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COMPENSATION OF LONGITUDINAL INSTABILITY IN THE PS : THE INFLUENCE OF PHASE-LOCK AND BUNCH TO BUNCH FREQUENCY SPREAD

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1. Introduction

As discussed in earlier notes [1], longitudinal instability occurs at high intensities in the PS. A cure has to be found because the resulting bunch blow-up would affect the luminosity obtainable in the ISR.

Reference [2] gives a first summary of the experimental and theoretical work which was undertaken with the aim of finding antidotes. One of the cures which was suggested is to decouple the bunches by making their synchrotron oscillation frequencies slightly different. The bunch to bunch frequency spread required for stabilization is calculated in reference [3].

These studies have been further pursued and a damping method has been successfully tested.

We have found it important to extend the theory to include the combined effect of the phase-lock beam control system and a bunch to bunch spread in the synchrotron frequencies and in the bunch population.

This degree of refinement of the theory was necessary to explain some experimental observations and to arrive at cures.

One of the new results is that in the presence of phase-lock the stabilizing influence of a bunch to bunch frequency spread depends critically on the pattern of frequency variation. In section 2 of the present report this is demonstrated for the simple case of a machine with 4 bunches (rf harmonic number h = 4). This "h=4 - model" is simple enough for an analytical treatment and still general enough to obtain the same principle results as for the h=20 case.

We discuss three modulation patterns out of which only one - the "meander" pattern - turns out to be suitable for stabilization in the presence of phase-lock.

For the more general case of $h=20$ the same results were obtained by means of a computer programme which is described in section 3 of this report.

In section 5 we summarize the experimental results which were obtained since ref. [2)] was written.

2. Analytical calculations

2.1 General considerations

The analytical approach used in the present section is a "root locus method". Starting from the characteristic equation for the case of equal bunches, we evaluate the displacement of the roots for a small frequency spread.

We start from the characteristic matrix, equ. (46) of ref. [2)]. The roots ω_i of the corresponding characteristic polynominal $H(\omega)$ determine the stability of the system.

We are interested in the behaviour of the roots when one perturbs the free synchrotron frequency Ω of some bunches by a small amount $d\Omega$. Let us define δ by :

$$(\Omega + d\Omega)^2 = \Omega^2 + \delta \qquad\qquad \delta \simeq 2\,\Omega\, d\,\Omega.$$

Expanding the polynominal $H(\omega, \delta)$ in a Taylor series around the point $\omega = \omega_i$, $\delta = o$ (ω_i is one of the roots of $H(\omega,o)=o$) one has:

$$H(\omega,\delta) = H(\omega_i,o) + \left(\frac{\partial H}{\partial \omega}\right)_{\omega_{i,o}} \cdot d\omega + \left(\frac{\partial H}{\partial \delta}\right)_{\omega_{i,o}} \cdot \delta + \quad \ldots \tag{1}$$

If ω_i is a single root of $H(\omega)$, the coefficient $\frac{\partial H}{\partial \omega}$ does not vanish, and one can drop the $(d\omega)^2$, $(d\omega.\delta)$ ... terms if δ is small.

Now, if the modulation pattern which describes the bunch frequencies Ω_i is symmetric (for instance odd number of equal bunches with equal coupling terms, and symmetric frequency spread), the $d\omega$ obviously does not depend on the sign of δ, and therefore the $\frac{\partial H}{\partial \delta}$ term vanishes. One can then write to first order in δ^2

$$d\omega = - \frac{\frac{1}{2}\left(\frac{\partial^2 H}{\partial \delta^2}\right)_{\omega_{i,o}} \cdot \delta^2}{\left(\frac{\partial H}{\partial \omega}\right)_{\omega_{i,o}}} \tag{2}$$

2.2 The "h=4 model"

The preceding considerations are valid in the general case. We shall now turn to the simple case of a machine with 4 equal bunches (rf harmonic number h = 4). We assume coupling via "short range wake fields" so that each bunch acts only on the subsequent one.

In the notation used in ref. 2) the characteristic matrix is

$$H(\omega,o) = \begin{pmatrix} \Omega^2-\omega^2-\beta & o & o & \beta & -\Omega^2 \\ \beta & \Omega^2-\omega^2-\beta & o & o & -\Omega^2 \\ o & \beta & \Omega^2-\omega^2-\beta & o & -\Omega^2 \\ o & o & \beta & \Omega^2-\omega^2-\beta & -\Omega^2 \\ A & A & A & A & -1 \end{pmatrix} \tag{3}$$

The quantity A characterizes the influence of the beam control loop. In the present case $A = \frac{1}{4}(1+jG\omega)$, $G\,\Omega \approx 5 \times 10^{-2}$ for the PS.

