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# COMPARISON OF THE PREDICTIONS OF THE STATISTICAL BREAKDOWN MODEL WITH EXPERIMENT

by

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## Introduction

At the Grenoble conference on dielectric liquids in 1968 two of the authors presented a theoretical analysis of the influence of various experimental parameters on the measured impulse breakdown strength of n-hexane[1]. From a few very simple assumptions based on earlier experimental work, predictions were established for the strength as a function of pulse-waveform, gap-geometry and electrical test method. The purpose of that study was to show how important changes in the apparent behaviour of an idealised test-liquid could be wrought by variations of procedure whose existence and importance had hitherto been ignored, and no supporting experimental evidence was submitted.

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At the same conference, however, the third author presented a separate paper giving the results of an independent and largely experimental study of some aspects of the same problem[2]. Unfortunately, the values chosen for the experimental parameters, particularly the gap length, differed in the two papers, and it was not possible at the time to make a comparison. Nevertheless, such an unwitting conjunction of theory and experiment offers a rare opportunity to test the validity of both the theory and the experimental technique.

## 1. The Statistical Model

The model originally assumed was based on a conditional failure rate $\phi(E)$, such that the probability of a breakdown occuring between times (t) and (t + dt), given that a breakdown has not occured between times (o) and (t), is equal to $\phi(E)dt$. In order to develop a model for the geometric effect, a further assumption was made that, for a given nominal stress $E_o$, $\phi(E_o)$ could be established by integrating a conditional failure rate per unit area $\eta(E)$, so that the effect of variation of the local stress E could be considered[3], and

$$\phi(E_o) = \int_A \eta(E)dA$$

It has since been established experimentally[4] that in a planar gap the conditional failure rate is proportional to area and independent of gap length. The further assumption that $\eta(E)$ varies exponentially with stress has also been verified.

Given these assumptions, it is only necessary to know the value of $\phi(E)$ for one particular gap between spheres and two values of stress, to be able to derive a theoretical prediction of the value at any gap and stress. Hence the probability of breakdown during a pulse of length $\tau$ and amplitude E can be calculated from :

$$F(\tau,E) = 1 - \exp\left[- \phi(E)\tau\right]$$

the formative time being negligible for the range of values considered here.

Given $F(\tau,E)$ it is a relatively straightforward matter to evaluate the electric strength obtained from the application of an ascending sequence of pulses, and an analytical expression has been obtained for the distribution of breakdown values, from which the mean strength can be obtained[5].

At the time it was put forward, the most conjectural aspect of this theory was the suggestion that $\phi(E)$ could be established by means of a surface integral on the electrodes. Previously, a similar thoery has been applied to conduction measurements in n.hexane and it was found that the entire gap and stress dependence could be accurately predicted from a knowledge of current at two values of stress and a single value of gap[3]. The method took advantage of the fact that good approximation for the current density is $J = \exp|\alpha E + B|$ so that the total current appeared as a function of the nominal applied stress $E_o$, the area of the sphere A and the gap/diameter ratio x. Thus

$$J = A.\exp(B).S(\alpha E_o, x)$$

The function S, which is the average value of an exponential function of the surface stress taken over one sphere can be calculated through knowledge of the stress distribution in a sphere gap.

Conditional failure rates are additive in the same way as current density, so the assumption $\eta = \exp(\alpha E + B)$ yields

$$\phi(E_o) = A.\exp(B).S(\alpha E_o, x).$$

## 2. The Experimental Basis

The original motivation for the experimental work was a desire to test a prediction that $\phi(E)$ should decay with time of stress application. This was evident from the fact that its value[5] in the microsecond region is $10^4\ s^{-1}$, but in the minute region it is only $0.2\ s^{-1}$. The measurement of variations of $\phi$ is somewhat difficult as it involves use of the so-called Laue plot, which is a plot of $-\ell n\ |1 - F\ (t)|$ versus t. This follows from the fact that the cumulative distribution of time-lags is given by

$$F(t) = 1 \exp \left[- \int_o^t \phi(E)\ dt\right]$$

so that

$$\frac{-d}{dt} \ell n \left[1 - F(t)\right] = \phi(t)$$

because it uses F(t) this is a method of order statistics, and the variance of each point on the plot increases with its rank in the distribution. Even for a constant value of $\phi$ (exponential distribution) the Laue plot is not a good extimator compared with the Best-Linear-Unbiased-Extimators of order statistics, but for a varying $\phi$ there seems to be no alternative.

