium orbit upwards or downwards. The approximate value of this shift is the distance for which the mean focusing force equals the force from the clearing field

$$F_{cf} = - e_o E_{cf} \tag{35}$$

The z-component of the focusing force in smooth approximation is

$$(F_f)_z = \frac{m_o \gamma \beta^2 c^2 Q_z^2}{R^2} z \tag{36}$$

By comparing Eqs. (35) and (36) we obtain for the shift of the equilibrium orbit

$$\Delta z = \frac{e_o R^2}{m_o \gamma \beta^2 c^2 Q_z^2} E_{cf} \tag{37}$$

If $(\Delta z)_{max}$ is the maximum shift which can be tolerated, the upper limit for the clearing field will be

$$(E_{cf})_{max} = \frac{\beta^2 E_o Q_z^2}{e_o R^2} \gamma (\Delta z)_{max} \tag{38}$$

where $E_o = m_o c^2$ is the rest energy of the electron.

Using the same parameters and approximations as before

$$\left.\begin{aligned} R &= 5 \text{ m} \\ Q_z &= 3.75 \\ \beta &\simeq 1 \end{aligned}\right\} \tag{21}$$

we obtain numerically

$$(E_{cf})_{max} = (45 \frac{\text{V/cm}}{\text{cm}}) \gamma (\Delta z)_{max} \tag{39}$$

This means that, for an electron energy $\gamma = 4$ and a maximum tolerable shift $(\Delta z)_{max} = 2.5$ mm, the maximum strength of the clearing field is 45 V/cm.

For a given clearing field the stacked current is limited; hence by combining Eqs. (34) and (39) we obtain an upper limit for the current proportional to the maximum shift of the equilibrium orbit.

$$\left(\frac{I}{2a}\right)_{max} = \frac{\varepsilon_o\, c\, \pi\, \beta^2\, w_o\, Q_z^2}{2\, e_o\, R^2}\, \gamma\, (\Delta z)_{max} \tag{40}$$

or numerically

$$\left(\frac{I}{2a}\right)_{max} = \left(1.9\, \frac{A/cm}{cm}\right) \gamma\, (\Delta z)_{max} \tag{41}$$

This means, a constant clearing field limits the stacked current to about 2 A per cm radial beam width, if we have electrons with $\gamma = 4$ and assume a maximum tolerable shift of the equilibrium orbit of 2,5 mm.

6. Alternating Clearing Field.

Another possible arrangement is a clearing field, constant in time, but alternating azimuthally in sign.

The most reasonable periodicity for an alternating clearing field would be ~~that of the~~ each magnet field, in other words, one positive and one negative clearing field sector in each magnet field period. An alternating clearing field has the advantage that the median plane of the beam remains unchanged. On the other hand it causes a vertical scalloping of the equilibrium orbit.

An approximation can be worked out for the scalloping amplitude, the maximum deviation of the disturbed equilibrium orbit from the medium plane, as a function of the field strength of the alternating clearing field.

From Fig. 2 it can be seen that the disturbed equilibrium orbit inside each half sector is part of a betatron oscillation about a plane which is shifted away from the median plane by the distance Δz. The magnitude of Δz is the same as described by Eq. (37) from section 5. For reasons of symmetry the disturbed orbit must intersect the median plane between two adjacent half sectors of the clearing field.

The disturbed orbit in the positive half sector can be described by

$$z(\Theta) = -\Delta z + (\Delta z + \delta Z) \cos Q\Theta \tag{42}$$

where the azimuth angle starts in the centre of the half sector.