The roots of $H(\omega,o) = 0$ are given by (ref. [2]):

$$\begin{aligned}
\omega_1^2 &= \Omega^2 - 2\beta \\
\omega_2^2 &= \Omega^2 - \beta\,(1+j) \\
\omega_3^2 &= \Omega^2 - \beta\,(1-j) \\
\omega_4 &= 0 \\
\omega_5 &= -\,j\,G\,\Omega^2
\end{aligned} \tag{4}$$

Depending on the sign of β, $\omega = +\,\omega_2$ or $\omega = +\,\omega_3$ is the unstable root. Let us assume that ω_2 is unstable ($\beta > o$), and examine the behaviour of this root. We expand $H(\omega,\delta)$ around the point $\omega = \omega_2$, $\delta = 0$.

As $H(\omega,\delta)$ is a polynominal, one has:

$$\left.\frac{\partial H}{\partial \omega}\right)_{\omega=\omega_2,\,\delta=o} = \left.\frac{\partial H(\omega,o)}{\partial \omega}\right)_{\omega=\omega_2}. \tag{5}$$

$H(\omega,o)$ can be written according to (3) and (4) in product form:

$$H(\omega,o) = -\,\omega(\omega+jG\Omega^2)(\omega^2-\omega_1^2)\;(\omega^2-\omega_2^2)\;(\omega^2-\omega_3^2) \tag{6}$$

The first derivative $\frac{\partial H}{\partial \omega}$ is a sum of product terms which all but one contain a factor $(\omega - \omega_2)$. Therefore, for $\omega = \omega_2$:

$$\left.\frac{\partial H}{\partial \omega}\right)_{\omega=\omega_2} = -\,\omega_2\ (\omega_2+jG\Omega^2)\ (\omega_2{}^2-\omega_1{}^2)\ (\omega_2{}^2-\omega_3{}^2)\ (2\ \omega_2) \tag{7}$$

Using the roots (4) and assuming $G\ \Omega$ small compared to unity, one finds:

$$\left.\frac{\partial H}{\partial \omega}\right)_{\omega_2,o} = 4\ \Omega^3\ \beta^2\ (1+j) \tag{8}$$

To arrive at (8), we have assumed small coupling terms so that $\omega_2 \simeq \Omega$.

Next we have to calculate the quantity $\frac{\partial^2 H}{\partial \delta^2}$. This term depends on the modulation pattern.

a) "Sinusoidal" pattern

The bunch frequencies are "perturbed" in the following way:

$$\begin{array}{ll} \text{1st bunch} & \Omega^2 \longrightarrow \Omega^2 + \delta \\ \text{2nd bunch} & \Omega^2 \longrightarrow \Omega^2 \\ \text{3rd bunch} & \Omega^2 \longrightarrow \Omega 2 - \delta \\ \text{4th bunch} & \Omega^2 \longrightarrow \Omega^2 \end{array} \tag{9}$$

Now the determinant of the matrix (3) may be expressed as a sum of determinants.

$$H(\omega,\delta) = H(\omega,o) + \delta \begin{pmatrix} \Omega^2-\omega^2-\beta & & & & -\Omega^2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & -1 & 0 & +1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} + \delta \begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} + \delta^2 \begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & -1 & 0 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} \tag{10}$$

As we have chosen a symmetric pattern, the coefficient $\frac{\partial H}{\partial \delta}$ of δ (see equ. (1)) vanishes for $\omega = \omega_2$. The second derivative $\frac{\partial^2 H}{\partial \delta^2}$ is given by the last determinant which, for $\omega = \omega_2$ $(\Omega^2 - \omega^2 - \beta = j\beta)$ becomes:

$$\frac{1}{2}\frac{\partial^2 H}{\partial \delta^2} = \begin{vmatrix} 1 & 0 & 0 & 0 & -1 \\ \beta & j\beta & 0 & 0 & -\Omega^2 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & \beta & j\beta & -\Omega^2 \\ A & A & A & A & -\Omega^2 \end{vmatrix} \qquad (11)$$

Expanding this determinant, one finds:

$$\frac{1}{2}\left.\frac{\partial^2 H}{\partial \delta^2}\right)_{\omega=\omega_2} = j\,\beta\,\left(2A(\beta - j\beta - \Omega^2) + j\,\beta\right) \qquad (12)$$

which reduces to: $-\,j\,\beta\,\frac{\Omega^2}{2}$ using the approximations $\beta \ll \Omega^2$ and $G\,\Omega \ll 1$ mentioned above.

b) "Meander" pattern

"Perturbations" alternate from one bunch to the next as follows:

$$\begin{array}{ll} \text{1st bunch} & \Omega^2 \longrightarrow \Omega^2 + \delta \\ \text{2nd bunch} & \Omega^2 \longrightarrow \Omega^2 - \delta \\ \text{3rd bunch} & \Omega^2 \longrightarrow \Omega^2 + \delta \\ \text{4th bunch} & \Omega^2 \longrightarrow \Omega^2 - \delta \end{array} \qquad (13)$$

The second order terms in δ result now from determinants having two lines of the form $[1 \quad 0 \quad 0 \quad 0 \quad -1]$ and so on.

$$\frac{1}{2}\frac{\partial^2 H}{\partial \delta^2} = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 1 & 0 & -1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & -1 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

$$+ \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} + \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & -1 & 0 & 0 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & -1 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} + \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} \qquad (14)$$