Thus, the fact that $\phi$ varies very little over the range of time covered by one experiment is unfortunate from the point of view of detecting the effect. It is, however, fortunate from the point of view of comparing the results with the constant $\phi$ model discussed previously.

## 3. Comparison of Results

As stated, the experimental results yielded values of $\phi$ as a function of time $\phi$ (E,t) The author, however, chose to transform them to a more familiar form - the probability distribution of breakdown for rectangular pulses of various widths - by an independently derived process of summation. So it is of interest to compare the result in this form and also to compare the mean strength predicted in the case.

We wish to emphasize that the geometric gap dependence in one case is determined purely by physical measurement and in the other case by a purely theoretical procedure, and that unknowingly the two pieces of work were carried out simultaneously in separate laboratories.

Firstly in Figs. 1 and 2 two graphs are shown of mean strength versus pulse width for four gaps. The following principal changes have been made from the previous forms : (a) the mean is plotted instead of the original mode in the upper graph, and (b) the values of gap in the theoretical computer program have been changed to align them with the experimental values. Apart from the remarkable agreement between the two sets of curves, the most interesting feature is the complete occultation of the effect of the decaying failure rate, which adds further weight to criticisms of the type of test which produces this form of plot.

This effect is shown clearly in the distributions of Figs. 3, 4 and 5. Fig. 3 is obtained from the measured values of $\phi$ (E,t) while Fig. 4 is obtained for $\phi$ (E) set to the value for short pulses ignoring the decay. Fig. 5 is the theoretical prediction. The parameters assumed are $E_{lim}$ = 0.9 MV/CM and $\lambda$ = 20. A more detailed comparison of the probability values obtained is given elsewhere, together with a discussion of the effects of experimental parameters[6].

## 3. Conclusion

The principal result of this comparison is the justification of a theoretical procedure which might have seemed highly conjectural at the time it was put forward - namely the evaluation of overall breakdown probability by integration over the surface of one of a symmetrical pair of electrodes. This in turn confirms the original assumption that the breakdown weak-link (initiating event) occurs on the surface of one of the electrodes.

Also, the comparison underlines the high degree of experimental control which may be achieved by careful measurement, particularly as the thoeretical calculations quoted were based on an isolated measurement of $\phi$ made by Lewis and Ward[7] in 1962. The authors feel that a more useful measure of breakdown strength is given by a simple quotation of $\phi$ for a given stress, but, since this conditional failure rate does not yet seem to have gained general acceptance as the most direct measure of the tendency to breakdown, they independently adopted the artifice of constructing the consequent curves for pulse breakdown, and to preserve the comparison with the cited papers, this procedure has been followed here.

The statistical model provides a powerful and accurate description of breakdown in liquids, and it is capable of predicting effects as complicated as changes in geometry.

