Minutes of
CERN-PS Staff Meeting
February 3rd, 1959

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CLOSED ORBIT ANALYSIS

Schoch reported on Closed Orbit analysis.

1. Introduction

In the CPS the particle orbits have to stay in a vacuum chamber whose width is only 1/1000 of the mean radius of the machine and this results in tolerance requirements of $\lesssim 10^{-3}$ for almost all important parameters.

The path of the proton beam may be displaced or deformed by imperfections of the guiding field and errors in energy. Beam observation can however be used to improve a considerably distorted beam by using the collected information to act on devices which can remove or compensate the perturbations provoking the distortions. Closed Orbit analysis is particularly connected with the determination of the perturbations from the observed beam shape.

2. Particle orbits in the A.G. synchrotron

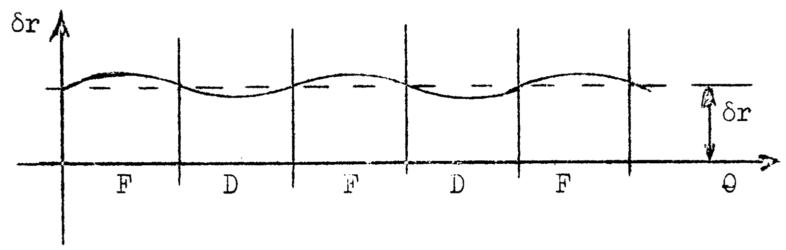
An "ideal" particle moves on a "closed orbit" consisting of arcs of a circle and straight sections. If the particle has a slightly wrong energy, there still exists a closed orbit for it, but this closed orbit is displaced to a different mean radius by an amount

$$\frac{\overline{\delta r}}{r_m} = \frac{1}{Q^2} \frac{\delta p}{p_o}$$

(momentum compaction).

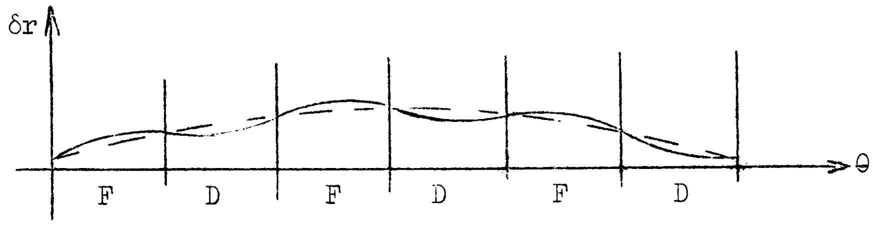
In any A.G. accelerator a radially displaced closed orbit shows "wriggles" due to the A.G. structure (fig. 1).

Fig. 1



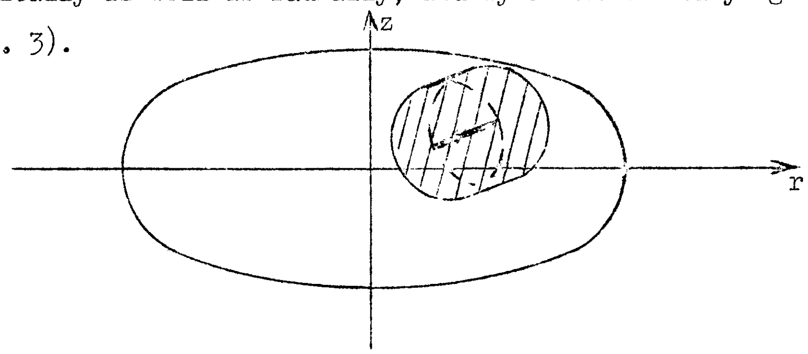
Particles having a transverse component of motion will perform (betatron) oscillations about the closed orbit belonging to the same energy. This oscillation is sinusoidal, except again for some modulation due to the A.G. structure (fig. 2).

Fig. 2



A beam consists of many particles distributed over a range of energies and betatron amplitudes. The width of the particle beam is twice the maximum amplitude of the betatron oscillation, centred on the closed orbits. The latter are spread over a certain radial range due to energy spread. Imperfections in the guiding field and frequency programme may furthermore shift the closed orbit off the centre of the vacuum chamber, vertically as well as radially, and by an amount varying round the machine (fig. 3).

Fig. 3



In the average over many revolutions the closed orbits (or the family of closed orbits) determine the beam centre, and the aim must be to straighten out the closed orbits and keep them as well as possible in the central region of the vacuum chamber. One is therefore faced with the problem of finding out where the closed orbits actually are.

3. Perturbations influencing the closed orbit

(i) Deviations in energy.

In the acceleration process the particles assume such an energy that their revolution frequency coincides with the radio-frequency f (or rather f/M, M being the harmonic number). Errors in the frequency programme therefore cause errors in energy. Furthermore, the particles carry out phase oscillations about the synchronous energy, leading to a certain energy spread.

