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STUDY AND COMPENSATION OF COHERENT LONGITUDINAL INSTABILITY IN THE CERN PS

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Summary

Coherent longitudinal instabilities, similar to those observed in electron storage rings and in the Brookhaven AGS[1], occur in the CERN PS. The resulting dilution in longitudinal phase space would critically affect the luminosity obtainable in the intersecting storage rings (ISR). A theoretical study of the mechanism of this instability is presented, including the influence of the beam control loop. A compensation technique, based on introducing a difference in the synchrotron frequencies of different bunches has been studied. We find that in the presence of beam control the effectiveness of this cure depends critically on the pattern in which the frequency varies between the bunches.

I. Introduction

Coherent bunch oscillations become unstable in the CPS when intense short bunches are accelerated. The instability has been attributed to a coupling from bunch to bunch via high frequency resonators[2]. The most harmful pieces of equipment were found to be the accelerating cavities which have a parasitic resonance in the 50 MHz region. A bunch passing through a cavity induces an oscillating voltage which decays relatively slowly and perturbs the motion of the following bunches. The system being closed after h=20 (harmonic number) bunches may be unstable.

II. Mathematical description of the instability

The perturbed synchrotron equations for bunch m are:

$$\left.\begin{aligned} \Delta\dot{p}_m &= a[V(\sin\phi - \sin\phi_s) + v_m(\phi)] \\ \dot{\phi} &= b\Delta p_m \end{aligned}\right\} \qquad (1)$$

where V is the accelerating and v_m the perturbing potential, a and b are constants, $\Omega=\sqrt{ab}$ is the synchrotron frequency.

a) Simple approach[1,2]

Linearizing (1) and writing

$$v_m = v_{om} + \sum_{n=1}^{h} \frac{\partial v_{mn}}{\partial\varphi}(\varphi_n - \varphi_m) \qquad (2)$$

where φ_n is the phase deviation of the centre of mass of bunch n, we get

$$\ddot{\varphi}_m + \Omega^2\varphi_m + \sum_{n=1}^{h} \beta_{mn}(\varphi_n - \varphi_m) = 0 \qquad (3)$$

$$\beta_{mn} = \frac{\Omega^2}{V\cos\phi_s}\frac{\partial v_{mn}}{\partial\varphi}$$

Looking for harmonic solutions of the form $\Phi_m \exp(\pm j\omega t)$, eq. (3) becomes algebraic

$$(-\omega^2+\Omega^2-\sum_{n=1}^{h}\beta_{mn})\Phi_m + \sum_{n=1}^{h}\beta_{mn}\Phi_n = 0 \qquad (4)$$

The h solutions for ω^2 can be found from the eigenvalues of the coefficient matrix of (4). The growth rates are $1/\tau = \mathrm{Im}(\omega)$. In the absence of frequency spread, adjacent bunches oscillate with a phase difference $k(2\pi/h)$, k = 1,2, ... h. For low Q coupling impedances, the mode with k closest to h/4 (k=5 in the PS) has the fastest growth.

b) Influence of the beam control system

In the case of beam controlled acceleration, the phase ϕ_{RF} of the RF voltage enters in (1) under the form:

$$\Delta\dot{p}_m = a[V(\sin(\phi-\phi_{RF}) - \sin\phi_s) + v_m(\phi)] \qquad (5)$$

and (4) becomes now:

$$(-\omega^2+\Omega^2-\sum_{n=1}^{h}\beta_{mn})\Phi_m + \sum_{n=1}^{h}\beta_{mn}\Phi_n - \Omega^2\Phi_{RF} = 0 \qquad (6)$$

These h equations must be completed by the beam control equation which says that the RF phase is the average of bunch phases corrected by the radial error signal.

$$\Phi_{RF} = \frac{1}{h}\sum\Phi_m + \frac{jG\omega}{h}\sum\Phi_m \qquad (7)$$

G is the gain of the radial loop, including some dynamical constants. (The factor $j\omega$ means that the radial position is proportional to the time derivative of the phase.) We have now h+1 equations, and the roots of the associated determinant will give us the oscillation modes of the system.

It can be demonstrated that these roots are exactly the same as those of the system (4) except for two trivial ones. Let us show this result in the case of equal bunches. For all the modes except for k=0 the average oscillation of all the bunches has zero amplitude and therefore equations (6) and (7) are satisfied with $\Phi_{RF}=0$. Thus, the corresponding eigenvalues are still solutions of the system. As the degree in ω of the determinant did not change, we have found all the solutions except two. But it is easy to check that $\omega=0$ and $\omega=-jG\Omega^2$ are also solutions of (6) and (7). One then can conclude that the beam control system does not affect the stability conditions of the system.

c) The dispersion relation[3]

Using a Vlasov equation approach and assuming that the h bunches are equal, the problem can be reduced to an eigenvalue problem, similar to (4) but where $\Omega^2-\omega^2$ has to be replaced by λ

$$\frac{1}{\lambda} = \pi\int\frac{\partial f_0}{\partial a}\frac{a^2\,da}{\Omega^2-\omega^2} \qquad (8)$$

f_0 being the stationary distribution and a the amplitude of the incoherent synchrotron oscillation.