Under the same assumptions as before one finds:

$$\frac{1}{2}\frac{\partial^2 H}{\partial \delta^2} \simeq \beta\,\Omega^2\,(1 - j) \qquad (15)$$

c) "Square" pattern characterized by :

$$\begin{array}{ll} \text{1st bunch} & \Omega^2 \longrightarrow \Omega^2 + \delta \\ \text{2nd bunch} & \Omega^2 \longrightarrow \Omega^2 + \delta \\ \text{3rd bunch} & \Omega^2 \longrightarrow \Omega^2 - \delta \\ \text{4th bunch} & \Omega^2 \longrightarrow \Omega^2 - \delta \end{array} \qquad (16)$$

In the h=4 case this pattern can also be considered as a sinusoidal pattern with a different phase.

Using the same method of calculation as above, one finds:

$$\frac{1}{2} \frac{\partial^2 H}{\partial \delta^2} = 0 \tag{17}$$

We are now able to calculate from equ. (2) the displacement of the unstable root, especially the shift of its imaginary component. Equations (2), (8), (12), (15) and (17) give the following results:

$$\begin{aligned} &\text{sinusoidal pattern:} && d\omega = \frac{\Omega}{4\beta} (1+j) (d\Omega)^2 \\ &\text{meander pattern:} && d\omega = j \frac{\Omega}{\beta} (d\Omega)^2 \\ &\text{square pattern:} && d\omega = 0. \end{aligned} \tag{18}$$

We compare these results to the frequency shift in the case of programmed acceleration (no beam control). Using the same method we find:

$$\begin{aligned} &\text{sinusoidal pattern:} && d\omega = j \frac{\Omega}{2\beta} (d\Omega)^2 \\ &\text{meander pattern:} && d\omega = j \frac{\Omega}{\beta} (d\Omega)^2 \\ &\text{square pattern:} && d\omega = j \frac{\Omega}{\beta} (d\Omega)^2 \end{aligned} \tag{19}$$

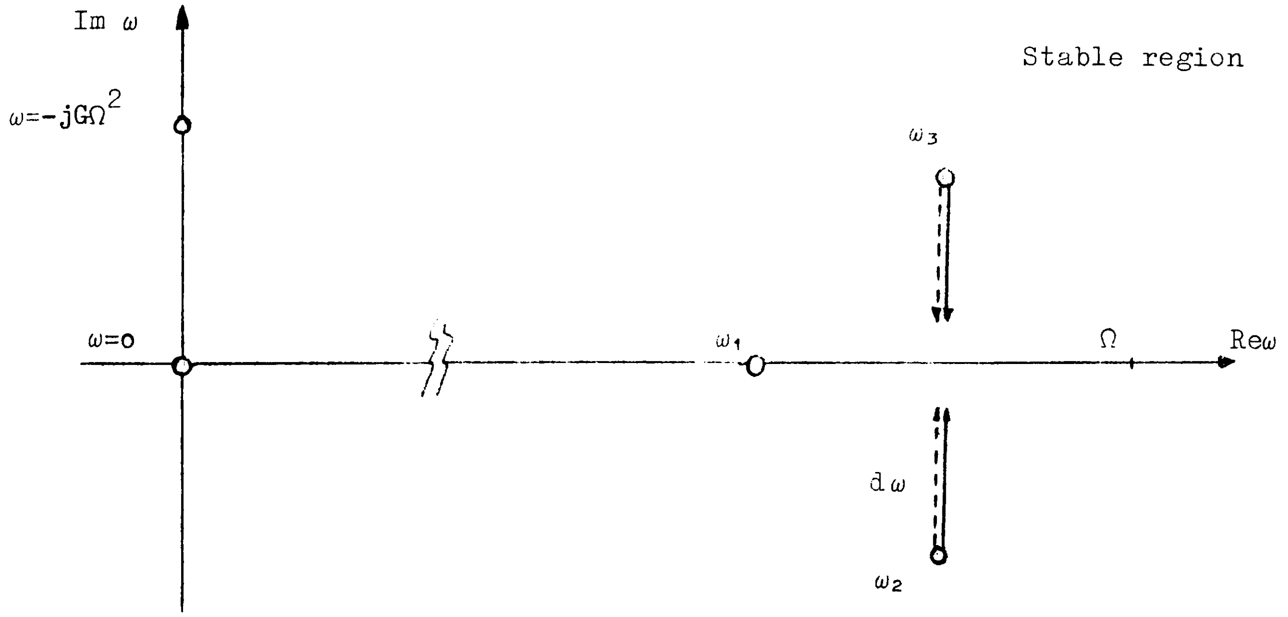
<u>Note</u>: 1) It is easy to check directly from the form of the characteristic matrix that without beam control the "meander" and the "square" pattern are equivalent.

2) One can verify that the stable root ($\omega = \omega_3$) and the unstable one remain complex conjugate (to first order) for the modulation patterns examined.