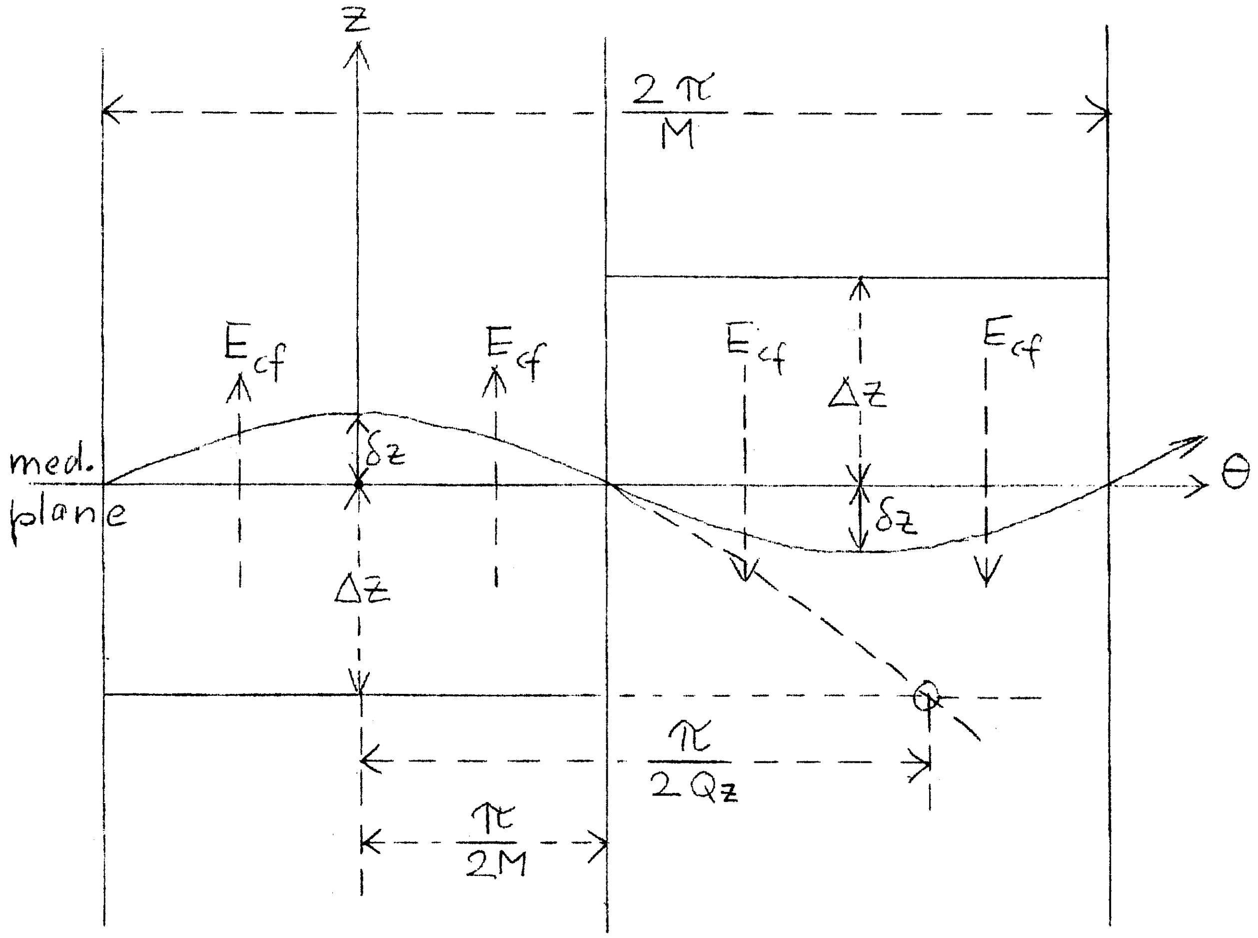

Fig. 2.

Scalloping equilibrium orbit in an alternating clearing field.

z axial displacement

θ azimuth angle

M number of sectors

The scalloping amplitude δz is now determined by the equation

$$z\left(\frac{\pi}{2M}\right) = 0 \tag{43}$$

$$-\Delta z + (\Delta z + \delta z)\cos\left(\frac{Q_z \pi}{2M}\right) = 0 \tag{44}$$

which leads to

$$\delta z = \left(\frac{1}{\cos\left(\frac{\pi Q_z}{2M}\right)} - 1\right)\cdot\Delta z \tag{45}$$

For $M = 8$ and $Q = 3.75$ we have

$$\left(\frac{1}{\cos\frac{\pi Q_z}{2M}} - 1\right) = 0.350 \qquad (46)$$

We see that the maximum deviation of the scalloping equilibrium orbit in an alternating clearing field is for the considered parameters only one third of the shift in a constant clearing field.

Correspondingly, the beam current limit, as given in Eq. (41), is a factor of three higher.

The beam current limit can furthermore be increased by using a higher periodicity of the clearing field. For instance, a clearing field with two positive and two negative sections on one focusing field sector will increase the current limit by a factor of four compared with the simple alternating clearing field as described before and by a factor of twelve compared with the limits from Eq. (41).

The last mentioned alternating clearing field fits well into the scheme of K. Johnsen's proposal 3c in PS/Int. AR/60-6, as can be seen from Fig. 3.