III. Spread in synchrotron frequencies

A very general method of damping the instability is to make the synchrotron frequencies of the bunches slightly different in order to reduce the influence of parasitic couplings. This method has first been proposed by C. Pellegrini and is being used in electron storage rings.

a) Analytical approach[5]

In the case of a machine without beam control a direct solution of the problem can be found for a sinusoidal modulation of bunch frequencies, using Chebyshev functions[4]. In the more complicated case of beam controlled acceleration we shall use a root locus method. Equations (6) and (7) are perturbed by symmetrical frequency shifts characterized by $\delta = (\Omega+d\Omega)^2-\Omega^2$, and the corresponding root displacement is given by:

$$d\omega = -\frac{1}{2}\left(\frac{\partial^2 H}{\partial\delta^2}\right)_{\omega_{i,o}}\cdot\delta^2 \Big/ \left(\frac{\partial H}{\partial\omega}\right)_{\omega_{i,o}} \qquad (9)$$

obtained by developing the determinant $H(\omega,\delta)$ around the root ω_i.

The denominator can be explicitely calculated, because we know the determinant. The numerator depends on the modulation pattern. For a machine with h=4, and assuming short range wake fields, so that every bunch acts on the subsequent one only, we find the following results:

Modul. pattern	Without beam control	With beam control
Sinusoidal +δ, 0, -δ, 0	$d\omega = j\frac{\Omega}{2\beta}(d\Omega)^2$	$d\omega = \frac{\Omega}{4\beta}(1+j)(d\Omega)^2$
+δ, +δ, -δ, -δ	$d\omega = j\frac{\Omega}{\beta}(d\Omega)^2$	$d\omega = 0$
Meander +δ, -δ, +δ, -δ	$d\omega = j\frac{\Omega}{\beta}(d\Omega)^2$	$d\omega = j\frac{\Omega}{\beta}(d\Omega)^2$

It is concluded that the beam control system reduces the stabilizing influence of a sinusoidal bunch to bunch frequency spread. However, the meander pattern for which frequency deviation alternates from one bunch to the other remains effective.

b) Computer calculation [5]

A computer programme has been written to find the growth rates by calculating the eigenvalues and eigenvectors of an h x h complex matrix. The action of phase lock, bunch to bunch spread in frequency and population can be taken into account. The coupling coefficients β_{mn}, for any measured coupling impedance and bunch shape, are calculated by a Fourier analysis method. An auxiliary programme calculates the dispersion integral and permits Landau damping calculation for any given density distribution.

The computer programme was used to calculate the growth rates for various modulation patterns in the presence of the beam control. The results for an h=20 machine confirm the conclusion drawn from the h=4 model. With beam control and for short range wake fields the meander pattern remains stabilizing, whereas the sinusoidal pattern (1 modulation period covers 1 turn) becomes ineffective.

IV. Experimental results

Different observation techniques were used:

- Direct observation of bunches on a fast oscilloscope with "mountain range" display (photo 1)
- Observation of the phase difference between bunches and the RF voltage, by means of phase discriminators. This permits easy measurements of the phase shift between bunches or growth time measurements.

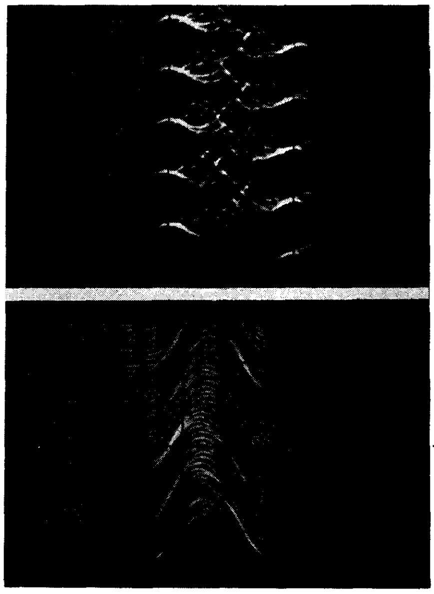
Clean oscillations of the first moment of the 13 ns long bunches are observed soon after transition, reaching a maximum peak to peak amplitude of 7 ns before filamentation and higher order instabilities come into play. The mode number currently observed is 5. The e-folding time may vary from 40 to 150 ms. Using the measured frequency distribution of the parasitic resonances of the 14 cavities as well as the shunt impedance (800 Ω) and the quality factor (24) (measured on a spare cavity), the calculated e-folding time (including Landau damping) is 80 ms, in good agreement with the growth time measurements made at the same time.

b) Compensation

A simple way to produce the meander pattern is to drive one cavity at half the RF frequency. Experiments were first made on a magnetic flat top at 10 GeV/c. Photo 1 shows typical bunch shapes at the end of the 500 ms flat top without (a) and with (b) one cavity fed at RF/2. (10 kV, corresponding to 7% of the main RF voltage, i.e. $\Delta\Omega/\Omega \sim \pm 3.5$ %.)

The scheme was found to work also during acceleration, though with reduced efficiency.

Experiments with a sinusoidal amplitude modulation of the RF at the revolution frequency (producing the sinusoidal pattern) clearly showed the ineffectiveness of this technique in the presence of beam control.



a) without RF/2

(bunches are already distorted)

b) with RF/2

Photo 1 5 ns/cm, horiz.
0.8 synchrotron period/cm, vert.

c) The missing bunch experiment [5]

Another observation, which emphasizes the importance of the beam control, was made during the missing bunch experiment. Since the instability is ascribed to a relatively low Q impedance, it was expected that making a gap of 4 or more bunches in the machine would open the loop, and suppress the instability. The experiment was done and showed that this was not true. Theoretical investigation with the techniques described above showed that, provided there exists small bunch to bunch spread (~ 1‰) in synchrotron frequencies, the phase lock system "bridges the gap".

Conclusion

The beam control system has an important influence on dipole longitudinal instability. Although usually designed to act only on the k=0 mode, it introduces an auxiliary coupling between bunches which becomes apparent for the other modes provided that the free synchrotron frequencies of the bunches are different.

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