The displacement of the roots is sketched in Fig. 1. From this figure as well as by comparison between equations (18) and (19) we conclude that the stabilizing influence of the sinusoidal and the square pattern is widely removed by the presence of the phase-lock system.

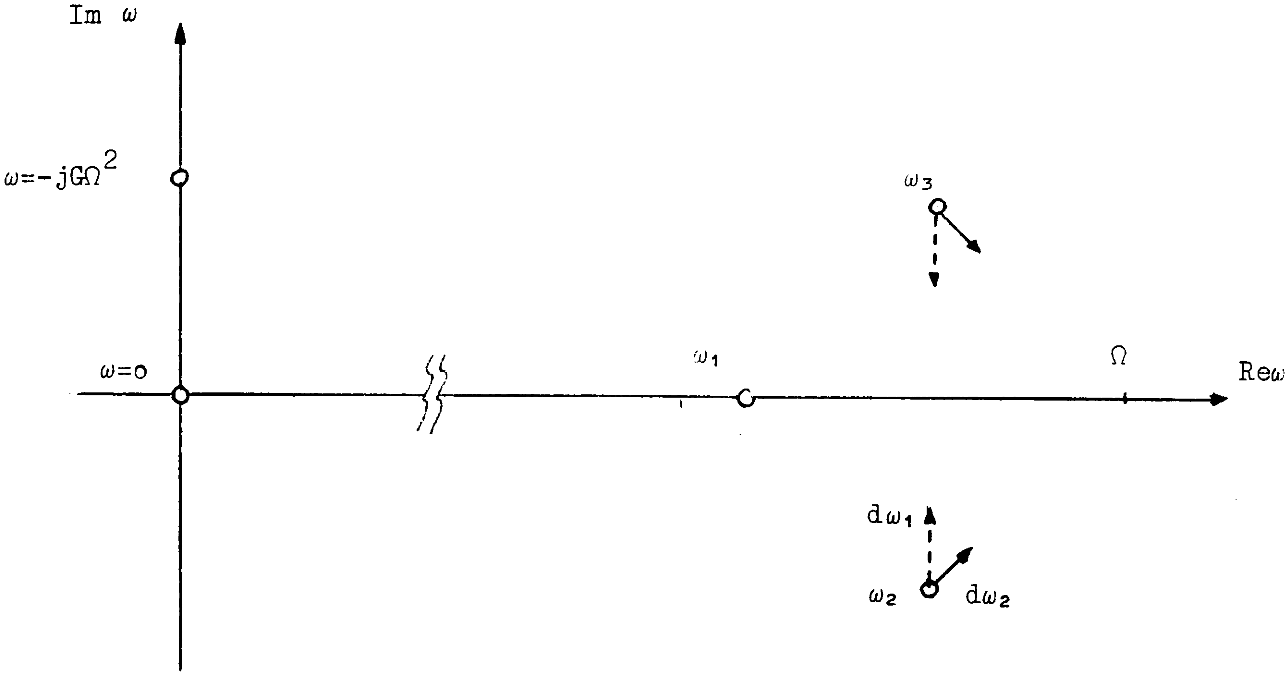
However, the stabilizing effect produced by the meander pattern is not affected by the influence of the beam control loop.

In addition we conclude from the examples of the present section that the radial loop gain is not a critical parameter. Therefore, it seems a reasonable approximation to replace the $jG\omega$ term in A by $jG\Omega$. With this approximation, which, of course, is not valid for the two "trivial roots" $\omega=0$ and $\omega = -jG^{2}$, the stability problem, even with beam control, reduces to an eigenvalue problem which can be solved by numerical calculations using a subroutine of the CERN computer library.



Unstable region

"Meander" pattern $d\omega = j\,\frac{\Omega}{\beta}\,(d\Omega)^2$



"Sinusoidal" pattern $d\omega_1 = j\,\frac{\Omega}{2\beta}\,(d\Omega)^2$

$$d\omega_2 = \frac{\Omega}{4\beta}\,(1+j)(d\Omega)^2$$

Fig. 1 - Root displacements

without beam control

with beam control

3. Numerical calculations

3.1 General description

A computer programme has been written, which finds the growth rates in the more general case of a machine with any RF-harmonic number, especially for the PS with h = 20. The programme is based on the CERN library subroutine F 203 which calculates the eigenvalues and eigenvectors of a complex matrix. Phase-lock, radial control and bunch to bunch spreads in frequency and bunch population are taken into account.

An approximation is used to include the influence of the radial control loop (see section 3.2).

In principle, the programme can be used to simulate the situation for any wake field and any pattern of bunch frequencies. Calculations have been performed for the two wake field models described in ref. [3].

Short range coupling (low Q-wakes) : every bunch acts on the subsequent one only.

Long range coupling (high Q-wakes) : every bunch acts on every other one in the same way except for a phase factor.