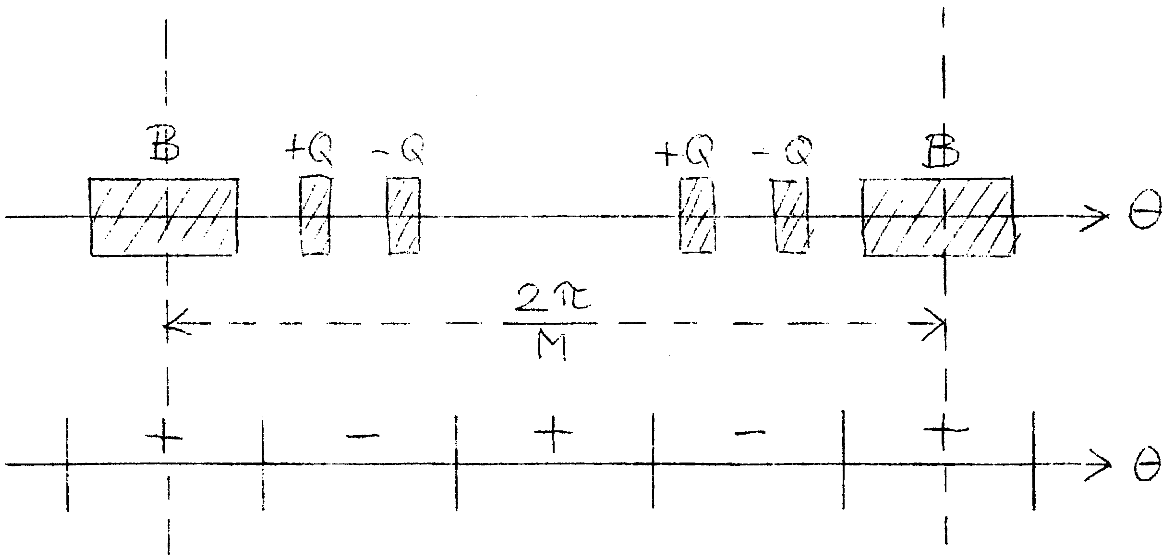

Fig. 3.

First line: bending and focusing magnets

Second line: Clearing field.

7. Gaps in the Clearing Field.

It could be necessary or useful to have parts of the orbit free from clearing electrodes, for instance sections which are occupied by R.F. cavities, pick-up electrodes, inflection devices and so on. The influence of such a gap is certainly negligible, if it is not longer than the vertical distance of the clearing field electrodes, especially if the clearing field has the same sign on both sides of the gap.

If the field free region is longer, the clearing of this region relies only on the thermal velocity of the ions.

The mean velocity of ions with mass number 28 at 20° Celsius is about 3×10^4 cm/sec. The energy transfer by electron collisions can be neglected.

Let us assume that inside each sector of the length $2\pi R/M$ there is a field-free gap of the length ℓ, which is not inside the magnetic field. The mean time, necessary to cross this gap with the thermal velocity, is $\ell \bar{v}$. This time has to be compared with the critical build-up time from Eqs. (29), (30) and Table III. But one has to keep in mind that a space charge, filling only a fraction of the orbit, shifts the betatron frequencies correspondingly less. Consequently the build-up time for a critical space charge is longer by roughly the factor $2\pi R/M\ell$. For the gap length we obtain finally the condition

$$\ell < \left(\frac{\bar{v}\ \tau_{crit}\ 2\pi R}{M}\right)^{1/2} \tag{47}$$

or, using $R = 4 \times 10^2$ cm, $M = 8$ and Eq. (30)

$$\ell < \frac{2 \times 10^3 \text{ cm}}{\gamma} \tag{48}$$

We see that a storage ring for energies up to $\gamma = 20$ can have rather large field free sections.

An interrupted clearing field has of course the effect of scalloping the equilibrium orbit vertically. If necessary, the disturbed equilibrium orbit can be constructed similar to the case of the continuous alternating clearing field, as described in Fig. 2.

8. Drift Velocity inside a Bending Magnet.

The insertion of clearing electrodes increases the total vertical dimensions of the vacuum chamber and therewith the gap of the bending magnets. It could be questioned whether it is possible to keep the vacuum chamber inside the bending magnet free from clearing electrodes. In this case the thermal velocity is not able to drive the positive ions out of the bending magnet. But the combination of the electrostatic field of the beam and the external magnetic field causes the ions to move on cycloid orbits with a main drift velocity parallel to the beam axis.

The drift velocity is given by

$$v_d = \left| \frac{E_y}{B} \right| \tag{49}$$

Using Eq. (3)

$$v_d = \frac{e_o n_e}{\varepsilon_o |B|} (1 - f) \, |y| \tag{50}$$

The factor $(1 - f) = \frac{1}{1 + (a/b)^2}$ equals 0.5 for a circular cross-section of the beam and approaches zero for a flat beam $\left[(a/b)^2 \gg 1\right]$. The drift velocity is furthermore proportional to the radial distance y from the equilibrium orbit. In order to obtain the mean drift velocity we put y equal $a/2$.

$$\overline{v_d} = \frac{e_o n_e a (1 - f)}{2 \varepsilon_o |B|} , \tag{51}$$

where the bar means the average over all electrons.

If we finally express the electron density by the current I

$$I = e_o n_e \beta c \pi ab \tag{52}$$

we obtain

$$\overline{v_d} = \frac{(1 - f)}{\pi \varepsilon_o \beta c} \frac{I/2b}{|B|} \tag{53}$$

The mean drift velocity is proportional to the vertical current density (2b is the vertical width of the beam) and reciprocal to the external magnet field.