## Acknowledgement

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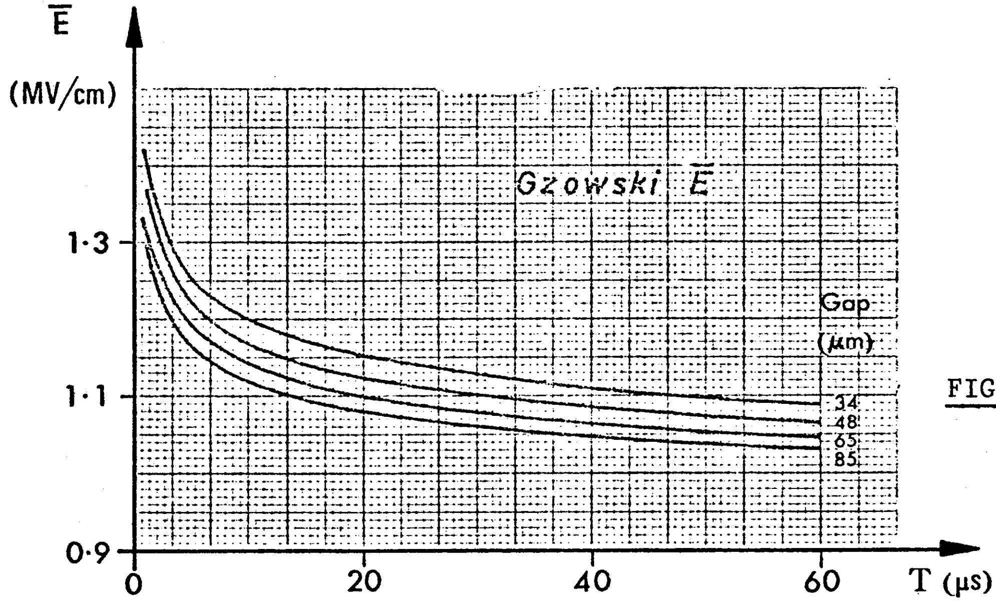


FIG. 1

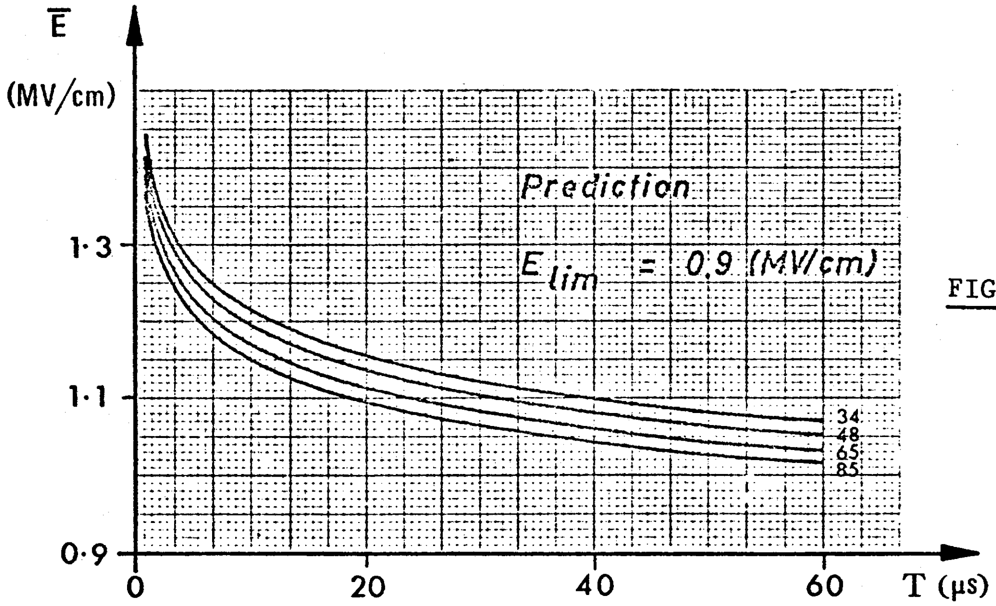


FIG. 2

Experimental and theoretical curves of Mean Breakdown strength vs. Pulse width for several sphere-electrode spacings.