The bunch frequency patterns which were already discussed in the preceeding section have been studied:

a) "Sinusoidal" pattern:

$$\Omega_\ell = \bar{\Omega} + \Delta\Omega \sin\left(\frac{\ell\, 2\pi}{h} + \text{const.}\right)$$

(bunch number ℓ out of h bunches)

b) "Meander" pattern:

$$\Omega_\ell = \begin{cases} \bar{\Omega} + \Delta\Omega & \text{for } \ell = 1, 3, 5 \ldots\ldots h-1 \\ \bar{\Omega} - \Delta\Omega & \text{for } \ell = 2, 4, 6 \ldots\ldots h \end{cases}$$

c) Rectangular pattern:

$$\Omega_\ell = \begin{cases} \bar{\Omega} + \Delta\Omega & \text{for } \ell = 1, 2, 3, \ldots\ldots h/2 \\ \bar{\Omega} - \Delta\Omega & \text{for } \ell = h/2 + 1,\ h/2 + 2, \ldots\ldots h \end{cases}$$

3.2 Equations of motion

For the numerical calculations we have assumed that the longitudinal motion of the centre of bunch ℓ is characterized by

$$\ddot{\Phi}_\ell + \Omega_\ell^2 \Phi_\ell = \underbrace{\Omega_\ell^2 (P - jG\bar{\Omega}) \frac{1}{h} \sum_{m=1}^{h} n_m \Phi_m}_{\text{beam control}} + \underbrace{\sum_{k=1}^{h} n_m \beta_{\ell m} (\Phi_\ell - \Phi_m)}_{\text{wake fields}} \qquad (20)$$

Ω_ℓ : free synchrotron frequency of bunch ℓ

$\bar{\Omega} = \frac{1}{h} \sum_{\ell=1}^{h} \Omega_\ell$: average synchrotron frequency

n_m : relative number of particles in bunch m ($\frac{1}{h} \sum_{m=1}^{h} n_m = 1$); for equal population $n_i = 1$.

$(P - jG\bar{\Omega})$ describes the influence of the beam control loop, $P = 1$ was used assuming infinite gain of the phase-lock loop at frequencies around $\bar{\Omega}$, $G\bar{\Omega}$(≈ 0.05 in the PS) is included as an approximate description of radial control, the exact expression would be $G\omega$ where ω is the corresponding eigenfrequency.

$\beta_{\ell m}$: coupling coefficient describing the influence of bunch m on bunch ℓ due to wake fields.

Short range wakes:

$$\beta_{\ell m} = \begin{cases} W\, 2\bar{\Omega} & \text{for } \ell = m + 1 \\ 0 & \text{else} \end{cases}$$

Long range wakes:

$$\beta_{\ell m} = \frac{2\bar{\Omega}}{h} \operatorname{Re} \left\{ W\, j \exp\left[j \frac{2\pi}{h} k\, (\ell - m) \right] \right\}$$

W is a wake field coefficient which determines the growth rate, k is a mode number. W and k are input parameters of the programme.

Equation (20) is basically the same as equs. (42), (43) of ref. [2] except that the radial control loop is included approximating $G\dot{\Phi} = j\, G\omega\, \Phi$ by $j\, G\, \bar{\Omega}\, \Phi$.

For small wake fields ($\beta \ll \bar{\Omega}^2$) this approximation is valid except for the two "trivial" eigenvalues (mode zero). The stability of the system (20) is determined by the eigenvalues of a matrix ((M)) with elements

$$M_{\ell m} = - n_m \left[\frac{1}{h} \Omega_\ell^2 (P - j G \Omega) + \beta_{\ell m} \right] \qquad \text{for } \ell \neq k$$

$$M_{\ell\ell} = \Omega_\ell^2 - \bar{\Omega}^2 - n_{\ell\ell} \frac{1}{h} \Omega_\ell^2 (1 - j G\bar{\Omega}) - n_{\ell\ell} \beta_{\ell\ell} + \sum_{m=1}^{h} n_m \beta_{\ell m} \tag{21}$$

The average bunch frequency $\bar{\Omega}$ is substracted from the diagonal terms for numerical convenience. The eigenfrequencies ω of (2) are related to the eigenvalues λ of ((M)) by

$$\omega^2 - \bar{\Omega}^2 = \lambda$$

$$\omega \approx \bar{\Omega} + \frac{\lambda}{2\bar{\Omega}} \qquad \text{for} \quad \lambda << \Omega^2 \tag{22}$$

As we have assumed a time dependence $e^{j\omega t}$ the growth rate is

$$\frac{1}{\tau} \approx - \frac{\mathrm{Im}(\lambda)}{2\bar{\Omega}} \tag{23}$$

modes with $1/\tau < 0$ are damped.

The eigenvectors of ((M)) describe the oscillation amplitudes of the bunches.

3.3 Results

The numerical results obtained for a machine with h = 20 are in full agreement with the conclusions which were drawn in section 2 from the simpler h=4 model.

For short range wake fields the stabilizing influence of the sinusoidal and the rectangular pattern is overridden by the phase-lock system. Unpractically large frequency spreads (50 % voltage modulation in the PS) are required for a significant reduction of the growth.