In terms of momentum these deviations are in the CPS

$\frac{\delta p}{p_o}$ due to phase oscillations $\lesssim 3.10^{-3}$, corresponding to a radial displacement $\delta r \lesssim 8$ mm.

$\frac{\delta p}{p_o}$ due to frequency errors at injection $= \frac{\delta f}{f_o} \lesssim 10^{-2}$, corresponding to a radial displacement $\lesssim 2{\cdot}5$ cm.

(ii) Imperfections of the guiding field.

These may be of a) magnetic origin, or due to b) current distribution, or c) magnet misalignments.

a) Imperfections of the guiding field of magnetic origin are field fluctuations round the circumference due to remanent field deviations, eddy currents in the vacuum chamber, saturation, etc.

b) Non-uniform current distributions in the magnets might arise from the transient state of the magnet regarded as a transmission line and from the voltage ripple of the magnet power supply.

c) Magnet displacements might occur as a result of unavoidable survey errors.

The tolerances for these various imperfections are more or less known by now.

For a random distribution round the machine the field error $\frac{dB}{B}$ should be $\lesssim 10^{-3}$ for a closed orbit displacement of 1 cm.

If an electrical resonance frequency of the magnet circuit should coincide with the ripple frequency (600 c/s), a ripple of $\frac{\delta V}{V} \approx 10^{-4}$ could produce closed orbit distortions of several cms.

Alignment tolerances of about 0·5 mm from magnet to magnet are required in order to keep the closed orbit distortion less than 1-2 cm.

Finally it should be kept in mind that most of these perturbations change with time and so will therefore the closed orbit.

4. Relation between perturbations and closed orbit

To draw conclusions from the shape of the closed orbit on the perturbations themselves one has to consider the relation between the perturbations and the closed orbit.

In a perfect machine the smooth part $\bar{x}$ of the betatron oscillation obeys (in linear approximation) the equation

$$\frac{d^2\bar{x}}{d\theta^2} + Q^2\bar{x} = 0$$

In a machine with imperfections a forcing term appears on the r.h.s. and the equation becomes

$$\frac{d^2x}{d\theta^2} + Q^2\bar{x} = r_m \left(\frac{\delta p}{p_o} - \frac{\delta B(\theta)}{r_o B_o / r_m}\right) + \left[Q^2 - \left(\frac{r_m}{r_o}\right)^2 n(\theta)\right] \xi(\theta)$$

Here

$\bar{x} = \bar{r} - r_m$

$d\theta = ds/r_m$

r_m = mean radius of CPS

r_o = radius of curvature inside magnets

Q = number of betatron oscillations per revolution

$\delta B\ (\Theta)$ = error in guide field

$\xi\ (\Theta)$ = magnet displacement

The perturbations being periodic in Θ (with the period 2π) it is convenient to decompose the perturbation and also the motion into Fourier components. The r.h.s. of the previous equation can then be written

$$f\ (\Theta) = \sum_{-\infty}^{+\infty} f_k\ e^{ik\Theta}$$

and the solution can be represented in the form

$$\bar{x}\ (\Theta) = \bar{C}\ (\Theta) + A \cos\ (Q\Theta + \varphi)$$

In this expression $\bar{C}\ (\Theta)$ is a "forced" oscillation with period 2π (closed orbit) and the remainder is a "free" betatron oscillation about the closed orbit.

Supposing now a situation where one knows the closed orbit $\bar{C}\ (\Theta)$, the perturbation forces can be calculated as

$$\frac{d^2\bar{C}}{d\Theta^2} + Q^2\ \bar{C} = f\ (\Theta)$$

In principle, therefore, it seems simple to find the perturbation from the knowledge of the closed orbit. The difficulty is that one can observe the closed orbit only in a limited number of points.

The relation between the smooth part $\bar{x}$ and the real particle excursion x (including the wriggles) is of a linear form, the coefficients being determined by the machine parameters. In the CPS one finds

$$x = (1 + 0{\cdot}17 \cos 50\ \Theta - 0{\cdot}001 \cos 100\ \Theta + \dots)\ \bar{x} - (0{\cdot}0067 \sin 50\ \Theta + \dots)\ \frac{d\bar{x}}{d\Theta}$$

$$\frac{dx}{d\Theta} = -\ Q\ (1{\cdot}3 \sin 50\ \Theta + \dots)\ \bar{x} + (1 - 0{\cdot}17 \cos 50\ \Theta + \dots)\ \frac{d\bar{x}}{d\Theta}$$

Here θ is measured from the middle of an F-section. Now, by a fortunate coincidence, all pick-up electrodes in the CPS are in the middle of either F or D sectors; so $\sin 50\,\theta = 0$ for all of them, and the reduction from full coordinates to smooth motion coordinates is easily done by

$$x = (1 + 0{\cdot}17 \cos 50\,\theta)\,\bar{x}$$

Similar relations hold for the vertical oscillations.