For a beam of circular cross-section the factor in Eq. (53) is

$$\frac{1 - f}{\pi \, \varepsilon_o \, \beta \, c} \simeq 60 \frac{V}{A} \tag{54}$$

In the same manner as in the previous section 8 , concerning a field-free section, the time necessary to travel the length ℓ_B of a bending magnet has to be compared with the build-up time for a critical space charge. This leads for $\ell_B = 50$ cm to the condition

$$\frac{I}{2\,b} > (2.1 \times 10^{-8} \,\frac{A/cm}{Gauss})\, \gamma^2 \,|B| \tag{55}$$

For $\gamma = 4$ and $B = 70$ Gauss we obtain for instance

$$\frac{I}{2b} > 2.3 \times 10^{-6} \text{ A/cm} \tag{56}$$

This estimate is made under the assumption of a circular cross-section. For an elliptic cross-section the limit from Eq. (56) has to be multiplied by the factor

$$\frac{1}{2(1 - f)} = \frac{1}{2} \left[1 + (a/b)^2 \right] ,$$

where a/b is as before the ratio of the radial width to the vertical width of the beam.

Even for a flat beam the result of Eq. (56) is surprisingly good: the electric field of only a few microamps of stacked electrons is sufficient to drive the positive ions fast enough out of the bending magnets.

9. Radial Clearing Field in the Bending Magnets.

In spite of the good result from Eq. (54) it is somewhat unsatisfactory to rely for the removal of positive ions only on the electric field of the stacked beam itself.

One would be independent of the field of the beam, without increasing the magnet gap, if the electrostatic field, necessary for the cycloidal motion, is produced by radial clearing electrodes as shown in Fig. 4.

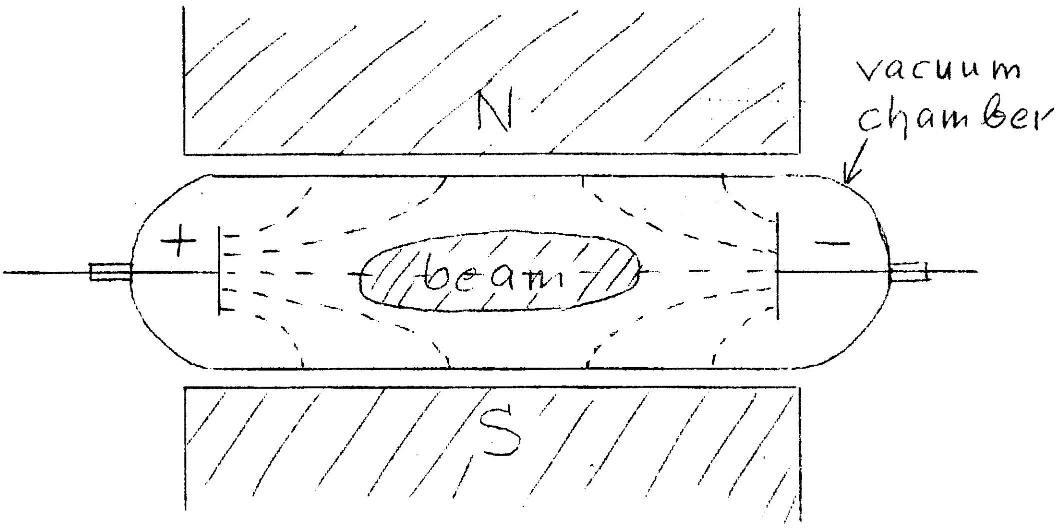

Fig. 4.

The necessary field strength is very small. It can be calculated from the condition

$$E > \frac{\ell_B \, |B|}{\tau_{crit}} \qquad (57)$$

Using Eq. (30), $\ell_B = 50$ cm, $\gamma = 4$ and $B = 70$ Gauss, we obtain

$$E \gtrsim 2 \times 10^{-4} \text{ V/cm} \qquad (58)$$

This condition is very easy to maintain, even considering a great distance between the two electrodes and the fact that the main part of the field will be deflected to the chamber wall.

10. Conclusion.

A clearing field is necessary if any important electron density should be stacked. The design of the clearing field depends of course on the kind of storage ring system to be built. But it seems to be clear from the foregoing, that no difficult problems exist for the design of a suitable clearing field array for any storage ring system, at least for energies up to 10 MeV. The D.C. voltages are not too high. The perturbation of the equilibrium orbit can be kept small by the use of an alternating clearing field. Azimuthal gaps in the clearing field are tolerable.

It is possible to have the bending magnets either free from clearing electrodes or with radial clearing electrodes, which does not increase the gap of the bending magnets.

E. Fischer

References:

1) Landolt-Börnstein, Vol. I, p. 343.

2) O'Neill, Panovski; An experiment on the limits of quantum electrodynamics.

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