However, the meander pattern remains stabilizing in despite of the presence of beam control. The "meander spread" required for a given reduction of the growth is practically the same with and without phase-lock.

For long range wake fields the stabilizing effect of all three patterns is almost unaltered by the presence of phase-lock.

The radial control loop is in most practical cases of little importance. It is mainly the phase-lock system that matters. A spread in the number of particles per bunch seems to have little influence.

4. Application to the PS

It is concluded from the results of sections 2 and 3 that a meander spread looks promising for reducing coherent oscillations of the bunch centre in proton machines. Let us, therefore, calculate the growth rate as a function of the "meander spread".

Assuming low Q-wake fields and neglecting phase-lock, we obtain from the characteristic equation (8) of ref. [3])

$$\omega = \bar{\Omega} \pm \sqrt{(\Delta\Omega)^2 - \left[\left(\frac{\beta}{2\bar{\Omega}}\right)^h\right]^{2/h}} \tag{24}$$

Equation (24) yields h different eigenfrequencies because

$\left[\left(\frac{\beta}{2\bar{\Omega}}\right)^h\right]^{2/h}$ has $h/2$ roots $x_n = \left|\frac{\beta}{2\bar{\Omega}}\right|^2 \exp\left(j\, 2\,\pi\, n\,\frac{2}{h}\right)$

Assuming $(\Delta\Omega)^2 >> \left|\frac{\beta}{2\bar{\Omega}}\right|^2$

the fastest growth rate is

$$1/\tau = I_m(-\omega) \approx \left(\frac{\beta}{2\,\bar{\Omega}}\right)^2 \frac{1}{2\,\Delta\,\Omega} \tag{25}$$

It is useful to compare $1/\tau$ to the growth rate $1/\tau_0 \approx \frac{|\beta|}{2\,\bar{\Omega}}$ in the absence of frequency spread:

$$1/\tau \leqq 1/\tau_0\, \frac{1/\tau_0}{2\,\Delta\,\Omega} \tag{26}$$

These results were derived neglecting phase-lock. However, the results of sections 2 and 3 suggest that equations (25) and (26) remain valid in the presence of phase-lock.

It is interesting to note that without phase-lock equations (24) - (26) pertain equally to the rectangular pattern.

Let us now put in numbers for the PS. We take

$$1/\tau_0 = (50 \text{ msec})^{-1}$$

$$\frac{2\Delta\Omega}{\Omega} \approx \frac{\Delta U_{rf}}{U_{rf}} = \pm 5\ \%$$

$$\bar{\Omega} = 2\ \pi/(3 \text{ msec})$$

free synchrotron frequency in the PS at high energy.

With these numbers we find a reduction of the growth rate

$$\frac{1/\tau_0}{1/\tau} = \frac{\Delta U_{rf}}{U_{rf}} \frac{\Omega}{1/\tau_0} \approx 5.2$$

or $\tau \gtrsim 260$ msec.

This growth time is long enough that we may regard the situation as practically stable.

In conclusion, a meander spread $\frac{\Delta\Omega}{\Omega} = \pm 2.5\ \%$ (voltage modulation $\frac{\Delta U}{U} = \pm 5\ \%$) should be sufficient to ensure longitudinal dipole stability in the present PS.

5. Experimental results

5.1 Instrumentation and measurements

Three different observation techniques were used:

a) Direct observation of bunches on a fast oscilloscope with a "Cappi type" trigger ("mountain range display"; see for instance photo 3)

b) Observation of radial oscillations of one or two bunches. The clamped signal of a radial PU station is gated at f_{rev}, and the low frequency component is filtered out and displayed on an oscilloscope.

c) Observation of the phase oscillation of one or two bunches. The wide-band PU signal is gated at f_{rev} in order to select one bunch and its RF component is filtered out using a tunable filter working in the range 9.0 MHz - 9.6 MHz. A phase discriminator then measures the phase difference between RF voltage and the selected bunch.

Photo 1 obtained by method c) shows the oscillation of two consecutive bunches. The phase shift between the two bunches is about 90° which corresponds to the most unstable mode ($k = 5$) in the PS. Using a slower sweep (photo 2A) one can measure the growth time of the instability. Measured values are typically in the range of 80 - 150 ms. Note that the decay observed later can be explained by the filamentation process.

In order to compare the measured growth rate with theoretical values calculated from the frequency response of the cavities, it was decided to measure the frequency of the first parasitic resonance of all 14 PS cavities.

Measurements were performed on October 28, 1970, during a machine stop. Cavities were driven through their RF power amplifier by a sweeped RF generator set around 50 MHz. Resonance curves were picked off via the coupling loops on both the upstream and the downstream side of the cavity. The precision of these frequency measurements is limited because resonance curves differ (by an amount as large as 0.7 MHz) from one side to the other of the same cavity. Therefore assumptions have to be made in order to average the data coming from each side of the cavity. The impedance at resonance, not measured by this method, was assumed to be 800 ohms. Using the computer programme described in [2] we have calculated the theoretical growth times corresponding to the actual data.