5. Restrictions due to the limited number of pick-up electrodes

20 stations for beam observation were planned (2 per superperiod, each for radial and vertical displacement). Actually only 19 exist because one had to be removed to free space for the injection inflector. Furthermore, two stations have been displaced from their regular position in the magnet superperiod.

The pick-up electrodes are placed in straight sections

3, 8, 13, 18, etc.

with 28 missing (pulsed electrostatic inflector)
68 shifted to 67
78 shifted to 77.

Now 19 points of the closed orbit are certainly not sufficient for its complete determination. On the other hand, a specification of the closed orbit which is perfect for all practical purposes does not require an infinity of data. In practice the perturbations have only as many degrees of freedom as there are magnet units in the machine, i.e. 100, and the contribution of a single magnet can conveniently be described by a kick perturbation equivalent to the combined effect of the field error $\frac{\delta B}{B}$ and the displacement ξ ; there is no sense in a finer analysis. The equations of motion then establish a linear relation between 100 magnet perturbations and the closed orbit in 100 points

$$C_i = \sum_{j=i}^{100} g_{ij}\; f_j \qquad\qquad i = 1,\ldots\ldots.100$$

which can, in principle, be solved unambiguously for the perturbations $f_1 \ldots\ldots f_{100}$, the matrix g_{ij} being determined by the parameters of the machine. If only a fraction of the C_i's are known, an unambiguous solution is no longer possible. In fact, there are then infinitely many sets of perturbations, all leading to the same subset of observed C_i's .

6. Ways of making a guess at the complete closed orbit

What one can do in this situation is to look for a way of making a good guess at the missing information. Several possibilities can be considered:

a) One could argue that 20 closed orbit values are sufficient to determine the combined perturbation of 5 magnets. However, 5 magnets cover about 1/3 of a betatron oscillation in the CPS and it is doubtful whether the assumption of a lumped perturbation can be considered as a satisfactory working approximation.

b) Assuming a probability distribution for the single magnet perturbations, a "most probable" closed orbit going through the $<$ 100 observed points and compatible with the perturbation probability distribution can be calculated. This is a feasible, unambiguous and systematic procedure. After some consideration it has however been discarded in the hope that the simpler approach c) might be good enough in practice.

c) A complete closed orbit is constructed by interpolation. The closed orbit substitute found in this way is then used for calculating the perturbation. This procedure has actually been tested by means of an electronic computer.

7. Electronic computer for interpolation and for perturbation analysis

The closed orbit excursions are supposed to be read from photographs of pick-up oscillograms. This human link appears to be essential to ensure reasonable interpretation of observational data.

The information is fed into the "interpolator" in the form of 19 proportional voltages. An additional set of 81 voltages, corresponding to the same number of intermediate pick-up positions, are tapped from a network which effects a third order fit through 4 consecutive observation stations. The 100 voltages, representing 100 points of the interpolated closed orbit substitute, are scanned periodically. Thus the closed orbit at the output appears as a step function in time. The steps are smoothed out by a filter, cutting off high frequencies.

Using a number of calculated or analogue closed orbit examples, it was checked that the interpolator produces in general a fair approximation.

The electronic calculation of the perturbation was not done as outlined above by double differentiation following directly the differential equation $\left[f\ (\Theta) = d^2\overline{C}/d\Theta^2 + Q^2\overline{C}\right]$, but by means of a filter designed to distort the closed orbit into the generating perturbation. In order to understand this, one can write the closed orbit equation in Fourier representation

$$\sum (Q^2 - k^2)\ C_k\ e^{ik\Theta} = \sum_k\ f_k\ e^{ik\Theta}$$

$$f_k = (Q^2 - k^2)\ C_k$$

This calculation would be effected by a filter having a frequency response $Q^2 - \omega^2$. No real filter can give such a characteristic, but an approximation has been achieved by means of a filter having two zeros at $\omega = \pm Q$ and the unavoidable poles at imaginary frequencies sufficiently far off to minimize their effect. The idea was to have a device which is not too accurate (the primary data are not too accurate anyhow), but which could be used for fast experiments.

The computer has been tested by means of calculated closed orbits, or closed orbits obtained using the electromechanical betatron oscillation model which can be perturbed by adjustable kicks. Generally a fair picture of the perturbations was obtained.

8. Limitations

The knowledge of Q is required for applying the procedure outlined above. There are not too many possibilities to measure Q . Possibly one could measure Q by the computer itself, by adding a known perturbation; if one knows the perturbation and the closed orbit, one can determine the Q-value. At present several experiments are in progress to check this and other possibilities.

Various measuring errors from the pick-up electrode oscillograms may affect the results. One also has to keep in mind that the closed orbit is not sharply defined because of energy spread; this also causes a spread in the Q-value.

In general it is likely than random perturbations will be difficult to analyze, but there is a good chance of detecting a unique perturbation, e.g. a magnet displaced by a few mm, or a quasisinusoidal perturbation voltage in the magnet power supply.

Various possibilities are being considered to improve the interpolator.

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