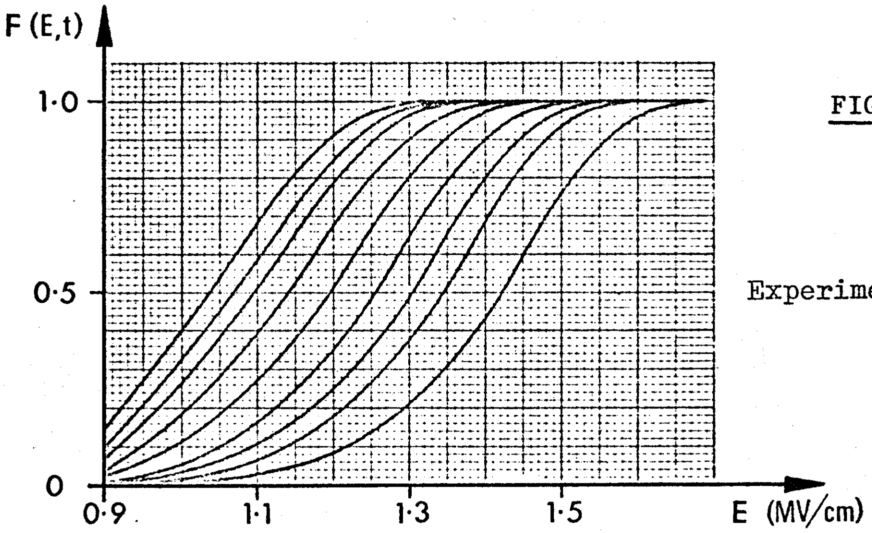


FIG. 3

Experiment

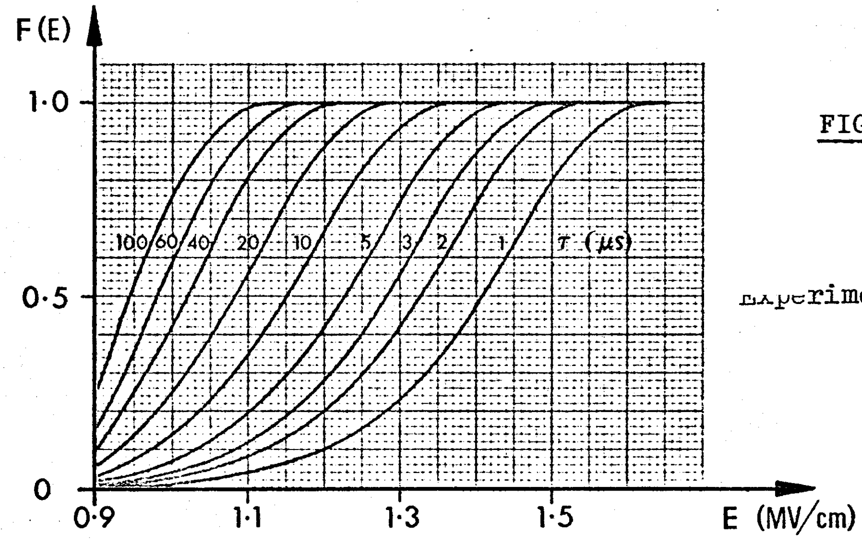


FIG. 4

Experiment

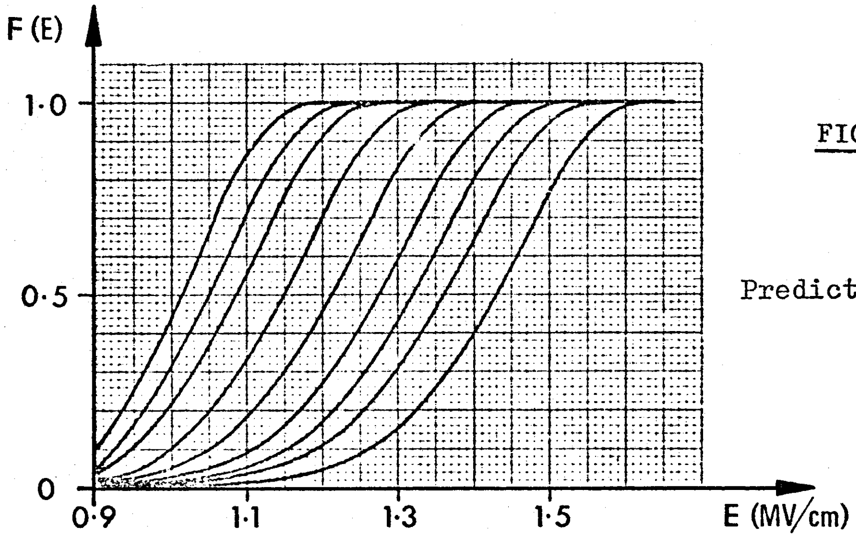


FIG. 5

Prediction

Experimental and theoretical curves of cumulative probability vs. Stress for several pulse widths at a gap of 34 μm.

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