For $N = 170 \times 10^{10}$ p/p we find a growth time of 80 msec for the fastest mode, which agrees with measurements made during the last MD sessions. However, it should be pointed out that shorter growth times have been observed during earlier MD's (ref. [2]).

5.2 Compensation

The simplest way to produce the "meander" pattern is to drive one (or more) cavity at half the RF frequency. The modulation of the synchrotron frequency depends on the amplitude and on the phase of the RF/2 voltage relative to the main RF. In order to avoid a changing phase relationship, the RF/2 voltage was only applied during the later part of the cycle where the revolution frequency is almost constant ($f_{RF} > 9.4$ MHz).

a) Experiments on a 10 GeV/c flat top (MD's on 24/10 and 14/11). Photo 2A shows typical oscillations growing during the 500 ms flat top, whereas photo 3A displays the bunch shapes just before the end of the flat top.

The oscillograms 2B, 3B display the same situation when the full RF/2 voltage is applied (10 kV corresponding to ≈ 7 % of the main RF, i.e. $\Delta\Omega/\Omega \approx \pm 3.5$ %). These pictures clearly show the effectiveness of this technique.

It was verified that the stabilizing influence depends on the phase difference between the RF- and RF/2-voltages; optimum stability was observed for values of the RF/2 phase differing by 180°.

b) Experiments during acceleration (MD on 14/11).
The RF/2 voltage was switched on just after transition. The "Cappi type" display photos 4A (without RF/2) and 4B (with RF/2 on) confirm that the meander spread is stabilizing during acceleration. These pictures were taken when the RF/2 cavity was working at about 5 kV.

When the cavity was driven at full amplitude (10 kV), very strong oscillations appeared. At present we do not have a clear explanation of this effect. It may be that the reduced size of the buckets produced by the superposition of the main RF and the RF/2 voltage complicated the situation.

Similar experiments were already performed (MD's on 6/8/70 and 8/10/70), both during acceleration and on a 10 GeV/c flat top, using modulation of the RF voltage at the revolution frequency. This produces the "sinusoidal" pattern discussed in sections 2 and 3. Although these measurements were done under somewhat different conditions of the machine, they seem to indicate that the "sinusoidal" pattern is not very effective for stabilization.

Acknowledgements

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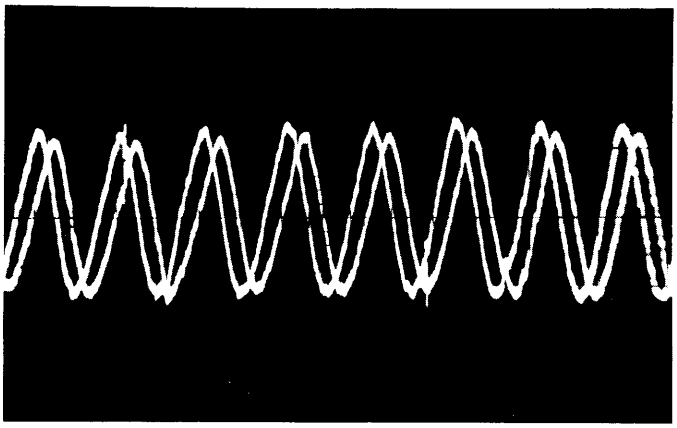
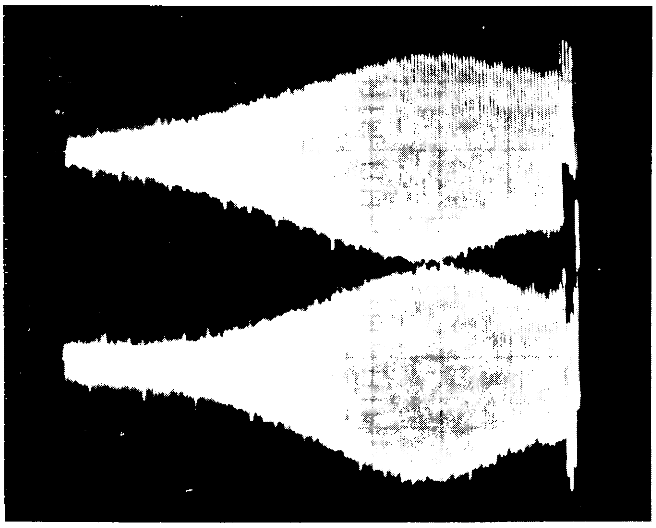


Photo 1

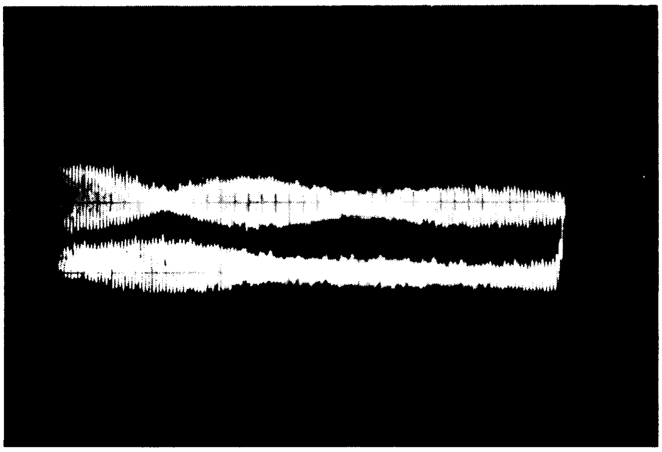
Oscillation of the centre of two consecutive bunches with respect to the RF.

Photo 2

Oscillation of the centre of two consecutive bunches on the flat top (10 GeV/c), 50 ms/cm.



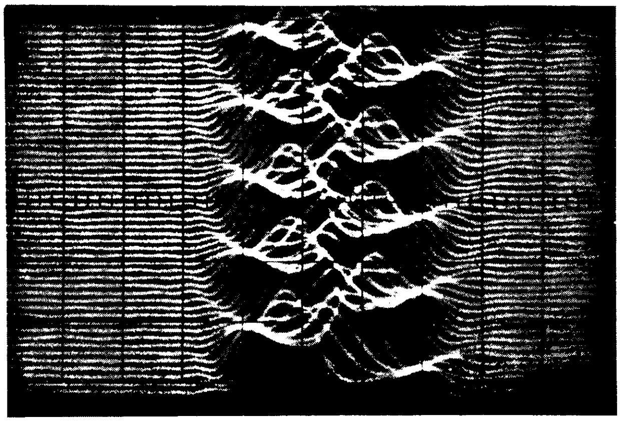
A. Without RF/2 voltage



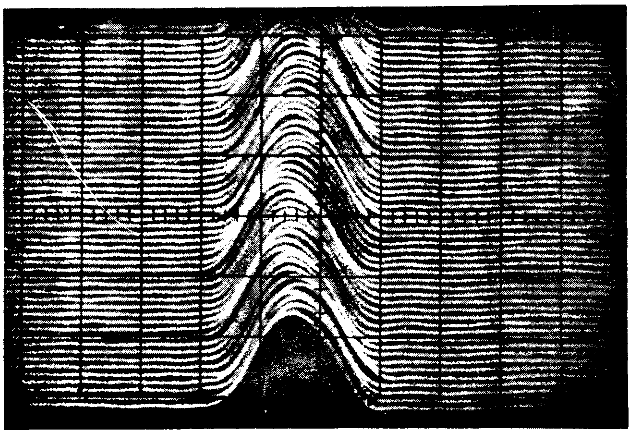
B. With full RF/2 voltage (10 kV)

Photo 3   Mountain range display of bunch shape

5 ns/cm, M 264 (10 GeV/c flat top)



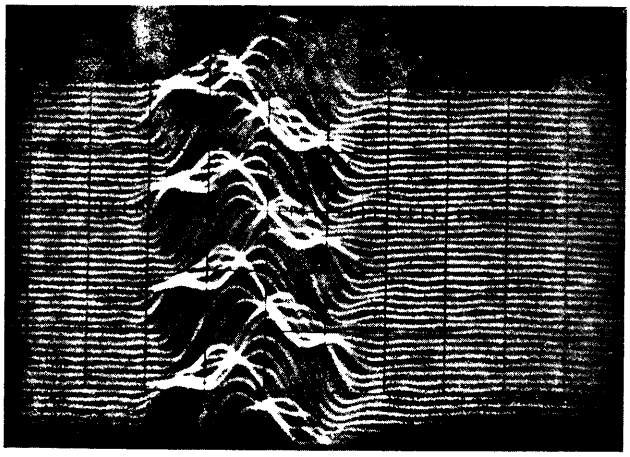
A.   Without RF/2



B.   With full RF/2 (10 kV)

Photo 4 Mountain range display of bunch shape

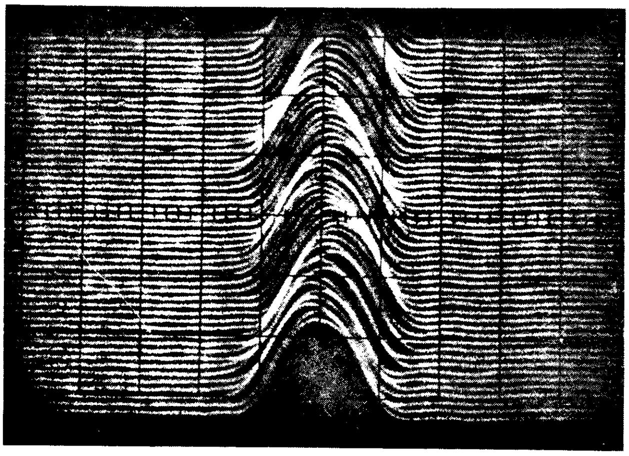
5 ns/cm normal field rise.



A. B 614 (rise)

12 GeV

RF/2 off



B. B 874 (rise)

17 GeV

RF/2 on (5 kV)